

Chen's Formulas of the Fine-structure Constant (viXra:2002.0203v9)

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper gives two series of formulas of the fine-structure constant α which are reasonable, precise, smart and elegant. It also demonstrates there are two values of α , i.e., $\alpha_1=1/137.035999037435$ and $\alpha_2=1/137.035999111818$, which are consistent with but much more accurate than those experiment measured values. The formulas consist of 2π -e formulas and some factors related to nucleon numbers of nuclides. A brief explanation of the fine-structure constant shows $1/\alpha \approx 137.036$ is the equal ratio factor between 112 and 168 (more precisely $168-1/3$). Based on these, all 119th to 170th ideal extended elements were predicted, the speed of light in atomic units was mathematically calculated by $c_{au}=1/(\alpha_1\alpha_2)^{1/2}=137.035999074627$, Schrödinger equation of hydrogen atom was simplified and correlated with α_1/α_2 , classical electron radius was calculated to be 2.81794032658(43) fm and proton charge radius was hypothetically calculated to be 0.833027202999(13) fm. In the end, it was found that the approximate rational numbers of 2π marvelously related to nuclides, a mathematic shell model of nuclides was established and a picture of elements and ideal extended elements was depicted.

Keywords: formulas; the fine-structure constant; the ideal extended elements; the speed of light; Schrödinger equation of hydrogen atom; the proton charge radius; 2π .

1. Introduction

The fine-structure constant (Sommerfeld's constant) is a critical dimensionless constant in physics, it is a century mystery of physics, it has been one of the biggest enigmas in physics since it was introduced by Arnold Sommerfeld in 1916. Its definition, some interpretations and the latest measured values are as follows^{1,2}:

$$\alpha = \frac{\lambda_e}{2\pi a_0}, \quad \alpha = \frac{2\pi r_e}{\lambda_e}, \quad \frac{a_0}{r_e} = \frac{1}{\alpha^2}; \quad \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{v_e}{c}, \quad \frac{c}{v_e} = \frac{1}{\alpha}$$

in atomic units, the speed of light $c_{au} = \frac{1}{\alpha}$

the 2014 CODATA recommended value: $\alpha = 1/137.035999139(31)$

the 2018 CODATA recommended value: $\alpha = 1/137.035999084(21)$

Science 13 April 2018 reported value: $\alpha = 1/137.035999046(27)$

The ratio of Bohr radius of hydrogen atom a_0 to the classical electron radius r_e is $1/\alpha^2$. The ratio of the speed of light c to the line velocity of ground state electron in hydrogen atom v_e is $1/\alpha$, this means in atomic units $c=1/\alpha$ and $E=mc^2=m/\alpha^2$ or $\alpha^2=m/E$. In quantum electrodynamics it substantially characterizes the strength of electromagnetic interaction between elementary charged particles such as electron and proton, so it is the coupling constant of electric charges. It is one of the 25 fundamental constants (could not be calculated theoretically, could only be determined by experiments) in Standard Model of physics and should be the most important one. As it is dimensionless, it could be called the proportional ruler of the nature or the bridge of mathematics and physics. However, to our knowledge, up to now (except this work), no one knows how it comes from, no one could give reasonable explanations to it or formulas of it since it was introduced.

In 2016 Paul Davis gave the following comment³: “Physicists have long wondered where this number, $1/137.035999$, comes from. Is there a deep reason why α has to be precisely this number for the world to function as it does? There is a long history of attempts to derive α from physical theory or to concoct a mathematical formula that has this value. For a brief time in the 1920s, when it looked as if α might be exactly $1/137$, astronomer Arthur Eddington searched for a theory that would throw up the numbers naturally, but his ideas ultimately led nowhere. Then in 1969 a young Swiss mathematician, Armand Wyler, pointed out that $(9/16\pi^3)(\pi/5!)^{1/4}$ comes close to $1/137.036$, which matched the value of α to the precision known at the time. However, his formula was not accompanied by any credible theory and was regarded as little more than a numerical curiosity. Several other attempts at α numerology have been made since, none of which have gained traction in the physics community.”

As for the fascination of the fine-structure constant, in the middle of 1980s, Richard Feynman stated⁴: “It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the hand of God wrote that number, and we don't know how He pushed his pencil.”

This paper shows how God pushed his pencil to write the fine-structure constant and how God used it to coordinate elements.

2. 2π -e formula(s)

2π -e formula, its related formulas and their preliminary applications were deduced independently by us from April to December of 2013.

Fig. 1. Diagram of $y=1/x$.

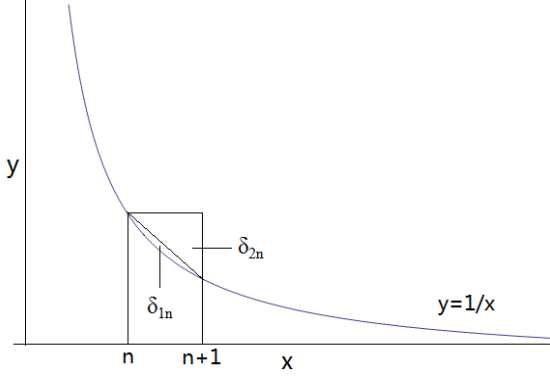
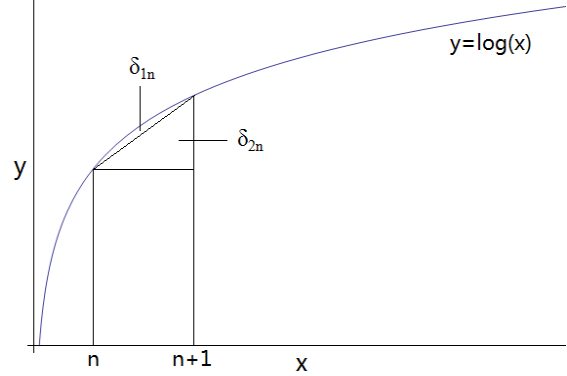


Fig. 2. Diagram of $y=\log(x)$.



$$\text{Euler-Mascheroni constant } \gamma : \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{\infty} = \ln \infty + \gamma$$

$$\text{As for } y = 1/x \text{ (Fig. 1), } \gamma = 0.577215\dots = 0.5 + 0.077215\dots = \sum_{n=1}^{\infty} \delta_{2n} + \sum_{n=1}^{\infty} \delta_{1n} = \frac{1}{2} + \gamma_1$$

$$\gamma_1 = \sum_{n=1}^{\infty} \delta_{1n} = \lim_{N \rightarrow \infty} \left(\sum_{n=1}^{N-1} \frac{1}{n} - \int_1^N \frac{1}{x} dx \right) - \frac{1}{2}, \text{ Generally } \gamma_s = \lim_{N \rightarrow \infty} \left(\sum_{n=1}^{N-1} \frac{1}{n^s} - \int_1^N \frac{1}{x^s} dx \right) - \frac{1}{2}, \quad s \in \mathbb{N}$$

$$\begin{aligned} \text{As for } y = \log(x) \text{ (Fig. 2), } \delta_{1,n} &= \int_n^{n+1} \ln x dx - \frac{1}{2} \ln \frac{n+1}{n} - \ln n = (x \ln x - x) \Big|_n^{n+1} - \frac{1}{2} \ln(n+1)n \\ &= (n+1) \ln(n+1) - n \ln n - 1 - \frac{1}{2} \ln(n+1) - \frac{1}{2} \ln n = \left(n + \frac{1}{2}\right) \ln\left(1 + \frac{1}{n}\right) - 1 \end{aligned}$$

$$\gamma_{c,N} = \sum_{n=1}^N \delta_{1,n} = \sum_{n=1}^N \left[\left(n + \frac{1}{2}\right) \ln\left(1 + \frac{1}{n}\right) - 1 \right] = \sum_{n=1}^N \ln \frac{\left(1 + \frac{1}{n}\right)^{\left(n + \frac{1}{2}\right)}}{e} = \ln \prod_{n=1}^N \frac{\left(1 + \frac{1}{n}\right)^{\left(n + \frac{1}{2}\right)}}{e}$$

$$\gamma_c = \gamma_{c,\infty} = \sum_{n=1}^{\infty} \delta_{1,n} = \lim_{N \rightarrow \infty} \left(\int_1^{N+1} \log(x) dx - \sum_{n=1}^N \log(n) - \frac{\log(N+1)}{2} \right)$$

$$= \sum_{n=1}^{\infty} \left[\left(n + \frac{1}{2}\right) \ln\left(1 + \frac{1}{n}\right) - 1 \right] = \sum_{n=1}^{\infty} \ln \frac{\left(1 + \frac{1}{n}\right)^{\left(n + \frac{1}{2}\right)}}{e} = \ln \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^{\left(n + \frac{1}{2}\right)}}{e}$$

$$\ln N! = \sum_{n=1}^N \ln n = \int_1^{N+1} \ln x dx - \sum_{n=1}^N \delta_{1,n} - \sum_{n=1}^N \delta_{2,n} = (x \ln x - x) \Big|_1^{N+1} - \ln e^{\gamma_{c,N}} - \sum_{n=1}^N \frac{\ln(n+1) - \ln n}{2}$$

$$= (N+1) \ln \frac{(N+1)}{e} + \ln \frac{e}{e^{\gamma_{c,N}}} - \frac{1}{2} \ln(N+1) = \ln \left[\frac{e^{1-\gamma_{c,N}}}{\sqrt{N+1}} \left(\frac{N+1}{e} \right)^{(N+1)} \right]$$

$$N! = \frac{e^{1-\gamma_{c,N}}}{\sqrt{N+1}} \left(\frac{N+1}{e} \right)^{(N+1)}, \text{ compared to Stirling formula : } N! \sim \sqrt{2\pi N} \left(\frac{N}{e} \right)^N$$

$$(N+1)! = (N+1)N! \sim \sqrt{2\pi(N+1)} \left(\frac{N+1}{e} \right)^{N+1}, \quad N! \sim \frac{\sqrt{2\pi}}{\sqrt{N+1}} \left(\frac{N+1}{e} \right)^{N+1}$$

$$\text{Compared to previous formula, gives } \sqrt{2\pi} \sim e^{1-\gamma_{c,N}} \text{ or } 2\pi = \left(\frac{e}{e^{\gamma_c}} \right)^2$$

$$2\pi - e \text{ formula(s): } 2\pi = \left(\frac{e}{e^{\gamma_c}} \right)^2 = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots, \quad (2\pi)_k = \left(\frac{e}{e^{\gamma_{c,k}}} \right)^2 = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}$$

$$\gamma_c = 0.0810614668, \quad e^{\gamma_c} = 1.0844375$$

2π -e formula is an expanding form of Stirling formula. To our knowledge, it was first deduced by us. If it was new, it could be named Chen's 2π -e formula.

3. Some Formulas Related to 2π -e Formula

The following formulas which correlate each other and has similar form could be called Chen's natural group formulas, and the form is called natural group.

$$\begin{aligned}
1 &= 4\gamma_c + \frac{4\gamma_1}{1(1+1)} + \frac{4\gamma_2}{2(2+1)} + \frac{4\gamma_3}{3(3+1)} + \dots \\
&= |B| \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{|B_{2n}|(\pi/2)^{2n}}{(2n)!} = \sum_{n=1}^{\infty} \frac{|B_{2n}|\pi^{2n}}{(2n)!} = -|B| \frac{3\pi}{2} + \sum_{n=1}^{\infty} \frac{|B_{2n}|(3\pi/2)^{2n}}{(2n)!} \\
N &\sim -\frac{3}{2}|B| + \sum_{n=1}^N \frac{|B_{2n}|(2\pi)^{2n}}{2(2n)!} \\
e &= 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \\
2\pi &= \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots
\end{aligned}$$

B, B_{2n} : the Bernoulli numbers such as $-\frac{1}{2}, -\frac{1}{6}, -\frac{1}{30}, \frac{1}{42}, -\frac{1}{30}, \dots$

$$\gamma_c = \lim_{N \rightarrow \infty} \left(\int_1^{N+1} \log(x) dx - \sum_{n=1}^N \log(n) - \frac{\log(N+1)}{2} \right) = 0.0810614668$$

$$\gamma_s = \lim_{N \rightarrow \infty} \left(\sum_{n=1}^{N-1} \frac{1}{n^s} - \int_1^N \frac{1}{x^s} dx \right) - \frac{1}{2}, \quad s \in \mathbb{N}$$

$\gamma_1 = 0.077215, \gamma_2 = 0.144934, \gamma_4 = 0.24899, \gamma_8 = 0.36122, \gamma_{16} = 0.433349, \dots, \gamma_{\infty} = 0.5$

$\gamma_c, \gamma_1, \gamma_2, \gamma_3, \dots$ are called Chen's natural group constants (analogue to Bernoulli numbers).

The following are some other formulas related to 2π -e Formula.

$$\begin{aligned}
\sqrt{2\pi} &= e^{1-\gamma_c}, \quad e = \sqrt{2\pi} e^{\gamma_c} = \sqrt{2\pi} \left(1 + \sum_{n=1}^{\infty} \frac{\gamma_c^n}{n!}\right) \\
\gamma_c &= \sum_{n=1}^{\infty} \left[\left(n + \frac{1}{2}\right) \ln\left(1 + \frac{1}{n}\right) - 1 \right] = \sum_{n=1}^{\infty} \frac{(2^{2n}-1)|B_{2n}|\pi^{2n} - 2(2n)!}{2(2n+1)!} = \frac{1}{4} - \sum_{s=1}^{\infty} \frac{\gamma_s}{s(s+1)} \\
\gamma_g &= \sum_{n=1}^{\infty} \left(n + \frac{1}{2}\right) \ln\left(1 + \frac{1}{n}\right) - \int_1^{\infty} \left(x + \frac{1}{2}\right) \ln\left(1 + \frac{1}{x}\right) dx \\
\gamma_{cg} &= \frac{1}{2} \lim_{N \rightarrow \infty} \left[\sum_{n=1}^N \frac{(2^{2n}-1)|B_{2n}|\pi^{2n}}{(2n+1)!} - \ln N \right] \\
\frac{\pi}{2} &= \left(\frac{e}{e^{\gamma_g}}\right)^2, \quad e = \sqrt{\frac{\pi}{2}} e^{\gamma_g} = \sqrt{\frac{\pi}{2}} \left(1 + \sum_{n=1}^{\infty} \frac{\gamma_g^n}{n!}\right); \quad \frac{\pi}{2} = \left(\frac{e^{\gamma/2}}{e^{\gamma_{cg}}}\right)^2, \quad \gamma = \ln \frac{\pi}{2} + 2\gamma_{cg} \\
\gamma_c &= \gamma_g - \ln 2 = 1 - \frac{\gamma}{2} + \gamma_{cg} - \ln 2, \quad \gamma_{cg} = \frac{1}{2} + \sum_{s=2}^{\infty} \frac{\gamma_s}{s(s+1)} - \ln 2 \\
\gamma_c &= 0.0810614668, \quad \gamma_g = 0.7742086474, \quad \gamma_{cg} = 0.0628164798 \\
\frac{\pi}{2} &= \sum_{n=1}^{\infty} \frac{|B_{2n}|\pi^{2n}}{2n(2n)!}; \quad \sum_{n=1}^{\infty} [\zeta(2n) - 1] = \frac{3}{4}, \quad \zeta(2n) = \sum_{k=1}^{\infty} \frac{1}{k^{2n}} \\
\sum_{n=1}^{\infty} \frac{1}{n} &= \sum_{n=1}^{\infty/2} \frac{|B_{2n}|(2\pi)^{2n}}{2n(2n)!} = \sum_{n=1}^{\infty} \frac{|B_{2n}|(2n2^{2n}+1)\pi^{2n}}{2n(2n+1)!}
\end{aligned}$$

4. Some Applications of 2π -e Formula and its Related Formulas

(1). 2π -e formula is basically an algebraic expanding of Stirling formula, but it is more meaningful, it exhibits the relationship between 2π and e. In 2π -e formula, γ_c is a real constant with geometric definition like Euler-Mascheroni constant γ . With 2π -e formula and its related formulas, 2π can be calculated from e and vice versa. So it is the real 2π -e relationship formula.

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \prod_{n=1}^{\infty} \frac{e^2}{\left(1 + \frac{1}{n}\right)^{2n+1}} = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots$$

$$e = \sqrt{2\pi} e^{\gamma_c} = \sqrt{2\pi} \left(1 + \sum_{n=1}^{\infty} \frac{\gamma_c^n}{n!}\right), \quad \gamma_c = \sum_{n=1}^{\infty} \frac{(2^{2n}-1)|B_{2n}|\pi^{2n} - 2(2n)!}{2(2n+1)!}$$

(2). 2π -e formula demonstrates 2π is a natural constant rather than π . $\pi/2$ is somewhat fundamental but not as complete as 2π . π is neither fundamental nor complete. In 2001 mathematician Bob Palais said “ π is wrong”⁵. 2π -e formula and the Taylor expansion of e have similar form (natural group form), this should give a conclusive proof that 2π is a real natural constant and π is not.

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \Rightarrow 2\pi \text{ or } \sqrt{2\pi} \text{ is a natural constant}$$

$$\frac{\pi}{2} = \left(\frac{e}{e^{\gamma_g}}\right)^2 = \left(\frac{e^{\gamma/2}}{e^{\gamma_{cg}}}\right)^2 \Rightarrow \frac{\pi}{2} \text{ or } \sqrt{\frac{\pi}{2}} \text{ is almost a natural constant}$$

$$\pi = \left(\frac{e}{e^{\gamma_c} \sqrt{2}}\right)^2 = \left(\frac{e\sqrt{2}}{e^{\gamma_g}}\right)^2 \Rightarrow \pi \text{ or } \sqrt{\pi} \text{ is not a natural constant}$$

Table 1 lists some points of view of Piist who support π is a natural constant, Tauist who support 2π is a natural constant and this work which supports the later.

Table 1. Comparison of points of view of Piist, Tauist and this work.

	Piist	Tauist	This work
Circumference of a circle	πd	$2\pi R$	$2\pi R$
Area of a circle	πR^2		$(1/2)(2\pi R)R$
Volume of sphere	$(4/3)\pi R^3$	$(2/3)(2\pi)R^3$	$(2\pi R^2/3)2R$
Volume of n-dimension sphere	$\frac{\pi^{n/2}}{\Gamma(n/2+1)}R^n$	$\frac{(2\pi)^{n/2}}{2^{n/2}\Gamma(n/2+1)}R^n$	$\frac{2\pi R^2}{n}V_{n-2}$
Euler's identity	$e^{i\pi}+1=0$	$e^{2\pi i}=1$	$e^{2\pi i}=1$
Gauss integral	$\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$		$\int_{-\infty}^{+\infty} e^{-x^2} dx = \frac{e}{e^{\gamma_c}} \frac{1}{\sqrt{2}}$

(3). As 2π is a square number, the frequent appearing of its square root in some

important equations such as Gaussian distribution (normal distribution) and Maxwell-Boltzmann distribution becomes reasonable and understandable. And the distributions can be transformed as follows.

$$\text{Standard Normal Distribution: } f(x, 0, 1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = e^{-\frac{x^2+2(1-\gamma_e)}{2}}$$

$$\text{Maxwell-Boltzmann Distribution: } f(v) = \frac{2}{\sqrt{2\pi}} v^2 \left(\frac{m}{kT}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}} = 2\left(\frac{m}{kT}\right)^{\frac{3}{2}} v^2 e^{-\left(\frac{m}{kT} \frac{v^2}{2} + 1 - \gamma_e\right)}$$

(4). Euler's identity (Euler's equation) $e^{i\pi} + 1 = 0$ is called God formula and the most beautiful formula in mathematics. However, as 2π is the real natural constant and π is not, $e^{2\pi i} = 1$ should be more beautiful.

(5). $\gamma = \ln(2\pi) + \gamma_{cg}$ may help to prove γ is an irrational number or even a transcendental number.

(6). The natural group formulas help us to establish "Chen's Periodic Table of Elements and Natural Group Theory"⁶ (2014-2017).

(7). The mathematic expression of chirality is $\pm 2\pi$. This concept is helpful for us to establish "Chirality and Poetry Model of Atomic Nuclei"⁷ (2017/12-2018/3).

(8). Based on the above theories, Chen's theory of the fine-structure constant was deduced (2018/4-6)⁸ and has been revised, modified and improved (2018/7-2020/1).

5. Original Inspiration for Formulas of the Fine-structure Constant

1. According to $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{\lambda_e}{2\pi a_0} = \frac{2\pi r_e}{\lambda_e} \approx \frac{1}{137.036}$, the formulas of α should relate to 2π .

$$2. \frac{137.036}{2\pi} = \frac{137.036}{6.28318} = 21.81, \quad 137.036 = 21.81 \times 2\pi$$

$$3. \text{According to } 2\pi\text{-e formula: } 2\pi = \left(\frac{e}{e^{\gamma_e}}\right)^2 = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots$$

2π is a square number, suppose $21.81 = x^2$, $x = 4.670 \approx 14/3$

$$\text{so: } \frac{1}{\alpha} \approx \left(\frac{14}{3}\right)^2 2\pi \text{ or } \alpha \approx \left(\frac{3}{14}\right)^2 \frac{1}{2\pi} \quad (\text{Discover: about 2 am on 2018/4/12})$$

4. Apply with $2\pi\text{-e}$ formula (in the afternoon of 2018/4/12, a meeting in the morning)

$$\alpha = \left(\frac{3}{14}\right)^2 \frac{1}{(2\pi)_{112}} = \left(\frac{3}{14}\right)^2 \frac{1}{e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{225}}} = 137.035781520, \text{ closest to the real value.}$$

As 112 is one of the most important stable numbers and the 112th element ${}_{112}^{285}\text{Cn}_{173}^*$ is the natural end of elements according to our Chen's Chirality and Poetry Model of Atomic Nuclei⁶.

$$\text{So: } \textit{Eureka!} \quad \text{Subsequently transformed to: } \alpha = \frac{6^2}{7(2\pi)_{112}} \frac{1}{112} = 137.035781520,$$

$$\text{Finally modified to: } \alpha = \frac{6^2}{7(2\pi)_{112}} \frac{1}{112 + \frac{1}{75^2}} = 137.035999037435$$

6. Logical Deduction of Chen's Formulas of the Fine-structure Constant

Physicist Richard Feynman noticed a hydrogen-like atom with Z protons and only one electron, according to Bohr model, the line velocity of the nth rank electron $v_{e/z/n}$ satisfies:

$$\frac{v_{e/z/n}}{c} = \frac{Ze^2}{n4\pi\epsilon_0\hbar c} = \frac{Z}{n}\alpha, \text{ as } v_{e/z/n} \leq c, \alpha = \frac{v_{e/z/n}}{c} \frac{n}{Z} \approx \frac{1}{Z_{\max\text{-ideal}}} = \frac{1}{Fy} = \frac{1}{137}$$

The 137th hydrogen-like element Fy (Feynmanium) is an ideal (imaginative) element,

in reality, the above formula should be modified to: $\alpha = f(Z_{\text{real}}) \frac{1}{Z_{\max\text{-real}}}$

According to Chen's Chirality and Peotry Model of Atomic Nuclei⁶,

$$Z_{\max\text{-real}} = 112 = 2 \cdot 56, \text{ so } \alpha = f(Z_{\text{real}}) \frac{1}{Z_{\max\text{-real}}} = f(Z_{\text{real}}) \frac{1}{112}$$

Compared to $\alpha = \frac{\lambda_e}{2\pi a_0}$, the formula should have a 2π factor:

$$\alpha = f(Z_{\text{real}}) \frac{1}{Z_{\max\text{-real}}} = \frac{n}{m(2\pi)_k} \frac{1}{Z_{\max\text{-real}}} = \frac{6^2}{7 \cdot (2\pi)_{112}} \frac{1}{112} = 1/136.8$$

Apply with 2π -e formula: $2\pi = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots$

the formula is transformed to:

$$\alpha = \frac{n}{m(2\pi)_k} \frac{1}{Z_{\max\text{-real}}} = \frac{6^2}{7 \cdot (2\pi)_{112}} \frac{1}{112} = \frac{6^2}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{225}}} \frac{1}{112} = 1/137.035782$$

Above deduction on 2018/4/12, only $(2\pi)_{112}$ gives the closest value to α , this coincidence of one part per infinity proves the formula itself is correct.

Added an calibration factor ($\delta=1/75^2$) on 2018/4/20, the accurate formula is:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{6^2}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{225}}} \frac{1}{112 + \frac{1}{75^2}}$$

Discover: 2018/4/12; Revise: 2018/4/20 (add $1/75^2$ factor)

By the same procedure but compared to $\alpha = \frac{2\pi r_e}{\lambda_e}$, the other formula is:

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{m(2\pi)_k}{n} \frac{1}{Z_{\max\text{-real}}} = \frac{13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{279}{278}\right)^{557}}}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} = 1/137.035999111818$$

Discover: 2018/4/24; Revise: 2018/9/18-20 ($280 \rightarrow 278$, $-\frac{1}{39^2} + \frac{1}{780^2} \rightarrow -\frac{1}{3 \cdot 29 \cdot 64}$)

Another amazing coincidence is 6^2 and 10^2 are square numbers in accordance with $2\pi = \left(\frac{e}{e^{\gamma c}}\right)^2$

This also demonstrates that α has two values with two kinds of formulas.

As $f(Z_{\text{real}}) = \frac{n}{m(2\pi)_k}$ or $f(Z_{\text{real}}) = \frac{m(2\pi)_k}{n}$, m n k δ should relate to nucleon numbers of nuclides.

7. The Two Most Important Formulas

The above two formulas for α_1 and α_2 were our first gained formulas and are the most important formulas among their serial formulas which will be given followed in this paper. Calculation to give the values of α_1 and α_2 is shown in **Fig. 3** and **Table 2**.

Fig. 3. Calculation diagram of α_1 and α_2 (2018/4-6).

