On the possible mathematical connections between some Ramanujan's equations and various formulas concerning several sectors of Theoretical Physics and Cosmology

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Abstract

In this paper we have described the possible mathematical connections between some Ramanujan's equations and various formulas concerning several sectors of Theoretical Physics and Cosmology

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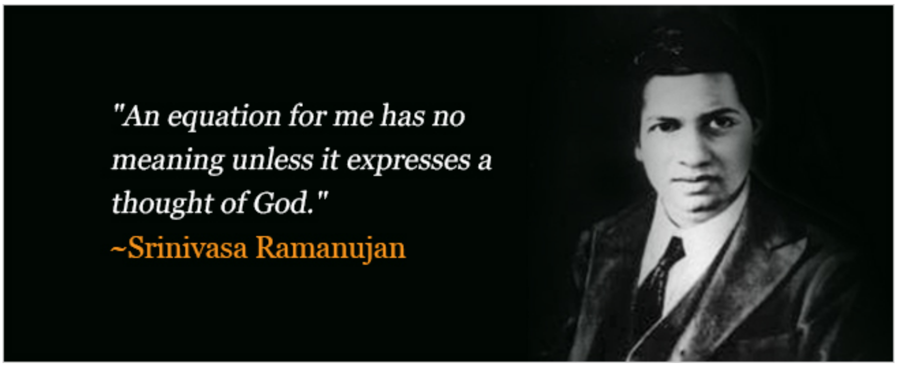


http://www.aicte-india.org/content/srinivasa-ramanujan

From:

The Fate of Massive F-Strings

Bin Chen, Miao Li, and Jian-Huang She - https://arxiv.org/abs/hep-th/0504040v2

In the following, we set $\alpha' = \frac{1}{2}$. Energy conservation gives

$$M = \sqrt{M_1^2 + k^2} + \sqrt{M_2^2 + k^2}, \tag{2.5}$$

with k the momentum in the noncompact dimension.

What we want to consider is the averaged semi-inclusive two-body decay rate. That is, for the initial string, we average over all states of some given mass, winding and KK momentum. For one of the two final strings, we sum over all states with some given mass, winding and KK momentum; only the other string's state is fully specified (by keeping explicit its vertex operator).

This decay rate can be written as

$$\Gamma_{semi-incl} = \frac{A_{D-d_c}}{M^2} g_c^2 \frac{F_L}{\mathcal{G}(N_L)} \frac{F_R}{\mathcal{G}(N_R)} k^{D-3-d_c} \prod_i^{d_c} R_i^{-1} \tag{2.6}$$

with closed string coupling g_c, compactification radius R_i, and numerical coefficient $A_p = \frac{2^{-p}\pi^{\frac{3-p}{2}}}{\Gamma(\frac{p-1}{2})}$, and F_L and F_R are given by

$$F_L = \sum_{\Phi_i|N_L} \sum_{\Phi_f|N_{2L}} \left|\langle\Phi_f|V_L(n_{1i}, w_{1i}, k)|\Phi_i\rangle\right|^2 \tag{2.7}$$

$$F_R = \sum_{\Phi_i|N_R} \sum_{\Phi_f|N_{2R}} \left|\langle\Phi_f|V_R(n_{1i}, w_{1i}, k)|\Phi_i\rangle\right|^2. \tag{2.8}$$

$D - 26$ is the full space-time dimension.

In the following, we shall calculate the decay rate (2.6). In the above discussions, we have fixed the levels of the incoming string states and one of the outgoing string states, which are $N_{L,R}, N_{2L,2R}$ respectively. From the mass-shell conditions, we know that once we fix the quanta of the incoming string states, the outgoing string states could have various kinds of masses, KK-momenta and windings, with respect to the energy condition (2.5), and conservations:

$$Q_- = Q_{1-} + Q_{2-}, \quad Q_+ = Q_{1+} + Q_{2+}. \tag{2.40}$$

One important observation is that we have inequality

$$\sqrt{N_{1L}} + \sqrt{N_{2L}} \leq \sqrt{N_L}. \tag{2.41}$$

The equality saturate when

$$k = 0, \qquad \frac{M}{Q_-} = \frac{M_2}{Q_{2-}}, \tag{2.42}$$

where k is the momentum in the noncompact directions and M_2, Q_{2-} are the quanta of the outgoing string states with fixed level N_{2L}. The same inequality holds in the right-mover.

Given a very massive initial string of high level, its state density has the asymptotic form

$$\mathcal{G}(N) \sim N^{-\frac{D+1}{4}} e^{a\sqrt{N}}, \qquad a = 2\pi\sqrt{\frac{D-2}{6}}. \tag{2.43}$$

The ratio between the first two terms in (2.37) can be estimated to be

$$\frac{\mathcal{G}(N_L - n)}{\mathcal{G}(N_L - 2n + m_L^2)} \sim e^{a\left(\sqrt{N_L - n} - \sqrt{N_L - 2n + m_L^2}\right)}. \tag{2.44}$$

Using the inequality(2.41), it is at most of order

$$\exp\left(a(\sqrt{N_L} - \sqrt{N_{2L}})\right), \quad \sqrt{N_{2L}} \geq \frac{\sqrt{N_L}}{2} \tag{2.45}$$

or

$$\exp(a(3\sqrt{N_{2L}} - \sqrt{N_L})), \quad \sqrt{N_{2L}} \leq \frac{\sqrt{N_L}}{2}. \tag{2.46}$$

In the extremal case $N_{2L} = N_L$, one can try to calculate $\mathcal{I}$ directly. Thus if generically $N_L > N_{2L} > \frac{N_L}{3}$, the first term dominates the whole summation in (2.37), and the other terms will be neglected to get

$$F_L \approx (N_L - N_{2L} - \frac{1}{2}m_L^2)\mathcal{G}(N_{2L}). \tag{2.47}$$

F_R can be carried out in the same way.

Note that in our approximation, $F_{L,R}$ do not depend on the details of the state specified in eq.(2.7) and (2.8) by the vertex operators, and all states of the same level are emitted with the same probability. Taking advantage of this, we can get the total decay rate for decays into arbitrary states of given mass, winding and KK momentum by simply multiplying eq.(2.6) by the state density $\mathcal{G}(N_1)$

$$\Gamma[(M,n_i,w_i)\to(m,n_{1i},w_{1i})+(M_2,n_{2i},w_{2i})]\approx A_{D-d_c}\frac{g_c^2}{M^2}\mathcal{N}_L\mathcal{N}_R\mathcal{G}_L\mathcal{G}_R k^{D-3-d_c}\prod_i^{d_c}R_i^{-1}, \tag{2.48}$$

where

$$\mathcal{N}_L=N_L-N_{2L}-\frac{1}{2}m_L^2\ ,\quad \mathcal{N}_R=N_R-N_{2R}-\frac{1}{2}m_R^2, \tag{2.49}$$

and

$$\mathcal{G}_L=\frac{\mathcal{G}(N_{1L})\mathcal{G}(N_{2L})}{\mathcal{G}(N_L)}\ ,\quad \mathcal{G}_R=\frac{\mathcal{G}(N_{1R})\mathcal{G}(N_{2R})}{\mathcal{G}(N_R)}. \tag{2.50}$$

Remember that

$$m_L^2=\frac{1}{4}(m^2-Q_{1-}^2)\approx 2N_{1L}. \tag{2.51}$$

As long as $N_{1L}\gg 1$ and $N_{2L}\gg 1$, we can use eq.(2.43) to write

$$\mathcal{G}_L\sim(2\pi T_H)^{-\frac{D-1}{2}}(\frac{N_{1L}N_{2L}}{N_L})^{-\frac{D+1}{4}}e^{-\sqrt{2}t_L/T_H}, \tag{2.52}$$

with the Hagedorn temperature $T_H=\frac{1}{\pi}\sqrt{\frac{3}{D-2}}$ and $t_L=\sqrt{N_L}-\sqrt{N_{1L}}-\sqrt{N_{2L}}$ in a sense coming from the kinetic energy released in the decay process. We have in the above restored the multiplicative constant in front of the state density. Note that for later convenience, in our notation we set $\mathcal{G}(M)dN=\mathcal{G}(N)dN$, a little different from usual sense $\mathcal{G}(M)dM=\mathcal{G}(N)dN$.

We can also compactify type II strings on the torus. According to the above calculations, we will obtain the same formulas as in (2.37), (2.47)-(2.50). Now the state density has the asymptotic form

$$\mathcal{G}(N_L)\approx 2^{-\frac{13}{4}}N_L^{-\frac{11}{4}}e^{\pi\sqrt{8N_L}}. \tag{4.1}$$

Consider first open strings and NS sector only. The state degeneracy $\mathcal{G}_{NS}(n)$ is given by

$$f_{NS}(w) = \mathrm{Tr}\frac{1+e^{i\pi F}}{2}w^N = \sum_{n=0}^{\infty}\mathcal{G}_{NS}(n)w^n 8\prod_{n=1}^{\infty}(\frac{1+w^n}{1-w^n})^8, \tag{A.1}$$

with N the summation of the bosonic and fermionic number operators. Generalization of Hardy-Ramanujan formula gives

$$\prod_{n=1}^{\infty}(\frac{1+w^n}{1-w^n})^{-1} = \vartheta_4(0|w) = (-\frac{\ln w}{\pi})^{-\frac{1}{2}}\vartheta_2(0|e^{\frac{\pi}{\ln w}}), \tag{A.2}$$

where the modular transformation of ϑ function

$$\vartheta_4(0|\tau) = (-i\tau)^{-\frac{1}{2}}\vartheta_2(0|-\frac{1}{\tau}) \tag{A.3}$$

has been used, with

$$\tau = -\frac{i\ln w}{\pi}. \tag{A.4}$$

As $w \to 1$, the second argument of ϑ_2, which now reads

$$\tau' = -\frac{1}{\tau} = -\frac{i\pi}{\ln w}, \tag{A.5}$$

approaches ∞. We know from the expansion

$$\vartheta_2(0|\tau') = \sum_{n=-\infty}^{\infty} e^{i\pi(n-\frac{1}{2})^2\tau'} \tag{A.6}$$

that

$$\vartheta_2(0|\tau' \to \infty) \to 2e^{\frac{i\pi}{4}\tau'} = 2e^{\frac{\pi^2}{4\ln w}}. \tag{A.7}$$

Thus (A.2) is asymptotically

$$\prod_{n=1}^{\infty}(\frac{1+w^n}{1-w^n})^{-1} \to (-\frac{\ln w}{\pi})^{-\frac{1}{2}} 2\exp(\frac{\pi^2}{4\ln w}). \tag{A.8}$$

¿From (A.1), the state degeneracy $\mathcal{G}_{NS}(n)$ can be expressed as a contour integral on a small circle around $w = 0$

$$\mathcal{G}_{NS}(n) = \frac{1}{2\pi i}\oint \frac{f_{NS}}{w^{n+1}}dw. \tag{A.9}$$

To compute the above integration, we make a saddle point approximation near $w = 1$. The power of w can be put on the exponential

$$\mathcal{G}_{NS}(n) = \frac{1}{2\pi i}\oint 8(-\frac{\ln w}{\pi})^4 2^{-8}\exp[-\frac{2\pi^2}{\ln w} - (n+1)\ln w]dw, \tag{A.10}$$

to get the saddle point at

$$\ln w_0 = \frac{\sqrt{2}\pi}{\sqrt{n+1}}, \tag{A.11}$$

where expansion can be made

$$\ln w = \ln w_0 + iu. \tag{A.12}$$

Then $\mathcal{G}_{NS}(n)$ is approximately

$$\mathcal{G}_{NS}(n) \sim \frac{1}{2\pi}\frac{1}{32}(\frac{\sqrt{2}}{\sqrt{n}})^4 e^{\pi\sqrt{8n}} \int_{-\infty}^{\infty} \exp(-\frac{\sqrt{2}n^{\frac{3}{2}}}{\pi}u^2)du. \tag{A.13}$$

Carrying out the integration over u we find

$$\mathcal{G}_{NS}(n) \sim 2^{-\frac{13}{4}} n^{-\frac{11}{4}} e^{\pi\sqrt{8n}}. \tag{A.14}$$

Or using $n \sim \alpha' m^2$, write it out in terms of mass

$$\mathcal{G}_{NS}(m) \sim 2^{-\frac{13}{4}} \alpha'^{-\frac{11}{4}} m^{-\frac{11}{2}} e^{\pi\sqrt{8\alpha'}m}. \tag{A.15}$$

Here we use the convention $\mathcal{G}_{NS}(m)dn = \mathcal{G}_{NS}(n)dn$, different from [17].

