Finding the Planck Length Multiplied by the Speed of Light Without Any Knowledge of G, c, or $\hbar$ Using a Newton Force Spring

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Abstract

In this paper, we show how one can find the Planck length multiplied by the speed of light, $l_p c$, from a Newton force spring with no knowledge of the Newton gravitational constant G, the speed of light c, or the Planck constant h. This is remarkable, as for more than a hundred years, modern physics has assumed that one needs to know G, c, and the Planck constant in order to find any of the Planck units. We also show how to find other Planck units using the same method. To find the Planck time and the Planck length, one also needs to know the speed of light. To find the Planck mass and the Planck energy in their normal units, we need to know the Planck constant, something we will discuss in this paper. For these measurements, we do not need any knowledge of the Newton gravitational constant.

It can be shown that the Planck length times the speed of light requires less information than any other Planck unit; in fact, it needs no knowledge of any fundamental constant to be measured. This is a revolutionary concept and strengthens the case for recent discoveries in quantum gravity theory completed by Haug [3].

Key words: Planck time, Newton force spring, Hooke's law, spring constant, gravitational constant.

1 Background on Planck Units

In 1899, Max Planck [1, 2] first published his hypothesis on the Planck units. He assumed there were three essential universal and fundamental constants, namely the speed of light c, the Newton gravity constant G, and the Planck constant h, which is equal to the reduced Planck constant multiplied by 2π. Based on dimensional analysis, he then derived what he thought could be fundamental units for mass, energy, length, and time; known as the Planck units, they were given by $m_p = \sqrt{\frac{\hbar c}{G}}$, $E_p = \sqrt{\frac{\hbar c^3}{G}}$, $l_p = \sqrt{\frac{G\hbar}{c^3}}$, and $t_p = \sqrt{\frac{G\hbar}{c^5}}$. What is important here is that we see all of these Planck units require the three universal constants, a view held to this day.

In this paper, we will challenge this view, and basically prove that in order to find the Planck length times the speed of light, we need no knowledge of these constants at all. Based on Planck's original analysis, if we take the Planck length times the speed of light, this would be $l_p c = \sqrt{\frac{G\hbar}{c}}$, so this is still dependent on the three universal constants.

Our finding, which we will show in the next sections, takes a divergent approach, in particular since the Planck scale is assumed to be essential for several theories of quantum gravity. However, most physicists have argued that the Planck scale cannot actually can be detected, or is so difficult to detect that we do not have any experiments showing evidence of the Planck scale, at least according to the standard view. Recently, Haug [3] has introduced a new quantum gravity theory, which seems to unify with quantum mechanics, which predicts that gravity itself is Lorentz symmetry breakdown at the Planck scale, and shows that the Planck scale therefore can be detected by almost any gravity observation.

This paper will give strong support to that theory by demonstrating that the Planck length multiplied by the speed of light can be extracted from a simple Newton force spring without any knowledge of G, $\hbar$, or even c. We will claim this is revolutionary, as it strongly points to a quantized quantum gravity theory that can be detected in basically any gravity phenomena. Naturally the reader is encouraged to be critical and check our arguments and derivations carefully. However, we also note that on unresolved questions and paradoxes in physics, in particular, it is important to consider new approaches and fresh thinking with an open mind. Rigid thinking and prejudice against alternative viewpoints can slow the progress of physics down, while robust and thoughtful criticism can, of course be quite useful. We hope the physics community will find this topic to be of interest for closer study; the findings can be tested experimentally and also evaluated from a theoretical standpoint.

2 Background on the Compton Wavelength

Since the Compton wavelength will be an essential input, it is important to understand some background on it and its relation to mass. Here we will cover the Compton wavelength in depth, based on investigations over many years. We will demonstrate how one can, in theory, find the Compton wavelength for any mass without knowledge of the Planck constant, or of the mass of the object in kg.