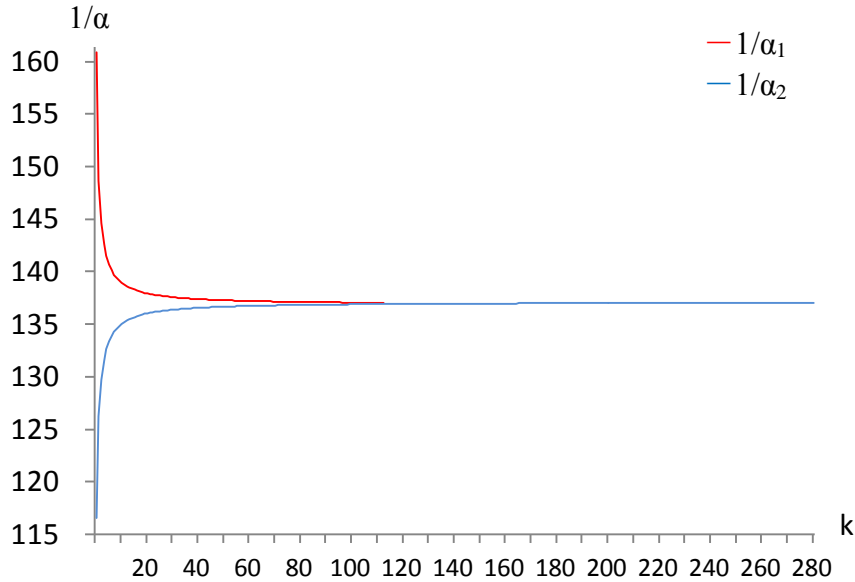


Table 2. Calculation of α_1 and α_2 (2018/4-6).

k	$(2\pi)_k$	$1/\alpha_1$	k	$(2\pi)_k$	$1/\alpha_2$
	7.389056099	160.917477134		7.389056099	116.596364743
1	6.824768754	148.628533230	1	6.824768754	126.236816375
2	6.640803185	144.622165589	2	6.640803185	129.733867427
3	6.549956514	142.643723845	3	6.549956514	131.533251879
4	6.49586908	141.465817857	4	6.49586908	132.628454999
5	6.46000004	140.684668634	5	6.46000004	133.364872233
6	6.434476503	140.128821836	6	6.434476503	133.893888578
7	6.415388754	139.713132398	7	6.415388754	134.292263980
8	6.400576029	139.390543654	8	6.400576029	134.603053878
9	6.388747203	139.132937708	9	6.388747203	134.852272701
10	6.379083388	138.922480953	10	6.379083388	135.056563407
14	6.353377324	138.362659116	14	6.353377324	135.603008624
28	6.319398093	137.622665802	28	6.319398093	136.332142298
56	6.301583891	137.234711452	56	6.301583891	136.717545138
110	6.29262658	137.039640822	112	6.292459356	136.915795771
111	6.292542221	137.037803660	224	6.28784124	137.016353814
112	6.292459356	137.035999037435	276	6.286966940	137.035408057
113	6.292377945	137.034226098	277	6.286953333	137.035704647
114	6.292297952	137.032484014	278	6.286939823	137.035999111818
			279	6.286926410	137.036291474
			280	6.286913093	137.036581756

In these two formulas (deduced from the modification of Z_{\max}), there are some factors which are essentially related to nucleon numbers of some nuclides especially some important stable numbers (stipulated by Chen's Chirality and Poetry Model of Atomic Nuclei⁷) such as 28, 42, 56, 83, 84, 112, 126, 166, 167, 168 *et al.* And these numbers correlate with each others. This kind of relationship is shown in the follows.

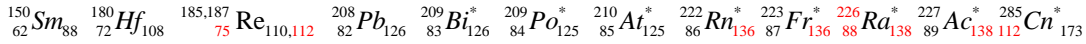
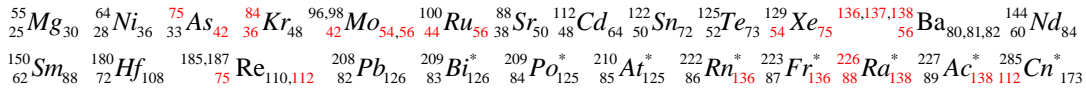
A brief illustration of the relationships between the fine-structure constant and nuclides:



Above nuclides indicate that 136–138, which can be called the fine-structure constant numbers, definitely relate to 112 and 166–168 (double of 56 and 83–84, the most stable numbers in nuclides).

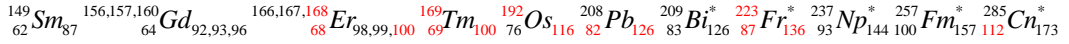
$$\alpha_1 = \frac{6^2}{7 \cdot (2\pi)_{112}} \frac{1}{112 + \frac{1}{75^2}} = \frac{6^2}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{225}}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

Relations to nuclides ($7 \cdot (2\pi)_{112} \approx 44$; $\frac{\text{nucleon}}{\text{proton}} X_{\text{neutron}}$): ${}^7_3\text{Li}_{4,4}$ ${}^9_5\text{Be}_{5,5}$ ${}^{11}_6\text{B}_{6,6}$ ${}^{12}_6\text{C}_{6,7}$ ${}^{14}_7\text{N}_{7,9}$ ${}^{19}_9\text{F}_{10}$ ${}^{24}_{12}\text{Mg}_{12,14}$ ${}^{28}_{14}\text{Si}_{14}$ ${}^{52}_{24}\text{Cr}_{28}$



$$\alpha_2 = \frac{13 \cdot (2\pi)_{278}}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} = \frac{13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{9 \cdot 31}{278}\right)^{557}}}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} = 1/137.035999111818$$

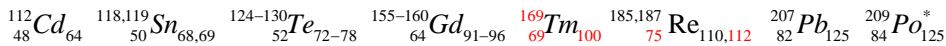
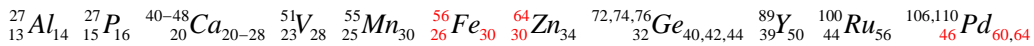
Relations to nuclides ($13 \cdot (2\pi)_{278} \approx 82$; $\frac{\text{nucleon}}{\text{proton}} X_{\text{neutron}}$): ${}^{20}_{10}\text{Ne}_{10}$ ${}^{27}_{13}\text{Al}_{14}$ ${}^{29}_{14}\text{Si}_{15}$ ${}^{55}_{25}\text{Mn}_{30}$ ${}^{54,56,57,58}_{26}\text{Fe}_{28,30,31,32}$



The value of the front part of each above formula is almost equal to $1/(3/2)^{1/2}$ (because 112 is the element natural proton end and 168 is the element natural neutron end as shown in ${}_{112}\text{Cn}_{168+5}$), so the formulas can be transformed to the follows.

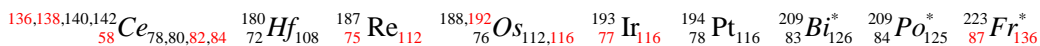
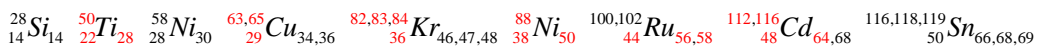
$$\alpha_1 = \alpha_{1-(3/2)} = \frac{1}{\left(\frac{3}{2} - \frac{1}{3 \cdot 112 + 1} + \frac{1}{2^2 \cdot 3 \cdot 5^3 \cdot 13 \cdot 23 - \frac{30}{64}}\right)^{1/2}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

2019/4/25 Relations to nuclides :



$$\alpha_2 = \alpha_{2-(3/2)} = \frac{1}{\left(\frac{3}{2} - \frac{1}{3 \cdot 112 + 1} + \frac{1}{2 \cdot 7 \cdot 11 \cdot 19 \cdot 29 + \frac{36}{75^2}}\right)^{1/2}} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} = 1/137.035999111818$$

2019/4/25 Relations to nuclides:



8. The Integrated Fine-structure Constant

Multiplication of α_1 and α_2 should almost divide out the 2π factors and give $3/2$ and 112×112 factors, this means $\alpha_1\alpha_2$ is almost equal to 112×168 , so we define $\alpha_c = (\alpha_1\alpha_2)^{1/2}$ as the integrated fine-structure constant or Chen's fine-structure constant.

$$\begin{aligned} \frac{1}{\alpha_c^2} &= \frac{1}{\alpha_1\alpha_2} = \frac{2\pi a_0}{\lambda_e} \frac{\lambda_e}{2\pi r_e} = \frac{a_0}{r_e} = \left(\frac{c}{v_e}\right)^2 \\ &= 112 \times \left(168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{6 \cdot 29 \cdot 53 \cdot 59 - 79/47}\right) \quad 2018/6/8-9, 9/18-19, 2019/4/19 \\ &= 136 \left(138 + \frac{1}{2} - \frac{1}{10 \cdot 29} + \frac{1}{12 \cdot 53 \cdot (6 \cdot 53 - 1) - 27/47}\right) \quad 2019/4/17-19 \\ &= 137 \left(137 + \frac{1}{13} - \frac{1}{7 \cdot 29} + \frac{1}{32 \cdot 33 \cdot 89 + 16/49}\right) \quad 2019/4/17-19 \\ &= 112 \cdot 167.668437878408 = 18778.865042381 \\ &\quad \begin{matrix} 27 & 29 & 47,49 & 53 & 54,56,58 & 59 & 58,60,61 & 63,65 & 79 & 87 \\ 13 & Al_{14} & Si_{15} & Ti_{25,27} & Cr_{29} & Fe_{28,30,32} & Co_{32} & Ni_{30,32,33} & Cu_{34,36} & Br_{44} & Sr_{49} \end{matrix} \\ &\quad \begin{matrix} 100,102 & 112 & 113 & 135-138 & 136,138 & 3-47 & 158,160 & 159 & 166,168 \\ 44 & Ru_{56,58} & Cd_{64} & In_{64} & Ba_{79-82} & Ce_{78,80} & Pr_{82} & Gd_{94,96} & Tb_{94} & Er_{98,100} \end{matrix} \\ &\quad \begin{matrix} 174 & 188 & 197 & 203 & 223 & 226 & 227 & 262 & 285 & 293 \\ 70 & Yb_{104} & Os_{112} & Au_{118} & Tl_{122} & Fr_{136}^* & Ra_{138}^* & Ar_{138}^* & Lr_{159}^* & Cn_{173}^* & Lv_{177}^{ie} \end{matrix} \\ \alpha_c^2 &= \alpha_1\alpha_2 = \left[\frac{6^2}{7 \cdot (2\pi)_{112}} \frac{1}{112 + \frac{1}{75^2}} \right] \left[\frac{13 \cdot (2\pi)_{278}}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} \right] \\ &= \frac{13 \cdot 3^2}{7 \cdot 5^2} \frac{e^2}{(2 \cdot 3 \cdot 19)_{227}} \frac{e^2}{(115)_{229}} \dots \frac{e^2}{(9 \cdot 31)_{557}} \frac{1}{112^2 - \frac{1}{30^2 \cdot 5} + \frac{1}{60^2 \cdot 15} - \frac{1}{120^2 \cdot 15 \cdot 29}} \\ &= 1/18778.865042381 \quad 2019/12/14 \\ &\quad \begin{matrix} 27 & 31 & 39 & 55 & 54,56,57,58 & 63,65 & 69,71 & 79,81 & 87 \\ 13 & Al_{14} & P_{16} & K_{20} & Mn_{30} & Fe_{28,30,31,32} & Cu_{34,36} & Ga_{38,40} & Br_{44,46} & Sm_{49} \end{matrix} \\ &\quad \begin{matrix} 89 & 93 & 112-120-124 & 135-138 & 139 & 136,138 & 144,145 & 157 \\ 39 & Y_{50} & Nb_{52} & Sn_{62-70-74} & Ba_{79-82} & La_{82} & Ce_{78,80} & Nd_{83,84} & Gd_{93} \end{matrix} \\ &\quad \begin{matrix} 200 & 209 & 223 & 237 & 278+7 & 284 \\ 80 & Hg_{120} & Bi_{126}^* & Fr_{136}^* & Np_{144}^* & Cn_{166+7}^* & Nh_{9,19}^{ie} \end{matrix} \\ \alpha_c^2 &= \alpha_1\alpha_2 = \frac{1}{\left(\frac{3}{2} - \frac{1}{3 \cdot 112 + 1} + \frac{1}{7 \cdot 19 \cdot 29 \cdot 37 - \frac{25}{44}}\right)} \frac{1}{112 + \frac{1}{75^2}} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} \\ &= 1/18778.865042381 \quad 2019/12/14 \\ &\quad \begin{matrix} 39 & 47,50 & 55 & 63,65 & 85,87 & 87,88 & 99,100,102,104 & 112 \\ 19 & K_{20} & Ti_{25,28} & Mn_{30} & Cu_{34,36} & Rb_{48,50} & Sr_{49,50} & Ru_{55,56,58,60} & Cd_{64} \end{matrix} \\ &\quad \begin{matrix} 112,114,115,116,120,124 & 5-37,11-17 & 223 & 226 \\ 50 & Sn_{62,64,65,66,70,74} & Re_{110,112} & Fr_{136}^* & Ra_{138}^* \end{matrix} \end{aligned}$$

9. A Brief Explanation of the Fine-structure Constant

According to Chen's Chirality and Poetry Model of Atomic Nuclei⁷, the ratio of neutron number N to proton number Z in nuclides increases from $1/1$ to $3/2$ (eventually slightly above $3/2$) along with the increasing of atomic number, for example, from ${}_{14}\text{Si}_{14}$, ${}_{26}\text{Fe}_{30}$, ${}_{29}\text{Cu}_{34,36}$, ${}_{56}\text{Ba}_{82}$, ${}_{84}\text{Po}_{125}$ to ${}_{112}\text{Cn}_{168+5}^*$. In this process, $(3/2)^{1/2}$ will act as a transition foothold. As for nuclide ${}_{112}\text{Cn}_{168+5}$ with $Z=112$, $N=168+5$ and $168/112=3/2$, 137 is just right their $(3/2)^{1/2}$ times intermediate stage. This should be why 137 exists and what's the real meaning of 137 .

electromagnetic wave or light, it should be reasonable to suppose the speed of light to be the integrated fine-structure constant, i.e., $c_{au}=1/\alpha_c=1/(\alpha_1\alpha_2)^{1/2}=137.035999074627$. It means we've theoretically/mathematically calculated the speed of light, the formula is intrinsically consistent with Maxwell's formula, and the value is much accurate.

In atomic units ($e = m_e = \hbar = 1$ and $\varepsilon_0 = \frac{1}{4\pi}$), $v_{e/au} = \alpha c_{au} = \frac{e^2}{4\pi\varepsilon_0\hbar} = 1$, so $c_{au} = \frac{1}{\alpha}$

There are two α (α_1 and α_2), but there shouldn't be two c or c_{au} ,

so it should be: $c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$ (au: atomic units)

Compared to Maxwell Formula $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, $c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$ should be reasonable.

$$c_{au} = \frac{1}{\sqrt{\mu_{0/au}\varepsilon_{0/au}}}, \mu_{0/au}\varepsilon_{0/au} = \alpha_1\alpha_2, \mu_{0/au} = 4\pi\alpha_1\alpha_2 \quad (2019/11/30)$$

So the theoretical formula of the speed of light in atomic units is as follows:

$$\begin{aligned} c_{au} &= \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}} = \frac{1}{\sqrt{\left(\frac{6^2}{7 \cdot (2\pi)_{112}} \frac{1}{112 + \frac{1}{75^2}}\right) \left(\frac{13 \cdot (2\pi)_{278}}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}}\right)}} \\ &= \frac{5}{3} \sqrt{\frac{7 \cdot (2\pi)_{112}}{13 \cdot (2\pi)_{278}} \left(112^2 - \frac{1}{30^2 \cdot 5} + \frac{1}{60^2 \cdot 15} - \frac{1}{120^2 \cdot 15 \cdot 29}\right)} \\ &= \sqrt{\frac{5 \cdot 17}{36} \frac{10}{7 \cdot 19} \frac{(2\pi)_{12389}}{(2\pi)_{28186}} \left(112^2 - \frac{23 \cdot (12 \cdot 41 - 1)}{10^{10}}\right)} \\ &= \frac{5}{3} \sqrt{\left(\frac{2^3 \cdot 17}{11 \cdot 23} + \frac{2 \cdot 17}{11 \cdot 23 \cdot 97}\right) \frac{(2\pi)_{34450}}{(2\pi)_{28186}} \left(112^2 - \frac{18 \cdot 97 + 1}{2 \cdot 10^9}\right)} \\ &= \sqrt{\left(\frac{3}{2} - \frac{1}{3 \cdot 112 + 1} + \frac{1}{7 \cdot 19 \cdot 29 \cdot 37 - \frac{25}{44}}\right) \left(112^2 - \frac{1}{30^2 \cdot 5} + \frac{1}{60^2 \cdot 15} - \frac{1}{120^2 \cdot 15 \cdot 29}\right)} \\ &= \sqrt{\frac{3}{2} \left(112 - \frac{1}{3^2} + \frac{1}{12^2 \cdot 13 - \frac{30 \cdot 19}{100} - \frac{1}{125 \cdot 100}}\right)} = \sqrt{\frac{3}{2} - \frac{1}{3 \cdot 112 + 1} + \frac{1}{14 \cdot 53 \cdot 193 - \frac{33}{2 \cdot 47}}} \times 112 \\ &= \sqrt{112 \times \left(168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{6 \cdot 29 \cdot 53 \cdot 59 - 79/47}\right)} \\ &= \sqrt{137.035999037435 \times 137.035999111818} = 137.035999074627 \end{aligned}$$

Note: $112/278 \approx 27/67$, $12389/28186 \approx 11/25$, $34450/28186 \approx 11/9 \approx 66/29$

Discover: 2019/12/16; Revise and Supplement: 2020/1/5-8, 2/24, 3/28-29

12. The Special 29 and 75 Factors

In the above formulas some factors especially 29 and 75 appear several times. This feature should be analyzed and explained. Accompanying N/Z ratio from 1/1 to slightly above 3/2 along with the increasing of atomic number, ${}_{29}\text{Cu}_{34,36}$ is the critical point of N/Z ratio approaching $(3/2)^{1/2}$ and ${}_{75}\text{Re}_{110,112}$ is the critical point of N/Z ratio approaching 3/2 (**Table 3**, **Fig. 4** and **Fig. 5**), so 29 and 75 are important factors and hence frequently appear in the formulas.

Table 3. N/Z ratios of the Elements (2019/4/23).

Z	N	N/Z	Z	N	N/Z	Z	N	N/Z	Z	N	N/Z				
H	1	0	0	Ga	31	38.80	1.25	Pm	61	84	1.38	Pa*	91	140	1.54
He	2	2.00	1.00	Ge	32	40.71	1.27	Sm	62	88.45	1.43	U*	92	146	1.59
Li	3	3.92	1.31	As	33	42	1.27	Eu	63	89.04	1.41	Np*	93	144	1.55
Be	4	5	1.25	Se	34	45.05	1.33	Gd	64	93.33	1.46	Pu*	94	150	1.60
B	5	5.80	1.16	Br	35	44.98	1.29	Tb	65	94	1.45	Am*	95	148	1.56
C	6	6.01	1.00	Kr	36	47.89	1.33	Dy	66	96.57	1.46	Cm*	96	151	1.57
N	7	7.00	1.00	Rb	37	48.56	1.31	Ho	67	98	1.46	Bk*	97	150	1.55
O	8	8.00	1.00	Sr	38	49.71	1.31	Er	68	99.33	1.46	Cf*	98	153	1.56
F	9	10	1.11	Y	39	50	1.28	Tm	69	100	1.45	Es*	99	153	1.55
Ne	10	10.19	1.02	Zr	40	51.32	1.28	Yb	70	103.11	1.47	Fm*	100	157	1.57
Na	11	12	1.09	Nb	41	52	1.27	Lu	71	104.03	1.47	Md*	101	157	1.55
Mg	12	12.32	1.03	Mo	42	54.04	1.29	Hf	72	106.54	1.48	No*	102	157	1.54
Al	13	14	1.08	Td	43	55	1.28	Ta	73	108	1.48	Lr*	103	159	1.54
Si	14	14.11	1.01	Ru	44	57.16	1.30	W	74	109.89	1.49	Rf*	104	161	1.55
P	15	16	1.07	Rh	45	58	1.29	Re	75	111.25	1.48	Db*	105	163	1.55
S	16	16.09	1.01	Pd	46	60.51	1.32	Os	76	114.27	1.50	Sg*	106	165	1.56
Cl	17	18.48	1.09	Ag	47	60.96	1.30	Ir	77	115.25	1.50	Bh*	107	163	1.52
Ar	18	21.99	1.22	Cd	48	64.52	1.34	Pt	78	117.12	1.50	Hs*	108	169	1.56
K	19	20.13	1.06	In	49	65.91	1.35	Au	79	118	1.49	Mt*	109	167	1.53
Ca	20	20.12	1.01	Sn	50	68.81	1.38	Hg	80	120.62	1.51	Ds*	110	171	1.55
Sc	21	24	1.14	Sb	51	70.86	1.39	Tl	81	123.41	1.52	Rg*	111	169	1.52
Ti	22	25.92	1.18	Te	52	75.70	1.46	Pb	82	125.24	1.53	Cn*	112	173	1.54
V	23	28	1.22	I	53	74	1.40	Bi*	83	126	1.52	Nh*	113	171	1.51
Cr	24	28.06	1.17	Xe	54	77.39	1.43	Po*	84	125	1.49	Fl*	114	175	1.54
Mn	25	30	1.20	Cs	55	78	1.42	At*	85	125	1.47	Mc*	115	173	1.50
Fe	26	29.91	1.15	Ba	56	81.42	1.45	Rn*	86	136	1.58	Lv*	116	177	1.53
Co	27	32	1.19	La	57	82	1.44	Fr*	87	136	1.56	Ts*	117	177	1.51
Ni	28	30.76	1.10	Ce	58	82.21	1.42	Ra*	88	138	1.57	Og*	118	176	1.49
Cu	29	34.62	1.19	Pr	59	82	1.39	Ac*	89	138	1.55				
Zn	30	35.45	1.18	Nd	60	84.41	1.41	Th*	90	142	1.58				

Z: atomic number, N: average neutron number or neutron number of the most stable isotope.

1. N/Z from 1/1 (${}_6\text{C}$) to slightly above 3/2 (such as ${}_{112}\text{Cn}$ which is the natural end of elements demonstrated by Chen's Chirality and Poetry Model of Atomic Nuclei⁷).
2. For ${}_{29}\text{Cu}$, N/Z ratio 1.19 is near to $(3/2)^{1/2}=1.22$, slightly less is because of stability effect.
3. For ${}_{75}\text{Re}$, N/Z ratio 1.48 is near to $3/2=1.50$, slightly less is because of stability effect.
4. From ${}_6\text{C}$ to ${}_{112}\text{Cn}$, the middle of N/Z 1.5 range is at $(76.5-5)/(112-5)=0.668\approx 2/3$ position.

Fig. 4 and **Fig. 5** shows that stability effect of nucleon number 64 makes the neutron numbers of ${}_{29}\text{Cu}$'s isotopes are relatively less (34 and 36) than normal so that its N/Z ratio is a little less than $(3/2)^{1/2}$ which is otherwise it should be. Also the

stability effect of nucleon numbers 110 and 112 make the neutron numbers of ^{75}Re 's nuclides are relatively less (110 and 112) than normal so that its N/Z ratio is a little less than $3/2$ which otherwise it should be.

Fig. 4. Complete Graph of N/Z Ratios of Elements (2019/4/23-24).

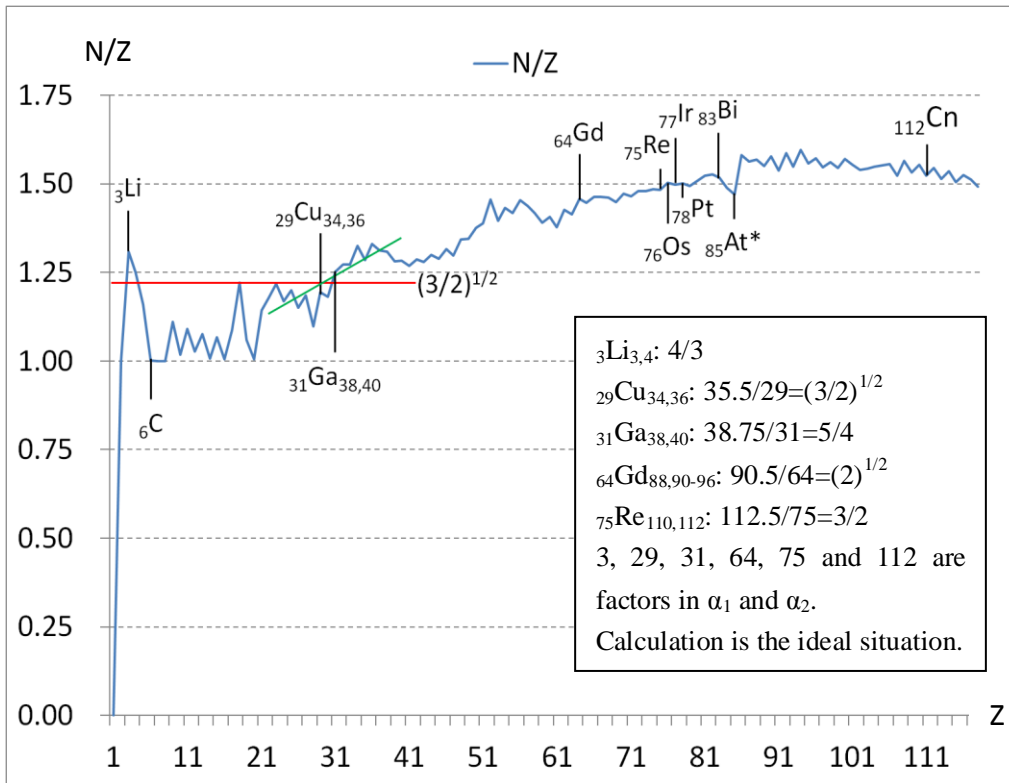
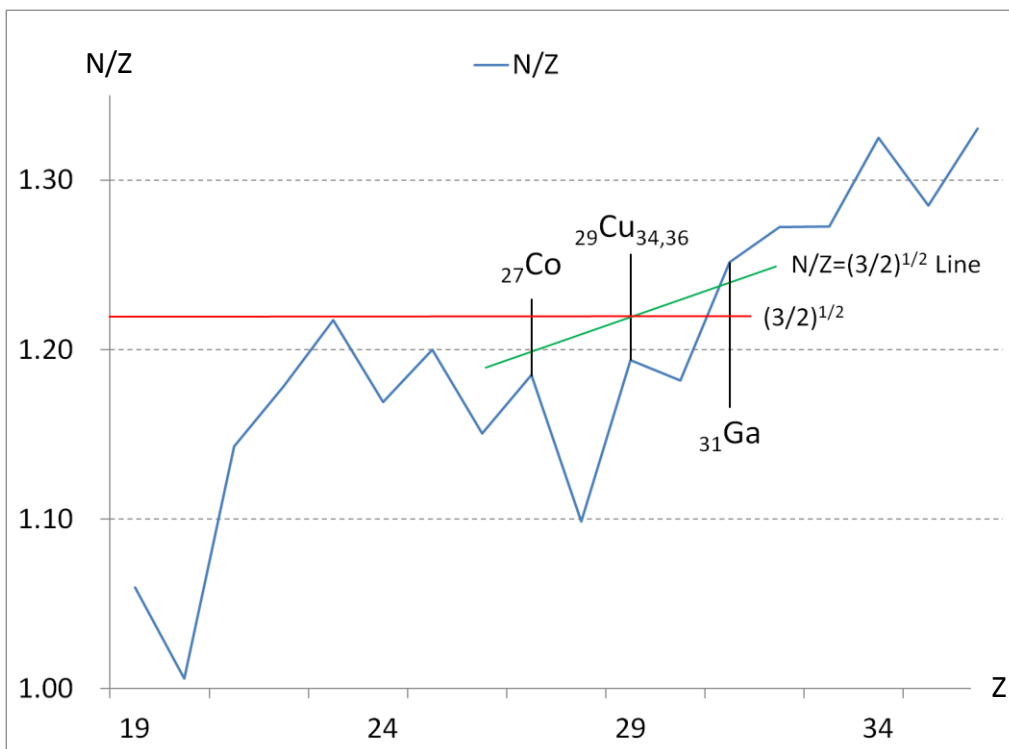


Fig. 5. Partially Amplified Graph of N/Z Ratios of Elements (2019/4/24).