At this point, we also note that R sector has the same expression. And combine the left and right pieces together we arrive at the expression for closed strings

$$\mathcal{G}^{cl}(n) = [\mathcal{G}^{op}(n)]^2 \sim 2^{-\frac{9}{2}} n^{-\frac{11}{2}} e^{4\pi\sqrt{2n}}. \tag{A.16}$$

Taking care of the difference between the mass shell conditions of open and closed strings ($\alpha' m^2 \sim 4n$ for closed strings), the state degeneracy for closed string as a function of mass reads

$$\mathcal{G}^{cl}(m) \sim 2^{\frac{13}{2}} \alpha'^{-\frac{11}{2}} m^{-11} e^{\pi\sqrt{8\alpha'}m}. \tag{A.17}$$

Thus open and closed strings have the same Hagedorn temperature

$$T_H = \frac{1}{\pi\sqrt{8\alpha'}}. \tag{A.18}$$

We know that (The Legacy of Srinivasa Ramanujan, RMS-Lecture Notes Series No. 20, 2013, pp. 261–279.**The Partition Function Revisited** - M. Ram Murty):

The partition function, denoted $p(n)$, is the number of ways of writing n as a non-decreasing sum of positive integers. Thus, $p(1) = 1$, $p(2) = 2$, $p(3) = 3$ and $p(4) = 5$ since

$$4, \quad 1+3, \quad 2+2, \quad 1+1+2, \quad 1+1+1+1+1$$

are the five partitions of 4. Thus, each partition can be "factored" uniquely as

$$1^{k_1} 2^{k_2} \ldots$$

where the notation symbolizes

$$n = \underbrace{1+1+\cdots+1}_{k_1} + \underbrace{2+2+\cdots+2}_{k_2} + \cdots$$

and that:

The question of the asymptotic behaviour of $p(n)$ was first answered in the 1918 paper of Hardy and Ramanujan [9]. They proved that

$$p(n) \sim \frac{e^{\pi\sqrt{2n/3}}}{4n\sqrt{3}}, \quad n \to \infty. \tag{4}$$

In their proof, they discovered a new method called *the circle method* which made fundamental use of the modular property of the Dedekind η-function. We see from the Hardy-Ramanujan formula that $p(n)$ has exponential growth.

We have, from (2.41), that:

$$\sqrt{N_{1L}} + \sqrt{N_{2L}} \leq \sqrt{N_L}.$$

$\sqrt{3} + \sqrt{5} \leq \sqrt{8}$; 1,732050807 + 2,23606797 ≤ 2,82842712;

3,968118777 ≤ 2,82842712

Thence, we have that:

$$p(n) \sim \frac{1}{4n\sqrt{3}} \cdot e^{\pi\sqrt{2n/3}} \quad (4)$$

$$\mathcal{G}(N) \sim \frac{1}{N^{(D+1)/4}} \cdot e^{2\pi\sqrt{(D-2)/6}\sqrt{N}} \quad (2.43)$$

We observe that the eq. (2.43) and the (4) is practically very similar.

Now, we have, from (2.43), for N = 10, α' = 1/2, and D = 26:

$$\mathcal{G}(N) \sim N^{-\frac{D+1}{4}} e^{a\sqrt{N}}, \qquad a = 2\pi\sqrt{\frac{D-2}{6}}.$$

that is:

$$\mathcal{G}(N) \sim \frac{1}{N^{(D+1)/4}} \cdot e^{2\pi\sqrt{(D-2)/6}\sqrt{N}}$$

$\mathcal{G}(N) \sim N^{-\frac{D+1}{4}} e^{a\sqrt{N}} = 10^{-27/4} e^{4\pi\sqrt{10}} = 10^{-6.75} e^{4\pi(3,1622776)}$ = (1,778279410038 * 10^{-7}) * (181195519824656285,625) = 32221626209,548572. Given a very massive initial string of high level, its state density has the above asymptotic form. We note that $(32221626209,548572)^{1/48}$ = 1,65546399123955

From the paper "**RAMANUJAN'S CLASS INVARIANTS,KRONECKER'S LIMIT FORMULA, AND MODULAR EQUATIONS** BRUCE C. BERNDT, HENG HUAT CHAN, AND LIANG-CHENG ZHANG", we have various expressions that can be related with some sectors of the string theory.

We take:

$$(130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}}=\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3,$$

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3.$$

that is equal to 1164,269601267364. We note that $(1164{,}269601267364)^{1/14} =$ 1,655784548804.
We have that:

$$\sqrt[48]{\frac{1}{N^{(26+1)/4}}\cdot e^{2\pi\sqrt{(26-2)/6}\sqrt{10}}} = 1{,}6554639\ldots$$

and

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}6557845$$

Thence a new possible mathematical connection between the asymptotic form of the state density of a very massive initial string of high level, and the above Ramanujan's class invariant. We have indeed:

$$\sqrt[48]{\frac{1}{N^{(26+1)/4}}\cdot e^{2\pi\sqrt{(26-2)/6}\sqrt{10}}} = \sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105\quad\sqrt{505}}{8}}\right)^3}$$

$$1{,}65546 \approx 1{,}65578$$

results that are very near to the mass of the proton $(1{,}672 * 10^{-27})$

We have, from (2.52), that:

$$\mathcal{G}_L \sim (2\pi T_H)^{-\frac{D-1}{2}} (\frac{N_{1L} N_{2L}}{N_L})^{-\frac{D+1}{4}} e^{-\sqrt{2} t_L / T_H},$$

with the Hagedorn temperature $T_H = \frac{1}{\pi}\sqrt{\frac{3}{D-2}}$ and $t_L = \sqrt{N_L} - \sqrt{N_{1L}} - \sqrt{N_{2L}}$

76,1095894446859531 * 0,01436287608 * 1659103,33006 = 1813653,1217337...
that is the total decay rate for decays into arbitrary states of given mass. We note that $(1813653,1217337)^{1/30}$ = 1,6166591858....

We have that:

$$\frac{1}{\sqrt{2}}(301 + 46\sqrt{43}) + \frac{1}{\sqrt{2}}\sqrt{7(25941 + 3956\sqrt{43})} = \left(\sqrt{\frac{46+7\sqrt{43}}{4}} + \sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3,$$

that is equal to: 852,2635597... We have that $(852,2635597)^{1/14}$ = 1,6192977355292

Thence, we have the following new mathematical connection:

$$\mathcal{G}_L \sim (2\pi T_H)^{-\frac{D-1}{2}} (\frac{N_{1L} N_{2L}}{N_L})^{-\frac{D+1}{4}} e^{-\sqrt{2} t_L / T_H},$$

$$\sqrt[30]{\mathcal{G}_L} \sim \sqrt[30]{(2\pi \cdot 0,1125395)^{-25/2} \left(\frac{15}{8}\right)^{-27/4} e^{-\sqrt{2}(-1,1396916/0,1125395)}} =$$

$$\sqrt[30]{1813653,1217337} = 1,6166591858 \ldots$$

$$\sqrt[14]{\left(\sqrt{\frac{46+7\sqrt{43}}{4}} + \sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3} = 1,6192977 \ldots$$

Thence:

$$\sqrt[30]{(2\pi \cdot 0{,}1125395)^{-25/2}\left(\frac{15}{8}\right)^{-2\ /4} e^{-\sqrt{2}(-1{,}1396916/0{,}1125395)}} \cong$$

$$\cong \sqrt[14]{\left(\sqrt{\frac{46+7\sqrt{43}}{4}}+\sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3}$$

1,61665 ≈ 1,61929

values very near to the electric charge of the electron.

Now, we have:

$$\mathcal{G}(N_L) \approx 2^{-\frac{13}{4}} N_L^{-\frac{11}{4}} e^{\pi\sqrt{8N_L}}.$$

That is equal to: 28390031,9 we note that $(28390031{,}9)^{1/34}$ = 1,65657369... and

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}6557845$$

Thence, we have:

$$\sqrt[34]{(2^{-13/4} \cdot 8^{-11/4} \cdot e^{8\pi})} = \sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3}$$

1,65657 ≈ 1,65578

results that are very near to the mass of the proton.

We have:

$$\mathcal{G}_{NS}(n) \sim 2^{-\frac{13}{4}} n^{-\frac{11}{4}} e^{\pi\sqrt{8n}}$$

That, for n = 4, is equal to: 121357,2462164 and $(121357{,}2462164)^{1/23}$ = 1,66359013

and:

$$\sqrt{\frac{1}{2}(135619+78300\sqrt{3})+\frac{3}{\sqrt{2}}(87+50\sqrt{3})}=\left(\sqrt{\frac{21+12\sqrt{3}}{2}}+\sqrt{\frac{19+12\sqrt{3}}{2}}\right)^3,$$

that is equal to: 736,53184348... We have that $(736{,}53184348)^{1/13}$ = 1,661702198... Thence:

$$\sqrt[23]{\left(2^{-1\ /4}\cdot 4^{-11/4}\cdot e^{\pi\sqrt{32}}\right)}=\sqrt[13]{\left(\sqrt{\frac{21+12\sqrt{3}}{2}}+\sqrt{\frac{19+12\sqrt{3}}{2}}\right)^3}$$

1,66359 ≈ 1,66170

results that are very near to the mass of the proton.

Now, we have:

$$\mathcal{G}_{NS}(m)\sim 2^{-\frac{13}{4}}\alpha'^{-\frac{11}{4}}m^{-\frac{11}{2}}e^{\pi\sqrt{8\alpha' m}}.$$

That is equal to: 121357,180534 with results similar as the expression obtained above.

We have:

$$\mathcal{G}^{cl}(n)=[\mathcal{G}^{op}(n)]^2\sim 2^{-\frac{9}{2}}n^{-\frac{11}{2}}e^{4\pi\sqrt{2n}}.$$

That is equal to: 58910324836,98435 and $(58910324836{,}98435)^{1/49}$ = 1,65882214636257.

We have that:

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3.$$

that is equal to 1164,269601267364. We note that $(1164{,}269601267364)^{1/14}$ = 1,655784548804

We obtain:

$$\sqrt[49]{2^{-9/2} \cdot 4^{-11/2} \cdot e^{4\pi\sqrt{8}}} = \sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}} + \sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3}$$

1,65882 ≈ 1,65578

results that are very near to the mass of the proton.

In conclusion, we have:

$$\mathcal{G}^{cl}(m) \sim 2^{\frac{13}{2}} \alpha'^{-\frac{11}{2}} m^{-11} e^{\pi\sqrt{8\alpha'}m}.$$

That is equal to: 2309101,7209 and $(2309101{,}7209)^{1/29}$ = 1,657406627...

We have that:

$$\sqrt[29]{2^{13/2} \cdot 0{,}5^{-11/2} \cdot 2{,}828427^{-11} \cdot e^{2\pi \cdot 2{,}828427}}$$

$$= \sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}} + \sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3}$$

1,65740 ≈ 1,65578

values that are very near to the mass of proton.

From:

Brane World of Warp Geometry: An Introductory Review
Yoonbai Kim, Chong Oh Lee, Ilbong Lee

https://arxiv.org/abs/hep-th/0307023v2

3.1 Pure anti-de Sitter spacetime

When the bulk is filled only with negative vacuum energy $\Lambda < 0$ without other matters $S_{\rm matter} = 0$ so that $T_{AB} = 0$, then the Einstein equations (2.14)∼(2.15) are

$$A'' = 0 \text{ and } A'^2 = -\frac{2\Lambda}{p(p+1)}. \tag{3.1}$$

Notice that $A(Z)$ can have a real solution only when Λ is nonpositive. General solution of Eq. (3.1) is given by

$$A_\pm(Z) = \pm\sqrt{\frac{2|\Lambda|}{p(p+1)}}Z + A_0, \tag{3.2}$$

where the integration constant A_0 can be removed by rescaling of the spacetime variables of p-brane, i.e., $dx^\mu \to d\bar{x}^\mu = e^{A_0}dx^\mu$. The resultant metric is

$$ds^2 = e^{\pm 2kZ}\eta_{\mu\nu}d\bar{x}^\mu d\bar{x}^\nu - dZ^2, \tag{3.3}$$

where $k = \sqrt{2|\Lambda|/p(p+1)}$ and a schematic shape of the metric $e^{2A(Z)}$ is shown in Fig. 2. Since the metric function $e^{2A_\pm}$ vanishes or is divergent at spatial infinity $Z = \mp\infty$ respectively, there exists coordinate singularity at those points. Despite of the coordinate singularity, the spacetime is physical-singularity-free everywhere as expected

$$R^{ABCD}R_{ABCD} = \frac{8(p+2)}{p^2(p+1)}|\Lambda|^2. \tag{3.4}$$

4.2 Gauge hierarchy from model I

As we explained briefly in the introduction, the gauge hierarchy problem is a notorious fine tuning problem in particle phenomenology of which the basic language is quantum field theory. So the readers unfamiliar to field theories may skip this subsection.

