Revival of MOND or the Gravity Law without Universalism

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Abstract In this note I argue that modified gravity can describe Dark Matter if one understands the modification of gravity as a tensor field $X^{\mu\nu}(t,x,y,z)$ in the Einstein equations, i.e. as an additional mathematical parameter filling the Universe without correspondence to new particles. Notably, there are many different fields in nature, e.g. the Higgs field, the inflaton field, and the temperature distribution field $T(t,x,y,z)$. My testable prediction that Dark Matter particles will never be discovered is well realized up to today; therefore, Popper's falsifiability criterion is satisfied, because the underground detectors could report the signal. On the other hand, the testable proof would be the indirect discovery of the sterile neutrino (because it does not interact with visible matter even weakly, it is an example of $X^{\mu\nu}$).

Keywords Dark Matter; Dark Energy; Alternative gravity theories; Geodesic; Cosmological Constant Problem; Information Paradox

1 Preface

What is the nature of dark matter? Is it a particle, or do the phenomena attributed to dark matter actually require a modification of the laws of gravity? In this first publication in a series of papers I deal with this question without applying mathematical tools. Nevertheless, all my points are backed up by evidence. The next two publications entitled "Broken Geodesics and Dark Matter" and "Energy Localization Problem points out the vanishing of matter in the First Order Deviation Equation" are highly mathematical applications of the theory described in this short note. My approach goes beyond standard ΛCDM cosmology, trying to find a solution for problems indicated in Ref. (Chen 2019). However, ΛCDM is contained as a special case in Eq. (1).

Modified Newtonian dynamics (MOND) is a hypothesis that proposes a modification of Newton's laws to account for observed properties of galaxies. It is an alternative to the hypothesis of dark matter in terms of explaining why galaxies do not appear to obey the currently understood laws of physics. Created in 1982 and first published in 1983 by the Israel physicist Mordehai Milgrom (Milgrom 1983), the hypothesis' original motivation was to explain why the velocities of stars in galaxies were observed to be larger than those expected by using Newtonian mechanics.

MOND is an example of a class of theories known as modified gravity, and it is an alternative to the hypothesis that the dynamics of galaxies are determined by massive, invisible dark matter halos. Since Milgrom's original proposal, MOND has successfully predicted a variety of galactic phenomena that are difficult to understand from a dark matter perspective (Banik 2018). However, MOND and its generalisations do not adequately account for observed properties of galaxy clusters, and no satisfactory cosmological model has been constructed from the hypothesis.

The accurate measurement of the speed of gravitational waves compared to the speed of light in 2017 ruled out many theories which used modified gravity to avoid dark matter (Boran 2018). However, according to the same study neither Milgrom's bi-metric formulation of MOND nor nonlocal MOND are ruled out.

2 Common feature of MOND proposals

Newton's law of universal gravitation usually states that every particle attracts every other particle in the

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universe with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. This is a general physical law derived from empirical observations by what Isaac Newton called inductive reasoning (Newton 1729).

The common feature of all MOND proposals is this universalism. Given the energy-momentum tensor for "visible" (e.g., baryonic) matter, one perfectly determines Dark Matter. However, that seems to be not true because galaxies without Dark Matter are discovered (Dokkum 2018). In contrast to this, I introduce a non-universal law of gravitation in Eq. (2). According to this, there are places and times in the universe where the gravitational force cannot be calculated just from the properties of visible matter. To fix the problems of MOND, I suggest to include a tensor field of Dark Matter.

