

Engineering the Albedo Solution in Global Warming

Part 1

Alec Feinberg[†]

Key Words: Re-Radiation Model, Global Warming Solution, Planck Parameter, Planck-Albedo Parameter

Abstract

In this paper, we model global warming with a re-radiation factor and use the Planck's parameter to verify consistency. The re-radiation factor is important in quantifying the relative global warming impact of the albedo effect compared to that of greenhouse gases (GHG). The potential reverse forcing due to a change in the Earth's global albedo compared to GHGs, is illustrated. Results of modeling support solar geoengineering solutions with two key parameters from modeling, an albedo-GHG and a Planck-Albedo feedback parameter. Using these, it is concluded that a 1.5% solar geoengineering change in the global albedo could result in a significant resolution to the global warming problem. Feasibility is discussed.

1 Introduction

In our race against time in global warming, it may be appropriate to ask the question, what are the best solutions rather than addressing what is viewed as the main problem. To address this question, we create an engineering model that uses a re-radiation factor, which helps to quantify differences between changes in the global albedo versus greenhouse gas forcing. The re-radiation parameter is obtained mainly in equilibrium modeling with appropriate interactions and constraints to aid in comparison; the re-radiation parameter is then found in the absence of warming feedbacks with a unique value of 0.612 (or $\beta=0.887$). The re-radiation factor is a redefined variable taken from the effective emissivity constant of the planetary system. Then, the Planck's feedback parameter is used to verify model consistency. The model illustrates a reasonable way to view the Earth's energy budget; simplifies estimates without the need for doubling theory, provides a number of useful insights in climatology sensitivity estimates and demonstrates the relative advantage of solar geoengineering solutions in global warming mitigation [1]. Specifically, a 1.6 engineering albedo-GHG factor along with a handy defined Planck-Albedo parameter (having a convenient value of $1\text{W/m}^2/\text{K}/\Delta\%\text{albedo}$) greatly simplify geoengineering [2, 3] calculations. This is used to exemplify a global warming albedo solution. Feasibility is discussed [1].

2. Data and Method

To introduce the re-radiation engineering model, we will often refer to the Planck parameter and its associated variables that play a key role in development and verifying this model. Therefore, we provide an overview in Appendix A which also includes a unique way to assess its value using an albedo approach (see A.1).

2.1 The Re-radiation Global Warming Model

In this model we define

$$P_{Total} = \sigma T_S^4 = \sigma \left(\frac{T_{TOA}}{\beta} \right)^4 \text{ and } P_{\alpha} = \sigma T_{\alpha}^4 = \sigma (\beta T_S)^4 \quad (1)$$

The definitions of T_{TOA} , T_S and β are provided in Appendix A (Eq. A-1, A-2, A-3). We consider a time when there is **no feedback issues** causing warming trends. Then by conservation of energy, the equivalent power re-radiated from GHGs in this model is dependent on P_{α}

$$P_{GHG} = P_{Total} - P_{\alpha} = \sigma T_S^4 - \sigma T_{\alpha}^4 \quad (2)$$

To be consistent with Eq. A-1, $T_{\alpha}=T_{TOA}$, since typically $T_{\alpha}\approx 255^{\circ}\text{K}$ and $T_S\approx 288^{\circ}\text{K}$, then in keeping with a common definition of Beta (see Eq. A-4) for the moment $\beta\approx T_{\alpha}/T_S\approx T_{TOA}/T_S$.

[†]A. Feinberg, Ph.D., DfRSoft Research, email: dfrsoft@gmail.com, ORCID: 0000-0003-4364-2460

57 This allows us to write the dependence
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$$59 \quad P_{GHG} = \sigma T_S^4 - \sigma T_\alpha^4 = \frac{\sigma T_\alpha^4}{\beta^4} - \sigma T_\alpha^4 = \sigma T_\alpha^4 \left(\frac{1}{\beta^4} - 1 \right) \quad (3)$$