First of all, any rest-mass in kg can be described as

$$m = \frac{h}{\lambda}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{1}$$

where h is the Planck constant, $\hbar$ is the reduced Planck constant, and λ and $\bar{\lambda}$ are the Compton wavelength and the reduced Compton wavelength, respectively. The Compton wavelength of an electron can be found by Compton scattering without any knowledge of the electron mass, see [4]. This is simply used to solve the Compton wavelength formula $\lambda = \frac{h}{mc}$ with respect to m. All masses are built of atoms that again consist of more elementary particles, so a composite mass will consist of many elementary particles that each have a Compton wavelength. These Compton wavelengths can be aggregated by the following formula

$$\lambda = \frac{1}{\sum_i^n \frac{1}{\lambda_i}} = \frac{\sum_i^n \lambda_i}{n} \tag{2}$$

and the same rule applies for the reduced Compton wavelengths

$$\lambda = \frac{1}{\sum_i^n \frac{1}{\bar{\lambda}_i}} = \frac{\sum_i^n \bar{\lambda}_i}{n} \tag{3}$$

where n is the number of fundamental particles in the composite mass. This addition rule is not in conflict with standard addition of mass; rather, it actually gives, or we can say is fully consistent with the standard addition of mass. So, the aggregated composite mass M consists of smaller masses m_i (particles) in the following standard way $M = \sum_i^n m_i$. This means if we know the Planck constant and the speed of light, we only need to know the Compton wavelength of that mass to know its mass in kg. Even if the Compton wavelength of a composite mass does not exist, it is a number that contains information about the aggregated Compton wavelengths of all particles in the mass.

Compton scattering consists of shooting a photon at an electron, and is based on measuring the wavelength of the photon before and after the impact on the electron. The original Compton scattering formula, as given by Compton, was

$$\lambda_1 - \lambda_2 = \frac{h}{mc}(1 - \cos\theta). \tag{4}$$

where λ_1 is the wavelength of the photon sent out, that is before it hits the electron, λ_2 is the wavelength of the photon after it has hit the electron, and θ is the angle between the incoming and outgoing beams of light. Further, m_e is the electron mass, and h is the Planck constant. However, since $\lambda = \frac{h}{mc}$, and also because any mass can be written as $m = \frac{h}{\lambda}\frac{1}{c}$, we can rewrite this as

$$\begin{aligned}
\lambda_1 - \lambda_2 &= \frac{h}{mc}(1 - \cos\theta) \\
\lambda_1 - \lambda_2 &= \frac{h}{\frac{h}{\lambda_e}\frac{1}{c}c}(1 - \cos\theta) \\
\lambda_1 - \lambda_2 &= \lambda_e(1 - \cos\theta) \\
\lambda_e &= \frac{\lambda_1 - \lambda_2}{1 - \cos\theta}
\end{aligned} \tag{5}$$

where λ_e is the Compton length of the electron that is found by shooting a photon at an electron. If we know the Planck constant h and the speed of light, then all we need to do to find the mass of an electron is a Compton scattering experiment, as shown by Prasannakumar et al. [5], for example. Yet, to find the Compton wavelength of the electron, we do not need knowledge of the Planck constant, or of the speed of light, as shown in formula 5.

Extending on this analysis, the cyclotron frequencies in masses are directly inversely proportional to their Compton wavelengths. A mass (a particle, for example) with twice the cyclotron frequency has half the Compton wavelength of the other particle. We have that

$$\frac{\bar{\lambda}_1}{\bar{\lambda}_2} = \frac{f_2}{f_2} = \frac{m_2}{m_1} \tag{6}$$

where f_2 and f_1 are the cyclotron frequencies of mass one and mass two. Thus, if one knows the cyclotron frequency ratio of different particles, such as protons and electrons, then one also knows their relative Compton wavelength ratios. For example, [6, 7] used cyclotron resonance experiments to find that the proton to electron mass ratio, m_P/m_e was about 1836.15247. The angular cyclotron velocity is given by

$$\omega = \frac{v}{r} = \frac{qB}{m} \tag{7}$$

and since electrons and protons have the same charge, the cyclotron ratio is given by

$$\frac{\omega_P}{\omega_e} = \frac{\frac{qB}{m_P}}{\frac{qB}{m_e}} = \frac{m_e}{m_P} = \frac{\lambda_P}{\lambda_e} \tag{8}$$