3 How to modify gravity

A general expression for modified gravity can be written as

$$\mathrm{G}^{*\mu\nu} = 8\pi\, T^{\mu\nu}\,, \tag{1}$$

where the left hand side is the modified Einstein tensor. $T^{\mu\nu}$ is the energy–momentum tensor of visible matter. Without loss of generality one can rewrite Eq. (1) using the definition $8\pi\, X^{\mu\nu} = \mathrm{G}^{\mu\nu} - \mathrm{G}^{*\mu\nu}$,

$$\mathrm{G}^{\mu\nu} = 8\pi\,(T^{\mu\nu} + X^{\mu\nu})\,, \tag{2}$$

where the unmodified Einstein tensor is on the left hand side. In the following I call $X^{\mu\nu}$ a virtual term, in particular Virtual Matter. This term cannot be detected in particle detectors, as it is not visible matter but rather a pure mathematical modification of Einstein's equations. In case the covariant divergence $X^{\mu\nu}_{;\nu}$ vanishes, we will call it Dark Matter. In this sense, Dark Energy is a class of Dark Matter because $(\Lambda\, g^{\mu\nu})_{;\nu} = 0$.

My proposal is to allow the 10 independent functions $X^{\mu\nu} = X^{\mu\nu}(t, x, y, z)$ not to be universal, i.e. being not always the most popular expression of Dark Matter (which is dust-like tensor $X^{\mu}_{\nu} = \mathrm{diag}(-\rho, 0, 0, 0)$), but different in any given task and problem. What determines the shape of $X^{\mu\nu}$? Is it theoretical physics or the experiment or observation? My answer is, that it is both, as e.g. in Section IV the introduction of $X^{\mu\nu}$ turns out to be a solution to particular theoretical problems. Therefore, one can not blame my proposal for having no predictive power – despite the fact that the absolute generality of Eq. (2) fits any possible experiment or observation.

4 Evidences of the necessity of $X^{\mu\nu}$ for fixing problems

4.1 Fixing singularities

Using known facts from General Relativity, it is indeed possible and easy to solve the mystery. Any singularity is simply a mathematical blow up of the theory of Relativity. To fix this and to make the theory physical rather than mathematical, I am using a virtual term $\psi(r)$ in the Schwarzschild Black Hole metrics,

$$ds^2 = -\left(1 - \frac{2M}{r + \psi(r)}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r+\psi(r)}} + r^2\, d\Omega^2\,, \tag{3}$$

where $\psi(r > 2M) = 0$, $\psi(r \leq 2M) = \epsilon\,(2M - r)$ for $0 \leq r < \infty$ and small $\epsilon > 0$.

The tensor $X^{\mu\nu}$ can be calculated from Eqs. (2) and (3) for $T^{\mu\nu} = 0$. The demand to fulfil the "energy conditions" (weak, strong, and others) is not applicable to the virtual matter $X^{\mu\nu}$, as it is not subject to measurements. So one would not measure a negative energy. Notably, the known concepts of "phantom fields" (Singh 2003) and "exotic matter" (Ivanov 2009; Morris 1988) have problems with energy conditions, but considering them as the examples of Virtual Matter, they have no such problems. By the way, any possible instability of my MOND proposal is simply removed by properly chosen variations of the arbitrary virtual term $X^{\mu\nu}$.

4.2 Fixing abrupt geodesics

If one releases a particle in Kerr, Kerr–Newman, or Reissner–Nordström spacetime with zero initial velocity $u^r = u^\theta = u^\phi = 0$ (in case of photon $u^\theta = u^\phi = 0$, $u^r < 0$), it will reach an abrupt end of the trajectory at the radius $r = r_m > 0$, because there is $(u^r)^2 < 0$ for $r < r_m$. The curvature singularity is at $r = 0$. Note that in case of a motion inside the equatorial plane $\theta = \pi/2$ the abrupt end geodesics are present for Kerr-Newman and Reissner-Nordström spacetimes. The abrupt end of geodesics mean the vanishing of matter.

Notably, Einstein's vision of a steady-state universe theory contains the "formation of matter from empty space", so that the density of matter in the expanding universe is kept constant. But it turned out that Einstein's equations are violated by such an assumption (O'Raifeartaigh 2014). Indeed, the appearance of matter in "forward time" is equivalent to the vanishing of matter in "backwards time". If so, one needs to use a properly chosen virtual term $X^{\mu\nu}$ to revisit the Einstein's proposal.

The details are found in Ref. (Martila 2019) which uses velocity expressions from Ref. (Lightman 1975). An evidence is presented in Appendix below.