60
61 We note that when $\beta^4 = 1$, there are no GHG contributions as required. We now define a re-radiation parameter $f = \beta^4$.
62 We know that some fraction of the blackbody radiation is re-radiated by the GHGs, so f is a re-radiation parameter.
63 That is, the energy, P_{GHG} , must be some fraction of P_α so that its dependence is also

$$64 \quad P_{GHG} = f P_\alpha = f \sigma T_\alpha^4 \quad (4)$$

65
66 This is a key difference in how we view the total effect from short wavelength absorption with the inclusion of re-
67 radiation [2]. Now in order for this to be true, we require from Equations 3 and 4

$$68 \quad P_{GHG} = \sigma T_\alpha^4 \left(\frac{1}{f} - 1 \right) = f \sigma T_\alpha^4 \quad (5)$$

69
70 This dependence leads us to the solution of the quadratic expression

$$71 \quad f^2 + f - 1 = 0 \text{ yielding } f_1 = 0.618034 = \beta^4, \beta = (0.618034)^{1/4} = 0.88664 \quad (6)$$

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73 This is very close to the common value estimated for β (Appendix A) and this has been obtained through energy
74 balance in the planetary system providing a completely self-determining assessment without approximations. In
75 Section 2.6, we double check this model in another way by balancing energy in and out and in Section 3 we will
76 apply the modeling to demonstrate its capability.

77 2.2 Re-radiation Model Applied to Two Different Time Periods

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79 Global warming can be modeled by looking at two different time periods. We can model the radiation for 1950
80 consistent with our model in Eq. 2 and 4

- 81 • we will assume no feedback issues causing a warming trend in 1950 so that from our model

$$82 \quad P_{Total_1950} = P_\alpha + P_{GHG} = P_\alpha + f_1 P_\alpha \quad (7)$$

83 where $P_\alpha = S_o \{0.25x(1 - Albedo)\}$ and $S_o = 1361 \text{ W/m}^2$. The equilibrium model is constrained by the energy balance
84 discussed in Section 2.3 and Eq. 5.

85 In 2019 due to global warming trends, this model is more complex and harder to separate out terms. However, it can
86 still be similarly modeled as

$$87 \quad P_{Total2019} = P_{\alpha'} + P_{GHG'+Feedback} \approx P_{\alpha'} + f_2 P_{\alpha'} \quad (8)$$

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89 Here, $P_{GHG'+Feedback}$ includes GHGs and its increase with feedbacks such as water-vapor, lapse rate effect and other
90 changes such as an increase in snow-ice albedo variations that are hard to separate out. That is, some of this
91 feedback is related to GHG forcing increases and some is related to albedo change. $P_{\alpha'}$ represents the 2019 point in
92 time with its albedo due to prior changes in UHI absorption, cloud absorption, ice and snow melting, and so forth
93 that can be discerned. The model does not demand rigid accountability in its application (see Sec.3). We note that f ,
94 a measure of the emissivity, is **not** constant, but must change since the amount of GHGs changes. However, f_2 is not
95 as accurate in terms of the actual emissivity value but is an approximation that in perhaps rigorous assessment could
96 be determined.

97 To be clear, f is just a fractional parameter related to the emissivity. In 1950 it was some function of the GHGs (with
98 no feedbacks). In 2019, it is more complex. The model is also constrained relative to f_1 as described in Section 2.3.2.
99 However, it is primarily related to GHG re-radiation since $P_{GHG} \approx P_{GHG'+Feedback}$ (see results in Section 3).

100 2.3 Balancing P_{out} and P_{in}

101 Although Eq. 7 with f_1 has the uniquely defined value found in Eq. 6. This should also result from balancing the
102 energy in and out of our global system.

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2.3.1 Balancing P_{out} and P_{in} in 1950

To balance the energy in with the energy out in 1950 with no global warming imbalance we can still start with Eq. 7. In equilibrium the radiation that leaves must balance what comes in P_{α} so that

$$\begin{aligned} \text{Energy}_{Out} &= (1-f_1)P_{\alpha} + (1-f_1)P = (1-f_1)P_{\alpha} + (1-f_1)\{P_{\alpha} + f_1P_{\alpha}\} \\ &= (1-f_1)\{2P_{\alpha} + f_1P_{\alpha}\} = 2P_{\alpha} - f_1P_{\alpha} - f_1^2P_{\alpha} = \text{Energy}_{In} = P_{\alpha} \end{aligned} \quad (9)$$