Let us assume that we live on the p-brane at $Z = r_c\pi$ and try a dimensional reduction of the Einstein gravity from the $D = p+2$-dimensional gravity to $p+1$-dimensional gravity on the p-brane at $Z = r_c\pi$. Then we have

$$\begin{aligned}
S_{\mathrm{EH}\,D} &= -\frac{M_*^p}{16\pi}\int d^Dx\sqrt{|g_D|}\,R &(4.23)\\
&= -\frac{M_*^p}{16\pi}\int d^{p+1}x\sqrt{|\det g_{\mu\nu}|}\int_{-r_c\pi}^{r_c\pi} dZ e^{-(p-1)k|Z|}(R_{p+1}+\cdots) &(4.24)\\
&= -\frac{M_*^p}{16k\pi}\left[1-e^{-(p-1)kr_c\pi}\right]\int d^{p+1}x\sqrt{|\det g_{\mu\nu}|}\,(R_{p+1}+\cdots) &(4.25)\\
&\equiv -\frac{M_{\mathrm{Planck}}^2}{16\pi}\int d^{p+1}x\sqrt{|\det g_{\mu\nu}|}\,(R_{p+1}+\cdots) &(4.26)\\
&= S_{\mathrm{EH}\,p+1}+\cdots. &(4.27)
\end{aligned}$$

We used $g_D = e^{-2(p+1)k|Z|}\det g_{\mu\nu}$ and $R = e^{2k|Z|}g^{\mu\nu}R_{\mu\nu}+\cdots = e^{2k|Z|}R_{p+1}+\cdots$ when we calculated the second line (4.24) from the first line (4.23). By comparing the third line (4.25) with the fourth line (4.27), we obtain a relation for 3-brane among three scales M_{Planck}, M_*, $|\Lambda|$ $(p=3)$:

$$M_{\mathrm{Planck}}^2 = \sqrt{\frac{p(p+1)}{2|\Lambda|}}\left[1-\exp\left(-\sqrt{\frac{8|\Lambda|}{p(p+1)}}r_c\pi\right)\right]M_*^{p=3}. \tag{4.28}$$

A natural choice for the bulk theory is to bring up almost the same scales for two bulk mass scales, i.e., $M_* \approx \sqrt{|\Lambda|}$. Suppose that the exponential factor in the relation (4.28) is negligible to the unity, which means r_c is slightly larger than $1/\sqrt{|\Lambda|}$. Then we reach

$$M_{\mathrm{Planck}} \approx M_* \approx \sqrt{|\Lambda|}. \tag{4.29}$$

A striking character of this Randall-Sundrum compactification I is that it provides an explanation for gauge hierarchy problem that *why is so large the mass gap between the Planck scale* $M_{\mathrm{Planck}} \sim 10^{19}\mathrm{GeV} \sim 10^{-38}\,M_\odot$ *and the electroweak scale* $M_{\mathrm{EW}} \sim 10^3\mathrm{GeV} \sim 10^{-54}\,M_\odot$ without assuming supersymmetry or others. As a representative example, let us consider a massive neutral scalar field H which lives on our 3-brane at $Z = r_c\pi$:

$$S_{\mathrm{scalar}} = \int_{-r_c\pi}^{r_c\pi} dZ\delta(Z-r_c\pi)\int d^4x\sqrt{g_5}\left[\frac{1}{2}g^{AB}\partial_A H\partial_B H - \frac{1}{2}M_{\mathrm{Planck}}^2H^2\right]$$

$$\begin{aligned}
&= \int_{-r_c\pi}^{r_c\pi} dZ e^{-4k|Z|}\delta(Z - r_c\pi) \int d^4x\sqrt{-\hat{g}_4} \\
&\qquad \times \left[\frac{1}{2}e^{2k|Z|}\hat{g}^{\mu\nu}\partial_\mu H\partial_\nu H - \frac{1}{2}M_{\rm Planck}^2 H^2 - \frac{1}{2}\hat{g}^{ZZ}\left(\partial_Z H\right)^2\right] \\
&= e^{-2r_c\pi k}\int d^4x\sqrt{-\hat{g}_4}\left[\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu H\partial_\nu H - \frac{1}{2}(e^{-r_c\pi k}M_{\rm Planck})^2 H^2\right] \qquad (4.30) \\
&= e^{-2r_c\pi k}\int d^4x\sqrt{-\hat{g}_4}\left[\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu H\partial_\nu H - \frac{1}{2}M_{\rm EW}^2 H^2\right], \qquad (4.31)
\end{aligned}$$

where $ds^2 = g_{AB}dx^A dx^B = e^{-2k|Z|}\hat{g}_{\mu\nu}dx^\mu dx^\nu - dZ^2$. The last two lines give us a relation:

$$\frac{M_{\rm EW}}{M_{\rm Planck}} = \exp\left(-\sqrt{\frac{2|\Lambda|}{p(p+1)}}r_c\pi\right) \sim 10^{-16}. \qquad (4.32)$$

Therefore, the radius r_c of compactified extra dimension of the Randall-Sundrum brane world model I is determined nearly by the Planck scale :

$$\frac{1}{r_c} \sim \frac{\pi}{16\sqrt{6}\ln 10}\sqrt{|\Lambda|} \sim \frac{M_{\rm Planck}}{30}. \qquad (4.33)$$

All the scales such as the fundamental scale of the bulk M_*, the bulk cosmological constant $\sqrt{|\Lambda|}$, the inverse size of the compactification $1/r_c$, are almost the Planck scales $M_{\rm Planck} \sim 10^{19}$ GeV together. The masses of matter particles on our visible brane at $Z = r_c\pi$ are in electroweak scale $M_{\rm EW} \sim 10^3$ GeV, however those on the hidden brane at $Z = 0$ in the Planck scale. Though the gauge hierarchy problem seems to be solved, it is actually not because a fine-tuning condition was urged in Eq. (3.25). However, it becomes much milder than that before.

Finally let us consider a fermionic field of which mass is provided by spontaneous symmetry breaking and its Lagrangian is

$$\mathcal{L}_{\text{fermion}} = \bar{\Psi}\gamma^A \nabla_A \Psi + g\phi\bar{\Psi}\Psi, \tag{4.36}$$

where g is the coupling constant of Yukawa interaction. If we neglect the quantum fluctuation $\delta\phi$ of ϕ, i.e. $\phi \equiv \langle\phi\rangle + \delta\phi$, the Lagrangian (4.36) becomes

$$\mathcal{L}_{\text{fermion}} = \bar{\Psi}\gamma^A \nabla_A \Psi + g\langle\phi\rangle\bar{\Psi}\Psi + \cdots, \tag{4.37}$$

where the second term is identified as mass term, and we neglected the vertex term $g\delta\phi\bar{\Psi}\Psi$ because we are not interested in quantum fluctuation. Again the fermion lives on our 3-brane at $Z = r_c\pi$, and then the action is

$$S_{\text{fermion}} = \int_{r_c\pi}^{r_c\pi} dZ\delta(Z - r_c\pi)\int d^4x\sqrt{g_5}\left[\bar{\Psi}\gamma^a e_a^A \nabla_A\Psi + M_{\text{Planck}}\bar{\Psi}\Psi\right], \tag{4.38}$$

where e_a^A is vielbein defined by $g_{AB} = \eta_{ab}e_A^a e_B^b$ and $M_{\text{Planck}} = g\langle\phi\rangle$ since the symmetry breaking scale should coincide with the fundamental scale. Subsequently, the action (4.38) becomes

$$\begin{aligned}
S_{\text{fermion}} &= \int_{r_c\pi}^{r_c\pi} dZ\delta(Z - r_c\pi)\int d^4x\sqrt{g_5}\left[\bar{\Psi}\gamma^a e_a^A \nabla_A\Psi + M_{\text{Planck}}\bar{\Psi}\Psi\right] \\
&= \int_{-r_c\pi}^{r_c\pi} dZ e^{-4k|Z|}\delta(Z - r_c\pi)\int d^4x\sqrt{-\hat{g}_4} \\
&\qquad \times\left[e^{k|Z|}\bar{\Psi}\gamma^a \hat{e}_a^\mu \nabla_\mu\Psi - \bar{\Psi}\gamma^a \hat{e}_a^Z \nabla_Z\Psi + M_{\text{Planck}}\bar{\Psi}\Psi\right] \\
&= e^{-3r_c\pi k}\int d^4x\sqrt{-\hat{g}_4}\left[\bar{\Psi}\gamma^a \hat{e}_a^\mu \nabla_\mu\Psi + (e^{-r_c\pi k}M_{\text{Planck}})\bar{\Psi}\Psi\right] \qquad (4.39) \\
&= e^{-3r_c\pi k}\int d^4x\sqrt{-\hat{g}_4}\left[\bar{\Psi}\gamma^a \hat{e}_a^\mu \nabla_\mu\Psi + m_{\text{fermion}}\bar{\Psi}\Psi\right]. \qquad (4.40)
\end{aligned}$$

Once again we obtain the same mass hierarchy relation $m_{\text{fermion}} = e^{-r_c\pi k}M_{\text{Planck}} = M_{\text{EW}}$ for the fermion from Eq. (4.39) and Eq. (4.40) with the help of Eq. (4.32).

We know that the mass Planck is defined as:

$$m_{\text{P}} = \sqrt{\frac{\hbar c}{G}},$$

where c is the speed of light in a vacuum, G is the gravitational constant, and $\hbar$ is the reduced Planck constant.

Substituting values for the various components in this definition gives the approximate equivalent value of this unit in terms of other units of mass:

1 $m_{\text{P}} \approx 1.220910\times10^{19}$ GeV/c^2 = $2.176470(51)\times10^{-8}$ kg.

Particle physicists and cosmologists often use an alternative normalization with the **reduced Planck mass**, which is

$$M_P = \sqrt{\frac{\hbar c}{8\pi G}}$$

$M_P \approx 2.435\times10^{18}$ GeV/c^2 = 4.341×10^{-9} kg

Now, from (4.28)

$$M_{\text{Planck}}^2 = \sqrt{\frac{p(p+1)}{2|\Lambda|}}\left[1 - \exp\left(-\sqrt{\frac{8|\Lambda|}{p(p+1)}}r_c\pi\right)\right] M_*^{p=3}.$$

we have that:

$M_{Planck}^2 = 1{,}4906212281 \times 10^{38}$ and E = $1{,}098819 * 10^{36}$ or:

$M_{Planck}^2 = 5{,}929225 \times 10^{36}$ and E = $2{,}1915 * 10^{35}$

And, from (4.33), we have that:

$$\frac{1}{r_c} \sim \frac{\pi}{16\sqrt{6}\ln 10}\sqrt{|\Lambda|} \sim \frac{M_{\text{Planck}}}{30}.$$

Thence: $\frac{1}{r_c} = 4{,}0697 * 10^{17}$ and $r_c = 2{,}4571835761 * 10^{-18}$

$r_c = 2{,}4571835761849767796152050519694 * 10^{-18}$ or:

$\frac{1}{r_c} = 8{,}11666 * 10^{16}$ and $r_c = 1{,}2320328542 * 10^{-17}$

We have that:

$M_{\text{EW}} \sim 10^3\text{GeV}$

and

$m_{\text{fermion}} = e^{-r_c\pi k}M_{\text{Planck}} = M_{\text{EW}}$

We have that:

$m_{\text{fermion}} = 1{,}22091 * 10^{19}$ GeV/c^2 or

$m_{\text{fermion}} = 2{,}435 \times 10^{18}$ GeV/c^2. The energy E is: $2{,}1915 * 10^{35}$

Now:

$(2{,}435 \times 10^{18})^{1/88} = 1{,}61784715017$ or $(2{,}435 \times 10^{18})^{1/89} = 1{,}609125347$ and for the value of the fermion energy E (for $E = mc^2$):

$(2{,}1915 * 10^{35})^{1/168} = 1{,}6231608397$ or $(2{,}1915 * 10^{35})^{1/169} = 1{,}6185153159$.