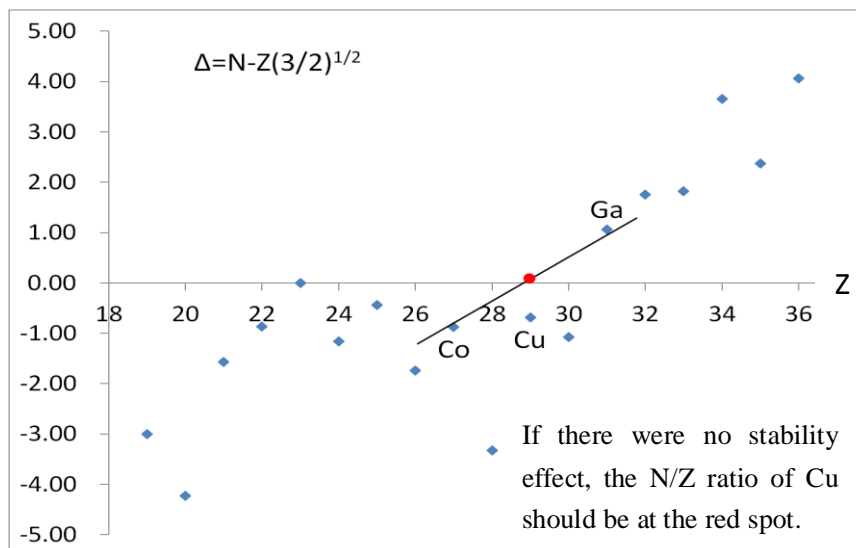


The general trend of N/Z ratio of elements is from 1/1 (${}^6\text{C}_6$) to slightly above 3/2 (${}^{112}\text{Cn}_{173}$) definitely. However, the increasing process is not smooth, the N/Z ratio rising fluctuates consecutively. According to Chen's Chirality and Poetry Model of Atomic Nuclei⁷, there are some stable numbers (magic numbers) which can bring about this kind of fluctuation (**Table 4** and **Fig. 6**).

Table 4. Effect of Stable Numbers on N/Z ratio's fluctuation (2019/4/22).

Element	Z	N(Average)	Z(3/2) ^{1/2}	N-Z(3/2) ^{1/2}	Stable Number
K	19	20.13	23.27	-3.17	20
Ca	20	20.12	24.49	-4.41	20+20
Sc	21	24	25.72	-1.74	
Ti	22	25.92	26.94	-1.07	22+26=48
V	23	28.00	28.17	-0.23	28
Cr	24	28.06	29.39	-1.39	28
Mn	25	30	30.62	-0.68	
Fe	26	29.91	31.84	-1.99	26+30=56
Co	27	32.00	33.07	-1.14	
Ni	28	30.76	34.29	-3.60	28+30=58、28+32=60
Cu	29	34.62	35.52	-0.97	64
Zn	30	35.45	36.74	-1.36	30+34=64、30+36=66
Ga	31	38.80	37.97	0.75	
Ge	32	40.71	39.19	1.44	32+40=72
As	33	42.00	40.42	1.50	
Se	34	45.05	41.64	3.30	34+46=80
Br	35	44.98	42.87	2.03	
Kr	36	47.89	44.09	3.71	36+48=84

Fig. 6. Effect of Stable Numbers on N/Z ratio's fluctuation (2019/4/22-23)



13. α_1/α_2 in Schrödinger Equation of Hydrogen Atom

Stationary Schrodinger Equation $-\frac{\hbar^2}{2m}\nabla^2\psi + U\psi = E\psi$, applied to hydron atom:

$$\nabla^2\psi + \frac{2m_e}{\hbar^2}(E + \frac{e^2}{4\pi\epsilon_0 r})\psi = 0, \quad E = -\frac{m_e e^4}{2n^2(4\pi\epsilon_0)^2\hbar^2}, \text{ do substitution and simplification:}$$

$$\frac{2m_e}{\hbar^2}(\frac{m_e e^4}{2n^2(4\pi\epsilon_0)^2\hbar^2} - \frac{e^2}{4\pi\epsilon_0 r})\psi = \nabla^2\psi, \quad [\frac{1}{n^2}(\frac{m_e e^2}{4\pi\epsilon_0\hbar^2})^2 - \frac{2}{r} \frac{m_e e^2}{4\pi\epsilon_0\hbar^2}]\psi = \nabla^2\psi,$$

$$[\frac{1}{n^2}(\frac{e^2}{4\pi\epsilon_0\hbar c} \frac{m_e c}{\hbar})^2 - \frac{2}{r} \frac{e^2}{4\pi\epsilon_0\hbar c} \frac{m_e c}{\hbar}]\psi = \nabla^2\psi,$$

$$\text{As } \sqrt{\alpha_1\alpha_2} = \frac{v_e}{c} = \frac{e^2}{4\pi\epsilon_0\hbar c}, \lambda_e = \frac{h}{m_e c} \text{ and } \alpha_1 = \frac{\lambda_e}{2\pi a_0}:$$

$$[\frac{1}{n^2}(\sqrt{\alpha_1\alpha_2} \frac{2\pi}{\lambda_e})^2 - \frac{2}{r} \sqrt{\alpha_1\alpha_2} \frac{2\pi}{\lambda_e}]\psi = \nabla^2\psi,$$

$$[\frac{1}{n^2(\lambda_e/2\pi/\sqrt{\alpha_1\alpha_2})^2} - \frac{2}{(\lambda_e/2\pi/\sqrt{\alpha_1\alpha_2})r}]\psi = \nabla^2\psi,$$

$$[\frac{1}{n^2 a_0^2 (\alpha_1/\alpha_2)} - \frac{2}{a_0 r \sqrt{\alpha_1/\alpha_2}}]\psi = \nabla^2\psi$$

$$\text{As } \alpha_1/\alpha_2 \approx 1, \text{ simplified to: } [\frac{1}{n^2 a_0^2} - \frac{2}{a_0 r}]\psi = \nabla^2\psi \text{ (factor 2 seems not beautiful)}$$

In atomic units (*au*: $e = m_e = \hbar = 1$ and $\epsilon_0 = \frac{1}{4\pi}$),

$$a_{0/au} = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 1, \quad v_{e/au} = \frac{e^2}{4\pi\epsilon_0\hbar} = 1, \quad c_{au} = \frac{v_{e/au}}{\alpha_c} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$$

$$[\frac{1}{n^2(\alpha_1/\alpha_2)} - \frac{2}{r_{au}\sqrt{\alpha_1/\alpha_2}}]\psi = \nabla_{au}^2\psi, \text{ or } (\frac{c_{au}^2}{\alpha_1^2 n^2} - \frac{2c_{au}}{\alpha_1 r_{au}})\psi = \nabla_{au}^2\psi$$

the above equation could be called Schrodinger-Chen equation of hydrogen atom, the later form of the equation shows factor 2 is still reasonable and beautiful.

$$\text{As } \alpha_1/\alpha_2 \approx 1, \text{ simplified to: } [\frac{1}{n^2} - \frac{2}{r_{au}}]\psi = \nabla_{au}^2\psi$$

Discover: 2018/4-6; Revise: 2019/12/13 (add *au* form)

$$\alpha_1/\alpha_2 = \frac{137.035999111818}{137.035999037435} = 1.0000000005428 = 1 + \frac{23 \cdot 59}{25 \cdot 10^{11}} = (1 + \frac{23 \cdot 59}{50 \cdot 10^{11}})^2$$

$$\sqrt{\alpha_1/\alpha_2} = 1 + \frac{23 \cdot 59}{50 \cdot 10^{11}} = 1.0000000002714$$

Relations to nuclides: ${}_{11}^{23}\text{Na}_{12}$ ${}_{23}^{50,51}\text{V}_{27,28}$ ${}_{25}^{55}\text{Mn}_{30}$ ${}_{44}^{99,100}\text{Ru}_{55,56}$ ${}_{46}^{105}\text{Pd}_{59}$ ${}_{56}^{137}\text{Ba}_{81}$
 ${}_{50}^{118+1}\text{Sn}_{69}$ ${}_{59}^{141}\text{Pr}_{82}$ ${}_{69}^{169}\text{Tm}_{100}$ ${}_{75}^{185,187}\text{Re}_{110,112}$ ${}_{88}^{169}\text{Ra}^*_{137}$

2019/8/28-29

Solution of Schrödinger equation of hydrogen atom gives some quantum numbers such as n , l and m_l which determine the electron shell structure and the chemical

properties of atoms. That means Schrödinger equation of hydrogen atom is the base of chemical periodicity of elements. On the other hand, from above analysis, we have already demonstrated the formulas of the fine-structure constant α are derived from Chen's Chirality and Poetry Model of Atomic Nuclei⁷ and hence mainly connected to the stability of atomic nuclei. So, a question is whether and how α is connected to Schrödinger Equation of hydrogen atom. This question should reveal the connection of the theory of electron shell of atoms and the theory of nuclei of elements. The above deduction provides the answer. The fine-structure constant α relates to Schrödinger Equation of hydrogen atom in α_1/α_2 way which is subtle and negligible but could show the equation is really reasonable and beautiful.

14. The Two Kinds of General Formulas of the Fine-structure Constant

Based on the above two formulas of α_1 and α_2 , it should be reasonable to assume there are two kinds of serial formulas of α_1 and α_2 which are listed in follows. Among these formulas, the above two first discovered formulas are the most fundamental and important. Some formulas both with a big m and an extra large k should be more important referring to the trend of the approximate values of α .

Approximate formulas:

$$\alpha_{1-m'} = \frac{n}{m \cdot (2\pi)_k} \frac{1}{112} = \frac{n}{m \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}} \frac{1}{112} \approx 1/137.036$$

$$\alpha_{2-m'} = \frac{m \cdot (2\pi)_k}{n} \frac{1}{112} = \frac{m \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}}{n} \frac{1}{112} \approx 1/137.036$$

Accurate Formulas:

$$\alpha_{1-m} = \frac{n}{m \cdot (2\pi)_k} \frac{1}{112 + \delta_1} = \frac{n}{m \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}} \frac{1}{112 + \delta_1}$$

$$= 1/137.035999037435$$

$$\alpha_{2-m} = \frac{m \cdot (2\pi)_k}{n} \frac{1}{112 - \delta_2} = \frac{m \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}}{n} \frac{1}{112 - \delta_2}$$

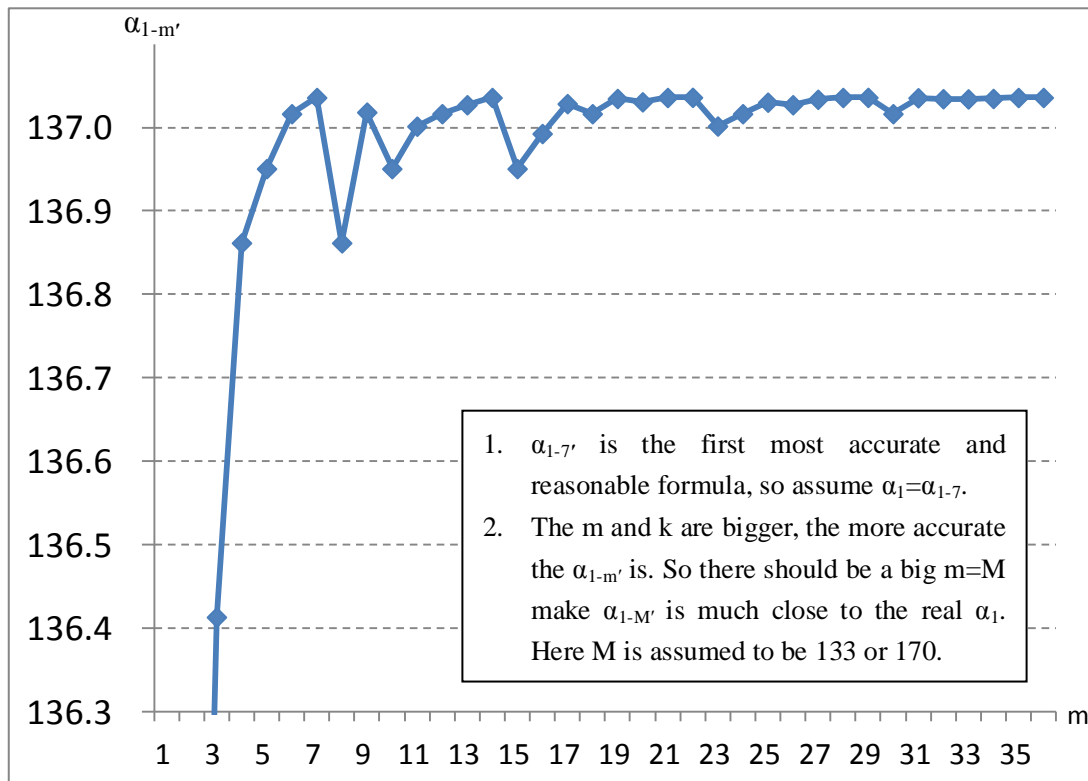
$$= 1/137.035999111818$$

Discover: 2019/6/27; Revise: 2019/7/2-3

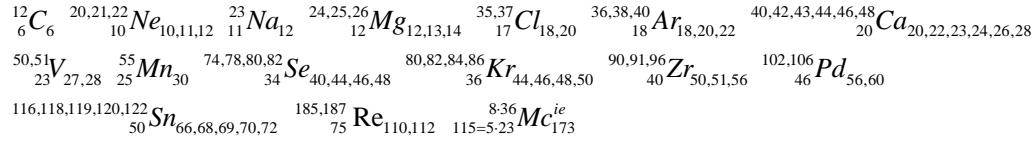
Table 5. Parameters and Results of Approximate Formulas of α_1 (2019/7/2).

m	n	k	$\alpha_{1-m'}$	m	n	k	$\alpha_{1-m'}$
1	6	1	122.265854937	24	124	27	137.016359405
2	11	2	135.230901223	25	129	34	137.030171763
3	16	4	136.413250690	26	134	46	137.027100696
4	21	7	136.861626741	27	139	66	137.033636049
5	26	13	136.950569252	28	144	112	137.035781520
6	31	27	137.016359405	29	149	321	137.035917078
7	36	112	137.035781520	30	155	27	137.016359405
8	42	7	136.861626741	31	160	32	137.035453560
9	47	9	137.018237882	32	165	40	137.034309209
10	52	13	136.950569252	33	170	52	137.034083409
11	57	18	137.001388822	34	175	72	137.034617877
12	62	27	137.016359405	35	180	112	137.035781520
13	67	46	137.027100696	36	185	236	137.035810961
14	72	112	137.035781520	43	221	200	137.035845637
15	78	13	136.950569252	50	257	181	137.035307038
16	83	16	136.992590996	59	303	2645	137.035986189
17	88	20	137.028423583	81	416	1605	137.035992406
18	93	27	137.016359405	96	493	5806	137.035998789
19	98	37	137.034579883	103	529	1310	137.035994308
20	103	58	137.030572071	133	683	12389	137.035999034
21	108	112	137.035781520	140	719	1923	137.035994882
22	113	782	137.035967638	155	796	3988	137.035997989
23	119	22	137.001596764	170	873	34450	137.035999031

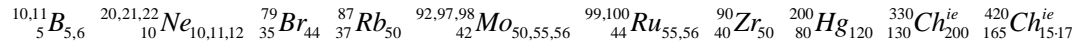
Fig. 7. Results of Approximate Formulas of α_1 (2019/7/2).



$$\alpha_{1-1} = \frac{6}{1 \cdot e^2 \left(\frac{2}{1}\right)^2} \frac{1}{112 + \frac{17}{2} - \frac{1}{40} + \frac{1}{6 \cdot 23 \cdot 25 - \frac{36}{55}}} = 1/137.035999037434$$

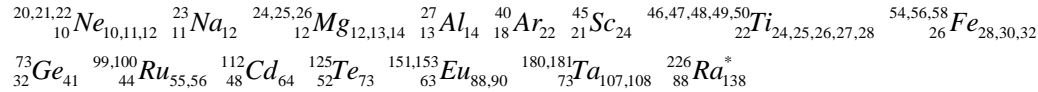


$$\alpha_{1-2} = \frac{11}{2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5}} \frac{1}{112 + \frac{3}{2} - \frac{1}{200} + \frac{1}{5 \cdot (3 \cdot 42 + 1) \cdot (6 \cdot 37 - 1) + \frac{2}{7}}} = 1/137.035999037435$$

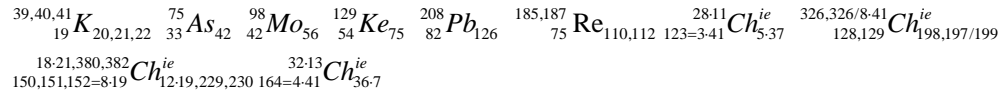


$$\alpha_{1-3} = \frac{4^2}{3 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \frac{e^2}{\left(\frac{5}{4}\right)^9}} \frac{1}{112 + 1 - \frac{1}{2} + \frac{1}{88} - \frac{1}{13 \cdot (2 \cdot 9 \cdot 5 \cdot 13 + 1) - \frac{2}{73}}}$$

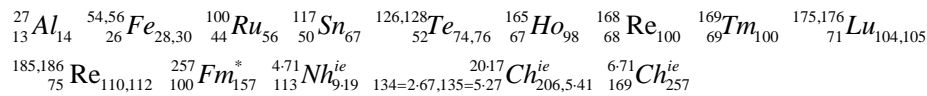
$$= 1/137.035999037435$$



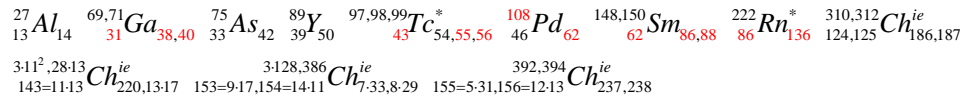
$$\alpha_{1-4} = \frac{21}{2^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{8}{7}\right)^{15}}} \frac{1}{112 + \frac{1}{7} - \frac{1}{8 \cdot 19 \cdot 41 - \frac{75}{98}}} = 1/137.035999037435$$



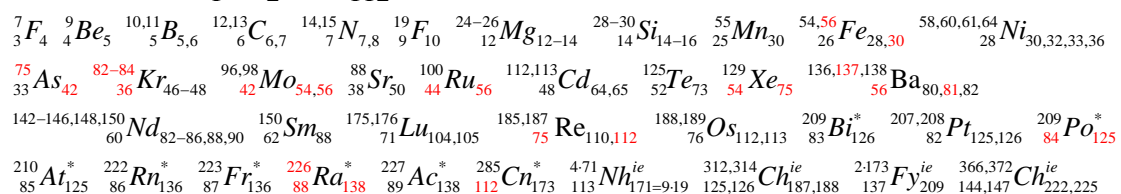
$$\alpha_{1-5} = \frac{26}{5 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{14}{13}\right)^{27}}} \frac{1}{112 + \frac{1}{14} - \frac{1}{9 \cdot 71} + \frac{1}{67 \cdot (75 \cdot 100 - 1) + \frac{1}{10}}} = 1/137.035999037435$$



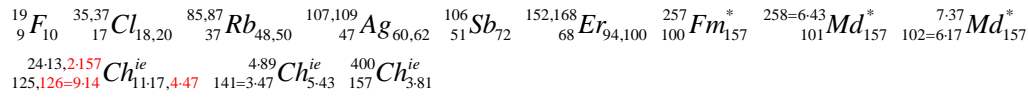
$$\alpha_{1-6} = \frac{31}{6 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{28}{27}\right)^{55}}} \frac{1}{112 + \frac{1}{2 \cdot 31} - \frac{1}{3 \cdot 11 \cdot 13 \cdot 31 - \frac{43}{4 \cdot 27}}} = 1/137.035999037435$$



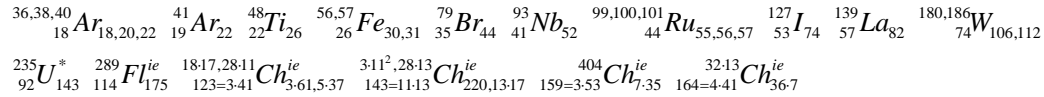
$$\alpha_1 = \alpha_{1-7} = \frac{6^2}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{225}}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$



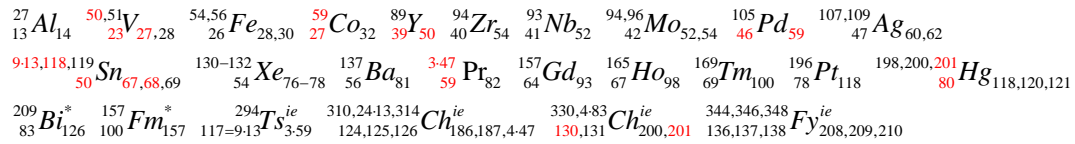
$$\alpha_{1-9} = \frac{47}{3^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{10}{9}\right)^{19}} 112 + \frac{1}{4 \cdot 17} - \frac{1}{2(8 \cdot 9 \cdot 37 - 1) + \frac{3 \cdot 17}{157}}} = 1/137.035999037436$$



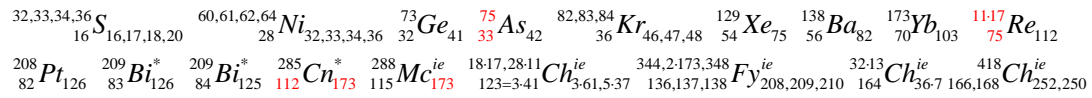
$$\alpha_{1-11} = \frac{3 \cdot 19}{11 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{19}{18}\right)^{37}} 112 + \frac{1}{35} - \frac{1}{88 \cdot 41 - \frac{5 \cdot 53}{22 \cdot 13}}} = 1/137.035999037435$$



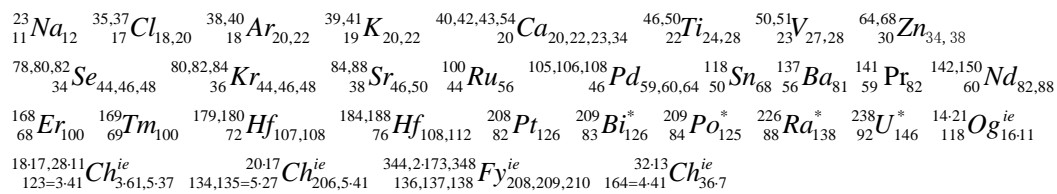
$$\alpha_{1-13} = \frac{67}{13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{47}{2 \cdot 23}\right)^{3 \cdot 31}} 112 + \frac{1}{137} - \frac{1}{6(2 \cdot 27 \cdot 59 + 1) + \frac{9}{50}}} = 1/137.035999037435$$



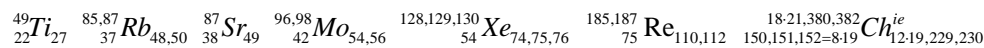
$$\alpha_{1-16} = \frac{83}{4^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{17}{16}\right)^{33}} 112 + \frac{1}{28} - \frac{1}{6 \cdot (18 \cdot 41 + 1) + \frac{173}{2 \cdot (2 \cdot 75 - 1)}}} = 1/137.035999037435$$