121
122 This is consistent with Eq. 6 so that in 1950, the value f solves the same quadratic equation as expected
123

$$f_1^2 + f_1 - 1 = 0 \text{ yielding } f_1 = 0.618 \quad (10)$$

125
126 Interestingly, this also says that
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$$P_{\alpha} = f_1P_{Total_1950} \text{ or } P_{\alpha} = f_1(P_{\alpha} + f_1P_{\alpha}) \text{ or } 1 = f_1(1 + f_1) \quad (11)$$

129
130 The RHS of Eq. 11 is Eq. 10 and Eq. 6. This illustrates why f_1 is unique. It is the fractional amount of total radiation
131 that is in equilibrium. As a final check, results will show in Section 3 and Table 1, that the value f_1 provides
132 reasonable results.
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2.3.2 Warming Imbalance in 2019

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135 The re-radiation parameters f_1 and f_2 , are connected and from Eq. 7 and 8 we have
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$$f_2 = f_1 + \left(\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_{\alpha}} \right) = f_1 + \Delta f \quad (12)$$

139 In this way f_2 is a function of $f_1=0.618$ and the differences in the global warming residuals that is identified in Eq. 12
140 as Δf .
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3.0 Results and Discussion

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143 Since the re-radiation parameter is fixed for $f_1=0.618$, to obtain $T_{1950}=13.89^{\circ}\text{C}$ (287.038°K), the only adjustable
144 parameter left in our model is the Earth's albedo. This value requires an albedo value of 0.3008 (see Table 1) to
145 obtain the correct value T_{1950} . This albedo numbers is reasonable and similar to values cited in the literature [4].
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148 In 2019, the average temperature of the Earth is $T_{2019}=14.84^{\circ}\text{C}$ (287.99°K). Here we are not sure of the albedo since
149 it likely changed due to UHI increase, snow and ice melting and cloud coverage changes. The IPCC value in AR5
150 [5] is 0.294118 (100/340) is given in AR5 [6]. However, this would represent a 3% change since 1950 which may be
151 an overestimation. In our assessment, we will assume a middle value of about 1.6% change. Another reason for this
152 choice will become apparent in the resulting analysis. Then, the f_2 parameter is adjusted to 0.6324 to obtain T_{2019} .
153 Results are provided in Table 1. The results yield $P_{Total_1950}=384.935 \text{ W/m}^2$ and $P_{Total_2019}=390.055 \text{ W/m}^2$. We find
154 that

$$\Delta P_{Total} = P_{2019} - P_{1950} = 5.121 \text{ W/m}^2 \quad (13)$$

156 and

$$\Delta T_{Total} = T_{2019} - T_{1950} = 0.95^{\circ}\text{C} \quad (14)$$

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159 which is the observed surface temperature increase since 1950.
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Table 1 Model results

Year	T($^{\circ}\text{K}$)	T_{α} ($^{\circ}\text{K}$)	f_1, f_2	α, α'	$P_{\alpha}, P_{\alpha'}$ (W/m^2)	P_{GHG} (W/m^2) $P_{\text{GHG}+\text{feedback}}$	P_{Total} (W/m^2)
2019	287.991	254.94	0.628354	29.599	239.540	150.516	390.056
1950	287.041	254.51	0.6180	30.08	237.903	147.032	384.935
$\Delta 2019-1950$	0.95	0.437	1.032%	-0.481 (1.6%)	1.638	3.484	5.12

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163 Table 1 summarizes model results for the specified albedos and observed Earth's surface temperatures. To show
 164 model consistency, the forcing change 5.121 W/m^2 , resulting in a 0.95°K rise, should agree with what is expected
 165 from Planck's feedback parameter. From Eq. A-1, A-14 and Eq. 6, it is evident that

$$166 \quad \beta^4 \Delta R_{\text{TOA}} = 5.12 \times \beta^4 = 3.164 \text{ W/m}^2 \quad (15)$$

168 This equation illustrates the consistency of the re-radiation model. Then, Planck's feedback parameter (3.3 W/m^2
 169 $^\circ\text{K}$) temperature rise is in agreement with what is observed by equilibrium modeling

$$170 \quad 3.164 \text{ W/m}^2 \times (1/3.3)^\circ\text{K/W/m}^2 = 0.959^\circ\text{K at } T_s \quad (16)$$

