Modeling the Albedo Advantage in Global Warming And an Albedo-Planck Parameter

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Abstract In this paper, we model global warming using a re-radiation factor and the Planck's parameter to verify consistency. The re-radiation factor is important in quantifying the relative global warming impact of the albedo effect compared to the greenhouse gas (GHG) effects. The forcing due to the change in the Earth's global albedo compared to GHGs is found to have a 2.6 times larger impact on global warming. In our simple model, we additionally define a handy Planck-Albedo feedback parameter having a convenient value of $1W/m^2/^oK/\Delta\%$albedo. Using these results, it is concluded that a 1.5% solar geoengineering change in the global albedo could result in -4.8 W/m^2 of forcing. An alternate way to assess the Planck parameter was also found.

1 Introduction

Although global warming is highly complex, often it is helpful to work with a simplified model. We create a model that uses a re-radiation factor which helps to quantify significant differences between changes in the global albedo versus greenhouse gas forcing. We use the Planck's feedback parameter to verify model consistency. This model illustrates a reasonable way to view the Earth's energy budget; it provides a number of useful insights in climatology sensitivity estimates and demonstrates the relative advantage of solar geoengineering solutions over GHG reduction in global warming mitigation [1]. In working the model, we also find a handy Planck-Albedo parameter that may be useful to climatologists [2].

2. Data and Method

In order to introduce the re-radiation surface model, it is helpful to initially look at the Planck parameter as it plays a key role in verifying modeling.

2.1 Overview of Planck Feedback Parameter

Estimates on Planck's feedback parameter are varied, typically between $-3.8W/m^2/^oK$ and $-3.21W/m^2/^oK$ with some values as large as $-7.1W/m^2/^oK$ [3]. The IPCC AR4 [4] list a value of $-3.21W/m^2/^oK$. Numerous authors have developed different expressions [3]. A typical estimate starts with

$$F_{TOA} = (1-\alpha)\,S_o/4 - \sigma(\beta T_S)^4 = (1-\alpha)\,S_o/4 - R_{LWR} \quad (1)$$

where $S_o=1361W/m^2$, F_{TOA} is the radiation budget at the top of the atmosphere, R_{LWR} is the outgoing long wave radiation (a function of surface temperature and albedo), σ is the Stefan-Boltzmann constant and β is described below. Then the Planck parameter λ_o can be calculated as

$$\lambda_o = \partial F_{TOA}/\partial T_S = -\partial R_{LWR}/\partial T_S \quad (2)$$

This result is

$$\lambda_o = -4\beta^4\sigma T_S^3 = -4\beta\sigma T_{TOA}^3 \quad (3)$$

where β varies from 0.876 to 0.887 (averaging=0.8815) and $Ts=288^oK$ [4]. This yields $-3.37W/m^2/^oK<\lambda_o<-3.21W/m^2/^oK$. However, from Eq. 3, β is often taken as the ratio

$$\beta = T_{TOA}/T_S = 255°K/288°K = 0.8854 \quad (4)$$

A common assessment uses $T_{TOA}=255^oK$, so that $\lambda_o=-3.33W/m^2/^oK$. Another expression developed by Schlesinger [5] is dependent on the albedo and surface temperature as

$$\lambda_o = S_o(1-\alpha)/T_S \quad (5)$$

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When S_o=1361, 0.294118<α<0.3, and Ts=288 °K then -3.308W/m^2/°K >λ_o>-3.3358W/m^2/°K , respectively.