4.3 Fixing the Cosmology Constant Problem

In cosmology, the "cosmological constant problem" or "vacuum catastrophe" is the disagreement in non-modified General Relativity (Einstein 1916) between the small value of spacetime curvature (taken at global scale) and the theoretical large value of the zero-point energy suggested by quantum field theories. Depending on the "Planck energy cutoff" and other factors, the discrepancy is as high as 120 orders of magnitude, a state of affairs described by physicists as "the largest discrepancy between theory and experiment in all of science" and "the worst theoretical prediction in the history of physics." (Adler 1995)

In my MOND proposal the "fixing" of this problem can be conducted by using the fixing of the spacetime metric tensor via $X^{\mu\nu} \neq 0$; hereby the friction-free inertial motion of visible matter inside the "sea" of (invisible) virtual particles of vacuum is explained in a most natural and easy way, because the virtual particles can be seen as an example of my Virtual Matter. It is an amazing historic coincidence that the virtual particles and my Virtual Matter concept both contain the word "virtual".

Here and in the following, "fixing" of spacetime means that we start by postulating the metric tensor $g_{\mu\nu}$, after that finding the behaviour of the visible matter in a given spacetime area (e.g. the motion of the test-particle below). Note that the "fixed" metric can be non-stationary. One indeed can notice that although we are used to read Eq.(1) from right to left (i.e. given the tensor $T^{\mu\nu}$ one finds the metric tensor by solving the differential equations), one can perform the following trick: starting with some considerations for $g_{\mu\nu}$ and taking the partial derivatives one easily finds the Einstein tensor $G^{\mu\nu}$, then given some $X^{\mu\nu}$ one finds the parameters inside the $T^{\mu\nu}$.

4.3.1 Occam's razor and Dark Energy

Occam's razor is the problem-solving principle that states that "Entities should not be multiplied without necessity." (Schaffer 2015) The idea is attributed to English Franciscan friar William of Ockham (1287–1347), a scholastic philosopher and theologian who used a preference for simplicity to defend the idea of divine miracles. It is sometimes paraphrased by a statement like "the simplest solution is most likely the right one."

According to this principle, if the influence of vacuum energy is that much reduced (more than 10^{120} times), the most simple and natural theory is the complete absence of such an influence (due to a counterparting tensor $X^{\mu\nu}$). In such case we would be lacking of an undisputed proof for the Dark Energy. Amazingly, there are new papers which put the existence of Dark Energy in doubt (Kang 2020).

4.4 Fixing the test-particle formalism

The known geodesic equation $u^{\mu}_{;\nu}\, u^{\nu} = 0$ of a test-particle motion silently assumes that the background spacetime is fixed, i.e. that there is no backreaction from high-speed (e.g. near Black Holes) geodesic motion on the spacetime.

The absence of a backreaction is only possible if $X^{\mu\nu} \neq 0$ is present in Eq. (2). This would be the simplest solution for the topic of backreaction, while other solutions have major mathematical complications with accounting for the gravitational backreaction, e.g. in Refs. (Hawking 1974).

4.5 Fixing the static universe

I am talking not about the real Universe (the one we are living in), but about an imaginable one, which is constructed from the known laws of nature.

It is known that the pressure in the perfect fluid model allows us to have a static drop of fluid in empty spacetime. It is expected that pressure as the resistance of matter counterparts gravity, and so a static universe filled with a perfect fluid should be allowed. Hereby, in case of the "flat Friedmann universe" metric, $X^{\mu\nu} = -T^{\mu\nu} \neq 0$ can be necessary.

It is interesting that while trying to construct the static universe (Einstein 1917), Albert Einstein found an example for a non-zero $8\pi\, X^{\mu\nu} = -\Lambda\, g^{\mu\nu} \neq 0$, naming it later the "biggest blunder" (Bodanis 2016) of his life without even realizing the entire potential and usefulness of this discovery (e.g. the possibility of interstellar travel (Morris 1988)).