This means we can find the Compton wavelength of the proton without any knowledge of the mass of the proton, or even knowledge of the Planck constant. Next, in order to find the Compton wavelength of a larger mass, one kg, for example, we can count the number of protons in one kg. Naturally, this is far from easy in practice, but it is not impossible. If we know the Planck constant, we can do do this more easily by using the Compton wavelength formula for any mass, even composite masses. So, as an example, for one kg we have

$$\lambda_{1kg} = \frac{h}{1kg \times c} = \frac{h}{c} \approx 2.21 \times 10^{-42} \text{ m} \tag{9}$$

Again, this is a Compton wavelength that does not exist in practice; it is the sum of the Compton wavelengths for all the subatomic particles that make up one kg.

3 Finding the Planck Length Times the Speed of Light from a Newton Spring

In 1678, Robert Hooke [8] published the following relation for a spring

$$F = kx \tag{10}$$

where k is the spring constant and x is the amount by which the end of the spring was displaced from its "relaxed" position (when it is not being stretched). Further, the harmonic oscillator function of a spring is given by (the spring frequency)

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{11}$$

where m is the mass attached to the end of a spring. Based on Haug's [3] recent insight in quantum gravity, we can easily show that we must have

$$l_p c = R\sqrt{\frac{kx\bar{\lambda}}{2\pi m}} = R\sqrt{\frac{g\bar{\lambda}}{2\pi}} \tag{12}$$

that is. to find the Planck length multiplied by the speed of light, we only need to know the spring constant k, the spring displacement x, and the mass m; $\bar{\lambda}$ is the reduced Compton wavelength of the gravity object. Further, R is the radius of the gravity object, that is, from the center of the gravity object to where the measurement is performed. If it is done on the surface of the Earth, this is simply the Earth's radius. We have claimed that the Planck length times the speed of light can be found independent of knowledge of the value of G, $\hbar$, and c, so we need to demonstrate that the inputs can be found without any knowledge of them as well.

We can find the kg weight of the mass m simply by weighing it on a calibrated scale. We could take the old kg in Paris and both put them on an old fashioned scale, and thereby find the weight in kg of the mass attached to the end of the spring. Next we can measure the spring displacement x by hanging the mass m on the spring. We then will measure how much the spring extends. Such a spring, often known as Newton force spring, can be bought for a few dollars online, and even may include a measurement scale. But actually, any spring would do (an exception is springs made of materials easily deforming, such as rubber). Next, the spring constant is given by

$$k = \frac{F}{x} = \frac{G\frac{Mm}{R^2}}{x} = \frac{gm}{x} \tag{13}$$

where g is the gravitational acceleration, which is about 9.81 m/s^2. This can be found from the Newton force spring by

$$g = 4\pi^2 x f^2 \tag{14}$$

where x is the spring displacement and f is the spring frequency; both are easily measured without any knowledge of G, $\hbar$, or c. Here we have already found m and x, so now we also have the spring constant. The last thing we need to find is the reduced Compton wavelength of the Earth. This we can do independent of any knowledge of the mass of the Earth by using the methodology described in the first section of this paper. First, we will measure the Compton wavelength of an electron, simply by measuring the wavelength of a photon before and after it hits the electron and taking the angle between the ingoing and outgoing photon. Second, since the cyclotron frequency is proportional to the Compton wavelength, we can find the ratio of the Compton wavelength of the proton relative to that of the electron by a cyclotron experiment. This also requires no knowledge of the mass, or the Planck constant, or the speed of light as demonstrated in section one. We now have the Compton wavelength of a proton. Next, we can count the number of protons in the Earth, though obviously this would be a very costly, if not impossible task in practice. However, it is hypothetically possible; it is just a question of resources. Luckily, there is a much more practical way to accomplish this task. Since the Compton wavelength of two masses is inversely proportional to the gravitational acceleration field ratio of the masses, we must have