Further, we have:

$(1{,}2320328542 * 10^{-17})^{1/80} = 0{,}614656924537$ and the reciprocal is 1,626923833;

and

$(5{,}929225 \times 10^{36})^{1/168} = 1{,}65533879$.. and $(5{,}929225 \times 10^{36})^{1/178} = 1{,}609125347$.

All results very near to the value of the golden ratio

Now, we have that:

$$P^{-2} := (G_{205}G_{41/5})^4 = \left(\frac{\sqrt{5}+1}{2}\right)^8 \left(\frac{43+3\sqrt{205}}{2}\right)$$
$$= \left(\frac{7+3\sqrt{5}}{2}\right)^2 \left(\frac{3\sqrt{5}+\sqrt{41}}{2}\right)^2.$$

that is equal to: 46,9787137637477918 (42,9767315949145297) = 2018,991572...
and $(2018{,}991572)^{1/16} = 1{,}6090550645269$

Thence:

$$M^2_{\text{Planck}} - \sqrt{\frac{p(p+1)}{2|\Lambda|}}\left[1 - \exp\left(-\sqrt{\frac{8|\Lambda|}{p(p+1)}}r_c\pi\right)\right] M_*^{p=3}.$$

$= 5{,}929225 \times 10^{36}$ and $(5{,}929225 \times 10^{36})^{1/178} = 1{,}609125347$.

We have the following interesting mathematical connection:

$$\sqrt[178]{\left(\sqrt{\frac{p(p+1)}{2|\Lambda|}}\left(1-exp\left(-\sqrt{\frac{8|\Lambda|}{p(p+1)}}r_c\pi\right)\right)M_*^{p=3}\right)} =$$

$$= \sqrt[16]{\left(\frac{7+3\sqrt{5}}{2}\right)^2 + \left(\frac{3\sqrt{5}+\sqrt{41}}{2}\right)^2}$$

1,6091253 ≈ 1,609055

Values that are very near to the electric charge of positron.

Now:

$$\frac{1}{r_c} \sim \frac{\pi}{16\sqrt{6}\ln 10}\sqrt{|\Lambda|} \sim \frac{M_{\text{Planck}}}{30}.$$

$= 8,11666 * 10^{16}$ and $r_c = 1,2320328542 * 10^{-17}$

and $1 / (1,2320328542 * 10^{-17})^{1/80} = 1,626923833.$

We have that:

$$\frac{1}{\sqrt{2}}(301+46\sqrt{43}) + \frac{1}{\sqrt{2}}\sqrt{7(25941+3956\sqrt{43})}$$
$$= \left(\sqrt{\frac{46+7\sqrt{43}}{4}} + \sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3,$$

$= 852,2635597...$ and $(852,2635597)^{1/14} = 1,6192977355292.$

Thence:

$$\left(1/\sqrt[80]{\frac{M_{Planck}}{30}}\right) = \sqrt[14]{\left(\sqrt{\frac{46+7\sqrt{43}}{4}} + \sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3};$$

1,626923 ≈ 1,619297

Values that are a good approximations to the electric charge of the positron and to the value of the golden ratio.

Now:

$m_{\text{fermion}} = e^{-r_c \pi k} M_{\text{Planck}} = M_{\text{EW}}$

$= 2{,}435 \times 10^{18}$ and $(2{,}435 \times 10^{18})^{1/89} = 1{,}609125347$.

We have that:

$$\sqrt[16]{\left(\frac{7+3\sqrt{5}}{2}\right)^2 + \left(\frac{3\sqrt{5}+\sqrt{41}}{2}\right)^2} = 1{,}609055..$$

Thence:

$$\sqrt[89]{e^{-r_c \pi k} \cdot M_{Planck}} = \sqrt[16]{\left(\frac{7+3\sqrt{5}}{2}\right)^2 + \left(\frac{3\sqrt{5}+\sqrt{41}}{2}\right)^2}$$

$$1{,}60912 \approx 1{,}60905$$

values very similar and very near to the electric charge of the positron.

We calculate the following double integrals:

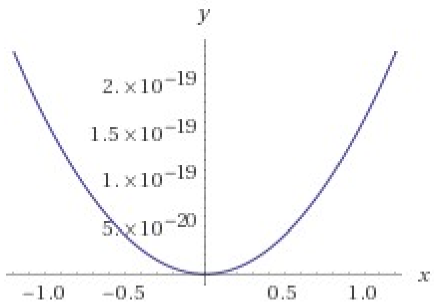
(4Pi/23) 1/(10^55) integrate integrate [5.929225 *10^36]

$$\left(4 \times \frac{\pi}{23}\right) \times \frac{1}{10^{55}} \int \left(\int 5.929225 \times 10^{36} \, dx \right) dx$$

Result:

$1.61976 \times 10^{-19} \, x^2$

Plot:



(x from −1.2 to 1.2)

and

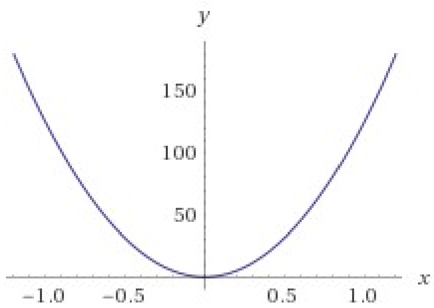
(2*21) * integrate integrate [5.929225]

$$2 \times 21 \int \left(\int 5.929225 \, dx \right) dx$$

Result:

$124.514 \, x^2$

Plot:



(x from -1.2 to 1.2)

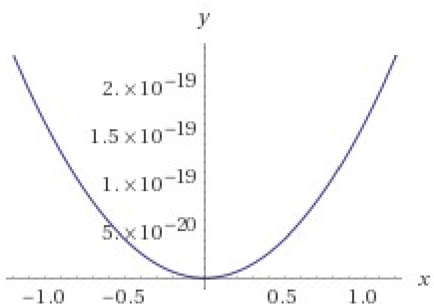
(4Pi/86) 1/(10^53) integrate integrate [2.1915 * 10^35]

$$\left(4\times\frac{\pi}{86}\right)\times\frac{1}{10^{53}}\int\left(\int 2.1915\times10^{35}\,dx\right)dx$$

Result:

$1.60112\times10^{-19}\,x^2$

Plot:



(x from -1.2 to 1.2)

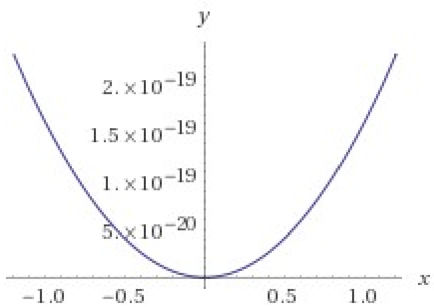
(4Pi/95) 1/(10^36) integrate integrate [2.435 × 10^18]

$$\left(4\times\frac{\pi}{95}\right)\times\frac{1}{10^{36}}\int\left(\int 2.435\times10^{18}\,dx\right)dx$$

Result:

$1.61048\times10^{-19}\,x^2$

Plot:



(x from -1.2 to 1.2)

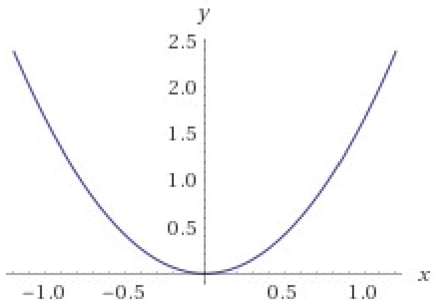
(24Pi/5) 1/(10^36) integrate integrate [2.1915 * 10^35]

$$\left(24\times\frac{\pi}{5}\right)\times\frac{1}{10^{36}}\int\left(\int 2.1915\times 10^{35}\,dx\right)dx$$

Result:

$1.65235\,x^2$

Plot:



(x from −1.2 to 1.2)

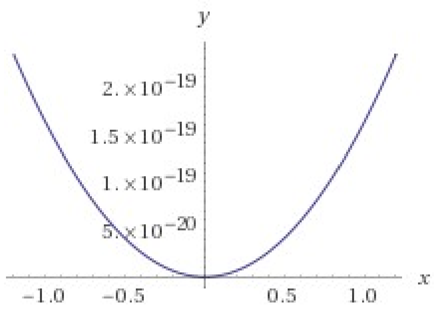
(3Pi^2/2) * 1/(10^55) * integrate integrate [2.1915 * 10^35]

$$\left(3\times\frac{\pi^2}{2}\right)\times\frac{1}{10^{55}}\int\left(\int 2.1915\times 10^{35}\,dx\right)dx$$

Result:

$1.62219\times 10^{-19}\,x^2$

Plot:



(x from −1.2 to 1.2)

All the results are good approximations to the value of the electric charge of positron

From:

Modular Relations for J-invariant and Explicit evaluations
M.S. Mahadeva Naika, D.S. Gireesh and N.P. Suman
Communicated by P. K. Banerji - Palestine Journal of Mathematics - Vol. 5(2) (2016) , 83–95

Theorem 4.2. *We have*

$$\text{(i)}\ J_{63} = \frac{5}{32}\left(\frac{3}{4}(a_1 + b_1)\right)^{\frac{1}{3}}, \tag{4.12}$$

$$\text{(ii)}\ J_{75} = 3\left(369830 - 165393\sqrt{5}\right)^{\frac{1}{3}}, \tag{4.13}$$

$$\textit{(iii)}\ J_{99} = \left(\frac{1147951079}{2} + \frac{199832633\sqrt{33}}{2}\right)^{\frac{1}{3}}, \tag{4.14}$$

$$\textit{(iv)}\ J_{147} = \left(531745995375 + 116036489250\sqrt{21}\right)^{\frac{1}{3}}, \tag{4.15}$$

$$\textit{(v)}\ J_{171} = \frac{1}{4}(a_2 + b_2)^{\frac{1}{3}}, \tag{4.16}$$

where

$a_1 = 180040533 + 39288067\sqrt{21},$

$b_1 = 273\sqrt{2\left(434925969567 + 94908627499\sqrt{21}\right)},$

$a_2 = 21187806942033 + 2806393586997\sqrt{57},$

and

$b_2 = 2\sqrt{\frac{4489231630128611072984 13933}{2} + \frac{594613255246519815126677 61\sqrt{57}}{2}}.$

We have:

$$\text{(ii)}\ J_{75} = 3\left(369830 - 165393\sqrt{5}\right)^{\frac{1}{3}},$$

that is equal to: 0,6239645246738... we have that: (1 / 0,6239645246738...) = = 1,60265521589... and (1,60265521589) * 8 = 12,821241..; (1,60265521589) * 4 = = 6,41062.. values that are good approximations to the mass of the SMBH87 and to the reduced Planck's constant.

$$J_{99} = \left(\frac{1147951079}{2} + \frac{199832633\sqrt{33}}{2}\right)^{\frac{1}{3}}$$

that is equal to: 1047,06697; $(1047{,}06697)^{1/14}$ = 1,64328337... and (1,64328337) * 8 = 13,146266997; (1,64328337) * 4 = 6,573133... values that are very near to the mass of the SMBH87 and to the reduced Planck's constant.

$$J_{147} = \left(531745995375 + 116036489250\sqrt{21}\right)^{\frac{1}{3}},$$

that is equal to: 10207,312425; $(10207{,}312425)^{1/19}$ = 1,6255313... and (1,6255313) * 8 = 13,00425; (1,6255313) * 4 = 6,5021252; values that are very near to the mass of the SMBH87 and to the reduced Planck's constant .

Now:

$$a_2 = 21187806942033 + 2806393586997\sqrt{57},$$
and
$$b_2 = 2\sqrt{\frac{448923163012861107298413933}{2} + \frac{594613255246519815126667761\sqrt{57}}{2}}.$$

$$(v)\ \ J_{171} = \frac{1}{4}(a_2 + b_2)^{\frac{1}{3}}$$

a_2 = 42375613884065,949055701411614262
b_2 = 42375613884065,967937986172212435

The result is : 10981,3400827; $(10981,3400827)^{1/19}$ = 1,63179677... and (1,63179677) * 8 = 13,0543742; (1,63179677) * 4 = 6,5271871...; values that are very near to the mass of the SMBH87 and to the reduced Planck's constant.