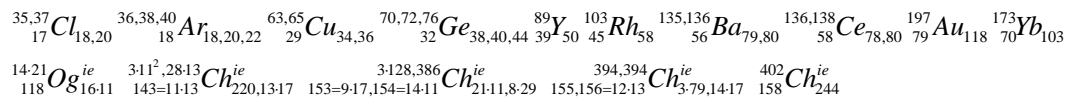
$$\alpha_{1-17} = \frac{2^2 \cdot 22}{17 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{21}{20}\right)^{41}} 112 + \frac{1}{137} - \frac{1}{2 \cdot 19 \cdot 23 \cdot 59 - \frac{30}{100}}} = 1/137.035999037435$$



$$\alpha_{1-19} = \frac{2 \cdot 7^2}{19 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{38}{37}\right)^{75}} 112 + \frac{1}{2 \cdot (8 \cdot 54 - 1) + \frac{54}{19^2}}} = 1/137.035999037440$$

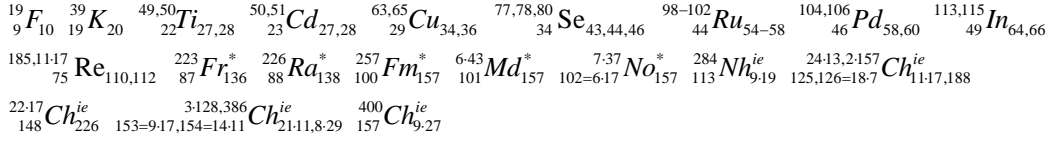


$$\alpha_{1-20} = \frac{103}{2^2 \cdot 5 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{59}{2 \cdot 29}\right)^{9 \cdot 13}} 112 + \frac{1}{32 \cdot 45 \cdot 79 + \frac{22}{3 \cdot 17}}} = 1/137.035999037435$$

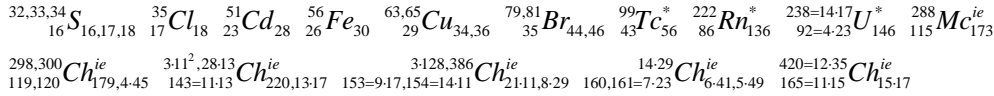


$$\alpha_{1-22} = \frac{113}{22 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{27 \cdot 29}{2 \cdot 17 \cdot 23}\right)^{5 \cdot (2 \cdot 157 - 1)}}} \frac{1}{112 + \frac{1}{2 \cdot [2 \cdot 3 \cdot 17 \cdot (10 \cdot 19 + 1) + 1] + \frac{29}{49}}}$$

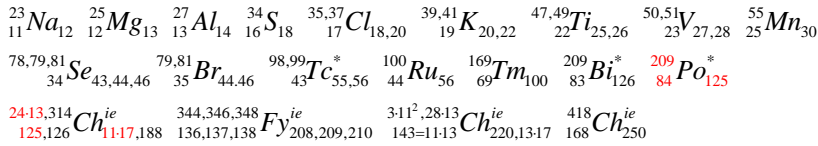
$$= 1/137.035999037435$$



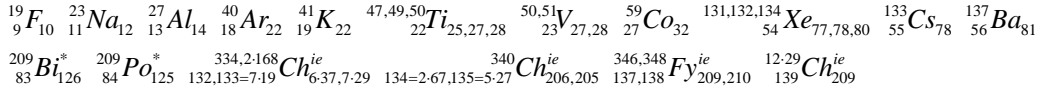
$$\alpha_{1-23} = \frac{7 \cdot 17}{23 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{23}{22}\right)^{45}}} \frac{1}{112 + \frac{1}{35} - \frac{1}{4 \cdot 13 \cdot 43 - \frac{2 \cdot 29}{16 \cdot 17 - 1}}}$$



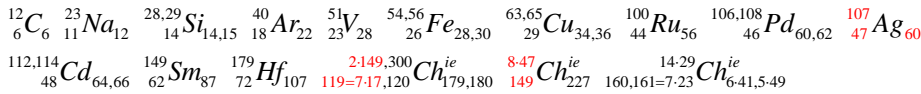
$$\alpha_{1-25} = \frac{3 \cdot 43}{5^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{35}{34}\right)^{3 \cdot 23}}} \frac{1}{11 \cdot 19 - \frac{1}{13^2(16 \cdot 17 - 1) + \frac{11}{25}}}$$



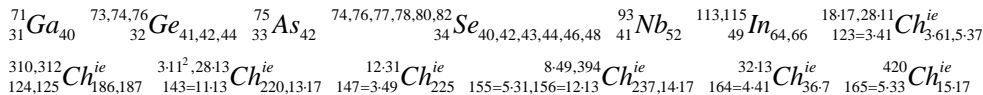
$$\alpha_{1-27} = \frac{139}{27 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{67}{66}\right)^{7 \cdot 19}}} \frac{1}{11 \cdot 47 + \frac{18}{23} + \frac{1}{138 \cdot 137}}$$



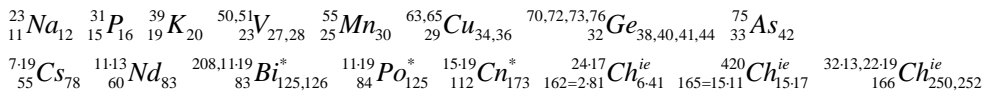
$$\alpha_{1-29} = \frac{149}{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{14 \cdot 23}{3 \cdot 107}\right)^{643}}} \frac{1}{6 \cdot 8 \cdot (12 \cdot 26 - 1) + \frac{11}{18}}$$



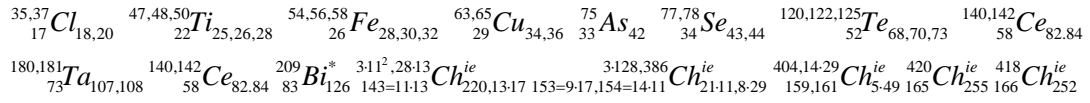
$$\alpha_{1-31} = \frac{4^2 \cdot 10}{31 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{33}{32}\right)^{5 \cdot 13}}} \frac{1}{12 \cdot 11 \cdot 17 - \frac{4 \cdot 49}{5 \cdot 41}}$$



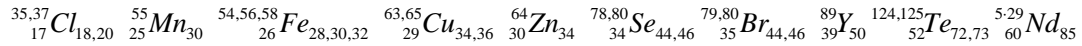
$$\alpha_{1-32} = \frac{15 \cdot 11}{2 \cdot 4^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{41}{40}\right)^{81}}} \frac{1}{25 \cdot 29 - \frac{5 \cdot 83}{19 \cdot 23}}$$



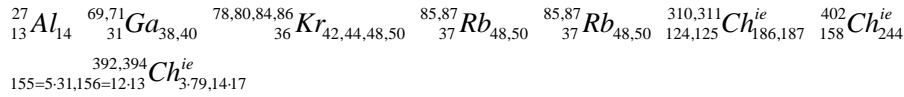
$$\alpha_{1-33} = \frac{170}{33 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{53}{4 \cdot 13}\right)^{105}} 112 + \frac{1}{22 \cdot 29 + \frac{4 \cdot 73}{5 \cdot 83}}} = 1/137.035999037436$$



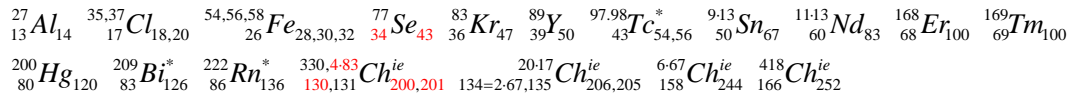
$$\alpha_{1-34} = \frac{7 \cdot 5^2}{34 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{73}{72}\right)^{5 \cdot 29}} 112 + \frac{1}{15 \cdot 59 + \frac{13}{15} + \frac{1}{3 \cdot (2 \cdot 15 \cdot 17 - 1)}}} = 1/137.035999037435$$



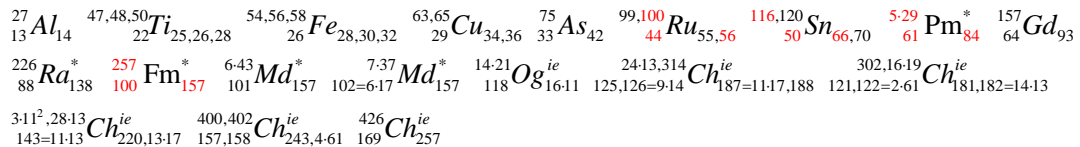
$$\alpha_{1-36} = \frac{5 \cdot 37}{6^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{3 \cdot 79}{4 \cdot 59}\right)^{11 \cdot 43}} 112 + \frac{1}{5 \cdot (31 \cdot 42 - 1) + \frac{3 \cdot 31}{14 \cdot 13}}} = 1/137.035999037436$$



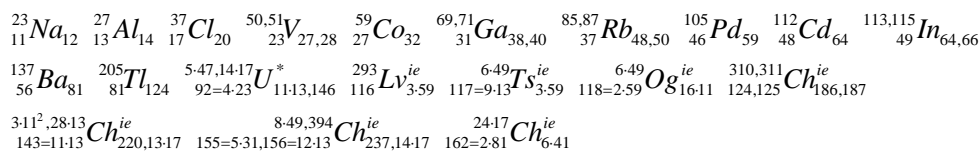
$$\alpha_{1-43} = \frac{13 \cdot 17}{43 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{3 \cdot 67}{200}\right)^{401}} 112 + \frac{1}{8 \cdot (12 \cdot 83 + 1) + \frac{4}{3 \cdot 13}}} = 1/137.035999037436$$



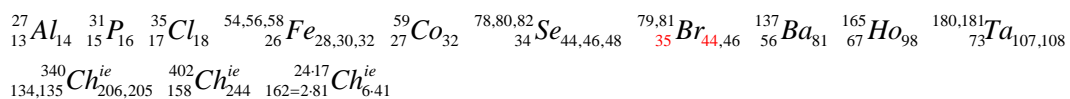
$$\alpha_{1-50} = \frac{2 \cdot 257}{100 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{14 \cdot 13}{181}\right)^{311^2}} 112 + \frac{1}{29 \cdot 61 + \frac{157}{16 \cdot 11}}} = 1/137.035999037436$$



$$\alpha_{1-59} = \frac{3 \cdot 101}{59 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{2 \cdot 27 \cdot 49}{5 \cdot 23^2}\right)^{11 \cdot 13 \cdot 37}} 112 + \frac{1}{48 \cdot 64 \cdot 31 - \frac{17}{81}}} = 1/137.035999037435$$

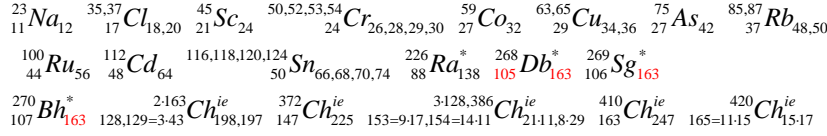


$$\alpha_{1-81} = \frac{4^2 \cdot 26}{9^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{22 \cdot 73}{15 \cdot 107}\right)^{13^2 \cdot 19}} 112 + \frac{1}{2 \cdot 81 \cdot 17 \cdot 67 + \frac{35}{88}}} = 1/137.035999037435$$



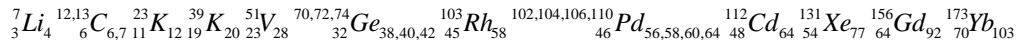
$$\alpha_{1-96} = \frac{17 \cdot 29}{4^2 \cdot 6 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{16 \cdot 3 \cdot 11^2 - 1}{27 \cdot 5 \cdot 43 + 1}\right)^{79 \cdot 147}}} \frac{1}{112 + \frac{163 \cdot (8 \cdot 21 \cdot 37 + 1)}{50 \cdot 10^{11}}}$$

$$= 1/137.035999037435$$



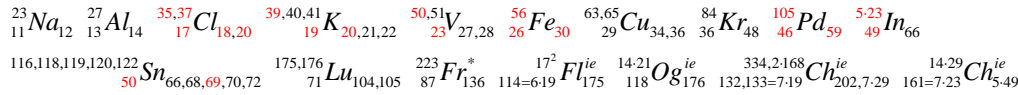
$$\alpha_{1-103} = \frac{23^2}{103 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{3 \cdot 19 \cdot 23}{7 \cdot 11 \cdot 17 + 1}\right)^{2621}}} \frac{1}{112 + \frac{1}{6 \cdot (12 \cdot (8 \cdot (64 \cdot 7 + 1) + 1) + 1) + \frac{3}{4}}}$$

$$= 1/137.035999037435$$



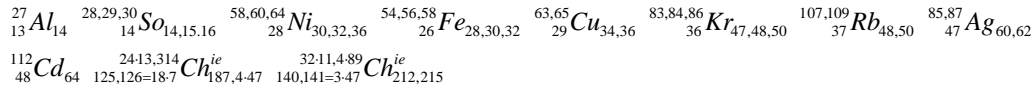
$$\alpha_{1-133} = \frac{683}{133 \cdot (2\pi)_{12389}} \frac{1}{112 + \frac{14651}{50 \cdot 10^{11}}} = \frac{6^2 \cdot 19 - 1}{7 \cdot 19 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{59 \cdot 210}{13 \cdot (8 \cdot 7 \cdot 17 + 1)}\right)^{71 \cdot (12 \cdot 29 + 1)}}} \frac{1}{112 + \frac{7^2 \cdot 13 \cdot 23}{50 \cdot 10^{11}}}$$

$$= 1/137.035999037435$$



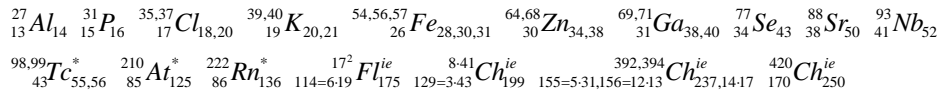
$$\alpha_{1-140} = \frac{6^2 \cdot 20 - 1}{140 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{4 \cdot 13 \cdot 37}{3 \cdot (64 \cdot 10 + 1)}\right)^{3847}}} \frac{1}{112 + \frac{1}{4 \cdot 9 \cdot (2 \cdot 3 \cdot 29 \cdot 47 + 1) + \frac{29}{54}}}$$

$$= 1/137.035999037435$$



$$\alpha_{1-155} = \frac{2^2 \cdot 199}{5 \cdot 31 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left[\frac{19 \cdot 210 - 1}{3^2 \cdot (2 \cdot 13 \cdot 17 + 1) + 1}\right]^{7977}}} \frac{1}{112 + \frac{1}{5 \cdot 17 \cdot 31 \cdot (2 \cdot 13 \cdot 17 + 1) - \frac{15}{43}}}$$

$$= 1/137.035999037435$$



$$\alpha_{1-170} = \frac{873}{170 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{34451}{34450}\right)^{68901}}} \frac{1}{112 + \frac{4171}{8 \times 10^{11}}}$$

$$= \frac{3^2 \cdot 97}{170 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left[\frac{47(12 \cdot 61 + 1)}{2 \cdot 25 \cdot 13 \cdot 53}\right]^{3 \cdot 7 \cdot 17 \cdot 193}}} \frac{1}{112 + \frac{43 \cdot 97}{8 \cdot 10^{11}}} = 1/137.035999037435$$

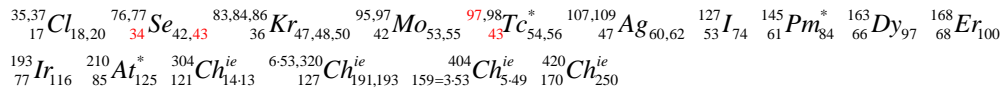
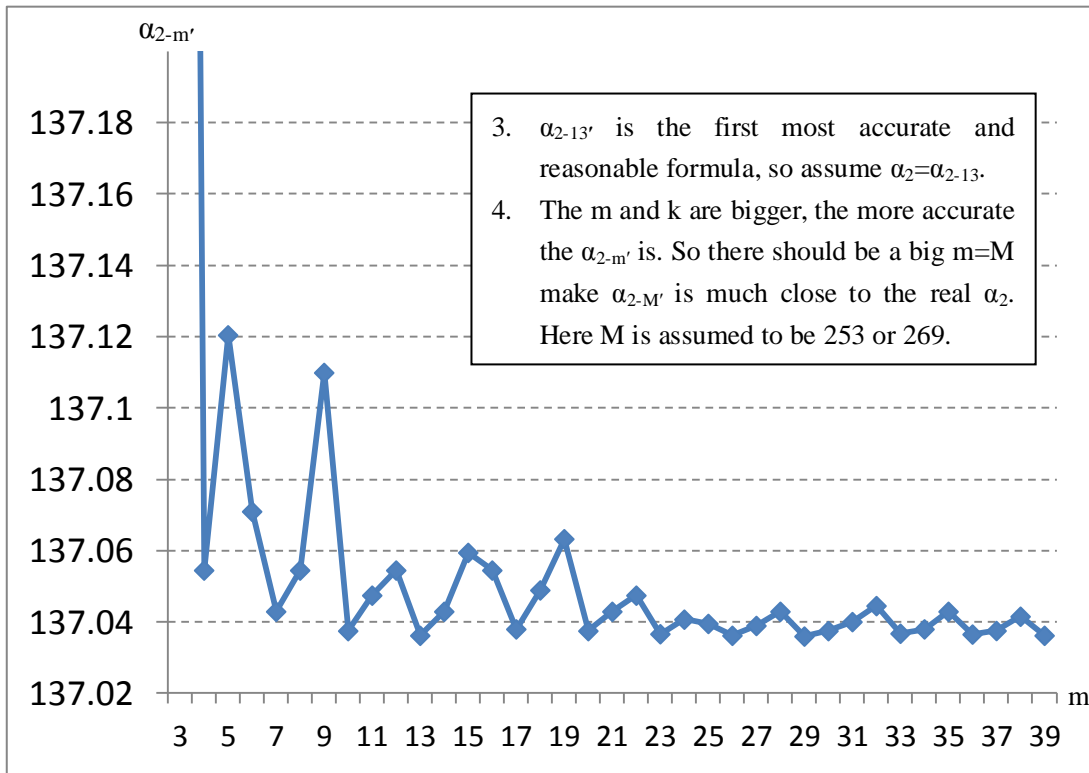


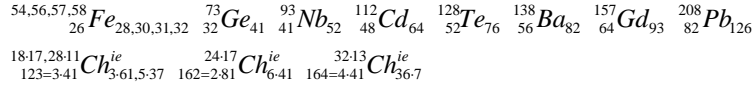
Table 6. Parameters and Results of Approximate Formulas of α_2 (2019/7/3).

m	n	k	$\alpha_{2-m'}$	m	n	k	$\alpha_{2-m'}$
1	8	4	137.933814383	22	170	32	137.047480404
2	16	4	137.933814383	23	177	161	137.036664793
3	24	4	137.933814383	24	185	62	137.040748949
4	31	20	137.054511358	25	193	39	137.039552569
5	39	11	137.120466691	26	200	278	137.036218856
6	47	8	137.070996332	27	208	80	137.038980680
7	54	48	137.042951195	28	216	48	137.042951195
8	62	20	137.054511358	29	223	655	137.036002235
9	70	14	137.109928583	30	231	104	137.037530964
10	77	104	137.037530964	31	239	58	137.040063944
11	85	32	137.047480404	32	247	41	137.044550585
12	93	20	137.054511358	33	254	138	137.036795730
13	100	278	137.036218856	34	262	70	137.038016730
14	108	48	137.042951195	35	270	48	137.042951195
15	116	28	137.059466839	36	277	190	137.036562950
16	124	20	137.054511358	37	285	85	137.037566566
17	131	70	137.038016730	38	293	56	137.041569603
18	139	37	137.048943854	39	300	278	137.036218856
19	147	26	137.063298933	125	961	4293	137.03599678
20	154	104	137.037530964	253	1945	28186	137.035999128
21	162	48	137.042951195	269	2068	41654	137.035999118

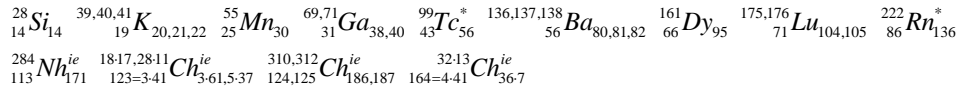
Fig. 8. Results of Approximate Formulas of α_2 (2019/7/3).



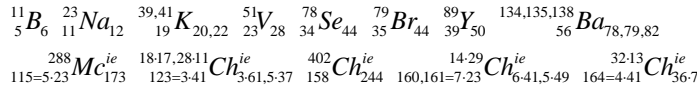
$$\alpha_{2-1} = \frac{e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \frac{e^2}{\left(\frac{5}{4}\right)^9}}{2 \cdot 2^2} \frac{1}{112 - 1 + \frac{1}{3} - \frac{1}{16} + \frac{1}{41 \cdot (12 \cdot 13 + 1) + \frac{13}{41}}} = 1/137.035999111816$$



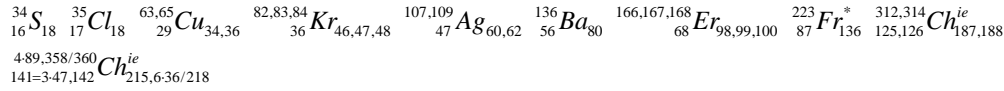
$$\alpha_{2-4} = \frac{2^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{21}{20}\right)^{41}}}{31} \frac{1}{112 - \frac{1}{66} + \frac{1}{71 \cdot (14 \cdot 43 - 1) - \frac{56}{95}}} = 1/137.035999111818$$



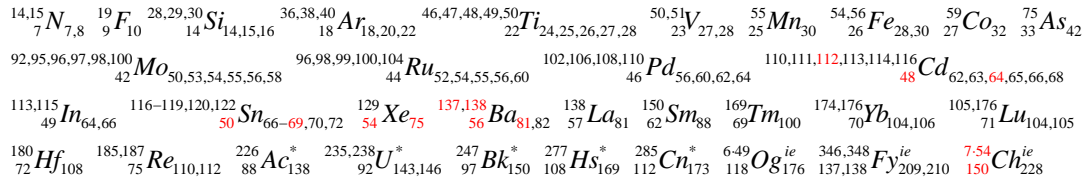
$$\alpha_{2-5} = \frac{5 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{12}{11}\right)^{23}}}{39} \frac{1}{112 - \frac{1}{14} + \frac{1}{10 \cdot 41} - \frac{1}{23 \cdot (14 \cdot 11 \cdot 79 + 1) + \frac{11}{16}}} = 1/137.035999111818$$



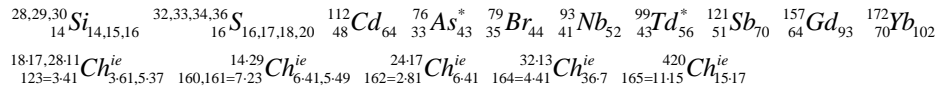
$$\alpha_{2-6} = \frac{6 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{9}{8}\right)^{17}}}{47} \frac{1}{112 - \frac{1}{2 \cdot 17} + \frac{1}{2 \cdot (36 \cdot 17 + 1) - \frac{4}{47}}} = 1/137.035999111818$$



$$\alpha_{2-7} = \frac{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{49}{48}\right)^{97}}}{6 \cdot 3^2} \frac{1}{112 - \frac{1}{11 \cdot 16 + \frac{2 \cdot 23}{3 \cdot 25 \cdot 137}}} = 1/137.035999111819$$



$$\alpha_{2-9} = \frac{3^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{15}{14}\right)^{29}}}{70} \frac{1}{112 - \frac{1}{16} + \frac{1}{11 \cdot 43} - \frac{1}{70 \cdot 17 \cdot (3 \cdot 64 - 1) - \frac{41}{70}}} = 1/137.035999111818$$



$$\alpha_{2-10} = \frac{10 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{5 \cdot 21}{8 \cdot 13}\right)^{11 \cdot 19}}}{77} \frac{1}{112 - \frac{1}{3 \cdot 14 \cdot 19} + \frac{1}{14 \cdot (4 \cdot 27 \cdot (2 \cdot 15 \cdot 19 + 1) - 1)}} = 1/137.035999111818$$

¹⁴₇N₇ ²⁷₁₃Al₁₄ ²⁸₁₄Si₁₄ ^{39,40,41}₁₉K_{20,21,22} ⁵⁶₂₆Fe₃₀ ^{97,99}₄₂Mo_{54,56} ^{131,136}₅₄Xe_{77,82} ^{138,139}₅₇La_{81,82} ¹⁹¹₇₇Ir_{2,57} ²⁰⁹₈₃Bi₁₂₆* ²⁰⁹₈₄Po₁₂₅* ²⁸⁵₁₁₂Cn₁₇₃*

$$\alpha_{2-11} = \frac{11 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{33}{32}\right)^{65}}}{85} \frac{1}{112 - \frac{1}{106} + \frac{1}{30 \cdot (4 \cdot 163 + 1) - \frac{35}{52}}} = 1/137.035999111818$$

⁷⁹₃₅Br₄₄ ⁷⁹₃₅Br₄₄ ^{96,99,100,104}₄₄Ru_{52,55,56,60} ¹²⁵₅₂Te₇₃ ¹²⁷₅₃I₇₄ ¹⁴⁵₆₀Nd₈₅ ²¹⁰₈₅At₁₂₅* ²⁶⁸₁₀₅Db₁₆₃* ²⁶⁹₁₀₆Sg₁₆₃* ²⁷⁰₁₀₇Bh₁₆₃*
^{2,163}_{128,129}Ch_{198=18-11,197}^{ie} ⁴¹⁰₁₆₃Ch₂₄₇^{ie} ⁴²⁰₁₆₅₌₅₋₃₃Ch₁₅₋₁₇^{ie} ⁴²⁰₁₇₀Ch₂₅₀^{ie}

$$\alpha_2 = \alpha_{2-13} = \frac{13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{9 \cdot 31}{2 \cdot 139}\right)^{557}}}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} = 1/137.035999111818$$