5 Conclusions

One should include such a concept as virtual terms, i.e. mathematical insertions into the equations and laws of nature which are made not from fundamental premises but "by hand" in order to fit the theory under observation. An example for such insertions are Dark Matter and Dark Energy. Therefore, these cannot be directly detected, but it is possible to measure their effect on nature. As a prime example, the Dark Matter anomaly has acted on the space-time grid in such an amount that it created an additional force of attraction of stars to the center of their galaxy. By the way, the proton radius measured by many experimenters was different in different years. This riddle did not find yet a solution (Karr 2019). I, personally, would solve this problem with a virtual insertion Ψ into the radius value, $r = R + \Psi$.

5.1 Definition and Status Quo of proton radius puzzle

The proton radius puzzle is an unanswered problem in physics relating to the size of the proton. (Krauth 2017) Historically the proton charge radius was measured by two independent methods, which converged to a value of about 0.877 femtometres. This value was challenged by a 2010 experiment using a third method, which produced a radius about 4% smaller than this, at 0.842 femtometres. (Pohl 2010) New experimental results reported in the fall of 2019 agree with the smaller measurement, and it has been proposed that the puzzle is now solved, (Hammer 2019) though this opinion is not yet common. (Karr 2019)

5.2 Fixing to missing antimatter

A virtual term at the initial conditions fixes the "missing antimatter paradox", for Dr. Christian Smorra comments the paradox as "All of our observations find a complete symmetry between matter and antimatter, which is why the universe should not actually exist. An asymmetry must exist here somewhere but we simply do not understand where the difference is. What is the source of the symmetry break?" (Smorra 2017)

5.3 Beauty and simplicity of my MOND proposal

Despite of a recently published book by Sabine Hossenfelder (Hossenfelder 2018) which blames the feeling of beauty for creating problems, one can show that my MOND proposal is so much beautiful that it could satisfy even the highest standards of scientific beauty posed by Albert Einstein.

By calling $X^{\mu\nu}$ matter (invisible matter) the author restores Einstein's original idea (Einstein 1916). For the scientific community there is no need to modify General Relativity if one accepts the existence of invisible (Virtual) matter. The "sterile neutrino" does not interact with visible matter even "weakly". As the sterile neutrino cannot be directly detected by neutrino detectors, it can be regarded as a theoretical example of my invisible matter concept. Already the confidence level of 6σ is reported in Ref. (Aguilar-Arevalo 2018).

Matter is defined to be invisible if it does not interact with visible matter (e.g. with baryonic matter). A word can have multiple meaning, I am adding a new meaning. Hereby, the gravitational interaction cannot be called a true interaction, because according to Albert Einstein the gravity is not a force. The examples of invisible matter are the sterile neutrino. Reality works on proper definitions and on the correct use of words. This is due to the First Law of Aristotle's Logic. Therefore, if gravity cannot be called a force, this is important to get to know about reality. In other words: according to the Equivalence Principle (Crépin 2019) the local physics is independent from spacetime curvature, thus, the gravity is not an interaction. However, in Eq. (2) the possibility of non-zero covariant divergence of the invisible matter tensor $X^{\mu\nu}$ is possible; this means, that the gravity can have much stronger influence than it is currently believed.

But even more beauty is possible if one considers the possibility $\Lambda = 0$ proposed in Refs. (Kang 2020). Virtual Matter would then be the known energy-momentum tensor, e.g. invisible pressure-free dust or an invisible perfect fluid with a properly chosen sign ($\pm X^{\mu\nu}$).

Hereby the singularity places (and other spacetime problems) could be simply cut out of our Universe map (if necessary), e.g. the event horizon is the edge of our Universe (Martila 2020). That can be the actual reason behind the "information loss paradox": the vanishing of matter, because a vanishing of matter is already shown (the "abrupt end geodesics" above).