Now:

$$a_1 = 180040533 + 39288067\sqrt{21},$$
$$b_1 = 273\sqrt{2\left(434925969567 + 94908627499\sqrt{21}\right)},$$

$$\text{(i)}\ \ J_{63} = \frac{5}{32}\left(\frac{3}{4}(a_1 + b_1)\right)^{\frac{1}{3}},$$

a_1 = 360081073,935996
b_1 = 360081088,577444

The result is: 127,2478775... $(127,2478775)^{1/10}$ = 1,6235477... (1,6235477) * 8 = 12,9883816; (1,6235477) * 4 = 6,4941908... values that are very near to the mass of the SMBH87 and to the reduced Planck's constant.

Now, we have that:

Theorem 4.3.

$$(i)\ \ J_{243} = 5\left(8704c_1 + \frac{72603983653110c_1}{1447318030184058288801} + 151022371885959\right)^{\frac{1}{3}}, \tag{4.22}$$

$$(ii)\ \ J_{275} = \frac{544127}{2} + 121667\sqrt{5} + \frac{\sqrt{592131660065 + 264809328784\sqrt{5}}}{2}, \tag{4.23}$$

where $c_1 = 522356752701807514228727100585 3^{1/3}$.

$$(i)\ \ J_{243} = 5\left(8704c_1 + \frac{72603983653110c_1}{1447318030184058288801} + 151022371885959\right)^{\frac{1}{3}},$$

5 (151022371885959,00037 + 8703,999 + 151022371885959)$^{1/3}$ =
= 5 (67095,0417540786) = 335475,2087...
We have that (335475,2087)$^{1/27}$ = 1,6019689349... (1,6019689349) * 8 = 12,81575...
(1,6019689349) * 4 = 6,407875 values that are very near to the mass of the SMBH87 and to the reduced Planck's constant.

We now calculate the following double integral:

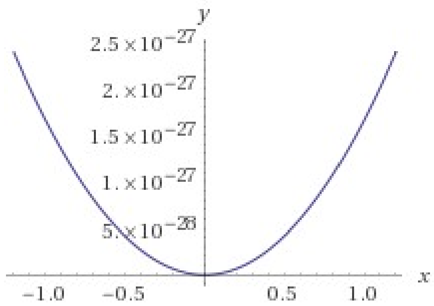
(Pi)^2 * (1/(10)^33) * integrate integrate [335475.2087]

$$\pi^2 \times \frac{1}{10^{33}} \int \left(\int 335\,475.2087\, dx \right) dx$$

Result:

$1.6555 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

Result that is a good approximation to the proton mass

Now:

$$(ii)\quad J_{275} = \frac{544127}{2} + 121667\sqrt{5} + \frac{\sqrt{592131660065 + 264809328784\sqrt{5}}}{2},$$

We have that: 272063,5 + 272055,682618 + 544119,316028 = 1088238,498646

We obtain: (1088238,498646)$^{1/29}$ = 1,61496420014; (1,61496420014) * 8 = 12,9197 and (1,61496420014) * 4 = 6,4598568 values that are a good approximation to the reduced Planck's constant and to the mass of the SMBH87

We calculate the following double integral:

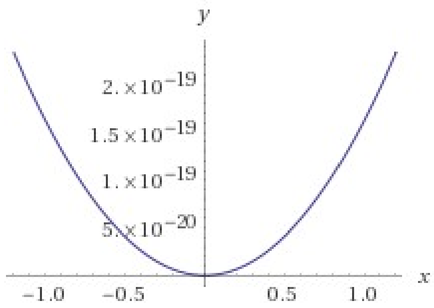
(Pi)/(13*0.25)^2 * (1/(10)^24) * integrate integrate [1088238.498646]

$$\frac{\pi}{(13 \times 0.25)^2} \times \frac{1}{10^{24}} \int \left(\int 1.088238498646 \times 10^6\, dx \right) dx$$

Result:

$1.61837\times 10^{-19}\, x^2$

Plot:



(x from −1.2 to 1.2)

Result that is a good approximation to the value of the electric charge of positron

Theorem 4.4. *We have*

$$\text{(i)}\quad J_{363} = 15\left(\frac{a_3}{4} + \frac{9\sqrt{2\,(b_3+c_3)}}{4}\right)^{\frac{1}{3}}, \tag{4.25}$$

$$\textit{(ii)}\quad J_{387} = \left(\frac{a_4}{4} + \frac{1}{2}\sqrt{\frac{b_4}{2} + \frac{c_4}{2}}\right)^{\frac{1}{3}}, \tag{4.26}$$

$$\textit{(iii)}\quad J_{475} = \frac{127580541}{2} + 28527876\sqrt{5} + a_5, \tag{4.27}$$

$$\textit{(iv)}\quad J_{603} = \left(\frac{a_6}{4} + \frac{1}{2}\sqrt{\frac{b_6}{2} + \frac{c_6}{2}}\right)^{\frac{1}{3}}, \tag{4.28}$$

where $a_3 = 8935875480904000075 + 1555536257627762 61\sqrt{33}$,

$b_3 = 9858008717311272244225627492154461$,

$c_3 = 1716059049900381797208659334764635\sqrt{33}$,

$a_4 = 211345136395511928131 25 + 1860790168869410611875\sqrt{129}$,

$b_4 = 446667666780375406374724355998383412241203125$,

$c_4 = 39326895204313325954377906531132680150421875\sqrt{129}$,

$a_6 = 9733193881239347414807 2097625 + 686526563243390788085932 5375\sqrt{201}$,

$b_6 = 94735063129795071747526692894932777233525896625488200015625$,

and

$c_6 = 6682096144668849090398550257920911897699497459405426718 75\sqrt{201}$.

We have that:

$$(ii)\ \ J_{387} = \left(\frac{a_4}{4} + \frac{1}{2}\sqrt{\frac{b_4}{2} + \frac{c_4}{2}} \right)^{\frac{1}{3}},$$

$$a_4 = 21134513639551192813125 + 1860790168869410611875\sqrt{129},$$
$$b_4 = 446667666780375406374724355998383412241203125,$$
$$c_4 = 39326895204313325954377906531132680150421875\sqrt{129},$$

We obtain:

(((((21134513639551192813125+(1860790168869410611875(sqrt(129)))/4)+0.5(sqrt((446667666780375406374724355998383412241203125)/2+((39326895204313325954377906531132680150421875*sqrt(129))/2)))))^0.33333

$$\left(\left(\frac{1}{4} \times 21\,134\,513\,639\,551\,192\,813\,125 + 1\,860\,790\,168\,869\,410\,611\,875 \times 11.3578\right) + 0.5\sqrt{\left(\frac{446\,667\,666\,780\,375\,406\,374\,724\,355\,998\,383\,412\,241\,203\,125}{2} + \frac{1}{2}\, 39\,326\,895\,204\,313\,325\,954\,377\,906\,531\,132\,680\,150\,421\,875 \times 11.3578\right)}\right)^{0.33333}$$

$3.33121\ldots \times 10^7$

33.312.064;

Note that: 3,33121 * 4 = 13,32484 and 3,33121 * 2 = 6,66242. Furthermore, $(33312064)^{1/35}$ = 1,6403309248 and (1,6403309248) * 8 = 13,12264739; (1,6403309248 * 4) = 6,56132369 values very near to the Planck's constant and to the mass of SMBH87.

We now calculate the following double integral:

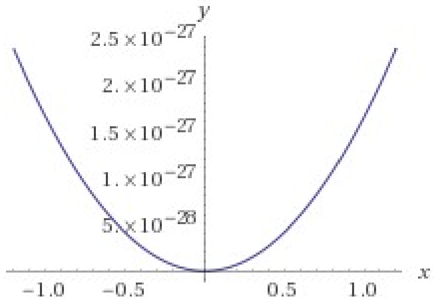
(Pi)^2 * (1/(10)^35) * integrate integrate [3.33121*10^7]

$$\pi^2 \times \frac{1}{10^{35}} \int \left(\int 3.33121 \times 10^7\, dx \right) dx$$

Result:

$1.64389 \times 10^{-27}\, x^2$

Plot:



(x from −1.2 to 1.2)

Result that is very near to the value of the proton mass

We have:

$$\text{(i)}\quad J_{363} = 15\left(\frac{a_3}{4} + \frac{9\sqrt{2(b_3+c_3)}}{4}\right)^{\frac{1}{3}}$$

$$\textit{where } a_3 = 893587548090400075 + 155553625762776261\sqrt{33},$$

$$b_3 = 9858008717311272244225627492154461,$$

$$c_3 = 1716059049900381797208659334764635\sqrt{33},$$

We obtain:

$$15\,((0.25 \times 893\,587\,548\,090\,400\,075 + 155\,553\,625\,762\,776\,261 \times 1.43614) + 2.25\,\sqrt{(2\,(9\,858\,008\,717\,311\,272\,244\,225\,627\,492\,154\,461 + 1\,716\,059\,049\,900\,381\,797\,208\,659\,334\,764\,635 \times 5.74456)))^{0.3333}$$

$1.44280... \times 10^7$

14428000;

Now: 1,4428 * 9 = 12,9852 and 12,9852 / 2 = 6,4926. Furthermore:

$(14428000)^{1/33}$ = 1,6479561079... and 1,6479561079 * 8 = 13,183648; 1,6479561079 * 4 = 6,591824... all values very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

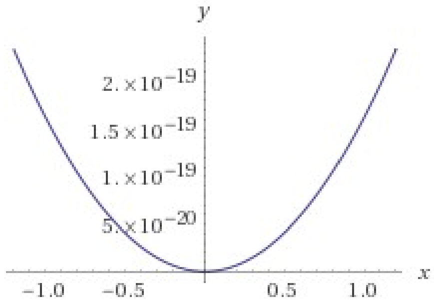
(Pi)/14 * (1/(10)^25) * integrate integrate [1.44280*10^7]

$$\frac{\pi}{14} \times \frac{1}{10^{25}} \int\left(\int 1.44280 \times 10^7 \, dx\right) dx$$

Result:

$1.61882 \times 10^{-19} x^2$

Plot:



(x from −1.2 to 1.2)

Result that is very near to the value of the electric charge of positron.

Now, we have:

$$(iv)\ \ J_{603} = \left(\frac{a_6}{4} + \frac{1}{2}\sqrt{\frac{b_6}{2} + \frac{c_6}{2}}\right)^{\frac{1}{3}},$$

$$a_6 = 97331938812393474148072097625 + 6865265632433907880859325375\sqrt{201},$$

$$b_6 = 94735063129795071747526692894932777233525896625488200015625,$$

and

$$c_6 = 668209614466884909039855025792091189769949745940542671875\sqrt{201}.$$

We obtain:

$$0.5\sqrt{\left(9\,473\,506\,312\,979\,507\,174\,752\,669\,289\,493\,277\,723\,352\,589\,662\,548\,820\,015\,625 / 2 + \frac{1}{2}\, 668\,209\,614\,466\,884\,909\,039\,855\,025\,792\,091\,189\,769\,949\,745\,940\,542\,671\,875 \times 14.1774\right)}$$

$4.86659... \times 10^{28}$

$(0.25 (97331938812393474148072097625 + 6865265632433907880859325375 \times 14.1774) + 4.86659 \times 10^{28})^{0.33333333}$

$4.59993... \times 10^{9}$

4.599.930.000;

We note that 4,59993 * 3 = 13,79979; 13,79979 / 2 = 6,899895. Furthermore: $(4.599.930.000)^{1/46}$ = 1,62203346153.. and 1,62203346153 * 8 = 12,976267692...; 1,62203346153 * 4 = 6,4881338; value very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

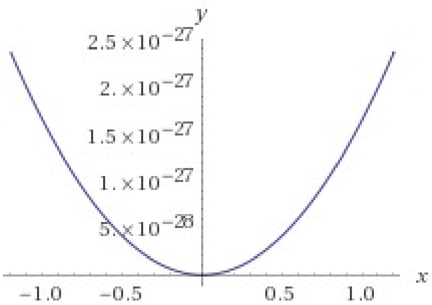
(Pi)/44 * (1/(10)^35) * integrate integrate [4.59993*10^9]

$$\frac{\pi}{44} \times \frac{1}{10^{35}} \int \left(\int 4.59993 \times 10^{9} \, dx \right) dx$$

Result:

$1.64217 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

Result that is very near to the proton mass.