⁷₃Li₄ ¹⁹₉F₁₀ ²⁷₁₃Al₁₄ ²⁹₁₄Si₁₅ ^{54,56,57,58}₂₆Fe_{28,30,31,32} ⁵⁸₂₈Ni₃₀ ^{63,65}₂₉Cu_{34,36} ^{69,71}₃₁Ga_{38,40} ⁷⁴₃₂Ge₄₂ ⁸⁹₃₉Y₅₀ ⁹³₄₁Nb₅₂ ⁹⁶₄₂Mo₅₄
¹⁰⁰₄₄Ru₅₆ ^{112,116}₄₈Cd_{64,68} ¹¹⁶₅₀Sn₆₆ ¹²⁸₅₂Te₇₆ ^{136,137,138}₅₆Ba_{80,81,82} ¹³⁹₅₇La₈₂ ^{136,138,140,142}₅₈Ce_{78,80,82,84} ¹⁴⁹₆₂Sm₈₇ ^{156,157,160}₆₄Gd_{92,93,96}
¹⁶²₆₆Dy₉₆ ^{166,167,168}₆₈Er_{98,99,100} ¹⁶⁹₆₉Tm₁₀₀ ¹⁷⁵₇₁Lu_{81,83} ¹⁸⁷₇₅Re₁₁₂ ^{188,189,190,192}₇₆Os_{112,113,114,116} ²⁰⁰₈₀Hg₁₂₀ ²⁰⁸₈₂Pb₁₂₆ ²⁰⁹₈₃Bi₁₂₆*
²⁰⁹₈₄Po₁₂₅* ²¹⁰₈₅At₁₂₅* ²²³₈₇Fr₁₃₆* ²³⁷₉₃Np₁₄₄* ²⁴⁷₉₆Cm₁₅₁* ²⁵⁷₁₀₀Fm₁₅₇* ¹⁵⁻¹⁹₁₁₂Cn₁₇₃* ⁴⁻⁷¹₁₁₃Nh₁₇₁^{ie} ^{310,312}_{124,125}Ch_{186,187,192}^{ie} ³¹⁸₁₂₆Ch₁₉₂^{ie} ³³⁰₁₃₀Ch₂₀₀^{ie}
^{344,346,348}_{136,137,138}Fy_{208,209,210}^{ie} ¹²⁻²⁹₁₃₉Ch₂₀₉^{ie} ⁴⁰⁰₁₅₇Ch₂₄₃^{ie} ⁶⁻⁷¹₁₆₉Ch₂₅₇^{ie}

$$\alpha_{2-15} = \frac{15 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{29}{28}\right)^{57}}}{2^2 \cdot 29} \frac{1}{112 - \frac{1}{4 \cdot 13} + \frac{1}{12 \cdot (36 \cdot 43 + 1) - \frac{1}{16}}} = 1/137.035999111818$$

^{24,25,26}₁₂Mg_{12,13,14} ³¹₁₅P₁₆ ⁵⁶₂₆Fe₃₀ ^{63,65}₂₉Cu_{34,36} ^{84,86}₃₆Kr_{48,50} ⁹⁹₄₃Tc₅₆* ²²²₈₆Rn₁₃₆* ²⁹³₁₁₆₌₄₋₂₉Ch₃₋₅₉^{ie}

$$\alpha_{2-17} = \frac{17 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{71}{70}\right)^{3 \cdot 47}}}{131} \frac{1}{112 - \frac{1}{6 \cdot 101} + \frac{1}{23 \cdot (30 \cdot 35^2 - 1) + \frac{6}{23}}} = 1/137.035999111818$$

³⁵₁₇Cl₁₈ ⁵¹₂₃V₂₈ ^{64,66,68}₃₀Zn_{34,36,38} ^{80,82}₃₄Se_{46,48} ⁸¹₃₅Br₄₆ ⁸³₃₆Kr₄₇ ¹⁰⁶₄₆Pd₆₀ ¹⁰⁷₄₇Ag₆₀ ^{121,122}₅₁Sb_{70,71} ¹⁴⁴₆₀Nd₈₄ ^{171,172}₇₀Yb_{101,102}
¹⁷⁵₇₁Lu₁₀₄ ²⁶²₁₀₃Lr₁₅₉* ^{312,314}_{125,126}Ch_{11-17,4-47}^{ie} ^{330,4-83}_{130,131}Ch_{200,201}^{ie} ^{4-89,358/360}_{141=3-47,142=2-71}Ch_{5-43,6-36/218}^{ie}

$$\alpha_{2-18} = \frac{18 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{38}{37}\right)^{75}}}{139} \frac{1}{112 - \frac{1}{2 \cdot 47} + \frac{1}{2 \cdot 31 \cdot (2 \cdot 136 - 1) + \frac{83}{137}}} = 1/137.035999111818$$

³¹₁₅P₁₆ ^{32,33,34,36}₁₆S_{16,17,18,20} ^{35,37}₁₇Cl_{18,20} ^{36,38}₁₈Ar_{18,20} ⁶⁹₃₁Ga₃₈ ^{70,72,74,76}₃₂Ge_{38,40,42,44} ^{74,76}₃₄Se_{40,42} ^{83,84}₃₆Kr_{47,48}
¹⁰⁹₄₇Ag₆₂ ^{148,150,152}₆₂Sm_{86,88,90} ^{130,132,136,137,138}₅₆Ba_{74,76,80,81,82} ^{185,187}₇₅Re_{110,112} ²⁰⁹₈₃Bi₁₂₆*
^{310,312,314}_{124,125,126}Ch_{186,187,188}^{ie} ^{344,346,348}_{136,137,138}Fy_{208,209,210}^{ie} ³⁴⁸₁₃₉Ch₂₀₉^{ie} ⁴¹⁸₁₆₆Ch₂₅₂^{ie}

$$\alpha_{2-19} = \frac{19 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{27}{26}\right)^{53}}}{3 \cdot 49} \frac{1}{112 - \frac{1}{44} + \frac{1}{16 \cdot (4 \cdot 37 + 1) - \frac{23}{6 \cdot 47 + 1}}} = 1/137.035999111818$$

²³Na₁₁ ^{39,40,41}K₁₉ ^{46,47,48,49,50}Ti₂₂ ⁵¹V₂₃ ^{54,56,57,58}Fe₂₆ ⁵⁹Co₂₇ ⁸³Kr₃₆ ^{85,87}Rb₃₇ ⁹⁵Mo₄₂
^{98,100}Ru₄₄ ^{107,109}Ru₄₇ ^{113,115}In₄₉ ¹²⁷I₅₃ ⁶⁻⁴⁹Og₁₁₈ ¹⁷⁶⁼¹⁶⁻¹¹ ^{32-11,4-89}Ch₁₁₈ ^{370,22-17}Ch₁₁₈ ^{404/14-29}Ch₁₁₈

$$\alpha_{2-23} = \frac{23 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{2 \cdot 81}{7 \cdot 23}\right)^{17 \cdot 19}}}{3 \cdot 59} \frac{1}{112 - \frac{1}{2 \cdot (40 \cdot 23 - 1) + \frac{9}{32 \cdot 10}}} = 1/137.035999111818$$

¹⁹F₉ ^{35,37}Cl₁₇ ³⁹K₁₉ ^{40,43}Ca₂₀ ^{50,51}V₂₃ ^{70,72}Ge₃₂ ^{80,82}Se₃₄ ^{90,91,94}Zr₄₀
^{102,105,110}Pd₄₆ ^{136,137,138}Ba₅₆ ⁶⁻⁴⁹Ts₁₁₇

$$\alpha_{2-24} = \frac{2^2 \cdot 6 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{63}{2 \cdot 31}\right)^{125}}}{5 \cdot 37} \frac{1}{112 - \frac{1}{257} + \frac{1}{10 \cdot (12 \cdot 13 \cdot 83 + 1) + \frac{23}{81}}} = 1/137.035999111818$$

^{24,24,26}Mg₁₂ ²⁷Al₁₃ ^{50,51}V₂₃ ^{50,52,54}Cr₂₄ ^{54,56,57}Fe₂₆ ^{85,87}Rb₃₇ ⁸⁹Rb₃₉ ¹⁰⁸Pd₄₆
^{110,111,112}Cd₄₈ ^{124,125,126,130}Te₅₂ ¹³⁷Ba₅₆ ^{184,186}W₇₄ ²⁰⁵Tl₈₁ ^{208,209}Bi₈₃ ²⁵⁷Fm₁₀₀
^{310,24-13}Ch₁₂₄ ^{32-13,418}Ch₁₆₆ ⁶⁻⁷¹Ch₁₆₉

$$\alpha_{2-25} = \frac{5^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{40}{3 \cdot 13}\right)^{79}}}{193} \frac{1}{112 - \frac{1}{8 \cdot 43} + \frac{1}{18 \cdot 23 \cdot (32 \cdot 27 - 1) - \frac{3}{7}}} = 1/137.035999111818$$

^{24,25,26}Mg₁₂ ⁴⁰Ar₁₈ ^{50,51}V₂₃ ⁵⁵Mn₂₅ ⁵⁹Co₂₇ ^{72,74}Ge₃₂ ^{92,96,98}Mo₄₂ ^{97,99}Tc₄₃ ¹⁹³Co₇₇
^{192,194}Pt₇₈ ²²²Rn₈₆ ²²⁶Ra₈₈ ^{24-13,314}Ch₁₂₅ ^{318,320}Ch₁₂₇ ^{8-43,348}Fy₁₂₇ ^{3-128,2-193}Ch₁₅₃

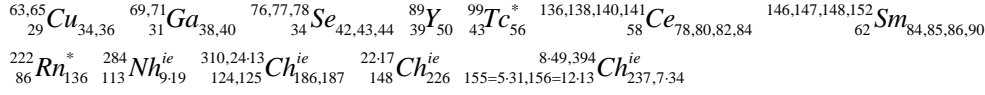
$$\alpha_{2-27} = \frac{3 \cdot 3^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{81}{80}\right)^{7 \cdot 23}}}{4^2 \cdot 13} \frac{1}{112 - \frac{1}{10 \cdot 41} + \frac{1}{2 \cdot 27 \cdot 43 \cdot (3 \cdot 64 + 1) - \frac{19}{26}}} = 1/137.035999111818$$

²⁷Al₁₃ ^{39,40,41}K₁₉ ^{50,51}V₂₃ ^{56,58}Fe₂₆ ⁵⁹Co₂₇ ^{86,87,88}Sr₃₈ ⁹³Nb₄₁ ^{97,99}Tc₄₃ ¹¹²Cd₄₈
^{126,128,130}Te₅₂ ¹⁶⁻¹³Pt₈₂ ^{8-43,346,348}Fy₁₂₆ ¹⁴⁻²⁹Ch₁₆₁ ²⁴⁻¹⁷Ch₁₆₂ ³²⁻¹³Ch₁₆₄

$$\alpha_{2-29} = \frac{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{16 \cdot 41}{5 \cdot 131}\right)^{3 \cdot 19 \cdot 23}}}{223} \frac{1}{112 - \frac{1}{29 \cdot 59 \cdot (12 \cdot 19 + 1) + \frac{19}{29}}} = 1/137.035999111818$$

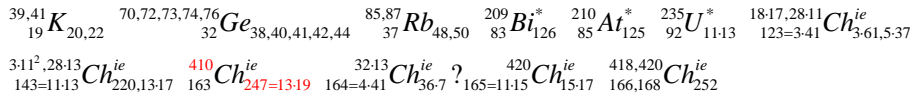
^{39,41}K₁₉ ^{63,65}Cu₂₉ ⁷³Ge₃₂ ¹³¹Xe₅₄ ¹³⁹La₅₇ ¹⁴¹Pr₅₉ ¹⁶⁹Tm₆₉ ^{9-23,208}Pb₈₂ ²²³Fr₈₇
^{330,332}Ch₁₃₀ ¹⁶⁻²³Ch₁₄₅ ¹⁴⁻²⁹Ch₁₆₁ ³²⁻¹³Ch₁₆₄

$$\alpha_{2-31} = \frac{31 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{59}{58}\right)^{9 \cdot 13}}}{7 \cdot 34 + 1} \frac{1}{112 - \frac{1}{7 \cdot 43 + \frac{9}{5 \cdot 113}}} = 1/137.035999111819$$

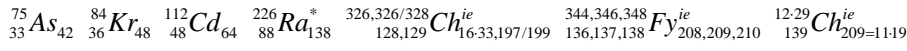


$$\alpha_{2-32} = \frac{2 \cdot 4^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{42}{41}\right)^{83}}}{13 \cdot 19} \frac{1}{112 - \frac{1}{11 \cdot 13} + \frac{1}{6 \cdot 37 \cdot (5 \cdot 210 - 1) + \frac{10}{11}}} = 1/137.035999111818$$

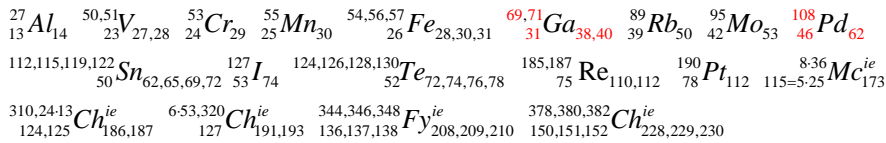
$$= 1/137.035999111818$$



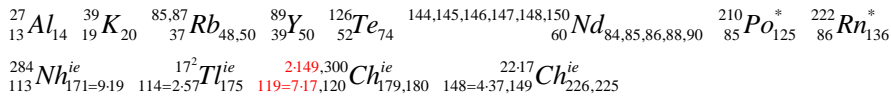
$$\alpha_{2-33} = \frac{33 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{139}{138}\right)^{277}}}{2 \cdot (2 \cdot 8^2 - 1)} \frac{1}{112 - \frac{1}{32 \cdot 48 - \frac{36}{35 \cdot 13}}} = 1/137.035999111818$$



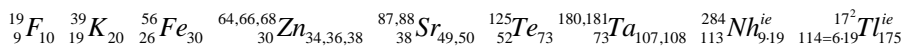
$$\alpha_{2-36} = \frac{6^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{191}{190}\right)^{3 \cdot 127}}}{2 \cdot 138 + 1} \frac{1}{112 - \frac{1}{10 \cdot 7 \cdot 31 + \frac{13}{25 \cdot 23}}} = 1/137.035999111818$$



$$\alpha_{2-37} = \frac{37 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{2 \cdot 43}{5 \cdot 17}\right)^{9 \cdot 19}}}{3 \cdot 5 \cdot 19} \frac{1}{112 - \frac{1}{4 \cdot 3 \cdot 5 \cdot 13} + \frac{1}{5 \cdot 37^2 \cdot 149}}} = 1/137.035999111818$$

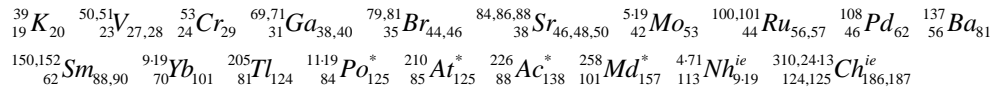


$$\alpha_{2-38} = \frac{2 \cdot 19 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{3 \cdot 19}{56}\right)^{113}}}{6 \cdot 7^2 - 1} \frac{1}{112 - \frac{1}{3 \cdot 73} + \frac{1}{30(8 \cdot 27 \cdot 17 + 1) - \frac{12}{13}}} = 1/137.035999111816$$



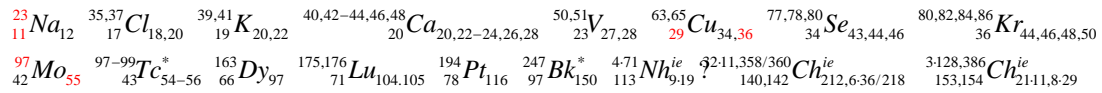
$$\alpha_{2-125} = \frac{5 \cdot 5^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{4294}{4293}\right)^{8587}}}{31^2} \frac{1}{112 - \frac{1}{2159481}}$$

$$= \frac{5 \cdot 5^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{2 \cdot 19 \cdot 113}{81 \cdot 53}\right)^{31 \cdot (12 \cdot 23 + 1)}}}{31^2} \frac{1}{112 - \frac{1}{3 \cdot 101 \cdot (8 \cdot 81 \cdot 11 - 1)}}} = 1/137.035999111818$$



$$\alpha_{2-253} = \frac{253 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{28187}{28186}\right)^{56373}}}{1945} \frac{1}{112 - \frac{10411}{8 \times 10^{11}}}$$

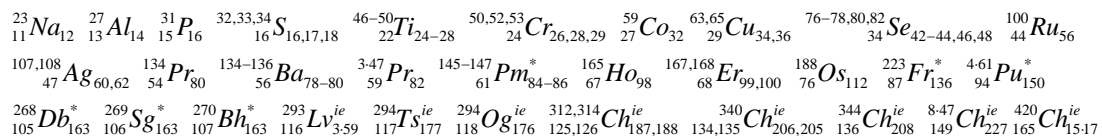
$$= \frac{11 \cdot 23 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left[\frac{71(36 \cdot 11 + 1)}{2 \cdot 17(36 \cdot 23 + 1)}\right]^{3 \cdot 19 \cdot 23 \cdot 43}}}{5(4 \cdot 97 + 1)} \frac{1}{112 - \frac{29(360 - 1)}{8 \times 10^{11}}} = 1/137.035999111818$$



$$\alpha_{2-269} = \frac{(270 - 1) \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{41655}{41654}\right)^{83309}}}{2068} \frac{1}{112 - 5.317 \times 10^{-9}}$$

$$= \frac{(4 \cdot 67 + 1) \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{15 \cdot (6 \cdot (16 \cdot 29 - 1) - 1)}{2 \cdot 59 \cdot (6 \cdot 59 - 1)}\right)^{227 \cdot (6 \cdot 61 + 1)}}}{4 \cdot 11 \cdot 47} \frac{1}{112 - \frac{13 \cdot (24 \cdot 17 + 1)}{10^{12}}}$$

$$= 1/137.035999111818$$



In above formulas, there are many amazing coincidences. As 136=8×17 and 138=6×23, 17 and 23 both appear in α₁₋₁, α₁₋₁₇, α₁₋₂₂, α₁₋₂₃, α₁₋₂₅, α₁₋₅₉, α₁₋₁₀₃, α₁₋₁₃₃, α₂₋₁₇ and α₂₋₂₃, 17 frequently appears in α₁ and 23 frequently appears in α₂. 157 and 257 in α₁₋₅₀ should relate to ¹⁰⁰Fm₁₅₇*, 173 in α₁₋₁₆ should relate to ¹¹²Cn₁₇₃*, and so on. As the factors in formulas of α are reasonably assumed to relate to nuclides, some ideal extended elements such as ^{136,137,138}Fy_{208,209,210} and ¹⁶⁹Ch₂₅₇ are predicted.

15. Radius of Electron and Proton

The classical electron radius r_e has been calculated very accurately. However, the proton charge radius r_p hasn't yet been determined precisely. Recent two experiments

measured r_p and had given the best results up to now which was $r_p=0.833(19) \text{ fm}^9$ and $r_p=0.831(19) \text{ fm}^{10}$, and hence CODATA revised its recommended data of r_p to $0.8414(19) \text{ fm}$. Here we give our calculation results of r_e and r_p . And it seems there is α_p similar to α . α_p could be called “the second fine-structure constant”.

Ratio of Bohr radius of hydrogen atom to classical electron radius:

$$\frac{a_0}{r_e} = \frac{1}{\alpha_c^2} = \frac{1}{\alpha_1 \alpha_2} = 112 \times \left(168 - \frac{1}{3} + \frac{1}{2^2 \cdot 3 \cdot 47} - \frac{1}{2 \cdot 3 \cdot 29 \cdot 53 \cdot 59 - 79 / 47} \right) = 18788.865042381$$

$$r_e = \alpha_c^2 a_0 = \alpha_1 \alpha_2 a_0 = \frac{5.29177210903(80) \times 10^{-11} \text{ m}}{18788.865042381} = 2.81794032658(43) \text{ fm}$$

Comparable to CODATA recommended value $r_e = 2.8179403262(13) \text{ fm}$ but more precise.

Ratio of Bohr radius of hydrogen atom to the proton charge radius should have the similar form, and is assumed to have the following hypothetical formulas:

$$\frac{a_0}{r_p} = \frac{1}{\alpha_{p/c}^2} = \frac{1}{\alpha_{p/1} \alpha_{p/2}} = 225 \cdot \left(282 + \frac{1}{3} - \frac{1}{12 \cdot 47} + \frac{1}{6 \cdot 29 \cdot 53 \cdot 59 - 79 / 47} \right) = 63524.60147736$$

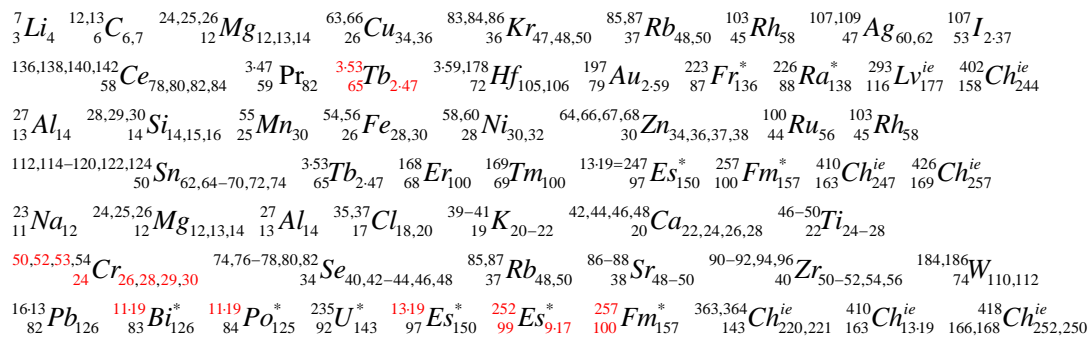
$$= 247 \cdot \left(257 + \frac{1}{5} - \frac{1}{5 \cdot 13} + \frac{1}{30 \cdot (28 \cdot (2 \cdot 100 - 1) + 1) + \frac{8}{45}} \right)$$

$$= \left(252 + \frac{1}{24} - \frac{1}{2 \cdot 17 \cdot 37} + \frac{1}{11 \cdot 13 \cdot 19 \cdot (2 \cdot 11 \cdot 19 + 1) + \frac{11}{20}} \right)^2 = 252.040872632515^2$$

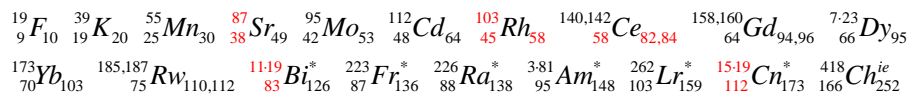
$$r_p = \alpha_{p/c}^2 a_0 = \alpha_{p/1} \alpha_{p/2} a_0 = \frac{5.29177210903(80) \times 10^{-11} \text{ m}}{63524.60147736} = 0.833027202999(13) \text{ fm}$$

$\alpha_{p/c} \approx \alpha_{p/1} \approx \alpha_{p/2} \approx 252.04$, α_p could be called the second fine-structure constant.