174 3.1 Why the Re-radiation Parameter is Significant

175 In Table 1, the measure of $\Delta f = 1.45\%$ fractional increase is mainly due to re-radiation change and associated
 176 feedbacks. This is significant. From Eq. 7, 8 and 12 we can illustrate this key characteristic of climate change

$$177 \quad \Delta f = \left(\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_{\alpha}} \right) = \left(\frac{P_{GHG'+F}}{P_{\alpha'}} - \frac{P_{GHG}}{P_{\alpha}} \right) \approx \left(\frac{P_{GHG'+F} - P_{GHG}}{P_{\alpha}} \right) \quad (17)$$

180 Therefore, f is an estimate of climate re-radiation and Δf an estimate of climate emissivity change. It is a measure of
 181 GHG forcing increase and the feedback relative to the initial 1950 radiation, and is generally helpful in looking at
 182 how our climate is working. Furthermore, we can deduce an albedo advantage.

184 3.2 The Albedo-GHG Factor

185 We can look at an important ratio, the power created by the albedo effect compared to GHGs in 1950. The initial
 186 radiation is P_{α} which heats the Earth to 254.51°K , and then according to Eq. 7 and Table 1, the P_{GHG} energy
 187 originates from a fraction of this original heating due to re-radiation as fP_{α}

$$188 \quad \frac{P_{\alpha} + P_{GHG}}{P_{GHG}} = \frac{P_{\alpha} + fP_{\alpha}}{P_{GHG}} = \frac{P_{\alpha} + fP_{\alpha}}{fP_{\alpha}} = \frac{1 + f_1}{f_1} = \frac{1.62}{0.62} = 2.62 \quad (18)$$

192 Here we include the eventual re-radiation that must occur after the initial short wavelength absorption in this
 193 assessment. In general, we note the important albedo-GHG factor, 1.62, obtained from initial absorption and the
 194 0.62 GHG re-radiation contribution.

195 In this engineering view, we can look at a change in albedo forcing compared with a change in GHGs. The change
 196 in the energy due to an albedo change and its re-radiation is from Table 1

$$197 \quad \Delta P_{\alpha} = \Delta P_{\alpha'} + f_2 \Delta P_{\alpha'} = 1.628 \Delta P_{\alpha'} \quad (19)$$

200 The change in GHGs can be written in terms of Δf and the absorbed energy that GHGs receive from solar absorption
 201 from Table 1 this is

$$202 \quad \Delta P_{GHG} = \Delta f P_{\alpha} = 1.032\% P_{\alpha'} \quad (20)$$

203 This ratio results in

$$204 \quad \frac{\Delta P_{\alpha'}}{P_{GHG}} = \frac{1.628 \times 1.638 \text{ W/m}^2}{0.01032 \times 239.54} \approx 1.08 \quad (21)$$

209 This ratio of course is dependent on the choice of the albedo in 2019. Here, we purposely used a value that would
 210 result in this ratio being close to unity. Although, true values of $\Delta\alpha$ and Δf are not easily obtained in 2019, it avoids
 211 CO_2 doubling estimates, which are also difficult to evaluate. The key point is that when we reverse engineer a global
 212 warming solution, it illustrates the importance of key values needed. In this view, the 1.6 albedo-GHG factor (which
 213 is reasonably accurate) is an important engineering value. It provides one of the significant numbers needed in
 214 reverse albedo forcing that takes into account the initial absorption change followed by the potential re-radiation.
 215 The other important number is described by the Planck-albedo parameter.