We have:

Theorem 4.5. *We have*

$$\text{(i)}\ J_{\frac{11}{9}} = -\left(\frac{11}{2}\right)^{\frac{1}{3}}\left(-104359189 + 18166603\sqrt{33}\right)^{\frac{1}{3}}, \tag{4.32}$$

$$\text{(ii)}\ J_{\frac{3}{25}} = 3\left(369830 - 165393\sqrt{5}\right)^{\frac{1}{3}}, \tag{4.33}$$

$$\text{(iii)}\ J_{\frac{11}{25}} = \frac{544127}{2} + 121667\sqrt{5} - \frac{\sqrt{11\left(53830150915 + 24073575344\sqrt{5}\right)}}{2}, \tag{4.34}$$

$$\text{(iv)}\ J_{\frac{3}{49}} = 15\left(157554369 - 34381182\sqrt{21}\right)^{\frac{1}{3}}. \tag{4.35}$$

Now:

$$\text{(iv)}\ J_{\frac{3}{49}} = 15\left(157554369 - 34381182\sqrt{21}\right)^{\frac{1}{3}}.$$

We obtain: $15(157554369 - 157554368{,}9970532)^{1/3} = 2{,}150505873...$ and

$5\ (2{,}150505873) = 10{,}7525293...$ $(10{,}7525293)^{1/5} = 1{,}60805954....$ Thence: $(1{,}60805954) * 8 = 12{,}864476$; $(1{,}60805954) * 4 = 6{,}432238...$ values that are a good approximation to the reduced Planck's constant and to the mass of the SMBH87.

We have:

$$\text{(iii)}\ J_{\frac{11}{25}} = \frac{544127}{2} + 121667\sqrt{5} - \frac{\sqrt{11\left(53830150915 + 24073575344\sqrt{5}\right)}}{2},$$

We obtain:

$(272063{,}5 + 272055{,}6826184 - 544119{,}31602868) = -0{,}13341028$; $-(0{,}13341028)^{1/4} = 0{,}60436224239$. Now we note that $1 / 0{,}60436224239 = 1{,}6546367887...$ and $1{,}6546367887 * 8 = 13{,}237094309$; $1{,}6546367887 * 4 = 6{,}618547...$ that are values very near to the Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

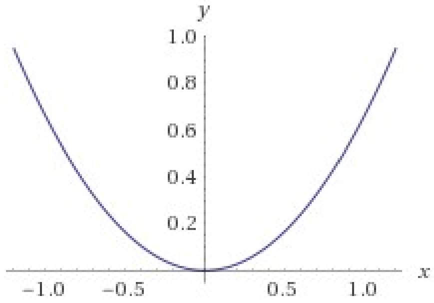
Pi^2 * integrate integrate [0.13341028]

$$\pi^2 \int\left(\int 0.13341028\, dx\right) dx$$

Result:

$$0.658353\, x^2$$

Plot:



(x from -1.2 to 1.2)

We note that the integral is given utilizing the following simple rules:

$$\int a\,dx = ax + C$$

$$\int x^a\,dx = \frac{x^{a+1}}{a+1} + C \qquad (\text{for } a \neq -1)$$ (Cavalieri's quadrature formula)

We have:

$$\text{(i)}\quad J_{\frac{11}{9}} = -\left(\frac{11}{2}\right)^{\frac{1}{3}} \left(-104359189 + 18166603\sqrt{33}\right)^{\frac{1}{3}},$$

We obtain: -0,35718823192 and $(0,35718823192)^{1/e}$ = 0,6847309205...

(0,6847309205) * 19 = 13,00988; 13,00988 / 2 = 6,504943 that are values very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

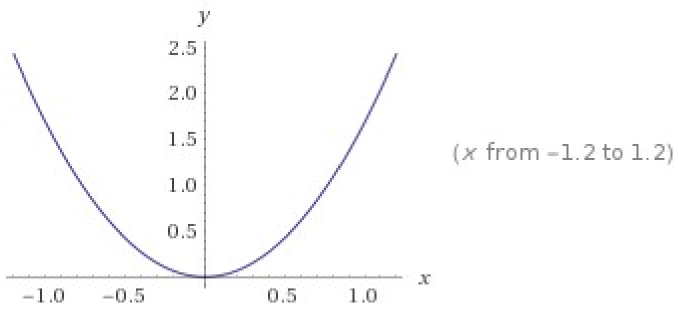
Pi * 3 integrate integrate [0.35718823192]

$$\pi \times 3 \int\left(\int 0.35718823192\,dx\right)dx$$

Result:

$1.6832098880\,x^2$

Plot:



We note that (1,6832098) * 8 = 13,4656784; (1.6832098) * 4 = 6,7328392 values very near to the reduced Planck's constant and to the mass of the SMBH87.

We have that:

Theorem 4.6. *We have*

(i) $\left[\left(13\sqrt{5}-25\right)^3+6^3\right]L_5^3-\left(13\sqrt{5}-25\right)^3R_5^2=0,$ (4.38)

(ii) $(5^3+4^3)\,L_7^3-5^3R_7^2=0,$ (4.39)

(iii) $(m_{13}^3+2^3)\,L_{13}^3-m_{13}^3R_{13}^2=0,$ (4.40)

where $m_{13}=-155+45\sqrt{13},$

(iv) $(45\,(a_1+b_1)+4^4)\,L_{63}^3-45\,(a_1+b_1)\,R_{63}^2=0,$ (4.41)

(v) $(m_{75}+8^{-3})\,L_{75}^3-m_{75}R_{75}^2=0,$ (4.42)

where $m_{75}=369830-165393\sqrt{5},$

(vi) $(8^3m_{147}+3^3)\,L_{147}^3-8^3m_{147}R_{147}=0,$ (4.43)

where $m_{147}=531745995375+116036489250\sqrt{21},$

(vii) $[2^7\,(a_2+b_2)+3^2]\,L_{171}^3-[2^7\,(a_2+b_2)]\,R_{171}^2=0,$ (4.44)

(viii) $\left[40^3\left(a_3+9\sqrt{2\,(b_3+c_3)}\right)+4\right]L_{363}^3-40^3\left(a_3+9\sqrt{2\,(b_3+c_3)}\right)R_{363}^2=0,$ (4.45)

(ix) $(m_{3/25}+8^{-3})\,L_{3/25}^3-m_{3/25}R_{3/25}^2=0,$ (4.46)

(x) $[40^3m_{3/49}+1]\,L_{3/49}^3-40^3m_{3/49}R_{3/49}^2.$ (4.47)

where $m_{3/25}=369830-165393\sqrt{5}$ *and* $m_{3/49}=\left(157554369-34381182\sqrt{21}\right).$

We have:

$$m_{147} = 531745995375 + 116036489250\sqrt{21},$$

We obtain: 1063491990740,054608; we note that: $(1063491990740{,}054608)^{1/55}$ = 1,6545042800359... and (1,6545042800359) * 8 = 13,2360342; (1,6545042800359)*4 = 6,61801712 that are values very near to the Planck's constant and to the mass of the SMBH87.

Now, we calculate the following double integral:

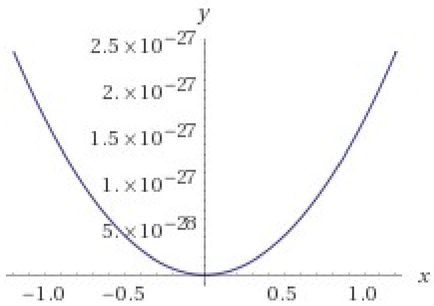
Pi * (1/(10)^39) * integrate integrate [1063491990740.054608]

$$\pi \times \frac{1}{10^{39}} \int\left(\int 1.063491990740054608 \times 10^{12}\, dx\right) dx$$

Result:

$$1.670529312630269987 \times 10^{-27}\, x^2$$

Plot:



(x from −1.2 to 1.2)

Result very near to the value of the proton mass

From:

MODULAR EQUATIONS FOR THE RATIOS OF RAMANUJAN'S THETA FUNCTION ψ AND EVALUATIONS

M. S. MAHADEVA NAIKA, S. CHANDANKUMAR AND K. SUSHAN BAIRY

(Received August 2010)

We have:

$$l_{9,8} = \frac{\sqrt{13 + 5\sqrt{6} + 7\sqrt{3} + 9\sqrt{2}}}{\sqrt{2}} + \frac{\sqrt{15 + 6\sqrt{6} + 9\sqrt{3} + 12\sqrt{2}}}{\sqrt{2}}, \qquad (4.6)$$

That is equal to

(5,004983837549 + 5,5792454002) = 10,5842292; $(10,5842292)^{1/5}$ = 1,60299381... and (1,60299381) * 8 = 12,82395; (1,60299381) * 4 = 6,41197... that are values very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

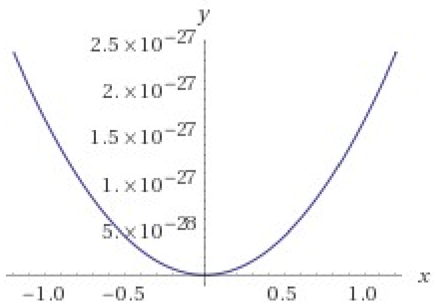
Pi * (1/(10)^28) * integrate integrate [10.5842292]

$$\pi \times \frac{1}{10^{28}} \int \left(\int 10.5842292 \, dx \right) dx$$

Result:

$1.66257 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

And (1,66257) * 8 = 13,30056; (1,66257) * 4 = 6,65028 values very near to the Planck's constant and to the mass of the SMBH87.

Now:

$$l'_{9,7} = \frac{3 + 2\sqrt{7} + 4\sqrt{3} + \sqrt{21} + \sqrt{9 + 2\sqrt{21}}\,(2 + 3\sqrt{3} - \sqrt{7})}{4}. \qquad (4.15)$$

That is equal to: 27,37560109; we note that $(27,37560109)^{1/7}$ = 1,604492407... and (1,604492407) * 8 = 12,835939; (1,604492407) * 4 = 6,41796... that are values very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

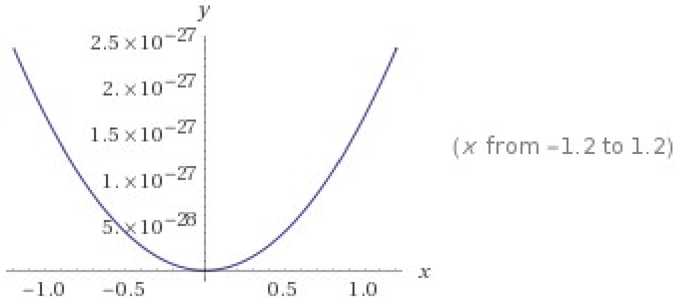
Pi^4/8 * (1/(10)^29) * integrate integrate [27.37560109]

$$\frac{\pi^4}{8} \times \frac{1}{10^{29}} \int \left(\int 27.37560109 \, dx \right) dx$$

Result:

$1.66665 \times 10^{-27} x^2$

Plot:



Result very near to the value of the proton mass.

Now:

$$l'_{9,28} = 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2+\sqrt{7})(2+\sqrt{3})\sqrt{9+2\sqrt{21}}. \quad (4.21)$$

18,52025917 + 19,052558 + 18,33030 + 18 + (4,64575131)(3,7320508)(4,2620595) = 147,79947... we note that $(147,79947)^{1/10}$ = 1,64803825... and (1,64803825) * 8 = 13,184306; (1,64803825) * 4 = 6,592153... that are values very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

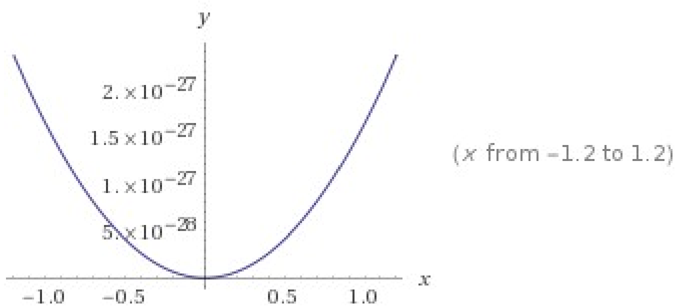
Pi^3/14 * (1/(10)^29) * integrate integrate [147.79947]

$$\frac{\pi^3}{14} \times \frac{1}{10^{29}} \int\left(\int 147.79947\, dx\right) dx$$

Result:

$1.63668 \times 10^{-27} x^2$

Plot:



Result that is very near to the value of the proton mass.