2019/12/19-23

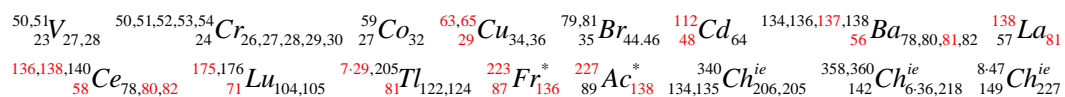


$$\alpha_{p/1} = \frac{5 \cdot 3^2}{8 \cdot (2\pi)_{58}} \frac{1}{225 + \frac{1}{4 \cdot 112} - \frac{1}{5 \cdot 19 \cdot 83 \cdot 103 + \frac{19}{20}}} = 1 / 252.040872632515$$



$$\alpha_{p/2} = \frac{23 \cdot (2\pi)_{227}}{2 \cdot 9^2} \frac{1}{225 - \frac{1}{3 \cdot 16 \cdot 29 - \frac{71}{2 \cdot 67}}} = 1 / 252.040872632514$$

2020/1/2



$$\alpha_{p/2} = \frac{22 \cdot (2\pi)_{164}}{5 \cdot 31} \frac{1}{225 - \frac{1}{7 \cdot 137 - \frac{7 \cdot 13}{197}}} = 1/252.040872632512 \quad 2020/1/3$$

47,48,49,50 ²²Ti_{25,26,27,28} 56,57 ²⁶Fe_{30,31} 69,71 ³¹Ga_{38,40} 89 ³⁹Y₅₀ 99,100 ⁴⁴Ru_{55,56} 134,136,137,138 ⁵⁶Ba_{78,80,81,82} 5-31 ⁶⁴Gd_{7,13} 164 ⁶⁶Dy₉₈ 196,198 ⁷⁸Pt_{118,120}

$$\alpha_{p/2} = \frac{21 \cdot (2\pi)_{126}}{2^2 \cdot 37} \frac{1}{225 - \frac{1}{16 \cdot 29} + \frac{1}{20 \cdot 13^2 \cdot 179 + \frac{8}{17}}} = 1/252.040872632515 \quad 2020/1/3$$

197 ⁷⁹Au₁₁₈ 206,207,208 ⁸²Pb_{124,125,126} 223 ⁸⁷Ra₁₃₆ 226 ⁸⁸Ra₁₃₈ 310,312 ⁸⁸Ch_{186,187}^{ie} 326,326/328 ⁸⁸Ch_{128,129}^{ie} 12-31 ¹⁴⁷Ch_{198,197/199}^{ie} 155=5-31,156=12-13 8-49,2-197 ¹⁶⁴Ch_{237,238}^{ie} 32-13 ¹⁶⁴Ch₂₅₂^{ie}

45 ²¹Sc₂₄ 63,65 ²⁹Cu_{34,36} 85,87 ³⁷Rb_{48,50} 126 ⁵²Te₇₄ 148 ⁶⁰Nd₈₈ 169 ⁶⁹Tm₁₀₀ 179 ⁷²Hf₁₀₇ 16-13 ⁸²Pb₁₂₆ 298,300 ^{119,120}Ch_{179,180}^{ie} 312,314 ^{125,126}Ch_{187,188}^{ie} 426 ¹⁶⁹Ch₂₅₇^{ie}

16. Direct Relationships between 2π and Nuclides

In Chen's formulas of the fine-structure constant, there are 2π-e formulas, in which k gets certain numbers and relate to nucleon numbers of some nuclides. So in the end of this paper we feel curious about whether 2π directly relate to nuclides.

$$2\pi = 6.2831853 \dots \approx \frac{4 \cdot 157}{100} = 6.28 \approx \frac{3 \cdot 7 \cdot 44 \cdot 68}{100^2} = 6.2832 \quad \begin{matrix} 7 \\ 3 \end{matrix} \text{Li}_4 \quad \begin{matrix} 100 \\ 44 \end{matrix} \text{Ru}_{56} \quad \begin{matrix} 157 \\ 64 \end{matrix} \text{Gd}_{93} \quad \begin{matrix} 168 \\ 68 \end{matrix} \text{Er}_{100} \quad \begin{matrix} 257 \\ 100 \end{matrix} \text{Fm}_{157}^* \quad \begin{matrix} 400 \\ 157 \end{matrix} \text{Ch}_{243}^{\text{ie}}$$

$$2\pi \approx \frac{4 \cdot 157}{100} = \frac{157}{25} = 6.28 \quad \begin{matrix} 55 \\ 25 \end{matrix} \text{Mn}_{30} \quad \begin{matrix} 100 \\ 44 \end{matrix} \text{Ru}_{56} \quad \begin{matrix} 157 \\ 64 \end{matrix} \text{Gd}_{93} \quad \begin{matrix} 118,119,120 \\ 50 \end{matrix} \text{Sn}_{68,69,70} \quad \begin{matrix} 168 \\ 68 \end{matrix} \text{Er}_{100} \quad \begin{matrix} 169 \\ 69 \end{matrix} \text{Tm}_{100} \quad \begin{matrix} 185,187 \\ 75 \end{matrix} \text{Re}_{110,112}$$

$$2\pi \approx \frac{16 \cdot 3 \cdot 7 \cdot 11 \cdot 17}{100^2} = \frac{48 \cdot 7 \cdot 11 \cdot 17}{100^2} = \frac{3 \cdot 7 \cdot 44 \cdot 68}{100^2} = \frac{3 \cdot 112 \cdot 11 \cdot 17}{100^2} = \frac{2 \cdot 168 \cdot 11 \cdot 17}{100^2} = \frac{2 \cdot 3 \cdot 7 \cdot 11 \cdot 136}{100^2} = \dots = 6.2832$$

⁷Li₄ ²⁰⁻²²Ne₁₀₋₁₂ ²³Na₁₂ ⁴⁵Sc₂₄ 46,47,49,50 ²²Ti_{24,25,27,28} 61 ²⁸Ni₃₃ ⁵⁵Mn₃₀ 54,56 ²⁶Fe_{28,30} 78,80 ³⁴Se_{44,46} 98,100 ⁴²Mo_{56,58} ¹⁰⁰Ru₅₆ ¹¹²Cd₆₄

¹³⁶Ba₈₀ ¹⁶⁸Er₁₀₀ ^{199,200}Hg_{119,120} 185,114,17 ⁷⁵Re_{110,112} 209 ⁸⁴Po₁₂₅ ²¹⁰At₈₅ ²²²Rn₁₃₆ ²²³Fa₁₃₆ ²²⁶Ra₁₃₈ ²⁸⁵Cn₁₇₃ ³⁴⁴⁻³⁴⁸Fy₂₀₈₋₂₁₀ ⁴²⁰Ch₂₅₂^{ie}

$$2\pi \approx \frac{44}{7} = \frac{2 \cdot 22}{7} = 6.2857 \dots \quad \begin{matrix} 50 \\ 22 \end{matrix} \text{Ti}_{28} \quad \begin{matrix} 61 \\ 28 \end{matrix} \text{Ni}_{33} \quad \begin{matrix} 100 \\ 44 \end{matrix} \text{Ru}_{56} \quad \begin{matrix} 136,137,138 \\ 56 \end{matrix} \text{Ba}_{80,81,82} \quad \begin{matrix} 226 \\ 88 \end{matrix} \text{Ra}_{138} \quad \begin{matrix} 294 \\ 118 \end{matrix} \text{Og}_{176} \quad \begin{matrix} 8-44 \\ 140 \end{matrix} \text{Ch}_{212}^{\text{ie}} \quad \begin{matrix} 22-17 \\ 148=4 \cdot 37 \end{matrix} \text{Ch}_{226}^{\text{ie}}$$

$$2\pi \approx \frac{201}{32} = \frac{3 \cdot 67}{32} = 6.2812 \dots \quad \begin{matrix} 32 \\ 16 \end{matrix} \text{S}_{16} \quad \begin{matrix} 59 \\ 16 \end{matrix} \text{Co}_{27} \quad \begin{matrix} 67 \\ 32 \end{matrix} \text{Zn}_{30} \quad \begin{matrix} 112 \\ 48 \end{matrix} \text{Cd}_{64} \quad \begin{matrix} 117 \\ 50 \end{matrix} \text{Sn}_{67} \quad \begin{matrix} 128,134 \\ 54 \end{matrix} \text{Xe}_{74,80} \quad \begin{matrix} 134 \\ 56 \end{matrix} \text{Ba}_{78} \quad \begin{matrix} 165 \\ 67 \end{matrix} \text{Ho}_{98} \quad \begin{matrix} 201 \\ 80 \end{matrix} \text{Hg}_{121} \quad \begin{matrix} 332 \\ 131 \end{matrix} \text{Ch}_{201}^{\text{ie}} \quad \begin{matrix} 402 \\ 158 \end{matrix} \text{Ch}_{244}^{\text{ie}}$$

$$2\pi \approx \frac{245}{39} = \frac{5 \cdot 7^2}{3 \cdot 13} = 6.2820 \dots \quad \begin{matrix} 7 \\ 3 \end{matrix} \text{Li}_4 \quad \begin{matrix} 27 \\ 13 \end{matrix} \text{Al}_{14} \quad \begin{matrix} 54,56 \\ 26 \end{matrix} \text{Fe}_{28,30} \quad \begin{matrix} 89 \\ 39 \end{matrix} \text{Y}_{50} \quad \begin{matrix} 79,81 \\ 35 \end{matrix} \text{Br}_{44,46} \quad \begin{matrix} 113,115 \\ 49 \end{matrix} \text{In}_{64,66} \quad \begin{matrix} 24-13,314 \\ 125,126 \end{matrix} \text{Ch}_{187,188}^{\text{ie}}$$

$$2\pi \approx \frac{289}{46} = \frac{17^2}{2 \cdot 23} = 6.2826 \dots \quad \begin{matrix} 3-17 \\ 23 \end{matrix} \text{V}_{28} \quad \begin{matrix} 78,80 \\ 34 \end{matrix} \text{Se}_{44,46} \quad \begin{matrix} 6-17 \\ 46 \end{matrix} \text{Pd}_{56} \quad \begin{matrix} 168 \\ 68 \end{matrix} \text{Er}_{100} \quad \begin{matrix} 169 \\ 69 \end{matrix} \text{Tm}_{100} \quad \begin{matrix} 136,137,138 \\ 56 \end{matrix} \text{Ba}_{80,81,82} \quad \begin{matrix} 11-17 \\ 75 \end{matrix} \text{Re}_{112} \quad \begin{matrix} 222 \\ 86 \end{matrix} \text{Rn}_{136}^*$$

²²³Fa₁₃₆ ²²⁶Ra₁₃₈ ²²⁷Ac₁₃₈ ²³⁸U₁₄₆ ¹⁷⁻¹⁷Fl₁₇₅^{ie} ^{344,346,348}Fy_{208,209,210}^{ie} ²²⁻¹⁷Ch₂₂₆^{ie}

$$2\pi \approx \frac{333}{53} = \frac{9 \cdot 37}{53} = 6.2830 \dots \quad \begin{matrix} 85,87 \\ 37 \end{matrix} \text{Rb}_{48,50} \quad \begin{matrix} 3 \cdot 37=111 \\ 48 \end{matrix} \text{Cd}_{7-9} \quad \begin{matrix} 127 \\ 53 \end{matrix} \text{I}_{74} \quad \begin{matrix} 180,184,189 \\ 74 \end{matrix} \text{W}_{106,110,112} \quad \begin{matrix} 222 \\ 86 \end{matrix} \text{Rn}_{136}^* \quad \begin{matrix} 269 \\ 106 \end{matrix} \text{Sg}_{163} \quad \begin{matrix} 280 \\ 111 \end{matrix} \text{Rg}_{169} \quad \begin{matrix} 22-17 \\ 148 \end{matrix} \text{Ch}_{226}^{\text{ie}}$$

$$2\pi \approx \frac{377}{60} = \frac{13 \cdot 29}{4 \cdot 3 \cdot 5} = 6.2833 \dots \quad \begin{matrix} 24,25,26 \\ 12 \end{matrix} \text{Mg}_{12,13,14} \quad \begin{matrix} 28,29,30 \\ 14 \end{matrix} \text{Si}_{14,15,16} \quad \begin{matrix} 31 \\ 15 \end{matrix} \text{P}_{16} \quad \begin{matrix} 54,56 \\ 26 \end{matrix} \text{Fe}_{28,30} \quad \begin{matrix} 63,65 \\ 29 \end{matrix} \text{Cu}_{34,36} \quad \begin{matrix} 116,120 \\ 50 \end{matrix} \text{Sn}_{66,70}$$

^{140,142}Ce_{82,84} ^{144,145,146,148,150}Nd_{84,85,86,88,90} ²⁰⁰Hg₁₂₀ ²²³Fa₁₃₆ ^{24-13,314}Ch_{187,188}^{ie}

$$2\pi \approx \frac{465}{74} = \frac{30 \cdot 31}{4 \cdot 37} = 6.2837 \dots \quad \begin{matrix} 31 \\ 15 \end{matrix} \text{P}_{16} \quad \begin{matrix} 67 \\ 30 \end{matrix} \text{Zn}_{37} \quad \begin{matrix} 69,71 \\ 31 \end{matrix} \text{Ga}_{38,40} \quad \begin{matrix} 6-31 \\ 74 \end{matrix} \text{W}_{112} \quad \begin{matrix} 85,67 \\ 37 \end{matrix} \text{Rb}_{48,50} \quad \begin{matrix} 4-37 \\ 60 \end{matrix} \text{Nd}_{88} \quad \begin{matrix} 157 \\ 64 \end{matrix} \text{Gd}_{93} \quad \begin{matrix} 243 \\ 95 \end{matrix} \text{Am}_{4-37}^* \quad \begin{matrix} 22-17 \\ 148=4 \cdot 37 \end{matrix} \text{Ch}_{226}^{\text{ie}}$$

$$2\pi \approx \frac{509}{81} = \frac{2 \cdot 3 \cdot 5 \cdot 17 - 1}{9^2} = 6.2839 \dots \quad \begin{matrix} 19 \\ 9 \end{matrix} \text{F}_{10} \quad \begin{matrix} 35,37 \\ 17 \end{matrix} \text{Cl}_{18,20} \quad \begin{matrix} 64,70 \\ 30 \end{matrix} \text{Zn}_{34,40} \quad \begin{matrix} 80,82 \\ 34 \end{matrix} \text{Se}_{46,48} \quad \begin{matrix} 136,137,138 \\ 56 \end{matrix} \text{Ba}_{80,81,82} \quad \begin{matrix} 203,205 \\ 81 \end{matrix} \text{Tl}_{61,62} \quad \begin{matrix} 210 \\ 85 \end{matrix} \text{At}_{125}^*$$

³⁻⁸¹Am₁₄₈ ^{344,346,348}Fy_{208,209,210}^{ie} ²²⁻¹⁷Ch₂₂₆^{ie} ⁴⁰⁰Ch₂₅₇^{ie}

$$2\pi \approx \frac{622}{99} = \frac{4 \cdot (24 \cdot 13 - 1)}{9 \cdot 22} = 6.2828 \dots \quad \begin{matrix} 23 \\ 11 \end{matrix} \text{Na}_{12} \quad \begin{matrix} 27 \\ 12 \end{matrix} \text{Al}_{13} \quad \begin{matrix} 46,48,49 \\ 22 \end{matrix} \text{Ti}_{24,26,27} \quad \begin{matrix} 50,52,54 \\ 24 \end{matrix} \text{Cr}_{26,28,30} \quad \begin{matrix} 54,56,58 \\ 26 \end{matrix} \text{Fe}_{28,30,32} \quad \begin{matrix} 99 \\ 44 \end{matrix} \text{Ru}_{55} \quad \begin{matrix} 167 \\ 68 \end{matrix} \text{Er}_{99} \quad \begin{matrix} 252 \\ 99 \end{matrix} \text{Es}_{153}^*$$

$$2\pi \approx \frac{2 \cdot 355}{113} = \frac{4 \cdot 5 \cdot 71}{2 \cdot 113} = 6.2831858 \dots \quad \begin{matrix} 71 \\ 31 \end{matrix} \text{Ga}_{40} \quad \begin{matrix} 112,113 \\ 48 \end{matrix} \text{Cd}_{64,65} \quad \begin{matrix} 113,115 \\ 49 \end{matrix} \text{In}_{64,66} \quad \begin{matrix} 120,122 \\ 50 \end{matrix} \text{Sn}_{70,72} \quad \begin{matrix} 2-71 \\ 60 \end{matrix} \text{Nd}_{82} \quad \begin{matrix} 171 \\ 70 \end{matrix} \text{Yb}_{101} \quad \begin{matrix} 175 \\ 71 \end{matrix} \text{Lu}_{74} \quad \begin{matrix} 186 \\ 74 \end{matrix} \text{W}_{112}$$

¹⁸⁷Re₁₁₂ ^{188,189}Os_{112,113} ²²⁶Ra₁₃₈ ²³²Th₉₀ ⁴⁻⁷¹Nh₁₇₁^{ie} ^{358/360}Ch₁₄₂₌₂₋₇₁^{ie} ²²⁻¹⁷Ch₂₂₆^{ie} ⁴⁰⁰Ch₂₅₇^{ie} 2020/1/8-10

The approximate rational numbers of 2π (could be called 2π formulas) relate to nuclides marvelously. This means 2π (along with 2π -e formula) plays important roles in atomic nuclei, and acts as a rational number rather than an irrational number in the world of atomic nuclei.

17. Correlations among α , 2π and nuclides

Some Chen's formulas of the fine-structure constant and 2π formulas correlate with each others with the same factors and all together relate to the same nuclides. For example, α_{1-9} and $2\pi \approx 4 \times 157/100$ have the same 157 and 100 factors, α_{1-50} and $2\pi \approx 3 \times 7 \times 44 \times 68/100^2$ have the same 100, 7, 11 and 16 factors, and they relate to the same corresponding nuclides. They also have common factors with α_{1-7} and α_{2-13} which should relate to $2\pi \approx 5 \times 7^2/3/13$ and $2\pi \approx 13 \times 29/4/3/5$.

$$\alpha_{1-9} = \frac{47}{3^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{10}{9}\right)^{19}} 112 + \frac{1}{4 \cdot 17}} - \frac{1}{2(8 \cdot 9 \cdot 37 - 1) + \frac{3 \cdot 17}{157}}$$

$$\alpha_{1-50} = \frac{2 \cdot 257}{100 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{14 \cdot 13}{181}\right)^{311^2}} 112 + \frac{1}{29 \cdot 61 + \frac{157}{16 \cdot 11}}}$$

$$2\pi \approx \frac{4 \cdot 157}{100} = \frac{157}{25} = 6.28 \quad 2\pi \approx \frac{3 \cdot 7 \cdot 44 \cdot 68}{100^2} = \frac{3 \cdot 7 \cdot 11 \cdot 17}{25^2} = 6.2832$$

$$\alpha_{1-7} = \frac{6^2}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{225}} 112 + \frac{1}{75^2}} \quad \alpha_{2-13} = \frac{13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{279}{278}\right)^{557}}}{10^2} - \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}}$$

$$2\pi \approx \frac{245}{39} = \frac{5 \cdot 7^2}{3 \cdot 13} = 6.2820 \dots \quad 2\pi \approx \frac{377}{60} = \frac{13 \cdot 29}{4 \cdot 3 \cdot 5} = 6.2833 \dots$$

α_{1-22} relates to $2\pi \approx 2 \times 22/7$, $2\pi \approx 17^2/7/23$ and $2\pi \approx 2 \times 355/113$ as follows. And $2\pi \approx 17^2/7/23$ also relates to α_{1-1} , α_{1-17} , α_{1-22} , α_{1-23} , α_{1-25} , α_{1-59} , α_{1-103} , α_{1-133} , α_{2-17} and α_{2-23} , in which both 17 and 23 factors appear.

$$\alpha_{1-22} = \frac{113}{22 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{27 \cdot 29}{2 \cdot 17 \cdot 23}\right)^{5 \cdot (2 \cdot 157 - 1)}} 112 + \frac{1}{2 \cdot [2 \cdot 3 \cdot 17 \cdot (10 \cdot 19 + 1) + 1] + \frac{29}{49}}}$$

$$2\pi \approx \frac{2 \cdot 22}{7} = 6.2857 \dots, \quad 2\pi \approx \frac{17^2}{2 \cdot 23} = 6.2826 \dots, \quad 2\pi \approx \frac{2 \cdot 355}{113} = \frac{4 \cdot 5 \cdot 71}{2 \cdot 113} = 6.2831858 \dots$$

α_{1-13} and α_{1-43} relate to $2\pi \approx 3 \times 67/32$, $2\pi \approx 5 \times 7^2/39$, $2\pi \approx 17^2/46$ and others as follows.

$$\alpha_{1-13} = \frac{67}{13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{47}{2 \cdot 23}\right)^{3 \cdot 31}}} \frac{1}{112 + \frac{1}{137} - \frac{1}{6(2 \cdot 27 \cdot 59 + 1) + \frac{9}{50}}}$$

$$\alpha_{1-43} = \frac{13 \cdot 17}{43 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{3 \cdot 67}{200}\right)^{401}}} \frac{1}{112 + \frac{1}{8 \cdot (12 \cdot 83 + 1) + \frac{4}{3 \cdot 13}}}$$

$$2\pi \approx \frac{3 \cdot 67}{32}, 2\pi \approx \frac{5 \cdot 7^2}{3 \cdot 13}, 2\pi \approx \frac{289}{46} = \frac{17^2}{2 \cdot 23}, 2\pi \approx \frac{13 \cdot 29}{60}, 2\pi \approx \frac{30 \cdot 31}{4 \cdot 37}$$

α_{1-11} , α_{1-36} , α_{2-24} , α_{2-23} , α_{2-37} and α_{2-125} relate to $2\pi \approx 9 \times 37/53$, $2\pi \approx 15 \times 31/2/37$ and $2\pi \approx (30 \times 17 - 1)/81$ as follows.

$$\alpha_{1-11} = \frac{57}{11 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{19}{18}\right)^{37}}} \frac{1}{112 + \frac{1}{35} - \frac{1}{88 \cdot 41 - \frac{5 \cdot 53}{22 \cdot 13}}}$$

$$\alpha_{1-36} = \frac{5 \cdot 37}{6^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{3 \cdot 79}{4 \cdot 59}\right)^{11 \cdot 43}}} \frac{1}{112 + \frac{1}{5 \cdot (31 \cdot 42 - 1) + \frac{3 \cdot 31}{14 \cdot 13}}}$$

$$\alpha_{2-24} = \frac{2^2 \cdot 6 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{63}{2 \cdot 31}\right)^{125}}}{5 \cdot 37} \frac{1}{112 - \frac{1}{257} + \frac{1}{10 \cdot (12 \cdot 13 \cdot 83 + 1) + \frac{23}{81}}}$$

$$\alpha_{2-23} = \frac{23 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{2 \cdot 81}{7 \cdot 23}\right)^{17 \cdot 19}}}{3 \cdot 59} \frac{1}{112 - \frac{1}{2 \cdot (40 \cdot 23 - 1) + \frac{9}{32 \cdot 10}}}$$

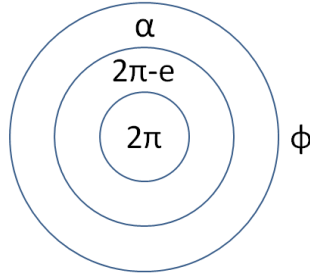
$$\alpha_{2-37} = \frac{37 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{2 \cdot 43}{5 \cdot 17}\right)^{9 \cdot 19}}}{3 \cdot 5 \cdot 19} \frac{1}{112 - \frac{1}{4 \cdot 3 \cdot 5 \cdot 13} + \frac{1}{5 \cdot 37^2 \cdot 149}}$$

$$\alpha_{2-125} = \frac{5 \cdot 5^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{2 \cdot 19 \cdot 113}{81 \cdot 53}\right)^{31 \cdot (12 \cdot 23 + 1)}}}{31^2} \frac{1}{112 - \frac{1}{101 \cdot (20 \cdot (12 \cdot 89 + 1) + 1)}}$$

$$2\pi \approx \frac{9 \cdot 37}{53} = 6.2830 \dots \quad 2\pi \approx \frac{15 \cdot 31}{2 \cdot 37} = 6.2837 \dots \quad 2\pi \approx \frac{30 \cdot 17 - 1}{81} = 6.2839 \dots$$

18. Chen's Mathematic Shell Model of Nuclides

In overall, there are multi-correlations among α , 2π and nuclides. It seems there should be a mathematical shell model of nuclides, in which the core is 2π formulas and the middle layer is 2π -e formulas and the outer layer is Chen's formulas of α (**Fig. 9**, ϕ is explained in **Section 21**). The nucleon numbers, stability and abundance of nuclides are regulated by these formulas, especially by their integer factors.



Chen's Mathematic Shell Model of Nuclides

Dr. Gang Chen (2020/1/12-13, 3/1)

Fig. 9

19. Ideal Extended Elements

In the deduction of Chen's formulas of the fine-structure constant, it was reasonably assumed the factors in them related to nucleon numbers of nuclides, and it seems this assumption is quite correct. So by somewhat correlation and decoding methodology, all 119th to 170th ideal extended elements were predicted (**Table 7**). In addition, nuclides can even relate to naked 2π 's approximate rational numbers (2π formulas). Some typical examples of correlations of ideal extended elements with formulas of α and 2π are listed as follows.

Example 1: Correlations of 100, 121, 125, 126, 157, 257, 169, *et al.*

$$\alpha_{1-9} = \frac{47}{3^2 \cdot e^2 \frac{e^2}{\binom{2}{1}^3} \frac{e^2}{\binom{3}{2}^5} \dots \frac{e^2}{\binom{10}{9}^{19}} 112 + \frac{1}{4 \cdot 17}} \frac{1}{2(8 \cdot 9 \cdot 37 - 1) + \frac{3 \cdot 17}{157}}$$

$$\alpha_{1-50} = \frac{2 \cdot 257}{100 \cdot e^2 \frac{e^2}{\binom{2}{1}^3} \frac{e^2}{\binom{3}{2}^5} \dots \frac{e^2}{\binom{14 \cdot 13}{181}^{3 \cdot 11^2}} 112 + \frac{1}{29 \cdot 61 + \frac{157}{16 \cdot 11}}}$$

$$\alpha_{2-24} = \frac{2^2 \cdot 6 \cdot e^2 \frac{e^2}{\binom{2}{1}^3} \frac{e^2}{\binom{3}{2}^5} \dots \frac{e^2}{\binom{63}{62}^{125}}}{5 \cdot 37} \frac{1}{112 - \frac{1}{257} + \frac{1}{10 \cdot (12 \cdot 13 \cdot 83 + 1) + \frac{23}{81}}}$$

$$2\pi \approx \frac{4 \cdot 157}{100} \quad 2\pi \approx \frac{16 \cdot 3 \cdot 7 \cdot 11 \cdot 17}{100} \quad 2\pi \approx \frac{4 \cdot 11}{7} \quad 2\pi \approx \frac{17^2}{2 \cdot 23} \quad 2\pi \approx \frac{30 \cdot 31}{4 \cdot 37} \quad 2\pi \approx \frac{4 \cdot 5 \cdot 71}{2 \cdot 113}$$

${}_{44}^{100}\text{Ru}_{56}$ ${}_{68,69}^{168,169}\text{Tm}_{100}$ ${}_{100}^{257}\text{Fm}_{157}^*$ ${}_{121=11^2}^{302}\text{Ch}_{181}^{ie}$ ${}_{125,126}^{24 \cdot 13 \cdot 2 \cdot 157}\text{Ch}_{117,4 \cdot 47}^{ie}$ ${}_{148=4 \cdot 37}^{2 \cdot 11 \cdot 17}\text{Ch}_{226}^{ie}$ ${}_{157,158}^{400,402}\text{Ch}_{3 \cdot 81,4 \cdot 61}^{ie}$ ${}_{169}^{6 \cdot 71}\text{Ch}_{257}^{ie}$

Example 2: Correlations of 83, 126, 84, 125, 209, 112, 173, 285, 115 and 137

$$\alpha_{1-16} = \frac{83}{4^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{17}{16}\right)^{33}} 112 + \frac{1}{28} - \frac{1}{6 \cdot (18 \cdot 41 + 1) + \frac{173}{2 \cdot (2 \cdot 75 - 1)}}$$

$$\alpha_{1-25} = \frac{3 \cdot 43}{5^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{35}{34}\right)^{3 \cdot 23}} 112 + \frac{1}{11 \cdot 19} - \frac{1}{13^2 (2 \cdot 136 - 1) + \frac{11}{25}}$$

$$\alpha_{1-32} = \frac{15 \cdot 11}{2 \cdot 2^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{41}{40}\right)^{81}} 112 + \frac{1}{25 \cdot 29} - \frac{1}{19 \cdot 23}$$

$$\alpha_{2-10} = \frac{10 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{5 \cdot 21}{8 \cdot 13}\right)^{11 \cdot 19}}}{77} \frac{1}{112 - \frac{1}{3 \cdot 14 \cdot 19} + \frac{1}{14 \cdot (4 \cdot 27 \cdot (2 \cdot 15 \cdot 19 + 1) - 1)}}$$

$$\alpha_{2-32} = \frac{2 \cdot 4^2 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{42}{41}\right)^{83}}}{13 \cdot 19} \frac{1}{112 - \frac{1}{11 \cdot 13} + \frac{1}{6 \cdot 37 \cdot (5 \cdot 210 - 1) + \frac{10}{11}}}$$

$$\alpha_{1-13} = \frac{67}{13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{47}{2 \cdot 23}\right)^{3 \cdot 31}} 112 + \frac{1}{137} - \frac{1}{6(2 \cdot 27 \cdot 59 + 1) + \frac{9}{50}}$$

$$\alpha_{1-17} = \frac{2^2 \cdot 22}{17 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{21}{20}\right)^{41}} 112 + \frac{1}{137} - \frac{1}{2 \cdot 19 \cdot 23 \cdot 59 - \frac{30}{100}}$$

$$\alpha_{1-27} = \frac{139}{27 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{67}{66}\right)^{7 \cdot 19}} 112 + \frac{1}{11 \cdot 47 + \frac{18}{23} + \frac{1}{6 \cdot 23 \cdot 137}}$$

$$\alpha_{2-18} = \frac{18 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{38}{37}\right)^{75}}}{139} \frac{1}{112 - \frac{1}{2 \cdot 47} + \frac{1}{2 \cdot 31 \cdot (16 \cdot 17 - 1) + \frac{83}{137}}}$$

$$2\pi \approx \frac{16 \cdot 3 \cdot 7 \cdot 11 \cdot 17}{100} \approx \frac{4 \cdot 11}{7} \approx \frac{17^2}{2 \cdot 23} \quad {}_{56}^{8-17,137,6-23}Ba_{80,81,82} \quad {}_{83}^{11-19}Bt_{126}^* \quad {}_{84}^{209}Po_{125}^* \quad {}_{85}^{210}At_{125}^* \quad {}_{112}^{15-19}Cn_{173}^* \quad {}_{137}^{2-173}Fy_{209}^{ie}$$

Table 7. Correlations of Ideal Extended Elements (**IEE**) with Formulas of α and 2π .