2.2 Estimating Planck's Parameter with an Albedo Method

Consider a global albedo change corresponding to 1°K rise from solar absorption. Since we are only concerned with an albedo change

$$F_{TOA} = 0 = (1-\alpha)E_o - \sigma(T_S)^4 \tag{6}$$

where $E_o=S_o/4$. Then a 1°K change is

$$\Delta T_S = T_2 - T_1 = \left(\frac{E_o}{\sigma}(1-\alpha_2)\right)^{1/4} - \left(\frac{E_o}{\sigma}(1-\alpha_1)\right)^{1/4} = 1°K \tag{7}$$

Here we will use the AR5 albedo starting value of 0.294118 [6]. We find that the corresponding albedo change is 0.28299 when E_o=340W/m^2. This corresponds to an absorption of

$$\Delta E_o = E_o\left\{(1-\alpha_2)-(1-\alpha_1)\right\} = E_o(\alpha_1-\alpha_2) = 3.784W/m^2 \tag{8}$$

Since this is for a 1°K rise, then it can also be written as

$$\lambda_{1K}=3.784\text{W/m}^2/°\text{K} \tag{9}$$

We note this is related to the surface value, then

$$\lambda_{1K} = -4\sigma T_s^3 \tag{10}$$

By comparison to above we have

$$\lambda_o=\lambda_{1K}\,\beta = -3.784\text{W/m}^2/°\text{K}=-3.349\text{W/m}^2/°\text{K} \tag{11}$$

This is very close to the -3.33 W/m^2/°K value obtained in the traditional manner.

2.3 Top of the Atmosphere and Beta

From Eq. 1

$$R_{LWR} = \sigma(\beta T_S)^4 = \sigma(T_S)^4 \tag{13}$$

giving

$$\beta^4 R_{TOA,T_S} = R_{TOA,T\,TOA} \tag{14}$$

We will need this expression later when showing model consistency with the Planck feedback parameter.

2.4 Re-radiation GHG GW Model

Global warming can be modeled by looking at two different time periods. We can model the radiation for 1950 as due to blackbody radiation with the addition of GHG re-radiation so

$$P_{Total\_1950} = P_\alpha + P_{GHG} = P_\alpha + f_1 P_\alpha \tag{15}$$

where $P_\alpha = S_o\left\{0.25x(1-Albedo)\right\}$ and S_o=1361W/m^2. Here we have a fraction of the blackbody radiation is re-radiated by the GHGs so f_1 is a re-radiation parameter. That is, the energy, P_{GHG}, must be some fraction P_α. In 2019 due to global warming, this model is more complex and harder to separate out terms. However, it can still be done in theory, so

$$P_{Total\,2019} = P_{\alpha'} + P_{GHG'+Feedback} = P_{\alpha'} + f_2 P_{\alpha'} \tag{16}$$

Here $P_{GHG'+Feedback}$ includes GHG and its increase comprising also of water-vapor increase, lapse rate effect and other effects such as an increase in snow-ice albedo changes that are hard to separate out. That is, some of this feedback is related to GHG increases and some is related to albedo change. $P_{\alpha'}$ represents any albedo change due to UHI absorption increases, cloud absorption change, ice and snow melting and so forth that can be discerned.

The re-radiation still must connect the absorption to re-radiation. We use a linear *f* parameter that indicates the fraction of P_α power that must be re-radiated back to obtain the observed temperature. To be clear, *f* is just a fractional parameter. In 1950 it is some function of the GHGs (with no feedbacks). In 2019 it is more complex and includes feedback effects. However, it primarily related to GHGs re-radiation since $P_{GHG} \approx P_{GHG'+Feedback}$.

We then write

$$P_{Total} = \sigma T^4 \text{ and } P_\alpha = \sigma T_\alpha^4 \tag{17}$$

We will find $T_\alpha / T \approx \beta$.

2.5 Balancing P_{out} and P_{in}

Although Eq. 15 is reasonably simple, it turns out that f_1 has a uniquely defined value obtained when balancing the energy.