5.4 Fixing to Black Hole Information Paradox

The black hole information paradox is a puzzle resulting from the combination of quantum mechanics and general relativity. Calculations suggest that physical information could permanently disappear in a black hole, allowing many physical states to devolve into the same state. This is controversial because it violates a core precept of modern physics. There are two main principles in play: (Hossenfelder 2019) "quantum determinism" means that given a present wave function, its future changes are uniquely determined by the evolution operator; "reversibility" refers to the fact that the evolution operator has an inverse, meaning that the past wave functions are similarly unique. The combination of the two means that information must always be preserved.

Such description of the information loss paradox implies, that the natural laws are not time-reversible, i.e. are not T-invariant. But that problem one can show without the complicated and imperfect Hawking's calculations, just understand, that while gravitational collapse of a dust cloud in co-moving coordinates the process becomes irreversible after the size of the cloud becomes smaller than Schwarzschild radius. It is because "nothing can escape the Black Hole".

As of November 2019, the paradox may have been resolved, at least for simplified models of gravity. (Penington 2019)

6 Appendix: abrupt end geodesics

The velocity components of a radial straightline falling of a neutral test-particle in Reissner–Nordström spacetime are given by

$$u^r \equiv \frac{d\,r}{d\tau} = -\frac{1}{r^2}\sqrt{B}\,, \quad u^\phi = u^\theta = 0\,, \tag{4}$$

where $B = E^2\,r^4 - (r^2 - 2\,M\,r + Q^2)\,r^2$.

In "geometrized" units (Q, M, r in meters) let us choose $Q = 1/5$ and $M = 1/2$. Zero initial velocity ($B = 0$ at $r = r_0 = 20$) determines the trajectory with

$$E = \frac{\sqrt{9501}}{100}\,. \tag{5}$$

Therefore

$$B = -\frac{499}{10000}\,r^4 + r^3 - \frac{1}{25}\,r^2\,, \tag{6}$$

which is negative in $r < r_m = 20/499$. Thus, at r_m one has $u^r = 0$.

This can mean a termination of the falling body. The terminations are present in a Kerr spacetime as well as in naked singularity regimes, and the "smooth correction" of metric (using my Virtual Matter like in Eq. (3)) can remove the singularity. Such terminations were never found yet. Indeed, it is not reported in Refs. (Lightman 1975; Hackmann 2013).

The value of E in Eq. (5) is slightly different for either slightly different parameter of spacetime (a, Q or M) or for slightly different initial velocities of the test particle. Therefore, my effect holds not for very specific parameters, but has a wide range of physically allowed parameters.

An anonymous referee of some top-ranked journal has written: "The author appears to be concerned about points where $u^r = 0$. However, even in the Schwarzschild solution this can happen and it is perfectly harmless. The geodesic equations in the Schwarzschild spacetime, when applied to light deflection, contain a point where $u^r = 0$, this is the point of closest approach and it makes perfect sense for this function to go through a minimum which is why its derivative is allowed to vanish. It is "turning point". This has nothing to do with the termination of a falling body."

Dear reader, you should not be satisfied by this bizarre report of the referee, as I am talking about a radial straightline falling into the Black Hole, while the referee talks about a curved "passing-by" orbit.

In case of Reissner–Nordström spacetime the direct radial falling is studied, so it has no turning point as imagined by the referee. Moreover, the Kerr spacetime is considered (Martila 2019) and falling goes along the axis of rotation $\theta = 0$, so it does not have a turning point as well. In case of the general falling with $\theta \neq 0$ the spacetime point with r_m has zero velocity for the falling body, $u^r = u^\theta = u^\phi = 0$, which points not to a turning point but to a complete stop. This is not the entrance into another universe, because the stationary observer at this point reports zero of kinetic energy despite the arguments of (Poisson 2004).

I have a Black Hole with nothing beyond the event horizon, only empty space-time. Is perfectly known that inside any Black Hole the matter is placed only at the singularity $r = 0$. Indeed, many authors, including the authors of Ref. (Hackmann 2013), do study the inner vacuum solution of the Black Hole.