IEE	Page	α	2π
${}_{113}\text{Nh}_{171}$	10 19 21 28 29 31	$\alpha_c^2 \alpha_{1-5,7} \alpha_{2-22,23,31,37,38,253}$	$2\pi \approx 4 \times 355/226$
${}_{114}\text{Fl}_{175}$	19 23 28 31	$\alpha_{1-11,133,155} \alpha_{2-37,38}$	$2\pi \approx 17^2/46$
${}_{115}\text{Mc}_{173}$	20 21 25 31	$\alpha_{1-1,16,23} \alpha_{2-5}$	$2\pi \approx 17^2/46$
${}_{116}\text{Lv}_{177} \quad {}_{117}\text{Ts}_{177}$	10 20 22 27 31	$\alpha_{1-13,59} \alpha_{2-23} 1/\alpha_c^2$	$2\pi \approx 622/99$
${}_{118}\text{Og}_{176}$	20 22 23 27	$\alpha_{1-17,20,50,59,133} \alpha_{2-19,269}$	$2\pi \approx 44/7$
${}_{119-122}\text{Ch}_{179-182}$	21-23 28 31 37 39 44	$\alpha_{1-23,29,50,170} \alpha_{2-37} \alpha_{p/2} C_{\text{au}}$	$2\pi \approx 44/7 \text{ et al.}$
${}_{123}\text{Ch}_{183/185}$	19 20 21 25 28	$\alpha_{1-4,11,16,17,31} \alpha_{2-1,4,5,9,32}$	$2\pi \approx 333/53 \approx 465/74 \text{ et al}$

The relationships between elements and ideal extended elements (the frontier of elements) and an overall picture of them were depicted as above.

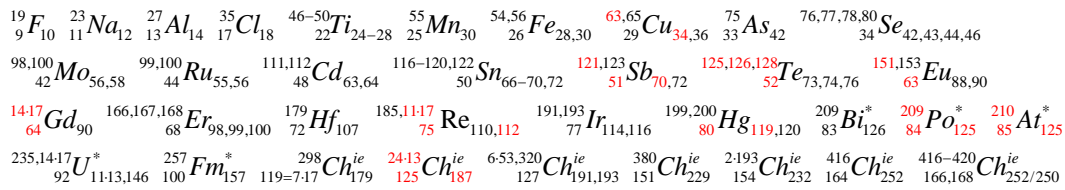
21. Some Supplements

Supplement 1:

$$2\pi \approx \frac{16 \cdot 3 \cdot 7 \cdot 11 \cdot 17}{100^2} = \frac{3 \cdot 7 \cdot 44 \cdot 68}{100^2} = \frac{3 \cdot 112 \cdot 11 \cdot 17}{100^2} = \frac{2 \cdot 168 \cdot 11 \cdot 17}{100^2} = \frac{2 \cdot 3 \cdot 7 \cdot 11 \cdot 136}{100^2} = \dots = 6.2832$$

Refer to Section 16; Supplements: ${}_{16}^{32,33,34}O_{16,17,18}$ and some of the follows

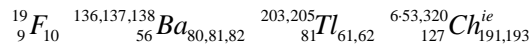
$$2\pi \approx \frac{9 \cdot 7 \cdot 127 \cdot (4 \cdot 13 \cdot 151 + 1)}{10^7} = \frac{63 \cdot 127 \cdot (52 \cdot 151 + 1)}{10 \cdot 100^3} = \frac{63 \cdot 127 \cdot (2 \cdot 3 \cdot 7 \cdot 11 \cdot 17 - 1)}{10 \cdot (8 \cdot 125)^2} = \dots = 6.2831853$$



2020/2/11-12

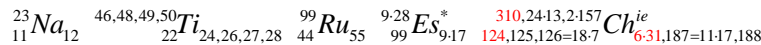
Supplement 2:

$$2\pi \approx \frac{509}{81} = \frac{4 \cdot 127 + 1}{9^2} \quad 2\pi \approx \frac{201}{32} = \frac{3 \cdot 67}{32} \quad 2\pi \approx \frac{333}{53} = \frac{9 \cdot 37}{53}$$



$$2\pi \approx \frac{622}{99} = \frac{4 \cdot (310 + 1)}{9 \cdot 22} \quad 2\pi \approx \frac{465}{74} = \frac{30 \cdot 31}{4 \cdot 37} \quad 2\pi \approx \frac{44}{7} = \frac{2 \cdot 22}{7}$$

$$2\pi \approx \frac{245}{39} = \frac{5 \cdot 7^2}{3 \cdot 13} \quad 2\pi \approx \frac{289}{46} = \frac{17^2}{2 \cdot 23} \quad 2\pi \approx \frac{377}{60} = \frac{13 \cdot 29}{4 \cdot 3 \cdot 5} \quad 2\pi \approx \frac{628}{100} = \frac{4 \cdot 157}{100}$$



2020/2/12

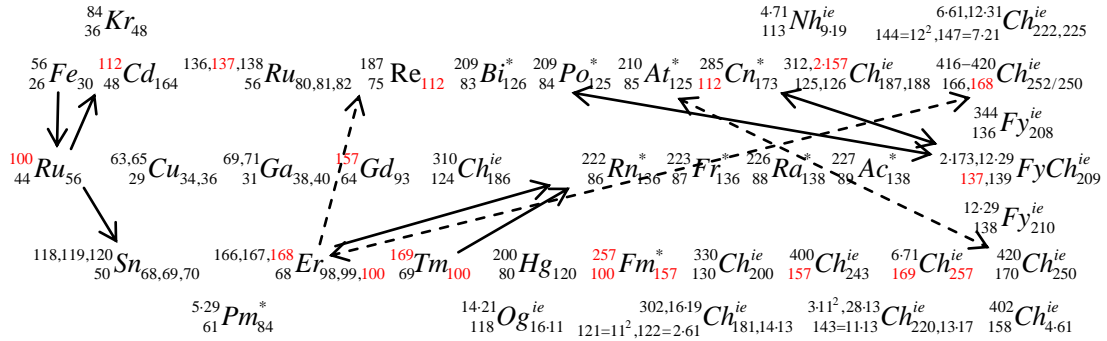
Supplement 3:

Table 8. Relationships of factors in α_{1-7} and α_{2-13} with primordial nuclides (2020/2/16-17).

Nuclides	${}_3Li_4$	${}_{29}Cu_{34}$	${}_{31}Ga_{40}$	${}_{64}Gd_{92}$	${}_{75}Re_{112}$
A	7	65=5×13	71	156=12×13	187=11×17
PN before	5	70	78	209	252
PN all	285	285	285	285	285
Ratios	1/57	14/57	26/95	11/15	84/95

- 3, 29, 31, 64, 75 and 112 are factors in α_1 and α_2 .
- PN: primordial nuclides; PN all: usually regarded as 286.
- Nucleon number 285 of ${}_{112}Cn_{173}$ would relate to PN all, or PN all should be 285 rather than 286, and ${}^{235}U$ should not be a primordial nuclide.
- ${}^{235}U_{143}$ should not be a primordial nuclide, its relative stability (but not much stable) should come from relative stable nucleon numbers $92=96-4$ and $143=11 \times 13$, so number of PN would become 285 from 286.

Supplement 4: Correlations of factors in α (α_{1-7} , α_{2-13} and α_{1-50}) and nuclides



Relationships between Formulas of α (α_{1-7} , α_{2-13} and α_{1-50}) and Nuclides

Dr. Gang Chen, 2020 / 2 / 18 – 19

In this scheme there are several important clues based on factors in the formulas of α_{1-7} , α_{2-13} and α_{1-50} such as 6 (36, 48, 138, 144, *et al*), 7 (56, 84, 112, 126, 166-168, 210, 252), 10 (30, 50, 70, 100, 120, 130, 170, 200, 210, 220, 250, 310, 330, 400, 420), 11 (44, 88, 121, 134, 176, 187, 209, 220, 330, 363), 13 (26, 143, 169, 221, 364), 29 (87, 145, 348), 25 (75, 100, 125, 200, 250, 400), 31 (93, 124, 186, 310, 372), 61 (122, 244), 64 (136 *et al*), 137 (68, 69, 136, 138), 139, 157 (314), 257, *et al*. And these clues correlate each others. These relationships are strong proofs that Chen’s formulas of the fine-structure constant are correct, otherwise so many coincidences couldn’t be explained.

In addition, numbers 7, 13 and 50 in α_{1-7} , α_{2-13} and α_{1-50} may have the following relationships: $(13+7)(13-7) = 50+70=120$ and ${}_{50}\text{Sn}_{70}$. And Sn is special, it has the most stable nuclides (up to 10) among which ${}_{50}\text{Sn}_{70}$ has the most relative abundance.

Supplement 5: Other two formulas of the fine-structure constant

$$\alpha_{1-9/11} = \frac{9}{11 + \frac{1}{84} - \frac{1}{11 \cdot 17 \cdot 53 + \frac{2 \cdot 191}{3 \cdot 157}}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

$$\frac{19}{9}\text{F}_{10} \quad \frac{23}{11}\text{Na}_{12} \quad \frac{35,37}{17}\text{Cl}_{18,20} \quad \frac{53}{24}\text{Cr}_{29} \quad \frac{75}{33}\text{As}_{42} \quad \frac{84}{36}\text{Kr}_{48} \quad \frac{95}{42}\text{Mo}_{53} \quad \frac{127}{53}\text{I}_{74} \quad \frac{129}{54}\text{Xe}_{75} \quad \frac{144}{60}\text{Nd}_{84}$$

$$\frac{157}{64}\text{Gd}_{93} \quad \frac{185,11,17}{75}\text{Re}_{110,112} \quad \frac{191}{77}\text{Y}_{114} \quad \frac{11,19}{84}\text{Po}_{125}^* \quad \frac{257}{100}\text{Fm}_{157}^* \quad \frac{7,37}{102=6,17}\text{No}_{157}^* \quad \frac{6,53}{127}\text{Ch}_{191}^{ie} \quad \frac{400}{157}\text{Ch}_{3,9^2}^{ie}$$

$$\alpha_{2-20/25} = \frac{20 + \frac{1}{2} - \frac{1}{14} + \frac{1}{251} - \frac{1}{8 \cdot (12 \cdot 43 \cdot 227 + 1) - \frac{8}{37}}}{25} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} = 1/137.035999111818$$

$$\frac{40,43,44,48}{20}\text{Ca}_{20,23,24,28} \quad \frac{55}{25}\text{Mn}_{30} \quad \frac{85,87}{37}\text{Rb}_{48,50} \quad \frac{90,96}{40}\text{Zr}_{50,56} \quad \frac{116,120,124}{50}\text{Sn}_{66,70,74} \quad \frac{99}{43}\text{Tc}_{56}^* \quad \frac{106,111,112,116}{48}\text{Cd}_{58,63,64,68}$$

$$\frac{191,193}{77}\text{Y}_{114,4,29} \quad \frac{200}{80}\text{Hg}_{120} \quad \frac{6,37}{86}\text{Rn}_{136}^* \quad \frac{223}{87}\text{Fr}_{136}^* \quad \frac{227}{89}\text{Ac}_{138}^* \quad \frac{251}{98}\text{Cf}_{153}^* \quad \frac{11,29}{127}\text{Ch}_{192=3,64}^{ie} \quad \frac{376}{149}\text{Ch}_{227}^{ie} \quad \frac{22,19}{167}\text{Ch}_{251}^{ie}$$

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Supplement 6: Other formulas of the speed of light c_{au}

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-9/11}\alpha_{2-20/25}}}$$

$$= \sqrt{\frac{(11 + \frac{1}{84} - \frac{1}{11 \cdot 17 \cdot 53 + \frac{2 \cdot 191}{3 \cdot 157}})(112 + \frac{1}{75^2}) \cdot 25 \cdot (112 - \frac{1}{3 \cdot 29 \cdot 64})}{9 \cdot (20 + \frac{1}{2} - \frac{1}{14} + \frac{1}{251} - \frac{1}{8 \cdot (12 \cdot 43 \cdot 227 + 1) - \frac{8}{37}})}}$$

$$= \frac{5}{3} \sqrt{\frac{(11 + \frac{1}{84} - \frac{1}{11 \cdot 17 \cdot 53 + \frac{2 \cdot 191}{3 \cdot 157}})(112^2 - \frac{7 \cdot 19}{2^2 \cdot 3^2 \cdot 25^2 \cdot 29} - \frac{1}{2^6 \cdot 3^3 \cdot 25^2 \cdot 29})}{20 + \frac{1}{2} - \frac{1}{14} + \frac{1}{251} - \frac{1}{8 \cdot (12 \cdot 43 \cdot 227 + 1) - \frac{8}{37}}}}$$

$$= \frac{5}{3} \sqrt{\frac{(11 + \frac{1}{84} - \frac{1}{11 \cdot 17 \cdot 53 + \frac{2 \cdot 191}{3 \cdot 157}})(112^2 - \frac{1}{6 \cdot 17 \cdot 47 + \frac{2}{3}})}{20 + \frac{1}{2} - \frac{1}{14} + \frac{1}{251} - \frac{1}{8 \cdot (12 \cdot 43 \cdot 227 + 1) - \frac{8}{37}}}}$$

$$= \sqrt{137.035999037435 \times 137.035999111818} = 137.035999074627$$

²³₁₁Cr₁₂ ²⁴⁻²⁶₁₂Mg₁₂₋₁₄ ²⁸⁻³⁰₁₄Si₁₄₋₁₆ ^{35,37}₁₇Cl_{18,20} ³⁹⁻⁴¹₁₉K₂₀₋₂₂ ⁴⁷₂₂Ti₂₅ ⁵³₂₄Cr₂₉ ⁵⁵₂₅Mn₃₀ ^{63,65}₂₉Cu_{34,36} ⁸⁴₃₆Kr₄₈
⁸⁴₃₇Rb₄₇ ^{85,87}₃₇Rb_{48,50} ^{90-92,94,96}₄₀Zr_{50-52,54,56} ⁹⁹₄₃Tc₅₆ ^{107,109}₄₇Ag_{60,62} ¹²⁷₅₃I₇₄ ⁷⁻¹⁹₅₅Cs₇₈ ⁵⁻²⁹₆₁Pm₈₄ ¹⁵⁷₆₄Gd₉₃
³⁻⁵³₆₅Tb₉₄ ^{5-37,11-17}₇₅Re_{110,112} ^{191,193}₇₇Y_{114,4-29} ¹¹⁻¹⁹₈₄Po₁₂₅ ²²³₈₇Fr₁₃₆ ²²⁷₈₉Ac₁₃₈ ²⁴⁴₉₄Pu₁₅₀ ²⁵¹₉₈Cf₁₇₇ ²⁵⁷₁₀₀Fm₁₅₇
⁶⁻⁴³₁₀₁Md₁₅₇ ⁷⁻³⁷₁₀₂No₁₅₇ ^{6-53,11-29,320}₁₂₇Ch^{ie}_{191,3-64,193} ^{32-11,4-89}_{140,141}Ch^{ie}_{4-53,5-43} ³⁻¹¹²₁₃₃Ch^{ie}₇₋₂₉ ⁸⁻⁴⁷₁₄₉Ch^{ie}₂₂₇ ⁴⁰⁰₁₅₇Ch^{ie}₂₄₃ ²²⁻¹⁹₁₆₇Ch^{ie}₂₅₁

2020 / 2 / 21 - 22

$$c_{au} = \frac{1}{\alpha_c} = \frac{25 \cdot 112}{3} \sqrt{\frac{37}{11 \cdot 12 \cdot 13} - \frac{1}{2 \cdot 17 \cdot 41 \cdot 163 + \frac{47}{6 \cdot 31}}} = 137.035999074628$$

²³₁₁Na₁₂ ^{24,25}₁₂Mg_{12,13} ^{35,37}₁₇Cl_{18,20} ⁵⁵₂₅Mn₃₀ ⁵⁶₂₆Fe₃₀ ^{69,71}₃₁Ga_{38,40} ^{74,77,78,82}₃₄Se_{40,43,44,48} ⁸⁴₃₇Rb₄₇ ^{85,87}₃₇Rb_{48,50} ⁹³₄₁Nb₅₂
¹⁰⁰₄₄Ru₅₆ ^{107,109}₄₇Ag_{60,62} ¹¹²₄₈Cd₆₄ ^{112,114-120,122,124}₅₀Sn_{62,64-70,72,74} ^{144,147,148,150,154}₆₂Sm_{82,85,86,88,92} ^{157,158}₆₄Gd_{93,94}
¹⁶³₆₆Dy₉₇ ¹⁶⁸₆₈Er₁₀₀ ¹⁶⁹₆₉Tm₁₀₀ ^{5-37,11-17}₇₅Re_{110,112} ^{204,206-1613}₈₂Pb_{122,124-126} ²³⁷₉₃Np₁₂₂ ²⁴⁷₉₇Bk₁₅₀ ²⁵⁷₁₀₀Fm₁₅₇ ²⁶⁸₁₀₅Db₁₆₃
²⁶⁹₁₀₆Sg₁₆₃ ²⁷⁰₁₀₇Bh₁₆₃ ²⁸⁵₁₁₂Cn₁₇₃ ³¹⁰₁₂₄Ch^{ie}₁₈₆ ³³⁴₁₃₂Ch^{ie}₂₀₂ ^{3-112,28-13}₁₄₃Ch^{ie}_{220,13-17} ³⁷⁸₁₅₀Ch^{ie}₂₂₈ ³⁹⁴₁₅₆Ch^{ie}₁₄₋₁₇ ⁴¹⁰₁₆₃Ch^{ie}₁₃₋₁₉

Note: $112 \times 5/3 \approx 187 = 11 \times 17$, $112 \times 25/3 \approx 5 \times 11 \times 17$

2020 / 2 / 24

$$c_{au} = \frac{1}{\alpha_c} = \frac{25 \cdot 112}{3} \sqrt{\frac{1}{47} + \frac{1}{40 \cdot 89} - \frac{1}{16 \cdot 9 \cdot (2 \cdot 21 \cdot 31 \cdot 43 + 1) + \frac{3}{4}}} = 137.035999074627$$

⁴⁵₂₁Ti₂₄ ⁴⁷₂₂Ti₂₅ ⁵⁵₂₅Mn₃₀ ^{64,66,70}₃₀Zn_{34,36,40} ^{69,71}₃₁Ga_{38,40} ⁷²₃₂Ge₄₀ ^{78,80,83,84,86}₃₆Kr_{42,44,47,48,50} ⁸⁹₃₉Y₅₀ ^{90,94,96}₄₀Zr_{50,54,56}
^{92,94-98,100}₄₂Mo_{50,52-56,58} ^{98,99}₄₃Tc_{55,56} ^{107,109}₄₇Ag_{60,62} ¹¹²₄₈Cd₆₄ ^{144,147,148,150,152}₆₂Sm_{82,85,86,88,90} ^{151,153}₆₃Eu_{88,90} ¹⁷⁸₇₂Hf₁₀₆
^{185,187}₇₅Re_{110,112} ²²²₈₆Rn₁₃₆ ²²⁷₈₉Ac₁₃₈ ²³⁷₉₃Np₁₄₄ ²⁴⁴₉₄Pu₁₅₀ ^{326,328}₁₂₉Ch^{ie}_{197/199} ³⁶⁶₁₄₄Ch^{ie}₂₂₂ ⁹⁻⁴²₁₅₀Ch^{ie}₂₂₈

$$c_{au} = \frac{1}{\alpha_c} = \frac{25 \cdot 112}{3} \sqrt{\frac{1}{46} - \frac{1}{55 \cdot 100} + \frac{1}{9 \cdot 25 \cdot 13 \cdot (20 \cdot 293 + 1) - \frac{4}{23}}} = 137.035999074627$$

^{50,51}₂₃Mn_{27,28} ⁵⁵₂₅Mn₃₀ ⁸⁹₃₉Y₅₀ ^{99,100}₄₄Ru_{55,56} ^{106,110}₄₆Pd_{60,64} ¹¹⁷₅₀Sn₆₇ ¹³³₅₅Cs₇₈ ¹⁶⁹₆₉Tm₁₀₀ ^{185,187}₇₅Re_{110,112} ¹⁹⁵₇₈Pd₁₁₇
^{5-47,238}₉₂U_{113,146} ²⁵⁷₁₀₀Fm₁₅₇ ²⁸⁵₁₁₂Cn₁₇₃ ²⁹³₁₁₆Lv₁₇₇ ²⁹⁴₁₁₇Ts₁₇₇ ⁴⁰⁰₁₅₇Ch^{ie}₂₄₃ ⁴²⁶₁₆₉Ch^{ie}₂₅₇

2020 / 2 / 25

Supplement 7: Comparison of formulas of 1, N, e, 2π , $\pi/2$, ϕ , α , α_c , c_{au} and $\alpha_{p/c}$

$$1 = 4\gamma_c + \frac{4\gamma_1}{1(1+1)} + \frac{4\gamma_2}{2(2+1)} + \frac{4\gamma_3}{3(3+1)} + \dots$$

$$= |B| \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{|B_{2n}| (\pi/2)^{2n}}{(2n)!} = \sum_{n=1}^{\infty} \frac{|B_{2n}| \pi^{2n}}{(2n)!} = -|B| \frac{3\pi}{2} + \sum_{n=1}^{\infty} \frac{|B_{2n}| (3\pi/2)^{2n}}{(2n)!}$$

$$N \sim -\frac{3}{2}|B| + \sum_{n=1}^N \frac{|B_{2n}| (2\pi)^{2n}}{2(2n)!}$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots, \quad \frac{\pi}{2} = \left(\frac{e}{e^{\gamma_s}}\right)^2 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

$$2\pi \approx \frac{4 \cdot 157}{100} \approx \frac{9 \cdot 37}{53} \approx \frac{4 \cdot 5 \cdot 71}{15^2 + 1} \approx \dots, \quad \frac{\pi}{2} \approx \frac{157}{25} \approx \frac{9(9+1/4)}{53} \approx \frac{5 \cdot 71}{15^2 + 1} \approx \dots$$

$$\phi_1 = \frac{\sqrt{5}-1}{2} = 0.618\dots, \quad \phi_2 = -\frac{\sqrt{5}+1}{2} = -1.618\dots$$

$$\sqrt{\frac{\sqrt{5}+1}{2} + 2} - \frac{\sqrt{5}+1}{2} = \frac{e^{-\frac{2\pi}{5}}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \dots}}}} \quad (\text{Ramanujan Formula})$$

$$\alpha_1 = \frac{6^2}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{225}}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{9 \cdot 31}{278}\right)^{557}}}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} = 1/137.0359991118181$$

$$\frac{1}{\alpha_c^2} = \frac{1}{\alpha_1 \alpha_2} = 112 \times \left(168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{6 \cdot 29 \cdot 53 \cdot 59 - 79/47}\right)$$

$$= 137.035999074627^2 = 18778.865042381$$

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = \frac{5}{3} \sqrt{\frac{7 (2\pi)_{112}}{13 (2\pi)_{278}} \left(112^2 - \frac{1}{30^2 \cdot 5} + \frac{1}{60^2 \cdot 15} - \frac{1}{120^2 \cdot 15 \cdot 29}\right)}$$

$$= \frac{25 \cdot 112}{3} \sqrt{\frac{1}{47} + \frac{1}{40 \cdot 89} - \frac{1}{16 \cdot 9 \cdot (2 \cdot 21 \cdot 31 \cdot 43 + 1)} + \frac{3}{4}}$$

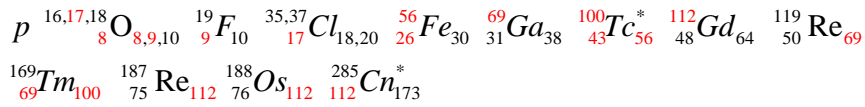
$$= \frac{25 \cdot 112}{3} \sqrt{\frac{1}{46} - \frac{1}{55 \cdot 100} + \frac{1}{9 \cdot 25 \cdot 13 \cdot (20 \cdot 293 + 1)} - \frac{4}{23}} = \dots = 137.035999074627$$

$$\frac{1}{\alpha_{p/c}^2} = \frac{1}{\alpha_{p/1} \alpha_{p/2}} = 252.040872632515^2 = 63524.60147736 \quad (\text{Supposed})$$

Ramanujan's formulas, so ϕ should connect to nuclides. And Fibonacci sequences are the integer presentations of ϕ , so Fibonacci sequences should connect to nuclides.

