

Optimal Binary Number System When Numbers Are Energy?

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Abstract

In this short note, we will quickly look at optimal binary number systems used in communication (or transactions) under the assumption that one must use energy to give away (send) numbers. We show that the current binary system is not the optimal binary number system as it can be arbitrated. We also show that there exist other optimal binary number systems in such a scenario. Naturally, one has to ask, “Optimal for whom” – For the one sending the number out, or for the one receiving the number? Alternatively, we can have a binary number system that, on average, is neutral for both sender and receiver. Numbers are typically only considered to have symbolic value, but if the money units were so small that they came in the smallest possible energy units, then we could be forced to switch to a number system where the physical value of each number was equal to its symbolic value. That is to say, the physical value of three must be higher than the physical value of two, for example. Numbers are always physical because storing or sending a number from a computer requires bits, and bits of information require energy.

Key Words: binary number systems, energy, time, value of numbers.

Binary Number Systems and the Value of Numbers

Let’s play a little game. In a four-bit binary system, we have 16 combinations that describe 16 numbers from zero to 15. If I asked you whether you would prefer the number three or the number eight in the form of binary numbers, which one would you choose? Or would you be neutral? If we assume 1 is an energy unit and zero is a no-energy unit, then actually the number three (0011) has twice the amount of energy as eight (1000), so the number three is naturally preferable.

Extending this idea, imagine a case where numbers are exchanged between two parties. Assume the numbers represent real value. The simplest model is to think of numbers as money, and that the higher the number the greater the value; therefore, 2 must be more valuable than 1 and 7 more valuable than 6, for example. Most numbers in computers and information systems are binary. Assume we are working with a four-bit binary system that can handle numbers from 0 to 15. In a standard binary system, zero is represented by 0000, and one is represented by 0001. Now assume these numbers are sent as discrete energy units. Let us choose to represent 1 with a unit of energy, and 0 as a unit of no energy (no energy basically means nothing is sent out), as posited previously. We can also say that the sender and receiver both have clocks ticking at a uniform rate. In each time unit one observes either nothing, which means 0, (this means zero sent or received energy units), or one observes 1, which means one energy unit has been sent or received. Clearly, energy is valuable, as it can be used for many other purposes besides sending and receiving information.

In a standard binary system, when we assume 1 represents energy and 0 represents no energy, then the number four that has a binary representation of 0100 contains less energy than the number 3. This means if the energy in the numbers were more valuable than other symbolic information they represented, then one would prefer receiving three instead of four; after all, three is now twice as valuable as four. Further, eight, which is represented by 1000, is much less valuable than seven, because seven contains three energy units, as it is represented by 0111. Even the number three contains twice as much energy as eight. This simply means that the standard binary system is not ordered by energy value; it is not a fair binary system in this way.

However, we can easily construct other types of binary number systems that can be more fair in the sense that higher numbers are more valuable. This is based again on the concept that the energy content in each number should be in the same order sequence as the numbers they are representing. A higher number should always contain more energy than a lower number. We all also know time is money. It is better to receive one dollar today than one dollar tomorrow, as you can put it to productive work today and will get return on the capital by tomorrow. In the same way, it is better to give away one energy unit tomorrow than today, and better

to receive one energy unit today than tomorrow. Let us take this analogy into our binary numbers. Assume we will represent the number one with 1000 and the number two with 0100. Then if the bits are sent with the bits from the right hand side first, then two is indeed more valuable than one for a receiver of the number, not because it contains more energy, but because we get the one energy unit in number two, one time unit earlier than for number one. Column three in Table 1 shows a binary number system, where higher numbers always have higher value for a receiver, based on taking energy and time into account. However, for the sender it is not so, as he would prefer to send the number two rather than one, as he would then keep the energy unit one time-unit longer. Column four is a binary number system that is what we can call “sender-friendly.” That is, when the value increases for higher numbers (for the sender), based on our assumption that 1 represents energy and the goal for the sender it to retain the energy for himself as long as possible. In the last column, we have tried to make a number system that is neutral. Here, the number one, represented by 0001, is more valuable than the number two, which is 1000 for a receiver, but the number two is more valuable than number three for a sender. So, on average, if all numbers are used in equal proportions, then both receiver and sender will be just as well off.

If we do not take the value of time into account, that is to get a bit of energy earlier rather than later, then we cannot compress the numbers in the way done in the systems described so far. But then, we can use a number system with a variable number of bits as described in the second column from the right in the table. The reason we start with a zero here is for the receiver (or sender to know there is a new number coming). The sender or receiver of numbers must still have a clock that checks, at each uniform time interval, if there is energy or no energy in the number “detector.” We can also use the 16-bit system, as described in the right-most column; this would require that one already knows the highest number one plans to use. If the highest number is 255, then one needs to use a 256-bit system for all numbers. In other words, if we want to have a number system where each number has a higher value in the physical world as well, we cannot compress the information in the way done in most of today’s number systems, where the numbers only have symbolical value. This is due to the fact that the symbolic value is much higher than the physical value.

| Number | Standard binary system | Receiver friendly | Sender friendly | “Neutral” on average | Energy ranked number system | Energy ranked 16 bits system |
|--------|------------------------|-------------------|-----------------|----------------------|-----------------------------|------------------------------|
| 0 | 0000 | 0000 | 0000 | 0000 | 00 | 0000000000000000 |
| 1 | 0001 | 0001 | 1000 | 0001 | 01 | 0000000000000001 |
| 2 | 0010 | 0010 | 0100 | 1000 | 011 | 0000000000000011 |
| 3 | 0011 | 0100 | 0010 | 0010 | 0111 | 0000000000000111 |
| 4 | 0100 | 1000 | 0001 | 0100 | 01111 | 000000000001111 |
| 5 | 0101 | 0011 | 1100 | 0110 | 011111 | 00000000011111 |
| 6 | 0110 | 0101 | 1010 | 1001 | 0111111 | 00000000111111 |
| 7 | 0111 | 1001 | 1001 | 0011 | 01111111 | 000000001111111 |
| 8 | 1000 | 0110 | 0110 | 1100 | 011111111 | 000000011111111 |
| 9 | 1001 | 1010 | 0101 | 1010 | 0111111111 | 000000111111111 |
| 10 | 1010 | 1100 | 0011 | 0101 | 01111111111 | 000001111111111 |
| 11 | 1011 | 0111 | 1110 | 1011 | 011111111111 | 000011111111111 |
| 12 | 1100 | 1011 | 1101 | 1101 | 0111111111111 | 000111111111111 |
| 13 | 1101 | 1101 | 1011 | 0111 | 01111111111111 | 001111111111111 |
| 14 | 1110 | 1110 | 0111 | 1110 | 011111111111111 | 011111111111111 |
| 15 | 1111 | 1111 | 1111 | 1111 | 0111111111111111 | 111111111111111 |
| 16 | | | | | 01111111111111111 | |
| 17 | | | | | 011111111111111111 | |
| 18 | | | | | ⋮ | |

Table 1: The table shows standard numbers with six corresponding binary number systems. The table assume the numbers are sent as single bit stream from left to right.

Conclusion

We have shown that the standard binary system is far from optimal if one considers numbers as energy units. If 1 is represent by an energy unit, and 0 is represented by no energy, then three will be more valuable than eight in a standard binary system. We have introduced an energy-linked binary system where higher numbers have higher values than lower numbers for a receiver of the numbers, and a similar system for the sender of binary

numbers. We have also suggested several other number systems where higher numbers are always more valuable (in terms of the energy they contain) than lower numbers.