Neutrino mixing and circulant mass operators

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Abstract

Under the neutrino CMB correspondence, low energy observables are analysed using quantum computation. Starting from the observed μ-τ symmetry, we discuss constraints on all neutrino masses and mixing parameters.

1 The neutrino CMB correspondence

There are many reasons [1] for thinking that quantum gravity employs the mathematics of quantum computation, rather than traditional Lagrangian techniques. In particular, neutrino rest masses are both local and non local, and neutrino oscillation models require physics beyond the Standard Model. In 2010, led by clues from quantum computation, we considered a triplet of right handed neutrino masses, which differed slightly from the active mass triplet [2]. Dungworth [3] immediately pointed out that the central mass in this triplet matched the present day temperature of the CMB under Wien's law. Since neutrino masses introduce a new infrared scale, it was natural to think of this neutrino CMB correspondence as a fundamental feature of quantum gravity [4], in which CMB temperature is the proper measure of cosmic time.

Combining the UV Planck scale and IR neutrino scale at 0.01 eV, we considered an inverse see-saw [5][6] rule $m_H = \sqrt{m_\nu m_{pl}}$ for the electroweak Higgs scale, under S duality. The introduction of a new IR scale is considered in condensate pictures for gravity with fermions [7][8][9], using two different Chern-Simons theories: one for neutrino gravity and one for QCD. For us, the Chern-Simons theories are about the knots and links that lie at the heart of ribbon categories in quantum computation.

Standard Model states are initially ribbon diagrams [10]. The quark $SU(3)$ color symmetry is given by three ribbon strands, and the twist in the strands determines the $U(1)$ electric charge. These symmetries are exact. An $SU(2)_L$ continuous symmetry emerges from the discrete braid group B_3, which is universal for qubit computation. Since a holographic principle [4] extends the $2+1$ dimensional theories to four dimensions, we expect mass splittings and mixings to exhibit a $2+1$ dimensional basis. In particular, the tribimaximal mixing matrix

$$T = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 & -\sqrt{2} & 0 \\ 1 & -1/\sqrt{2} & -\sqrt{3}/\sqrt{2} \\ 1 & -1/\sqrt{2} & \sqrt{3}/\sqrt{2} \end{pmatrix} \tag{1}$$

is known to approximate the PMNS mixing matrix, just as the identity approximates the CKM matrix for quarks.

An effective sterile neutrino at 1.29 eV was also derived [4][11] from the neutrino CMB correspondence, using the phases that appear below. The next section introduces the μ-τ symmetry for neutrino mixing, and section 3 defines the related rest mass triplets. The approximate separation of μ-τ from the ν_e flavor state is associated with the selection of a single time coordinate in the localisation to low energy atomic matter in a $(3,1)$ spacetime in motivic gravity. Section 4 introduces our motivic sterile neutrino, as an explanation for short baseline anomalies in oscillation experiments.

2 The μ-τ approximate symmetry

Within the quantum information paradigm, a simple mass matrix is a complex Hermitian circulant, which diagonalises under the discrete 3×3 Fourier transform to three rest mass eigenvalues. In the standard phenomenology of neutrino masses and mixing, this is not a primary consideration, but observation indicates the existence of an approximate symmetry between the ν_μ and ν_τ parameters [12], which we consider here from the circulant framework.

The μ-τ symmetry conditions are

$$\theta_{23} = \frac{\pi}{4}, \quad \delta_{CP} = \pm\frac{\pi}{2}, \tag{2}$$

which arise naturally as follows. We let the PMNS matrix [14][15]

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{3}$$

be approximated by three complex rotation factors in the form [4][13],

$$U = \frac{1}{N} \begin{pmatrix} a & i & 0 \\ i & a & 0 \\ 0 & 0 & a+i \end{pmatrix} \begin{pmatrix} b & 0 & i \\ 0 & b+i & 0 \\ i & 0 & b \end{pmatrix} \begin{pmatrix} c+i & 0 & 0 \\ 0 & c & i \\ 0 & i & c \end{pmatrix}, \tag{4}$$

where $N = N(a,b,c)$ is the normalisation and a, b and c are real. Such a matrix automatically has a maximal CP phase of $\delta_{CP} = -\pi/2$, derived from the observed Euler angles from a, b and c. Each factor is both a $SU(2) \times U(1)$ matrix and also an element of the finite group algebra for S_3, the permutation group on three letters.