$$\frac{\lambda_1}{\lambda_2} = \frac{g_2 R_2^2}{g_1 R_1^2} \tag{15}$$

where λ_1 and λ_2 are the Compton wavelengths of mass one and two, and g_1 and g_2 are the gravitational acceleration fields of the two masses, and R_1 and R_2 are the distance from the center of the two masses used in the gravity observations). We have already found the gravitational acceleration field of the Earth using the Newton force spring, and to find the gravitational acceleration from a small gravitational object, we can use a Cavendish apparatus (see Figure 1); the derivation gives

$$g_c = \frac{4\pi^2 L}{T^2}\theta \tag{16}$$

where L is the length between the two small balls in the Cavendish apparatus, T is the oscillation periodicity of the movable arm in the Cavendish apparatus, and θ is the measured displacement angle from the rest position to the position to which the arm moves and R_2 is the distance between the center of one of the the large balls to the center of the closest small ball in the apparatus, when the apparatus is in initial position . As $T = \frac{1}{f_2}$ where f_2 is the oscillation frequency in the Cavendish apparatus, we see that the main difference between formula 14 and formula 16 is the angle θ, and naturally that L is the length of the bar in the Cavendish apparatus, and x is the displacement of the spring. Still, we see that the two formulas are very similar structurally. That is, in the Newton force spring, we do not need to know any angle. In the small object, we can technically count the number of atoms; this is of particular relevance with the recent progress in silicon spheres being used count the number of atoms, see Becker [9] and Bartl et al. [10]. We then know the number of protons (and for simplicity we assume that neutrons have the same Compton wavelength), and we can find the Compton wavelength of the proton with no knowledge of $\hbar$ or c using the cyclotron frequency relative to the electron. All inputs to the Planck time formula (formula 12) can be found without knowledge of G, $\hbar$, and the speed of light.

4 Other Planck Units from the Newton Spring

The Planck length is given by

$$l_p = \frac{R}{c}\sqrt{\frac{kx\lambda}{2\pi m}} \tag{17}$$

which is trivial mathematically, but we see that in order to find the Planck length, we need to know one more constant compared to what we needed when finding the Planck length times the speed of light. Here, we also need to know the speed of light. We can naturally (and luckily) measure the speed of light independent of any gravity experiment. So, this means we can find the Planck length independent of G and $\hbar$. In keeping with the discussion above, Haug has shown how the Planck length can be found independent of G using a Cavendish apparatus, see in particular the appendix in [11]; this paper extends that work considerably further.

The Planck time is given by

$$t_p = \frac{R}{c}\sqrt{\frac{kx\lambda}{2\pi m}} \tag{18}$$

As we can measure the speed of light independent of gravity, this means we can find the Planck length with no knowledge of any fundamental constant except for the speed of light.

The Planck mass is given by

$$m_p = \frac{\hbar}{R}\sqrt{\frac{2\pi m}{kx\lambda}} \tag{19}$$

Interestingly, we see that to find the Planck mass, we need to know one more constant than to know the Planck length times the speed of light, we need to know the Planck constant as well. This is, in our view, simply because the mass definition is linked to an arbitrary clump of matter known as the kg. As explained by Haug in citetHau20UnifiedA a more fundamental mass measure can be used, but we will leave that discussion in the other paper.