Chronology of the universe
From Wikipedia

Electroweak symmetry breaking[edit]

10^{-12} seconds after the Big Bang

As the universe's temperature continued to fall below a certain very high energy level, a third symmetry breaking occurs. So far as we currently know, it was the final symmetry breaking event in the formation of our universe. It is believed that below some energies unknown yet, the Higgs field spontaneously acquires a vacuum expectation value. When this happens, it breaks electroweak gauge symmetry. This has two related effects:

1. Via the Higgs mechanism, all elementary particles interacting with the Higgs field become massive, having been massless at higher energy levels.
2. As a side-effect, the weak force and electromagnetic force, and their respective bosons (the W and Z bosons and photon) now begin to manifest differently in the present universe. Before electroweak symmetry breaking these bosons were all massless particles and interacted over long distances, but at this point the W and Z bosons abruptly become massive particles only interacting over distances smaller than the size of an atom, while the photon remains massless and remains a long-distance interaction.

After electroweak symmetry breaking, the fundamental interactions we know of – gravitation, electromagnetism, the strong interaction and the weak interaction – have all taken their present forms, and fundamental particles have mass, but the temperature of the universe is still too high to allow the formation of many fundamental particles we now see in the universe.

The quark epoch

Between 10^{-12} seconds and 10^{-6} seconds after the Big Bang

The quark epoch began approximately 10^{-12} seconds after the Big Bang. This was the period in the evolution of the early universe immediately after electroweak symmetry breaking, when the fundamental interactions of gravitation, electromagnetism, the strong interaction and the weak interaction had taken their present forms, but the temperature of the universe was still too high to allow quarks to bind together to form hadrons.

During the quark epoch the universe was filled with a dense, hot quark–gluon plasma, containing quarks, leptons and their antiparticles. Collisions between particles were too energetic to allow quarks to combine into mesons or baryons.

The quark epoch ended when the universe was about 10^{-6} seconds old, when the average energy of particle interactions had fallen below the binding energy of hadrons.

W and Z bosons

From Wikipedia, the free encyclopedia

The **W and Z bosons** are together known as the **weak** or more generally as the **intermediate vector bosons.** These elementary particles mediate the weak interaction; the respective symbols are W^+, W^-, and Z. The W bosons have either a positive or negative electric charge of 1 elementary charge and are each other's antiparticles. The Z boson is electrically neutral and is its own antiparticle. The three particles have a spin of 1. The W bosons have a magnetic moment, but the Z has none. All three of these particles are very short-lived, with a half-life of about 3×10^{-25} s. Their experimental discovery was a triumph for what is now known as the Standard Model of particle physics.

These bosons are among the heavyweights of the elementary particles. With masses of 80.4 GeV/c^2 and 91.2 GeV/c^2, respectively, the W and Z bosons are almost 80 times as massive as the proton:

W: 80.379±0.012 GeV/c^2 Z: 91.1876±0.0021 GeV/c^2

We note that: 80,379 / 48 = 1,6745625; 80,379 / 50 = 1,60758 and 91,1876 / 55 = 1,6579563; 91.1876 / 56 = 1,62835 values very near to the mass of the proton and the electric charge of the electron.

β^- decay (electron emission)

An unstable atomic nucleus with an excess of neutrons may undergo β^- decay, where a neutron is converted into a proton, an electron, and an electron antineutrino (the antiparticle of the neutrino).

This process is mediated by the weak interaction. The neutron turns into a proton through the emission of a virtual W^- boson. At the quark level, W^- emission turns a down quark into an up quark, turning a neutron (one up quark and two down quarks) into a proton (two up quarks and one down quark). The virtual W^- boson then decays into an electron and an antineutrino.

β^+ decay (positron emission)

Unstable atomic nuclei with an excess of protons may undergo β^+ decay, also called positron decay, where a proton is converted into a neutron, a positron, and an electron neutrino.

Beta Decay

β-decay, radioactive decay of an atomic nucleus accompanied by the escape of an electron or positron from the nucleus. This process is caused by a spontaneous transformation of one of the nucleons in the nucleus into a nucleon of another type—specifically, a transformation either of a neutron (n) into a proton (p) or of a proton into a neutron. In the former case, with an electron (e^+) escaping from the nucleus, so-called β^-decay takes place. In the latter case, with a positron (e^+) escaping from the nucleus, β^+ -decay takes place. The electrons and positrons emitted in beta decay are termed beta particles. The mutual transformations of the nucleons are accompanied by the appearance of still another particle—the neutrino *(v)* in the case of β^+ -decay, the antineutrino *(v)* in the case of β^- -decay. In β^- -decay, the number of protons (Z) in the nucleus increases by a unit and the number of neutrons decreases by a unit. The mass number *A* of the nucleus—equal to the total number of nucleons present in the nucleus—does not vary, and the product nucleus is an isobar of the original nucleus, standing on the right of the latter in the periodic table of the elements. Conversely, the number of protons in β^+ -decay decreases by a unit and the number of neutrons increases by a unit, so that an isobar standing to the left of the original nucleus is formed. The two beta decay processes are written symbolically as

$${}^{A}_{Z}X \rightarrow {}_{Z+1}{}^{A}X + e^- + \bar{\nu}$$

$${}^{A}_{Z}X \rightarrow {}_{Z-1}{}^{A}X + e^+ + \nu$$

where ${}^{A}_{Z}X$is the symbol of the nucleus, consisting of Z protons and *A* - *Z* neutrons.

The simplest example of β^- -decay is the transformation of a free neutron into a proton with the emission of an electron and an antineutrino (neutron half-life ≈ 13 min)

$${}^{1}_{0}n \xrightarrow{\beta^-} {}^{1}_{1}p + e^- + \bar{\nu}$$

Higgs Boson

The Higgs boson is an elementary, massive and scalar boson that plays a fundamental role within the standard model. It was theorized in 1964 and detected for the first time in 2012 in the ATLAS and CMS experiments, conducted with the LHC accelerator of CERN. Its importance is to be the particle associated with the Higgs field, which according to the theory permeates the universe by giving the mass to elementary particles.

Since the Higgs field is scalar, the Higgs boson has no spin. The Higgs boson is also its own antiparticle and is CP-even, and has zero electric and colour charge.

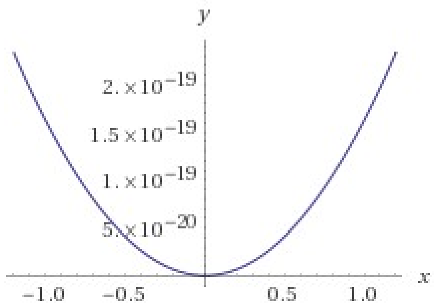
The Standard Model does not predict the mass of the Higgs boson. If that mass is between 115 and 180 GeV/c^2 (consistent with empirical observations of 125 GeV/c^2), then the Standard Model can be valid at energy scales all the way up to the Planck scale (10^{19} GeV).

We note that 1,602176 * 78 = 124,969728 and 1,672621 * 74 = 123,773954 where 1,602176 and 1,672621 are the electric charge of the positron and the mass of the proton respectively.

Furthermore, we have that:

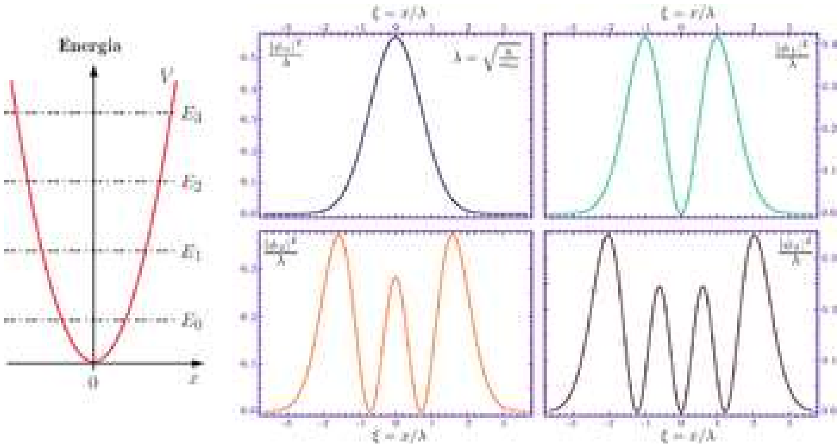
$125{,}09 * 9 * 10^{16} = 11258100000000000000$; $\quad (11258100000000000000)^{1/93} =$
$= 1{,}6027082167167$

Now, we want to analyze the parabola plots concerning the results of the various double integrals. Indeed, for all values i.e. for the electric charge of positron, the mass of the proton and the mass of the Higgs boson, the plot is always a parabola of this type:



(x from -1.2 to 1.2)

With regard the quantum harmonic oscillator, we note the following graphs:



which represent the potential energy and probability density associated with the ground state and the first excited states of the harmonic oscillator

From Wikipedia:

Quantum harmonic oscillator

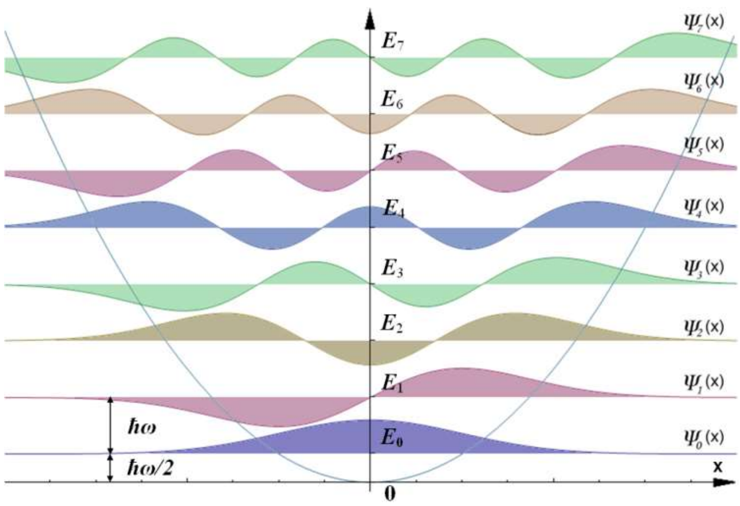


Fig.: Wavefunction representations for the first eight bound eigenstates, $n = 0$ to 7. The horizontal axis shows the position x. Note: The graphs are not normalized, and the signs of some of the functions differ from those given in the text. (Wave function representations for the first eight linked eigenstates).

This suggests that the graphs representing the parables associated with the particle type solutions of the integrals carried out, could mean that electrons / positrons, protons / neutrons and massive bosons, such as the Higgs one, are open strings. It is important to underline that from the graphs mentioned above, it is highlighted that these strings are in a sort of "ground state", therefore they are "static" strings. Subsequently, due to the quantum fluctuations of the false vacuum, these strings pass

from an inert state to a dynamic state, in which they begin to vibrate and behave like waves, as happens for the quantum harmonic oscillator.
So, this could mean that, the static parabola represented in the graphs, is the corpuscular nature of the electron, the proton and the Higgs boson, while the graph, always of the parabolic type, of the harmonic oscillator, their wave nature (dualism wave-particle)

One-dimensional harmonic oscillator

Hamiltonian and energy eigenstates

The Hamiltonian of the particle is:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 ,$$

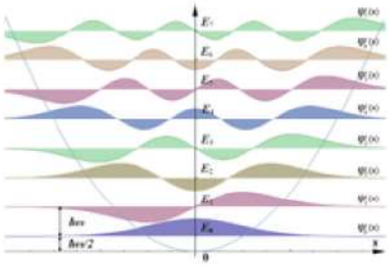
where m is the particle's mass, k is the force constant, $\omega = \sqrt{\frac{k}{m}}$ is the angular frequency of the oscillator, $\hat{x}$ is the position operator (given by x), and $\hat{p}$ is the momentum operator (given by $\hat{p} = -i\hbar\frac{\partial}{\partial x}$). The first term in the Hamiltonian represents the kinetic energy of the particle, and the second term represents its potential energy, as in Hooke's law.

One may write the time independent Schrödinger equation

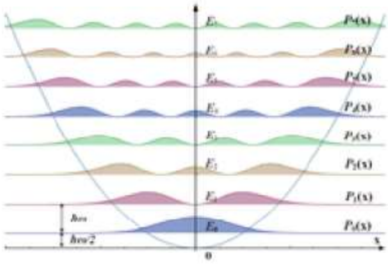
$$\hat{H}\,|\psi\rangle = E|\psi\rangle \;,$$

where E denotes a to-be-determined real number that will specify a time-independent energy level, or eigenvalue, and the solution $|\psi\rangle$ denotes that level's energy eigenstate.