2.5.1 Balancing P_{out} and P_{in} in 1950

In order to balance the energy in with the energy out in 1950 with no global warming imbalance we can still start with Eq. 15. In equilibrium the radiation that leaves must balance what comes in $P\alpha$ so that

$$\begin{aligned} Energy_{Out} &= (1-f_1)P_\alpha + (1-f_1)P = (1-f_1)P_\alpha + (1-f_1)\{P_\alpha + f_1 P_\alpha\} \\ &= (1-f_1)\{2P_\alpha + f_1 P_\alpha\} = 2P_\alpha - f_1 P_\alpha - f_1^2 P_\alpha = Energy_{In} = P_\alpha \end{aligned} \tag{18}$$

In 1950 the value of *f* solves the quadratic equation

$$f_1^2 + f_1 - 1 = 0 \tag{19}$$

This yields the unique value in 1950

$$f_1 = 0.618 \tag{20}$$

2.5.2 Warming Imbalance in 2019

The re-radiation parameters f_1 and f_2 are connected and from Eq. 15 and 16 we have

$$f_2 = f_1 + \left(\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_\alpha}\right) = f_1 + \Delta f \tag{21}$$

In this way f_2 is a function of f_1=0.618 and the differences in the global warming residuals can be defined as Δf.

3.0 Results and Discussion

Since the re-radiation parameter $f_1=0.618$, in order to obtain T_{1950}=13.89°C (287.038°K), the only adjustable parameter in our simple model is the Earth's albedo. This value requires an albedo value of 0.3008 (see Table 1) to obtain the correct value T_{1950}. This is a reasonable and similar to values cited in the literature [11].

In 2019, the average temperature of the Earth is T_{2019}=14.84°C (287.99°K). Here we are not sure of the albedo since it likely changed due to UHI increase, snow and ice melting and cloud coverage changes. The IPCC value in AR5 [6] is 0.294118. However, this would represent a 3% change since 1950 which may be an overestimation. In our assessment, we will assume a 1% change. Then the f_2 parameter is adjusted to *0.6324* in order to obtain T_{2019}. Results are provided in the Table 1. The results yields $P_{Total\_1950}=384.918\ W/m^2$ and $P_{Total\_2019}=390.024$ W/m^2. We find that

$$\Delta P_{Total} = P_{2019} - P_{1950} = 5.097 W/m^2 \tag{22}$$

and

$$\Delta T_{Total} = T_{2019} - T_{1950} = 0.95°C \tag{23}$$

which is the observed surface temperature increase since 1950.

Table 1 Model results

Year	T(°K)	T_α(°K)	f_1, f_2	α, α'	P_α, $P_{\alpha'}$ (W/m^2)	P_{GHG} (W/m^2) $P_{GHG'+feedback}$	P_{Total} (W/m^2)
2019	287.989	254.78	0.6324	29.779	238.927	149.870	390.024
1950	287.0395	254.51	0.618	30.08	237.903	147.024	384.918
Δ2020-1950	**0.95**	0.27	**1.44%**	-0.3 **(1%)**	**1.024**	2.846	**5.097**

The table below summarizes model results for the specified albedos and setting the model to the observed Earth's surface temperatures.

To show model consistency, the forcing change 5.097 W/m^2 resulting in a 0.95°K rise, should agree with what is expected from Planck's feedback parameter. From Eq. 14 it is evident that

$$\beta^4 \Delta R_{TOA} = 5.097 \text{ x } \beta^4 = 3.132 \text{W/m}^2 \quad (24)$$

This illustrates the consistency of the simple re-radiation model. Then Planck's feedback parameter temperature rise is in agreement with what is observed

$$3.139 \text{W/m}^2 \text{ x } (1/3.3)^{\circ}\text{K/W/m}^2 = 0.949^{\circ}\text{K at } T_s \quad (25)$$

3.1 Why the Re-radiation Parameter is Significant

In Table 1, the measure of *Δf=1.44%* fractional increase is due to re-radiation change. This is significant. From Eq. 15, 16 and 21 we can illustrate this key characteristic of the climate change

$$\Delta f = (\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_{\alpha}}) = (\frac{P_{GHG'+F}}{P_{\alpha'}} - \frac{P_{GHG}}{P_{\alpha}}) \approx (\frac{P_{GHG'+F} - P_{GHG}}{P_{\alpha}}) \quad (26)$$

Therefore f is an estimate of climate re-radiation and Δf an estimate of climate change from a different perspective. It is a measure of GHG increase and the feedback relative to the initial radiation, and is generally helpful in looking at how our climate is working. Furthermore, we can deduce an albedo advantage.