References

Adler R. J., Casey B., Jacob O. C., "Vacuum catastrophe: An elementary exposition of the cosmological constant problem." Am. J. Phys. 63, 620–626 (1995); Hobson M. P., Efstathiou G. P., Lasenby A. N., "General Relativity: An introduction for physicists" (Reprint ed.), Cambridge University Press, 2006, p. 187.

Aguilar-Arevalo A. A. *et al.* (MiniBooNE Collaboration), Significant Excess of ElectronLike Events in the MiniBooNE Short-Baseline Neutrino Experiment, Phys. Rev. Lett. 121, 221801 (2018).

Banik I., O'Ryan D., Zhao H., "Origin of the Local Group satellite planes", Monthly Notices Roy. Astr. Soc. 477, 4768–4791 (2018); McGaugh S., "A Tale of Two Paradigms: the Mutual Incommensurability of LCDM and MOND," Can. J. Phys. 93 (2), 250–259 (2015); Kroupa P., Pawlowski M., Milgrom M., "The failures of the standard model of cosmology require a new paradigm," Int. J. Mod. Phys. D 21 (14), 1230003 (2012).

Bodanis D., "Einstein's Greatest Mistake: A Biography", Houghton Mifflin Harcourt, 2016, 304 pages.

Boran S., Desai S., Kahya E., Woodard R., "GW170817 Falsifies Dark Matter Emulators," Phys. Rev. D 97, 041501 (2018).

Chen G. C.-F. *et al.*, "A SHARP view of H0LiCOW: H0 from three time-delay gravitational lens systems with adaptive optics imaging", Monthly Notices Roy. Astr. Soc. 490(2), 1743–1773 (2019); Chen H., Fishbach M., Holz D. E., "A two per cent Hubble constant measurement from standard sirens within five years", Nature 562, 545–547 (2018). See also the report of the W. M. Keck Observatory in phys.org, "A crisis in cosmology: New data suggests the universe expanding more rapidly than believed."

Crépin P.-P., Christen C., Guérout R., Nesvizhevsky V. V., Voronin A. Yu., Reynaud S., "Quantum interference test of the equivalence principle on antihydrogen", Phys. Rev. A 99, 042119 (2019); Touboul P. *et al.* "MICROSCOPE Mission: First Results of a Space Test of the Equivalence Principle", Phys. Rev. Lett. 119, 231101 (2017).

Dokkum P.-v., Danieli S., Cohen Y. *et al.*, "A galaxy lacking dark matter," Nature 555, 629–632 (2018).

Einstein A., "The Foundation of the General Theory of Relativity", Annalen der Physik 354, 769 (1916); Einstein A., "Die Feldgleichungen der Gravitation", Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin: 844–847 (1915).

Einstein A., "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie." Sitzungsb. König. Preuss. Akad. 142–152 (1917), reprinted and translated in "The Principle of Relativity" (Dover, 1952) 175–188; O'Raifeartaigh C., O'keeffe M., Nahm W., Mitton S., "Einstein's 1917 static model of the universe: a centennial review", Eur. Phys. J. H 42, 431–474 (2017).

Hackmann E., Xu H., "Charged particle motion in Kerr-Newmann space-times", Phys. Rev. D 87, 124030 (2013); Adamo T., Newman E. T., "The Kerr-Newman metric: A Review", Scholarpedia 9, 31791 (2014), arXiv:1410.6626; Cebeci H., Özdemir N., Sentorun S., "Motion of the charged test particles in Kerr-Newman-Taub-NUT spacetime and analytical solutions", Phys. Rev. D 93, 104031 (2016).

Hammer H.-W.; Mei*ner Ulf-G., "The proton radius: From a puzzle to precision", 2019, arXiv:1912.03881

Hawking S. W., "Black hole explosions?" Nature 248, 30–31 (1974); Hawking S. W., "Particle creation by black holes." Commun. Math. Phys. 43, 199–220 (1975); Hawking S. W., "Hawking on the Big Bang and Black Holes," Advanced Series in Astrophysics and Cosmology, 85–106 (1993); Mersini-Houghton L., "Backreaction of Hawking radiation on a gravitationally collapsing star I: Black holes?" Phys. Lett. B 738, 61–67 (2014).