The Planck energy is given by

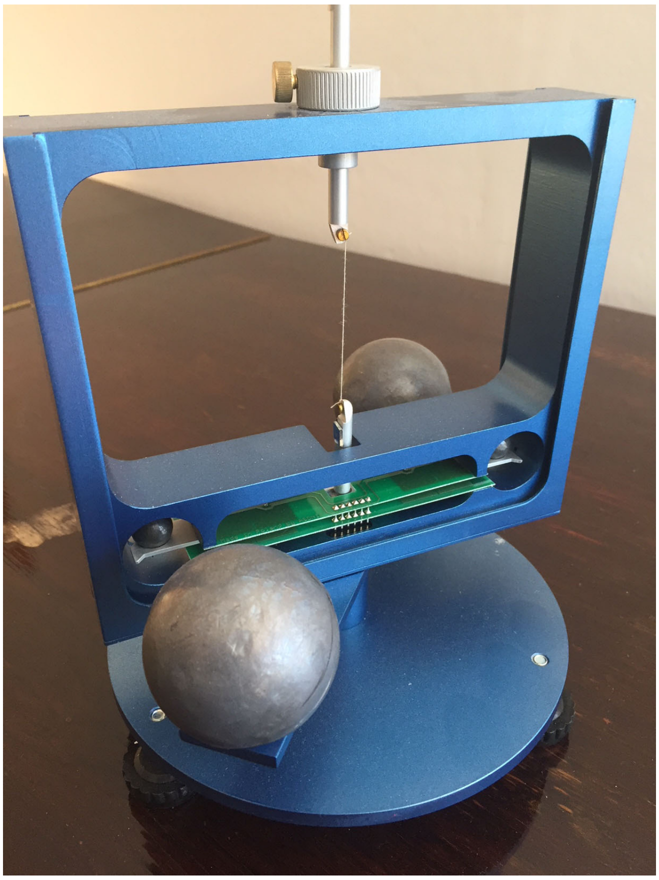

Figure 1: A low-budget modern Cavendish apparatus combining old mechanics with modern electronics that feeds directly to your computer through a USB cable. It is remarkable that with such an instrument we can measure the Planck time with only about 5% error from the kitchen table, or here from the top of my grand piano.

$$E_p = \frac{\hbar c^2}{R}\sqrt{\frac{2\pi m}{kx\lambda}} \tag{20}$$

Here we can see that to find the Planck energy, we need to know two more constants than we did to determine the Planck length times the speed of light; here we need to know the Planck constant and the speed of light.

The speed of light is given by

$$c = \frac{R}{l_p}\sqrt{\frac{kx\lambda}{2\pi m}} \tag{21}$$

and to find the speed of light (gravity), we need to know one more constant than we did to know the Planck length times the speed of light; here we need to know the Planck length as well. However to find the Planck length we need to know the speed of light, so for this last one there is in many way circular problem.

It is worth mentioning that the Planck length times the speed of light, $l_p c = R\sqrt{\frac{kx\lambda}{2\pi m}} = R\sqrt{\frac{g\lambda}{2\pi}}$ is the "unit? that needs the minimum amount of information to be found. It needs no information about any other physical constants. It does need the spring constant, but the spring constant itself needs no constants to be found, as it is given by $\frac{gm}{x}$, and we have shown how g can easily be measured by the same Newton spring force with no knowledge about any constants. The Planck length times the speed of light, we will claim, contains the two most important fundamental constants for gravity, namely the Planck length and the speed of gravity (light), c. We think this is highly significant for quantum gravity theories. This means that even in a weak gravitational field, we can measure the Planck length times the speed of gravity (light) with no knowledge of constants that are assumed to be important for knowledge of gravity, namely G and the speed of gravity $c_g = c$.

As indirectly shown previously in this paper, we do not even need knowledge of the small mass m, as the formulas above can be simplified further. We have that the Planck length times the speed of light is given by

$$l_p c = R\sqrt{\frac{kx\lambda}{2\pi m}} = R\sqrt{\frac{g\lambda}{2\pi}} = \frac{R}{f}\sqrt{2\pi x\lambda} \tag{22}$$

whereas before f is the spring frequency and x is the spring displacement.

The Planck length is given by

$$l_p = \frac{R}{c}\sqrt{\frac{kx\lambda}{2\pi m}} = \frac{R}{c}\sqrt{\frac{g\lambda}{2\pi}} = \frac{R}{cf}\sqrt{2\pi x\lambda} \tag{23}$$

The Planck time is given by

$$t_p = \frac{R}{c^2}\sqrt{\frac{kx\lambda}{2\pi m}} = \frac{R}{c^2}\sqrt{\frac{g\lambda}{2\pi}} = \frac{R}{c^2 f}\sqrt{2\pi x\lambda} \tag{24}$$