221 3.3 Planck-Albedo Parameter

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223 The albedo changes and ΔP_α in Table 1, are: $\% \Delta \alpha = 1.6\%$ and 1.638 W/m^2 , respectively. We note that we can define
224 a unique handy Planck-albedo parameter $\lambda_{\% \Delta \alpha} = \Delta P_\alpha / \% \Delta \text{albedo}$. To illustrate from Table 1

$$225 \lambda_{\% \Delta \alpha} = 1.024 \text{ W/m}^2 / \% \Delta \text{albedo} \quad (20)$$

227 This parameter can also be expressed per degree (noting the 0.95°K change in Table 1)

$$230 \lambda_{\% \Delta \alpha \Delta T} \approx 1 \text{ W/m}^2 / \Delta \% \text{albedo} / ^\circ \text{K} \quad (21)$$

231 The helpful parameter was first noted in Feinberg 2020 [3] but is featured here as a modeling tool. We term it the
232 Planck-albedo parameter, since it relates to blackbody (P_α) absorption. A simple numeric example is given in the
233 conclusion to illustrate how it provides helpful estimates. This interesting parameter simplifies from the basic
234 assessments of the two different time periods (see also Eq. A-8) as

$$237 \lambda_{\% \Delta \alpha} = \frac{(\Delta E_o)_\alpha}{\frac{\alpha_1 - \alpha_2}{\alpha_1} 100} = \frac{E_o (\alpha_1 - \alpha_2)}{\frac{\alpha_1 - \alpha_2}{\alpha_1} 100} = E_o \alpha_1 / 100 \approx 1 \text{ W/m}^2 / \% \Delta \text{albedo} \quad (22)$$

238 where $E_o = 340 \text{ W/m}^2$ and when α_1 is 0.294118, the value $1.000 \text{ W/m}^2 / \% \Delta \text{albedo}$ is obtained. We note the value
240 29.4118% ($100/340$) is given in AR5 [6]. The parameter's relationship to λ_α is

$$242 \lambda_\alpha = \lambda_{\% \Delta \alpha \Delta T} \times \% \Delta \alpha \quad (23)$$

244 and the feedback parameter could including the re-radiation factor f in 2019 [2] as

$$246 \lambda_\alpha^\dagger = \lambda_{\% \Delta \alpha \Delta T} \times \% \Delta \alpha \times 1.6 \quad (24)$$

247 The albedo-GHG and the Planck-Albedo feedback parameter may be combined for geoengineering solution
248 estimates using the following equation

$$251 P_{\text{Rev}} = -\lambda_{\% \Delta \alpha \Delta T} \times \% \Delta \alpha \times 1.6 \times A$$

252 Here P_{Rev} is the reverse forcing, and A is an estimate of the anticipated GW amplification reduction. An example is
253 provided in the conclusion.

256 4.0 Conclusion

257 In this paper, we provided a re-radiation global warming model. The model shows consistency with the Planck
258 parameter. We noted that the re-radiation parameter increased by about 1.45% due to global warming from 1950 to
259 2019, illustrating the warming from a different perspective. From the model, a helpful albedo-GHG parameter was
260 quantified having a value of 1.6.

262 We also found an engineering factor that we termed the Planck-albedo parameter, which is about
263 $\lambda_{\% \Delta \alpha \Delta T} \approx 1 \text{ W/m}^2 / \Delta \% \text{albedo} / ^\circ \text{K}$. These findings can be helpful in quickly estimating the effect of an albedo change on
264 global warming and in assessing λ_α . These results support added solar geoengineering solutions [3, 7-9].

266 For example, Feinberg 2020 [2] suggested a goal of 1.5% geoengineering albedo change. Using Equation 25, with a
267 decrease in water-vapor feedback anticipated, we might use a value of $A \approx 2$ [10], then

$$270 P_{\text{Rev}} = -1 \text{ W/m}^2 / \% \times 1.5\% \times 1.6 \times 2 = -4.8 \text{ Watt/m}^2$$

271 One can multiply this by β^f or compare this to the 5.12 W/m^2 results in Table 1 indicating a significant resolution to
272 the current warming trend. As one might suspect, a 1.5% albedo change requires a lot of modified area. Feasibility is
273 discussed in more detail in Feinberg's 2020 [2]. Results of this paper indicate the required area of change, if proper
274 hotspots are targeted, is 3.4-17 times smaller than current urbanization areas. Other solar geoengineering solutions
275 have been proposed [7-9].