3.2 The Albedo Advantage

We can look at an important ratio, the power created by the albedo effect compared to GHGs in 1950. The initial radiation is P_α which heats the Earth to 254.51K then according to Eq. 15 and Table 1, the P_{GHG} energy originates from a fraction of this original heating due to re-radiation as fP_α

$$\frac{P_\alpha + P_{GHG}}{P_{GHG}} = \frac{P_\alpha + fP_\alpha}{P_{GHG}} = \frac{P_\alpha + fP_\alpha}{fP_\alpha} = \frac{1+f_1}{f_1} = \frac{1.62}{0.62} = 2.62 \quad (27)$$

In general, this also means that albedo change has a higher impact factor in climate forcing, 2.6 times larger than ΔP_{GHG} as well, that is a change, ΔP_α compared with a change in ΔP_{GHG} would yield the same impact factor

$$\frac{\Delta P_\alpha + \Delta P_{GHG}}{\Delta P_{GHG}} \approx \frac{\Delta P_\alpha + f\Delta P_\alpha}{f\Delta P_\alpha} \approx \frac{1+f_1}{f_1} = \frac{1.62}{0.62} = 2.62 \quad (28)$$

This is a key reason that UHIs, cloud coverage, snow and ice melting, can create significant climate effects. Appendix A puts this important impact factor in layman's terms.

In this view, an albedo solution is advantageous having significant potential for reversing global warming or ignoring it, as in UHIs likely can create serious issues. Therefore, trying to control global warming by reducing GHGs is important. However, certainly an albedo approach is more advantageous. It reduces both initial absorption and its potential for its re-radiation. Its impact rating can be taken as 162% compared to re-radiation f with a 62% impact by comparison according to Eq. 27 and 28, yielding a 2.6 times higher advantage. It is important to realize that because the albedo solution can highly impact GW and reverse trends, it is also vital in preventing a tipping point from occurring.

3.3 Planck-Albedo Feedback Parameter

The albedo changes in Table 1, is: $\%\Delta\alpha = 1\%$. The albedo ΔP_α change in Table 1 is 1.024W/m^2. We note that we can define a unique Planck-Albedo parameter $\lambda_{\%\Delta\alpha} = \Delta P_\alpha / \%\Delta albedo$. To illustrate from Table 1

$$\lambda_{\%\Delta\alpha} = 1.024 \text{ W/m}^2/\Delta\%\text{albedo} = 1.024/1\% \quad (29)$$

This parameter can also be expressed per degree (noting the 0.95°K change in Table 1)

$$\lambda_{\%\Delta\alpha\Delta T} \approx 1W/m^2/\Delta\%albedo/^\circ K \quad (30)$$

The parameter was first noted in Feinberg 2020 [2] but is featured here as a modeling tool. We term it the Planck-Albedo parameter, since it relates to blackbody (P_α) absorption. A simple numeric example is given in the Conclusion to illustrate how it provides helpful estimates. This interesting parameter arises from the basic assessment

$$\lambda_{\%\Delta\alpha} = \frac{(\Delta E_o)_\alpha}{\frac{\alpha_1-\alpha_2}{\alpha_1}100} = \frac{E_o(\alpha_1-\alpha_2)}{\frac{\alpha_1-\alpha_2}{\alpha_1}100} = E_o\alpha_1/100 \approx 1W/m^2/\%\Delta albedo \quad (31)$$

where E_o=340 W/m^2 and when α_1 is 29.4118%, the value 1.000W/m^2/Δ%albedo is obtained. We note the value 29.4118% (100/340) is given in AR5 [6]. The parameter's relationship to λ_α is

$$\lambda_\alpha = \lambda_{\%\Delta\alpha\Delta T}\ x\ \%\Delta\alpha \quad (32)$$

and the feedback parameter including f re-radiation is in 2019

$$\lambda_\alpha^\dagger = \lambda_{\%\Delta\alpha\Delta T}\ x\ \%\Delta\alpha\ x 1.618 \quad (33)$$