The Planck mass is given by

$$m_p = \frac{\hbar}{R}\sqrt{\frac{2\pi m}{kx\lambda}} = \frac{\hbar}{R}\sqrt{\frac{2\pi}{g\lambda}} = \frac{\hbar}{Rf}\sqrt{\frac{1}{2\pi x\lambda}} \tag{25}$$

The Planck energy is given by

$$E_p = \frac{\hbar c^2}{R}\sqrt{\frac{2\pi m}{kx\lambda}} = \frac{\hbar c^2}{Rf}\sqrt{\frac{1}{2\pi x\lambda}} \tag{26}$$

The speed of light is given by

$$c = \frac{R}{l_p}\sqrt{\frac{kx\lambda}{2\pi m}} = \frac{R}{l_p}\sqrt{\frac{g\lambda}{2\pi}} = \frac{Rf}{l_p}\sqrt{2\pi x\lambda} \tag{27}$$

We think it is a significant point that to find the speed of light, we need to know the Planck length, if measured using a Newton force spring, and to find the Planck length, we only need to know the speed of light. This strongly supports Haug's quantum gravity theory that shows the speed of light is related to how far an indivisible particle can travel while two indivisible particles collide. The diameter of these indivisible particles are the Planck length, and they travel at the speed of light. Gravity contains them both (but not sepratley). To find both, we only need to find one of them.

We maintain that the Planck constant is not a true fundamental constant. It is linked to an arbitrary amount of mass, as discussed in [3]. If we instead use the more fundamental collision-time as mass and collision length as energy, then we do not need to know the Planck constant. The Planck constant is, however, very essential if we want to operate with kg. The Planck constant is indeed linked to quantization of energy (and mass) when, but only when, we want to compare this to one kg.

We could alternatively have done the "same" by for example using a pendulum "clock" instead of a spring. We then have

$$l_p c = \frac{R}{f_p}\sqrt{2\pi L\lambda} \tag{28}$$

whereas before f_p is the pendulum clock frequency and L is the length of the pendulum, R is the radius of the Earth as we do the measurements from the radius of the Earth.

The Planck length is given by

$$l_p = \frac{R}{cf}\sqrt{2\pi L\lambda} \tag{29}$$

The Planck time is given by

$$t_p = \frac{R}{c^2 f_p}\sqrt{2\pi L\lambda} \tag{30}$$

The Planck mass is given by

$$m_p = \frac{\hbar}{Rf_p}\sqrt{\frac{1}{2\pi L\lambda}} \tag{31}$$

The Planck energy is given by

$$E_p = \frac{\hbar c^2}{Rf_p}\sqrt{\frac{1}{2\pi L\lambda}} \tag{32}$$

The speed of light is given by

$$c = \frac{Rf_p}{l_p}\sqrt{2\pi L\lambda} \tag{33}$$

From the equations one see a Newton force spring and a Pendulum clock have very much in common, the difference is in one method one use the spring diceplacement and in the other the length of the pendulum. Both have a frequency (that is basically a form of gravity clock in both methods).

5 Conclusion

We have shown how the Planck length times the speed of light can be measured from a Newton force spring without any knowledge of G, c, or $\hbar$. This is remarkable, as it is assumed that the Planck length (times the speed of light) can only be found from dimensional analysis of of G, c, or $\hbar$. In order to find the Planck length times the speed of light, we need no knowledge of any fundamental constants; to find the Planck length and the Planck time, we need to know the speed of light in addition; to find the Planck mass, we need to know the Planck constant; and to find the Planck energy, we need to know the speed of light and the Planck constant. Finally, to find the speed of light, we only need to know the Planck length, but to find the Planck length we need to know the speed of light, so this last one basically confirm the Planck length is closely related to the speed of light (gravity). Our findings strongly support Haug's recent published unified quantum gravity theory, which predicts that all gravity is Lorentz symmetry break down at the Planck scale, and that the Planck units therefore can be extracted from simple gravity experiments with no knowledge of G. This also supports Haug's analysis that the speed of light (gravity) and the Planck length are closely connected.

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