The value $\theta_{23} = \pi/4$ is a feature of the tribimaximal mixing matrix (1) [16], which has parameters $(a,b,c) = (\sqrt{2},1,0)$. This θ_{23} remains consistent with global fits to oscillation data. Whatever the theoretical reason for the deformation to $\theta_{13} \neq 0$, we consider a mass matrix of the form [12]

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ M_{\tau e} & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix} = U D_\nu U^T, \tag{5}$$

whose eigenvalues are the neutrino rest masses. Under the exact μ-τ interchange, for which the μ and τ rows of U are equal, a circulant form for M_ν is tightly constrained to

$$M_\nu = \begin{pmatrix} X & Y & Y \\ Y & X & Y \\ Y & Y & X \end{pmatrix}, \tag{6}$$

where we assume that X and Y are real. The degenerate eigenvalues of this matrix are

$$m_1 = m_3 = X - Y, \quad m_2 = X + 2Y. \tag{7}$$

This is still roughly consistent with oscillation data, because the two small masses in the normal hierarchy have little effect on Δm^2 parameters. Moreover, it follows that $\tan 2\theta_{12} = 2\sqrt{2}$, so that $\theta_{12} = 35.26°$, which is also consistent with observation [17].

As in [13], we view $\theta_{13} \sim 9°$ as a perturbation to a three dimensional theory, from some degenerate but exact theory in two dimensions, for which tribimaximal mixing holds. In the 3×3 mass circulants of the next section, the phase parameter θ is expected to be related to θ_{13} [13]. In particular, a triality action on $\theta = 2/9$ radians in the Jordan algebra gives a phase $4/27 \sim 8.5°$ as a candidate for θ_{13}.

3 Circulant mass and mixing matrices

Allowing for CPT violation in the neutrino sector, Wolfenstein's [5] Dirac mass matrix for one generation takes the circulant form

$$M \begin{pmatrix} \nu_L \nu_R & \overline{\nu}_L \nu_R \\ \nu_L \overline{\nu}_R & \overline{\nu}_L \overline{\nu}_R \end{pmatrix} = \begin{pmatrix} c & d \\ d & c \end{pmatrix}, \tag{8}$$

and is diagonalised by the 2×2 Fourier matrix. The CPT theorem is recovered at $c = 0$. For a democratic mass matrix, with $c = d$, the sterile mass satifies $m_s \gg m_a \sim 0$.

Within the active triplet, we start with 3×3 circulants, which are diagonalised by the Fourier transform

$$F_3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \overline{\omega} \\ 1 & \overline{\omega} & \omega \end{pmatrix}, \tag{9}$$

where ω is the cubed root of unity. This F_3 is the unitary version of democratic mixing, for which all transition probabilities are equal. It's relation to (1) is given below. A 2×2 democratic mixing circulant is given by $(c,d)=(1,i)/\sqrt{2}$, which diagonalises as the transform

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = \begin{pmatrix} c & d \\ d & c \end{pmatrix} \begin{pmatrix} m_1c^2 - m_2d^2 & (m_1-m_2)cd \\ -(m_1-m_2)cd & m_2c^2 - m_1d^2 \end{pmatrix} \begin{pmatrix} c & -d \\ -d & c \end{pmatrix}. \tag{10}$$

For $m_1 \gg m_2$, this mass operator is the circulant $m_1(c,d)$.

In the 1980s, Koide [18][19] found a simple constraint on the charged lepton rest mass triplet, which accurately predicted the τ mass. In 2006, Brannen [20] realised that the neutrinos could obey a similar relation, if one worked with the square root of the mass matrix. When we formulate these relations using a 3×3 circulant, the neutrino phases differ from the charged lepton phase by $\pm\pi/12$, which is a fundamental arithmetic phase for modular geometry.

Let us start with the basic qutrit density matrices, which are the idempotents

$$B = \frac{1}{3}\begin{pmatrix} 1 & \omega & \overline{\omega} \\ \overline{\omega} & 1 & \omega \\ \omega & \overline{\omega} & 1 \end{pmatrix}, \quad C = \frac{1}{3}\begin{pmatrix} 1 & \overline{\omega} & \omega \\ \omega & 1 & \overline{\omega} \\ \overline{\omega} & \omega & 1 \end{pmatrix}, \quad A = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{11}$$

so that a Hermitian operator is a combination of idempotents, as in

$$\sqrt{M} = aA + bB + cC \tag{12}$$

for a,b,c real. Our masses are the squares of the three eigenvalues of $\sqrt{M}$. Without loss of generality, fix a mass scale by the rule $(a+b+c)^2 = 1$. The Koide rule follows from the eigenvalues of the complex charged lepton matrix

$$\sqrt{M} = \frac{\sqrt{\mu}}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & \theta & \overline{\theta} \\ \overline{\theta} & \sqrt{2} & \theta \\ \theta & \overline{\theta} & \sqrt{2} \end{pmatrix}, \tag{13}$$

where the scale $\mu = 4/3$ follows from $(a+b+c)=1$. For the charged lepton triplet, experimentally, the 4 in μ represents the mass of the proton m_p. In analogy to (10), a 3×3 circulant transform uses a basis that is mutually unbiased with respect to F_3, giving a non circulant Hermitian mass matrix with μ on the diagonal.