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Appendix A

Overview of Planck Feedback Parameter

Estimates on Planck's feedback parameter are varied, typically between $-3.8\text{W/m}^2/\text{K}$ and $-3.21\text{W/m}^2/\text{K}$ with some values as large as $-7.1\text{W/m}^2/\text{K}$ [11]. The IPCC AR4 [12] lists a value of $-3.21\text{W/m}^2/\text{K}$. Numerous authors have developed different expressions [11]. A typical estimate starts with

$$F_{TOA} = (1 - \alpha)S_o/4 - \sigma(\beta T_s)^4 = (1 - \alpha)S_o/4 - R_{LWR} \quad (\text{A-1})$$

where $S_o=1361\text{W/m}^2$, F_{TOA} is the radiation budget at the top of the atmosphere, R_{LWR} is the outgoing long wave radiation (a function of surface temperature and albedo), σ is the Stefan-Boltzmann constant and β is described in this section below and is redefined in terms of a re-radiation parameter in this paper. Then the Planck parameter λ_o can be calculated as

$$\lambda_o = \partial F_{TOA} / \partial T_s = -\partial R_{LWR} / \partial T_s \quad (\text{A-2})$$

This result is

$$\lambda_o = -4\beta^4 \sigma T_s^3 = -4\beta \sigma T_{TOA}^3 \quad (\text{A-3})$$

where β varies in the literature from 0.876 to 0.887 (averaging=0.8815) and $T_s=288^\circ\text{K}$ [12]. This yields $-3.37\text{W/m}^2/\text{K} < \lambda_o < -3.21\text{W/m}^2/\text{K}$. However, from Eq. A-3, β is often taken as the ratio

$$\beta = T_{TOA} / T_s = 255^\circ\text{K} / 288^\circ\text{K} = 0.8854 \text{ and } \beta^4 = 0.615 \quad (\text{A-4})$$

A common assessment uses $T_{TOA}=255^\circ\text{K}$, so that $\lambda_o = -3.33\text{W/m}^2/\text{K}$. Another expression developed by Schlesinger [6] is dependent on the albedo and surface temperature as

$$\lambda_o = S_o(1 - \alpha) / T_s \quad (\text{A-5})$$

When $S_o=1361$, $0.294118 < \alpha < 0.3$, and $T_s=288^\circ\text{K}$ then $-3.308\text{W/m}^2/\text{K} > \lambda_o > -3.3358\text{W/m}^2/\text{K}$, respectively.

A.1 Estimating Planck's Parameter with an Albedo Method

Consider a global albedo change corresponding to 1°K rise from solar absorption letting

$$F_{TOA} = 0 = (1 - \alpha)E_o - \sigma(T_s)^4 \quad (\text{A-6})$$

where $E_o=S_o/4$. Then a 1°K change is

$$\Delta T_s = T_2 - T_1 = \left(\frac{E_o}{\sigma} (1 - \alpha_2) \right)^{1/4} - \left(\frac{E_o}{\sigma} (1 - \alpha_1) \right)^{1/4} = 1^\circ\text{K} \quad (\text{A-7})$$

Here we will use the AR5 albedo starting value of 0.294118 [6]. We find that the corresponding albedo change is 0.28299 when $E_o=340\text{W/m}^2$. This corresponds to

$$\Delta E_o = E_o \{ (1 - \alpha_2) - (1 - \alpha_1) \} = E_o (\alpha_1 - \alpha_2) = 3.784\text{W} / \text{m}^2 \quad (\text{A-8})$$

Since this is for a 1°K rise, then it can also be written as

$$\lambda_{1K} = 3.784\text{W/m}^2/\text{K} \quad (\text{A-9})$$

We note this is related to the surface value, then

$$\lambda_{1K} = -4\sigma T_s^3 \quad (\text{A-10})$$

By comparison to above we have

$$\lambda_o = \lambda_{1K} \beta = -3.784\text{W/m}^2/\text{K} = -3.349\text{W/m}^2/\text{K} \quad (\text{A-11})$$

This is very close to the $-3.33\text{W/m}^2/\text{K}$ value obtained in the traditional manner.

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A.2 Top of the Atmosphere and Beta

From Eq. A-1

$$R_{LWR} = \sigma(\beta T_S)^4 = \sigma(T_{TOA})^4 \quad (\text{A-13})$$

giving

$$\beta^4 R_{TOA,T_S} = R_{TOA,T_{TOA}} \quad (\text{A-14})$$

We use this expression in our when showing model consistency with the Planck feedback parameter.

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