Hossenfelder S., "Lost in Math: How Beauty Leads Physics Astray", New York: Basic Books, 2018, 304 pages.

Hossenfelder S., "How do black holes destroy information and why is that a problem?" Back ReAction (23 August 2019).

Ivanov P., "On the dynamics of exotic matter: towards creation of Perpetuum Mobile of third kind", Phys. Lett. B 680, 212–216 (2009).

Kang Y., Lee Y.-W., Kim Y.-L., Chung C., Ree C. H., "Early-type Host Galaxies of Type Ia Supernovae. II. Evidence for Luminosity Evolution in Supernova Cosmology", ApJ 889, 8 (2020); Colin J., Mohayaee R., Rameez M., and Sarkar S., "Evidence for anisotropy of cosmic acceleration", Astron. Astrophys. 631, L13 (2019); Colin J., Mohayaee R., Rameez M., Sarkar S., A response to Rubin and Heitlauf: "Is the expansion of the universe accelerating? All signs still point to yes", arXiv:1912.04257 [astro-ph.CO] (2019).

Karr J.-P., Marchand D., "Progress on the proton-radius puzzle", Nature 575, 61–62 (2019); Castelvecchi D., "How big is the proton? Particle-size puzzle leaps closer to resolution", Nature 575 , 269–270 (2019); Hill H., "Proton radius puzzle may be solved", Physics Today (6 November 2019).

Krauth J. J., Schuhmann K., Abdou Ahmed M., Amaro F. D., Amaro P., *et al.* (2 June 2017). "The proton radius puzzle." 52nd Rencontres de Moriond EW 2017. La Thuile, Aosta Valley. arXiv:1706.00696

Lightman A. P., Press W. H., Price R. H., Teukolsky S. A., "Problem Book in Relativity and Gravitation," Princeton University Press, Princeton, 1975.

Martila D., "Broken Geodesics and Dark Matter", 2019, to be published.

Martila D., "Is Black Hole the hole?", 2020, to be published.

Milgrom M., "A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis", Astrophys. J. 270, 365–370 (1983); Milgrom M., "A modification of the Newtonian dynamics – Implications for galaxies", Astrophys. J. 270, 371–383 (1983); Milgrom M., "A modification of the Newtonian dynamics – Implications for galaxy systems". Astrophys. J. 270, 384–389 (1983).

Morris M. S., Thorne K. S., "Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity," Am. J. Phys. 56, 395 (1988).

Newton I.: "In [experimental] philosophy particular propositions are inferred from the phenomena and afterwards rendered general by induction," Citation taken from "Principia", Book 3, General Scholium, on page 392 in Volume 2 of Andrew Motte's English translation, published 1729.

O'Raifeartaigh C., McCann B., Nahm W., Mitton S., "Einstein's steady-state theory: an abandoned model of the cosmos", Eur. Phys. J. H 39, 353–367 (2014).

Penington G., Shenker S., Stanford D., Yang Z., "Replica wormholes and the black hole interior", 2019, arXiv:1911.11977; Almheiri A., Hartman T., Maldacena J., Shaghoulian E., Tajdini A., "Replica Wormholes and the Entropy of Hawking Radiation", 2019, arXiv:1911.12333.

Pohl R., Antognini A., Nez F., Amaro F. D., Biraben F., *et al.*, "The size of the proton", Nature 466, 213–216 (2010).

Poisson E., A Relativist's Toolkit: The Mathematics of Black-Hole Mechanics, 2004, look up pages 182 and 183.

Schaffer J., "What Not to Multiply Without Necessity", Australas. J. Philos. 93:4, 644–664 (2015).

Singh P., Sami M., Dadhich N., "Cosmological dynamics of the phantom field," Phys. Rev. D 68(2), 023522 (2003).

Smorra C., Sellner S., Borchert M. *et al.*, "A parts-per-billion measurement of the antiproton magnetic moment", Nature 550, 371–374 (2017).

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