4.0 Conclusion

In this paper we provided a simple re-radiation global warming model. The model shows consistency with the Planck parameter. We noted that the re-radiation parameter increased by about 1.44% due to global warming from 1950 to 2019, illustrating the warming from a different perspective. From the model, the albedo effect was quantified having an impact rating of 162% compared to GHGs with 62%. The albedo effect then yields a 2.6 times higher advantage upon comparison. These results strongly support moving forward with solar geoengineering solutions [2, 7-9].

We also found a handy parameter that we termed the Planck-Albedo parameter which is about $\lambda_{\%\Delta\alpha\Delta T} \approx 1W/m^2/\Delta\%albedo/^\circ K$. This can be helpful in quickly estimating the effect of an albedo change on global warming and in assessing λ_α. For example, Feinberg 2020 [1] suggested a goal of 1.5% geoengineering albedo change. Using this parameter, an impact of 1.5 Watts/m^2 warming reduction should result. Given a 1.62 reemission factor (Eq. 28), this is 2.4W/m^2 improvement. With a reduction in water-vapor feedback, often estimated by a factor of 2 [10], provides an overall resulting effect that could be as high as 4.8W/m^2. Feasibility is discussed in more detail in Feinberg's 2020 paper [1] and other solutions have been proposed [6-9].

Appendix A: Quantifying the Albedo Advantage in Layman's Terms

It may be helpful for the reader to have a layman's view of how the 2.62 factor comes about. Consider the Earth with a roof. The roof represents the GHGs over the Earth and only allows 40% of any energy leaves with the rest returning to Earth. Sunlight comes in and some is absorbed and heats the Earth's floor to 255^{o}K (-2.3^{o}F very cold). Let's say it takes 100 units of energy. The heat rises but only 40 units of energy can leave so 60 units comes back and warms the Earth's floor some more to 288^{o}K (57^{o}F average temp of Earth). On average the Earth's floor is warmed a total of 160 units. The Sun keeps warming the Earth's floor at 100 units on average and the roof keeps sending back 60. So the roof is responsible for 60 units on average of energy and the Earth's floor is warmed up to 160 units on average. We can write this as

Energy units: 160=100+60=100+100x0.6

We see the 100 units is in two places in the equation, while the 60 is only in one place. That is without the floor absorption first, the roof cannot keep the Earth warm. Therefore, the heat coming from the Earth's floor results in160 units and the roof is only 60 units by comparison. The impact factor is

- 160/60=2.66, that is the heat from the Earth's floor has this much larger impact.

Alternately, for every unit of energy given off, by the Earth's floor after absorption it is equivalent to causing 1.6 units of heating while the roof (GHG) is only responsible for 0.6.

How much heat leaves in equilibrium? There was the initial 40 leaving of the 100 units of energy absorbed and radiated. As well the Earth's floor received a total of 160 units but the roof only let 40% leave that is another 64 (=0.4 x 160) units of energy leaving. The total leaving is 104 units in equilibrium so roughly 100 units comes in and almost same goes out.

This can be refined to 61.8% (Eq. 20). Then 100 units is absorbed and radiated, then 38.2 units initially leave, and 61.8 units is radiated so the Earth's floor is heated to 161.8 units of energy. From this 0.382 x 161.8 leaves=61.8 units or energy. The total is 61.8+38.2=100 units of energy leaves and another 100 units comes and equilibrium is established. Any difference causes global warming.

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