One may solve the differential equation representing this eigenvalue problem in the coordinate basis, for the wave function $\langle x|\psi\rangle = \psi(x)$, using a spectral method. It turns out that there is a family of solutions. In this basis, they amount to Hermite functions,



Wavefunction representations for the first eight bound eigenstates, $n = 0$ to 7. The horizontal axis shows the position x. Note: The graphs are not normalized, and the signs of some of the functions difer from those given in the text.



Corresponding probability densities.

$$\psi_n(x) = \frac{1}{\sqrt{2^n\,n!}} \cdot \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot e^{-\frac{m\omega x^2}{2\hbar}} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right), \qquad n = 0,1,2,\ldots.$$

The functions H_n are the physicists' Hermite polynomials

$$H_n(z) = (-1)^n\;e^{z^2}\frac{d^n}{dz^n}\left(e^{-z^2}\right).$$

The corresponding enegy levels are

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) = (2n+1)\frac{\hbar}{2}\omega\,.$$

This energy spectrum is noteworthy for three reasons. First, the energies are quantized, meaning that only discrete energy values (integer-plus-half multiples of $\hbar\omega$) are possible; this is a general feature of quantum-mechanical systems when a particle is confined. Second, these discrete energy levels are equally spaced, unlike in the Bohr model of the atom, or the particle in a box. Third, the lowest achievable energy (the energy of the $n = 0$ state, called the ground state) is not equal to the minimum of the potential well, but $\hbar\omega/2$ above it; this is called zero-point energy. Because of the zero-point energy, the position and momentum of the oscillator in the ground state are not fixed (as they would be in a classical oscillator), but have a small range of variance, in accordance with the Heisenberg uncertainty principle

The ground state probability density is concentrated at the origin, which means the particle spends most of its time at the bottom of the potential well, as one would expect for a state with little energy. As the energy increases, the probability density peaks at the classical "turning points", where the state's enegy coincides with the potential enegy. (See the discussion below of the highly excited states.) This is consistent with the classical harmonic oscillator, in which the particle spends more of its time (and is therefore more likely to be found) near the turning points, where it is moving the slowest. The correspondence principleis thus satisfied. Moreover, special nondispersive wave packets, with minimum uncertainty, called coherent states oscillate very much like classical objects, as illustrated in the figure; they are*not* eigenstates of the Hamiltonian.

From:
http://www.umich.edu/~chem461/Ex5.pdf

1. For a classical harmonic oscillator, the particle can not go beyond the points where the total energy equals the potential energy. Identify these points for a quantum-mechanical harmonic oscillator in its ground state. Write an integral giving the probability that the particle will go beyond these classically-allowed points. (You need not evaluate the integral.)

2. Evaluate the average (expectation) values of potential energy and kinetic energy for the ground state of the harmonic oscillator. Comment on the relative magnitude of these two quantities.

3. Apply the Heisenberg uncertainty principle to the ground state of the harmonic oscillator. Applying the formula for expectation values, calculate

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \qquad \text{and} \qquad \Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

and find the product $\Delta x \Delta p$.

1. The turning points for quantum number occur where the kinetic energy equals 0, so that the potential energy equals the total energy. For quantum number n, this is determined by

$$\frac{1}{2}kx_{\max}^2 = \left(n + \frac{1}{2}\right)\hbar\omega$$

recalling that $\omega = \sqrt{k/m}$ and $\alpha = \sqrt{mk}/\hbar$, we find

$$x_{\max}^2 = (2n+1)\frac{\hbar}{\sqrt{km}} = \frac{(2n+1)}{\alpha}$$

Therefore

$$P(x_{\max} \le x \le \infty) = P(-\infty \le x \le -x_{\max}) = \int_{x_{\max}}^{\infty} |\psi_n(x)|^2 \, dx$$

[Optional: For $n = 0$,

$$P_{\text{outside}} = 2\int_{1/\sqrt{\alpha}}^{\infty} \left(\frac{\alpha}{\pi}\right)^{1/2} e^{-\alpha x^2} \, dx = \frac{2}{\sqrt{\pi}}\int_{1}^{\infty} e^{-\xi^2} \, d\xi = \text{erfc}(1) \approx 0.158$$

where erfc is the complementary error function. This result means that in the ground state, there is a 16% chance that the oscillator will "tunnel" outside its classical allowed region.]

2.

$$\psi_0(x) = (\alpha/\pi)^{1/4} e^{-\alpha x^2/2}, \qquad \alpha = (mk/\hbar^2)^{1/2}$$

Using integrals in Supplement 5,

$$\langle V \rangle = \int_{-\infty}^{\infty} \psi_0(x) \left(\frac{1}{2}kx^2\right) \psi_0(x) \, dx = \frac{k}{4\alpha} = \frac{1}{4}\hbar\omega = \frac{1}{2}E_0$$

$$\langle T \rangle = \int_{-\infty}^{\infty} \psi_0(x) \left(-\frac{\hbar^2}{2m}\right) \psi_0''(x)\, dx = \frac{1}{2}E_0$$

Thus the average values of potential and kinetic energies for the harmonic oscillator are equal. This is an instance of the virial theorem, which states that for a potential energy of the form $V(x) = \text{const}\, x^n$, the average kinetic and potential energies are related by

$$\langle T \rangle = \frac{n}{2}\langle V \rangle$$

3. The expectation values $\langle x \rangle$ and $\langle p \rangle$ are both equal to zero since they are integrals of odd functions, such that $f(-x) = -f(x)$, over a symmetric range of integration. You have already calculated the expectation values $\langle x^2 \rangle$ and $\langle p^2 \rangle$ in Exercise 2, namely

$$\langle x^2 \rangle = \frac{1}{2\alpha} \qquad \text{and} \qquad \langle p^2 \rangle = \frac{\hbar^2 \alpha}{2}$$

Therefore

$$\Delta x \Delta p = \frac{\hbar}{2}$$

which is its minimum possible value.

We know that the reduced Planck's constant is:

$\hbar = 1{,}054\,571\,726(47) \times 10^{-34}\ \mathbf{J \cdot s} = 6{,}582\,119\,28(15) \times 10^{-16}\ \mathbf{eV \cdot s}$

Thence, we have that the minimum possible value is:

$$\Delta x \Delta p = \frac{\hbar}{2} = \frac{6{,}582119 \times 10^{-16}}{2} = 3{,}2910595 \times 10^{-16}$$

Now, we calculate the following double integrals:

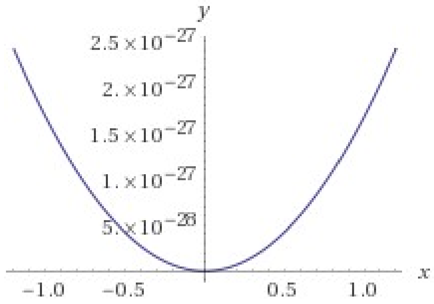
(2*1.618*Pi) * 1/(10^12) integrate integrate [0.00000000000000032910595]

$$(2 \times 1.618\, \pi) \times \frac{1}{10^{12}} \int \left(\int 3.2910595 \times 10^{-16}\, dx \right) dx$$

Result:

$1.67288 \times 10^{-27}\, x^2$

Plot:



(x from -1.2 to 1.2)

Or:

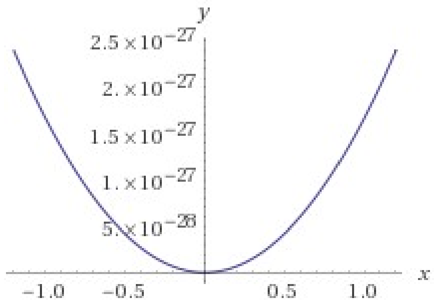
1.08643 * (8Pi/27) * 1/(10^11) integrate integrate [0.00000000000000032910595]

$$1.08643\left(8\times\frac{\pi}{27}\right)\times\frac{1}{10^{11}}\int\left(\int 3.2910595\times10^{-16}\,dx\right)dx$$

Result:

$1.66412\times10^{-27}\,x^2$

Plot:



(x from -1.2 to 1.2)

Or:

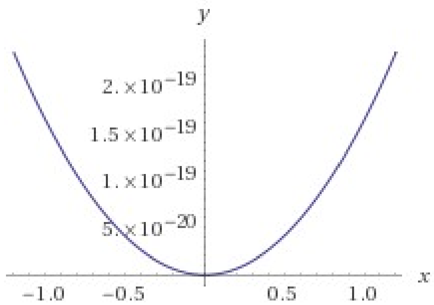
(Pi^2) * 1/(10^4) integrate integrate [0.00000000000000032910595]

$$\pi^2\times\frac{1}{10^4}\int\left(\int 3.2910595\times10^{-16}\,dx\right)dx$$

Result:

$1.62407\times10^{-19}\,x^2$

Plot:



(x from -1.2 to 1.2)

And

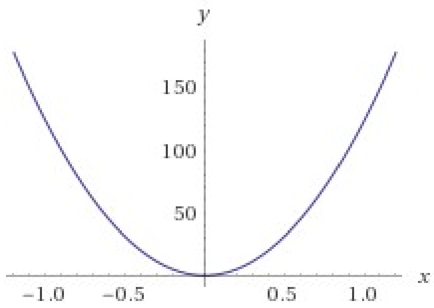
1.08643 (7*Pi^2) (10^16) integrate integrate [0.00000000000000032910595]

$$1.08643\,(7\pi^2)\times 10^{16}\int\left(\int 3.2910595\times 10^{-16}\,dx\right)dx$$

Result:

$$123.511\,x^2$$

Plot:



(x from −1.2 to 1.2)

Value very near to the mass of the Higgs boson, while the energy from the $E = mc^2$, considering the value 125,09 is $11{,}2581 * 10^{18}$. Note that $(11{,}2581 * 10^{18})^{1/89} =$ 1,63704797... value very near to the mass of the proton.

We now calculate the following double integral:

1.08643 (Pi/12) * 1/(10^37) * integrate integrate [11.2581*10^18]

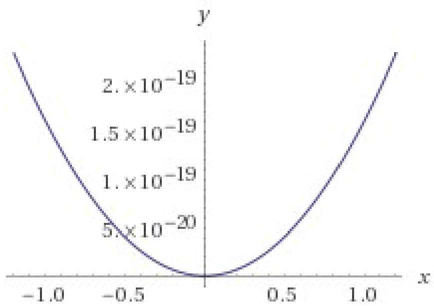
$$1.08643\times\frac{\pi}{12}\times\frac{1}{10^{37}}\int\left(\int 11.2581\times 10^{18}\,dx\right)dx$$

Result:

$$1.60105\times 10^{-19}\,x^2$$

$1.60105 * 10^{-19}$ result very near to the value of the electric charge of positron

Plot:



(x from −1.2 to 1.2)

Open code

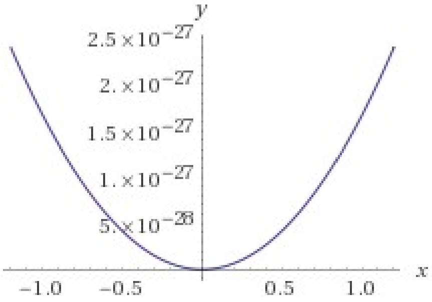
1.08643 (Pi^5/(3*37)) * 1/(10^46) * integrate integrate [11115990000000000000]

That is the value of the energy obtained from 123,511

$$1.08643 \times \frac{\pi^5}{3 \times 37} \times \frac{1}{10^{46}} \int \left(\int 11\,115\,990\,000\,000\,000\,000\, dx \right) dx$$

Result:

$$1.66474 \times 10^{-27} x^2$$



(x from -1.2 to 1.2)

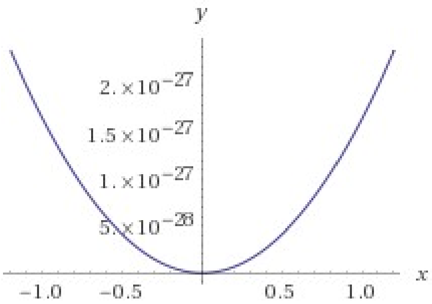
Or:

1.08643 * (e) * 1/(10^46) * integrate integrate [11115990000000000000]

$$1.08643\, e \times \frac{1}{10^{46}} \int \left(\int 11\,115\,990\,000\,000\,000\,000\, dx \right) dx$$

Result:

$$1.6414 \times 10^{-27} x^2$$



(x from -1.2 to 1.2)

Results 1.66474 * 10^{-27} and 1.6414 * 10^{-27} that are good approximations of the proton mass

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