The $\sqrt{2}$ diagonal parameter corresponds to $(X,Y)=(\sqrt{2},1)$ in (6), giving the tribimaximal Euler parameters. Thus the complex phase θ is interpreted as the deformation parameter into the third dimension. As noted above, the observed value of θ for charged leptons is close to $2/9$. The eigenvalues λ_i of (13) are expressed in terms of the tribimaximal coefficients in (1), in the form [13]

$$\lambda_1 = -\frac{1}{\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{3}}\cos\theta + 0, \tag{14}$$

$$\lambda_2 = \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta,$$

$$\lambda_3 = \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta.$$

Table 1: Neutrino masses (eV)

ν_L	0.0507	0.0089	0.0004
ν_R	0.0582	0.00117	0.0006

In the full μ-τ symmetry [12] the tribimaximal form also determines the mixing angle $\theta'_{12} = \tan^{-1}\sqrt{2}$, which is complementary to $\theta_{12} = 35.26°$ about $\pi/4$. We obtain the eigenvalues λ_i from the row sums of

$$\frac{\sqrt{2}}{\sqrt{3}} \begin{pmatrix} 1 & \cos 0 & \sin 0 \\ 1 & \cos\omega & \sin\overline{\omega} \\ 1 & \cos\overline{\omega} & \sin\omega \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & \sin\theta \end{pmatrix}, \tag{15}$$

making explicit the connection to the Fourier transform of (9).

4 The effective sterile neutrino

Fix the neutrino scale at $\mu = 0.01$ eV, so that the rest mass triplet in the top row of Table 1 matches the Δm^2 parameters for oscillation experiments. Whereas the active neutrinos use the phase $\theta + \pi/12$, the second mass triplet [1][11][21] uses the complementary $\theta - \pi/12$. This right handed triplet includes the present day CMB temperature at 0.00117 eV, because CMB photons are directly associated to the neutrino mass gap in a condensate mechanism for gravity.

Mavromatos [22] considers a direct decoherence argument for evading the apparent CPT violation, while we ignore CPT in the neutrino sector, which looks at degrees of freedom that exist prior to Lorentz symmetry. Here the $SU(2)$ component of compactified Minkowski space emerges from the discrete braid group B_3 that characterises neutrino states in the ribbon particle scheme.

Under the crude empirical hypothesis [11] that the CMB creation time is given by a ΛCDM redshift of $z = 1100$, the early universe CMB mass is 1.29 eV, in good agreement with sterile neutrino fits to anomalous data in short baseline (SBL) experiments [23]. In the SBL approximation, we may take a single active mass state m_a to represent all small masses, and oscillation data is fitted to a 2×2 mixing solution with a heavier (sterile) mass m_s, as in

$$\begin{pmatrix} m_a \\ m_s \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \nu_a \\ \nu_s \end{pmatrix}. \tag{16}$$

The matrix U is approximately the identity, with the observed $|U_{12}|^2 \simeq 0.01$ corresponding to a mixing angle of about 6°. A similar angle occurs in the 2×2 circulant for the right handed 0.00117 eV mixed with the active neutrino scale of 0.01 eV.

One preferred value [23] for the sterile mass is 1.14 eV, but this is a little lower than expected, and appears to disagree with the perfect 1.3 eV estimate of MiniBooNE from 2018 [24]. Small differences from the CMB value might be accomodated by a Dirac-Milne [25], or alternative, redshift history.

Our effective sterile can accomodate the null results of some experiments, by preferring the ν_e state as a partner to the proton, which represents the single time direction of ordinary matter. Vacuum polarisation [26] under the neutrino CMB correspondence [11] generates local inertial mass as a deformation of a *zero mass limit* on cosmological scales. This principle does away with the need for dark energy. Only for massless neutrinos in the Standard Model does CPT hold. Table 1 indicates the CPT violation for the Dirac spinors, with separate masses for ν and $\overline{\nu}$ shown to be phenomenologically viable in [27][28]. Moreover, the masses of Table 1, from June 2010, predicted quantitatively [2][3] the CPT violation, observed in later MINOS results [29].

The mathematician R. A. Wilson has also considered [30] the selection of a special low energy flavor in connection with the solar eccliptic plane, from the perspective of the $(3,3)$ metric. It will be interesting to investigate anomalies arising from multiple-time physics.

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