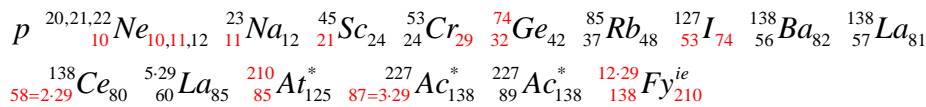
These connections are described as follows.

Fibonacci Sequence p_1 : 1 8 9 17 26 43 69 112



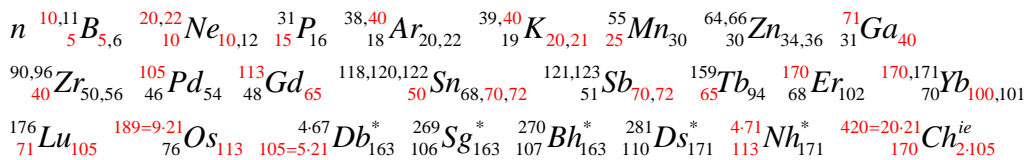
Note: $\begin{matrix} 56 \\ 26 \end{matrix} \text{Fe}_{30}$ $\begin{matrix} 100 \\ 43 \end{matrix} \text{Tc}_{56}^*$ $\begin{matrix} 169 \\ 69 \end{matrix} \text{Tm}_{100}$, relay of the numbers 56 and 100.

Fibonacci Sequence p_2 : 1 10 11 21 32 53 85 138



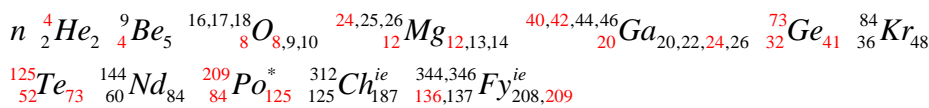
Note: $\begin{matrix} 74 \\ 32 \end{matrix} \text{Ge}_{42}$ $\begin{matrix} 127 \\ 53 \end{matrix} \text{I}_{74}$ $\begin{matrix} 210 \\ 85 \end{matrix} \text{At}_{125}^*$ $\begin{matrix} 12\cdot 29 \\ 138 \end{matrix} \text{Fy}_{210}^{ie}$, relay of the numbers 29, 74 and 210.

Fibonacci Sequence n_1 : 0 5 5 10 15 25 40 65 105 170



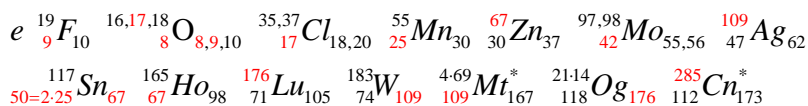
Note: $\begin{matrix} 170 \\ 68=4\cdot 17 \end{matrix} \text{Er}_{102=6\cdot 17}$ $\begin{matrix} 170,171 \\ 70 \end{matrix} \text{Yb}_{100=4\cdot 25,101}$ $\begin{matrix} 420 \\ 170 \end{matrix} \text{Ch}_{2\cdot 105}^{ie}$, relay of 21, 71 and 113.

Fibonacci Sequence n_2 : 0 4 4 8 12 20 32 52 84 136



Note: relay of the numbers 24, 41, 73, 125 and 209.

Fibonacci Sequence e : -1 9 8 17 25 42 67 109 176 285



Note: $\begin{matrix} 117 \\ 50=2\cdot 25 \end{matrix} \text{Sn}_{67}$, Fibonacci Sequence e is less relevant to specific nuclides.

Numbers of primordial nuclides before ${}_6\text{C}$ ${}_{11}\text{B}_6$ ${}_{10}\text{Ne}$ ${}_{14}\text{Si}$ ${}_{20}\text{Ca}_{22}$ ${}_{28}\text{Ni}_{33}$

${}_{40}\text{Zr}_{54}$ ${}_{56}\text{Ba}$ and ${}_{112}\text{Cn}^*$ are 9 8 17 25 42 67 109 176 and 285

2020 / 2 / 27 - 28, 3 / 3 (add Fibonacci Sequence p_2 and n_2)

As stated in **Section 4**, the mathematic expression of chirality is $\pm 2\pi$. There are 10 fingers in a pair of human hands, and there are 14 finger segments in a single hand. It was assumed by us that the numbers 10/20 and 28/56 stand for a pair of "hands" in the world of nuclides. So nuclides ${}_{10}\text{Ne}$ ${}_{14}\text{Si}$ ${}_{20}\text{Ca}$ ${}_{28}\text{Ni}$ ${}_{40}\text{Zr}$ ${}_{56}\text{Ba}$ and ${}_{112}\text{Cn}$ stand for two pairs of "hands" emerging gradually, and the numbers of primordial nuclides just before them are the numbers of Fibonacci Sequence e . This means chirality or $\pm 2\pi$ is the inner essence and ϕ or Fibonacci sequence is the outer expression of nuclides.

1014	56	112
${}_{5}\text{B}_6$ ${}_{10}\text{Ne}_{17}$ ${}_{14}\text{Si}_{25}$ ${}_{20}\text{Ca}_{42}$ ${}_{28}\text{Ni}_{67}$ ${}_{40}\text{Zr}_{109}$ ${}_{56}\text{Ba}_{176}$ ${}_{112}\text{Cn}_{285}$		
8	176	285

Natural End

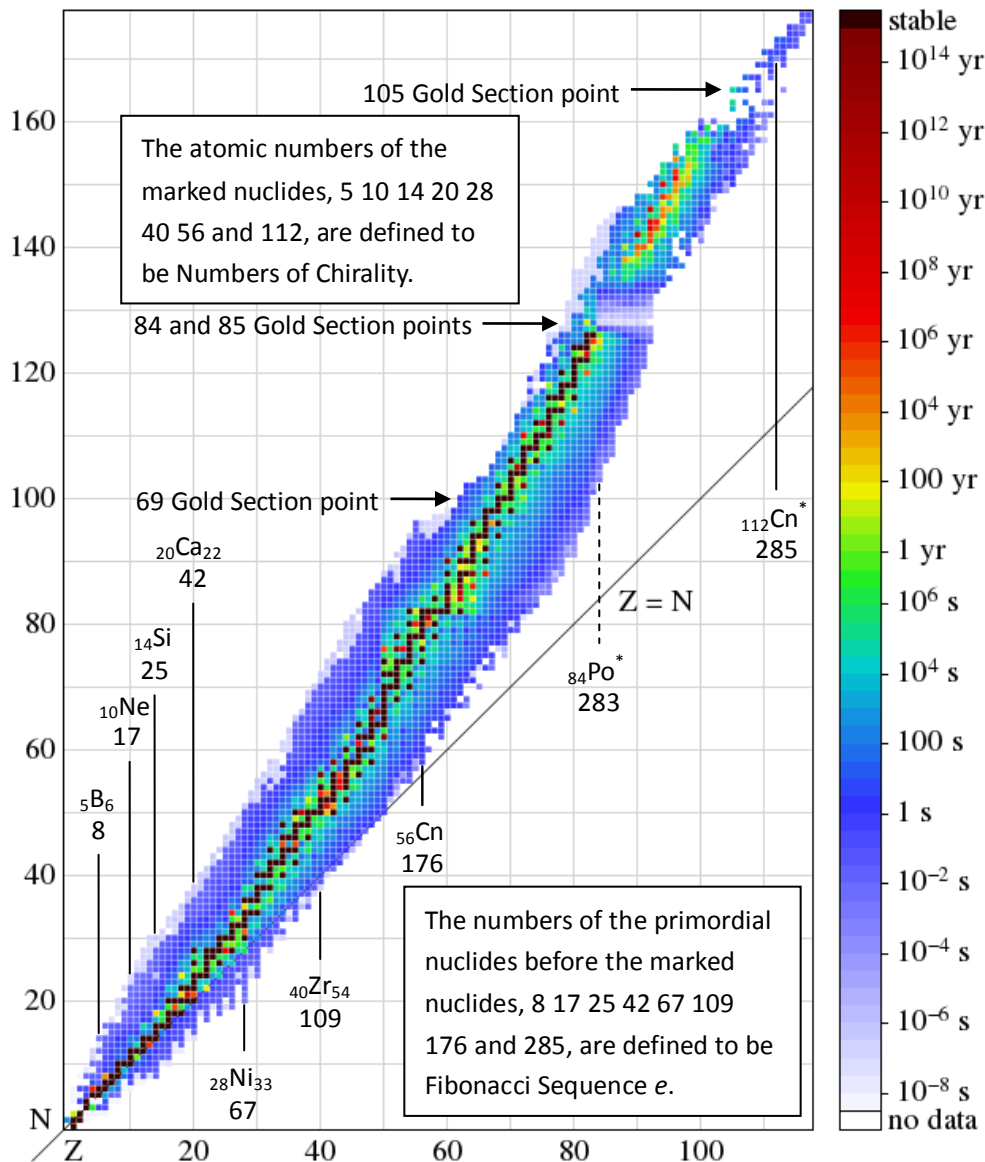
Primordial Nuclides

Primordial nuclides (PN) before Ba take about 0.618 part of all (176/285)
 Numbers of PN before ${}_{6}\text{C}$ ${}_{5}\text{B}_6$ ${}_{10}\text{Ne}$ ${}_{14}\text{Si}$ ${}_{20}\text{Ca}_{22}$ ${}_{28}\text{Ni}_{33}$ ${}_{40}\text{Zr}_{54}$ ${}_{56}\text{Ba}$ and ${}_{112}\text{Cn}$ are
 9 8 17 25 42 67 109 176 and 285 which is Fibonacci Sequence e

Primordial Nuclides and Fibonacci Sequence e

Dr. Gang Chen (2018/1-3, 2020/2/29-3/1)

Fig. 12



The Integrated Picture of Nuclides and Fibonacci Sequences

The Nuclide Picture was taken from Wikipedia

Dr. Gang Chen (2018/1-3, 2020/3/1-3)

Fig. 13

Why are there two pairs of “hands” in the world of nuclides? This should be because a pair of “hands” takes the right “hand” as priority and the other pair takes left “hand” as priority. So, 5 10 14 20 28 40 56 112 could be defined to be Numbers of Chirality, they are connected to Fibonacci Sequence e in nuclides (**Fig 13**).

Supplement 10: Other formulas of the speed of light c_{au}

$$c_{au} = \frac{1}{\alpha_c} = \frac{25 \cdot 112}{3 \cdot \sqrt{46 + \frac{1}{2} - \frac{1}{8} + \frac{1}{77} - \frac{1}{17 \cdot (2 \cdot 11 \cdot 19 + 1) - \frac{149}{13 \cdot 19}}}} = 137.035999074627$$

^{16,17}₈O_{8,9} ²³₁₁Na₁₂ ²⁷₁₃Al₁₄ ^{35,37}₁₇Cl_{18,20} ³⁹₁₉K₂₀ ⁵⁵₂₅Mn₃₀ ^{102,104,105,110}₄₆Pd_{56,58,59,64} ¹³¹₅₄Xe₇₇ ^{185,187}₇₅Re_{110,112} ^{191,192}₇₇Ir_{114,116}
²⁰⁹⁼¹¹⁻¹⁹₈₃Bi₁₂₆* ²⁰⁹₈₄Po₁₂₅* ²⁴⁷⁼¹³⁻¹⁹₉₆Cm₁₅₁* ²⁴⁷₉₇Cm₁₅₀* ^{344,346,348}_{136,137,138}Fy_{208,209,210}^{ie} ³⁷⁶₁₄₉Ch₂₂₇^{ie} ⁴¹⁰₁₆₃Ch₂₄₇^{ie}

$$c_{au} = \frac{1}{\alpha_c} = \frac{25 \cdot 112}{3 \cdot (7 - \frac{1}{5} + \frac{1}{4 \cdot 23} - \frac{1}{3 \cdot (16 \cdot 9 \cdot 17 \cdot 19 - 1) + \frac{11}{56} \text{ or } \frac{34}{173}})} = 137.035999074627$$

¹⁹₉F₁₀ ²³₁₁Na₁₂ ^{32,33,34,36}₁₆S_{16,17,18,20} ^{35,37}₁₇Cl_{18,20} ³⁹₁₉K₂₀ ⁴⁶⁻⁵⁰₂₂Ti₂₄₋₂₈ ^{50,51}₂₃V_{27,28} ⁵⁵₂₅Mn₃₀ ^{54,56-58}₂₆Fe_{28,30-32} ^{78,80,82}₃₄Se_{44,46,48}
¹⁰⁰₄₄Ru₅₆ ^{102,105,106,108,110}₄₆Pd_{56,59,60,62,64} ^{136,137,138}₅₆Ba_{80,81,82} ¹⁷³₇₀Yb₁₀₃ ^{185,187}₇₅Re_{110,112} ²⁰⁹₈₄Po₁₂₅* ²¹⁰₈₅At₁₂₅* ²²²₈₆Rn₁₃₆*
²²³₈₇Fr₁₃₆* ²²⁶₈₈Ra₁₃₈* ²²⁷₈₉Ac₁₃₈* ^{235,138}₉₂U_{143,146}* ²⁸⁵₁₁₂Cn₁₇₃* ²⁸⁴₁₁₃Nh₁₇₁* ²⁸⁸₁₁₅Mc₁₇₃* ^{344,346,348}_{136,137,138}Fy_{208,209,210}^{ie} ³⁶⁶₁₄₄Ch₂₂₂^{ie} ³⁸⁴₁₅₃Ch₂₃₁^{ie}

$$c_{au} = \frac{1}{\alpha_c} = \frac{4 \cdot 100}{3} (1 + \frac{1}{36} - \frac{1}{63 \cdot (8 \cdot 15 \cdot 17 - 1) - \frac{34}{173} \text{ or } \frac{157}{17 \cdot 47}}) = 137.035999074627$$

⁷₃Li₄ ^{14,15}₇N_{7,8} ¹⁹₉F₁₀ ³¹₁₅P₁₆ ^{35,37}₁₇Cl_{18,20} ^{63,65}₂₉Cu_{34,36} ^{64,66,68,70}₃₀Zn_{34,36,38,40} ^{78,80}₃₄Se_{44,46} ^{82-84,86}₃₆Kr_{46-48,50} ^{90,91,94}₄₀Zr_{50,51,54}
¹⁰⁰₄₄Ru₅₆ ^{107,109}₄₇Ag_{60,62} ¹¹¹₄₈Cd₆₃ ^{118,120,122}₅₀Sn_{68,70,72} ¹³⁶₅₆Ba₈₀ ^{144,145,150}₆₀Nd_{84,85,90} ^{151,153}₆₃Eu_{88,90} ¹⁵⁷₆₄Gd₉₃ ¹⁶⁸₆₈Er₁₀₀ ¹⁷³₇₀Yb₁₀₃
^{199,200,204}₈₀Hg_{119,120,124} ²²²₈₆Rn₁₃₆* ²²³₈₇Fr₁₃₆* ²⁵⁷₁₀₀Fm₁₅₇* ²⁵⁸₁₀₁Md₁₅₇* ²⁵⁹₁₀₂No₁₅₇* ³⁰⁰₁₂₀Ch₁₈₀^{ie} ³³⁰₁₃₀Ch₂₀₀^{ie} ^{344,345}₁₃₆Fy_{208,209}^{ie} ⁴⁰⁰₁₅₇Ch₂₄₃^{ie}

$$c_{au} = \frac{1}{\alpha_c} = \frac{4 \cdot 100}{3 \cdot (1 - \frac{1}{37} + \frac{1}{29 \cdot (10 \cdot 13 \cdot 36 - 1) + \frac{31}{81}})} = 137.035999074627$$

⁷₃Li₄ ¹⁹₉F₁₀ ^{20,22}₁₀Ne_{10,12} ²⁷₁₃Al₁₄ ^{24,25,26}₁₂Mg_{12,13,14} ^{40,44,46,48}₂₀Ca_{20,26,24,28} ^{54,56,57,58}₂₆Fe_{28,30,31,32} ^{63,65}₂₉Cu_{34,36} ^{69,71}₃₁Ga_{38,40} ⁸⁴₃₆Kr₄₈
^{85,87}₃₇Rb_{48,50} ⁸⁹₃₉Y₅₀ ^{90,91,92,94}₄₀Zr_{50,51,52,54} ¹³⁷₅₆Ba₈₁ ^{196,200,204}₈₀Hg_{116,120,124} ^{7-29,205}₈₁Tl_{122,4-31} ²²³₈₇Fr₁₃₆* ³⁰⁰₁₂₀Ch₁₈₀^{ie} ³³⁰₁₃₀Ch₂₀₀^{ie} ⁴⁰⁰₁₅₇Ch₂₄₃^{ie}

$$c_{au} = \frac{1}{\alpha_c} = \frac{4 \cdot 100}{3} \sqrt{1 + \frac{1}{17} - \frac{1}{2 \cdot 199} + \frac{1}{50 \cdot (14 \cdot 43 \cdot 173 + 1) + \frac{5}{17}}} = 137.035999074627$$

⁷₃Li₄ ^{14,15}₇N_{7,8} ^{20,22}₁₀Ne_{10,12} ^{28,29,30}₁₄Si_{14,15,16} ^{35,37}₁₇Cl_{18,20} ^{40,42,43,44,48}₂₀Ca_{20,22,23,24,28} ⁵⁰₂₂Ti₂₈ ^{76,77}₃₄Se_{42,43} ⁸⁶₃₆Kr₅₀ ^{90,91}₄₀Zr_{50,51}
⁹⁹₄₃Tc₅₆* ¹⁰⁰₄₄Ru₅₆ ^{118,120}₅₀Sn_{68,70} ¹⁶⁸₆₈Er₁₀₀ ¹⁷³₇₀Yb₁₀₃ ^{199,200}₈₀Hg_{117,120,124} ²⁰⁹₈₄Po₁₂₅* ²¹⁰₈₅At₁₂₅* ²²²₈₆Rn₁₃₆* ²⁸⁵₁₁₂Cn₁₇₃* ³⁰⁰₁₂₀Ch₁₈₀^{ie}
³¹²₁₂₅Ch₁₁₇^{ie} ³²⁸₁₂₉Ch₉₉^{ie} ³³⁰₁₃₀Ch₂₀₀^{ie} ^{344,2-173,348}_{136,137,138}Fy_{208,209,210}^{ie} ⁴⁰⁰₁₅₇Ch₂₄₃^{ie} ⁴²⁰₁₇₀Ch₂₅₀^{ie}

$$c_{au} = \frac{1}{\alpha_c} = \frac{4 \cdot 100}{3 \cdot \sqrt{1 - \frac{1}{18} + \frac{1}{5 \cdot 89} - \frac{1}{75 \cdot 10 \cdot 19 \cdot 79 + \frac{11}{20}}}} = 137.035999074627$$

⁷₃Li₄ ⁹₄Be₅ ^{10,11}₅B_{5,6} ^{12,13}₆C_{6,7} ¹⁹₉F₁₀ ^{20,21,22}₁₀Ne_{10,11,12} ²³₁₁Na₁₂ ^{24,25,26}₁₂Mg_{12,13,14} ^{36,38,40}₁₈Ar_{18,20,22} ^{39,40,41}₁₉K_{20,21,22}
^{40,42,44,48}₂₀Ca_{20,22,24,28} ⁴⁶⁻⁵⁰₂₂Ti₂₄₋₂₈ ⁵⁵₂₅Mn₃₀ ^{54,56,57}₂₆Fe_{28,30,31} ^{66,68,70}₃₀Zn_{36,38,40} ⁷⁵₃₃As₄₂ ⁷⁹₃₅Br₄₄ ⁸⁹₃₉Y₅₀ ^{116,120,122}₅₀Sn_{66,70,72}
^{90,94,96}₄₀Zr_{50,54,56} ^{99,100,101,102}₄₄Ru_{55,56,57,60} ^{129,130,131,134,136}₅₄Xe_{75,76,77,80,82} ¹³⁵₅₆Ba₇₉ ^{138,139}₅₇La_{81,82} ¹⁵⁰₆₂Sm₈₈ ^{151,153}₆₃Eu_{88,90}
¹⁶¹₆₆Dy₉₅ ¹⁹⁷₇₉Au₁₁₈ ^{185,187}₇₅Re_{110,112} ¹⁹⁰₇₆Os₁₁₄ ^{198,200,201}₈₀Hg_{118,120,121} ²⁰⁹₈₄Po₁₂₅* ²¹⁰₈₅At₁₂₅* ²²⁶₈₈Ra₁₃₈* ²²⁷₈₉Ac₁₃₈* ²⁴³₉₅Am₁₄₈*
²⁴⁷₉₇Bk₁₅₀* ³⁰⁰₁₂₀Ch₁₈₀^{ie} ³¹²₁₂₅Ch₁₈₇^{ie} ³³⁰₁₃₀Ch₂₀₀^{ie} ³⁷⁸₁₅₀Ch₂₂₈^{ie} ⁴⁰⁰₁₅₇Ch₂₄₃^{ie} ⁴¹⁸₁₆₈Ch₂₅₀^{ie} ⁴²⁰₁₇₀Ch₂₅₀^{ie}

Supplement 11: Construct formulas of the fine-structure constant with Wallis

formula of $\pi/2$ instead of 2π -e formula

Wallis Formula of $\pi / 2$:

$$\text{Traditional format: } \frac{\pi}{2} = \frac{2}{1} \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots = \prod_{n=1}^{\infty} \left(\frac{2n}{2n-1} \frac{2n}{2n+1} \right)$$

$$\text{Natural group format: } \frac{\pi}{2} = 2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots = 2 \prod_{n=1}^{\infty} \left(\frac{2n}{2n+1} \frac{2n+2}{2n+1} \right)$$

$$\left(\frac{\pi}{2} \right)_k = 2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{2k}{2k+1} \frac{2k+2}{2k+1} = 2 \prod_{n=1}^k \left(\frac{2n}{2n+1} \frac{2n+2}{2n+1} \right)$$

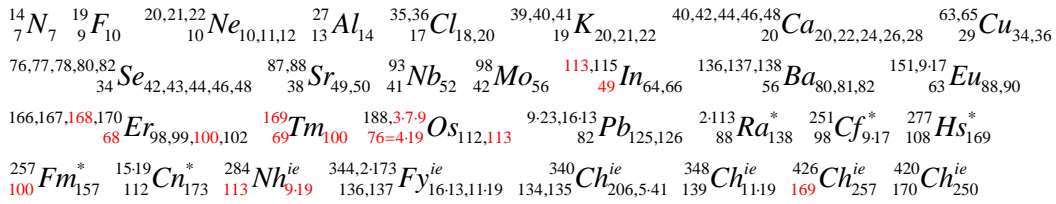
$$\text{Comparable and similar to } 2\pi - e \text{ formula: } (2\pi)_k = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}$$

So, Wallis Formula of $\pi / 2$ could be used to construct formulas of α .

Note: There should be $(2\pi)_k \sim \left(\frac{\pi}{2}\right)_{3k/2}$, or $(2\pi)_k$ and $\left(\frac{\pi}{2}\right)_{3k/2}$ keep the same accuracy.

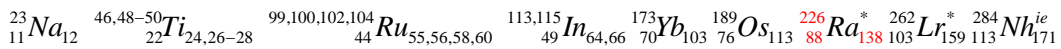
$$\alpha_{1-7\text{-Wallis}} = \frac{9}{7 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{338}{339} \frac{2 \cdot 10 \cdot 17}{2 \cdot 169 + 1} \right) 112 + \frac{1}{11 \cdot 137 + \frac{13 \cdot 41}{2 \cdot 19 \cdot 49 - 1}}}$$

$$= 1/137.035999037435$$



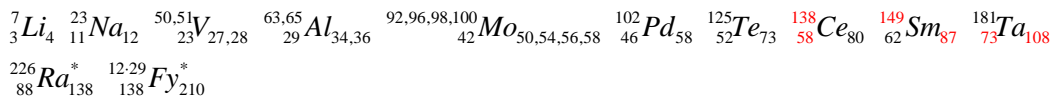
$$\alpha_{1-22\text{-Wallis}} = \frac{113}{4 \cdot 22 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{2346}{2347} \frac{4 \cdot (12 \cdot 49 - 1)}{2 \cdot 3 \cdot 17 \cdot 23 + 1} \right) 112 + \frac{1}{2 \cdot 103 \cdot (24 \cdot 13 + 1) + \frac{1}{14}}}$$

$$= 1/137.035999037435$$



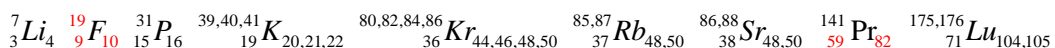
$$\alpha_{1-29\text{-Wallis}} = \frac{149}{4 \cdot 29 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{964}{965} \frac{6 \cdot 7 \cdot 23}{2 \cdot 2 \cdot (2 \cdot 11^2 - 1) + 1} \right) 112 + \frac{1}{27 \cdot 7 \cdot 73 - \frac{5}{9}}}$$

$$= 1/137.035999037435$$



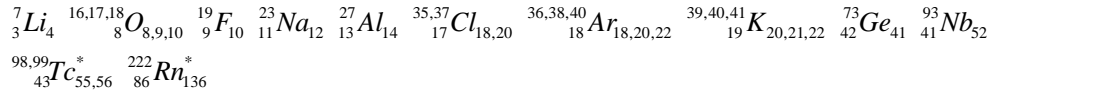
$$\alpha_{1-36\text{-Wallis}} = \frac{5 \cdot 37}{16 \cdot 9 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{708}{709} \frac{10 \cdot 71}{12 \cdot 59 + 1} \right) 112 + \frac{1}{15 \cdot (2 \cdot 19 \cdot 41 + 1) + \frac{4}{3 \cdot 15}}}$$

$$= 1/137.035999037435$$



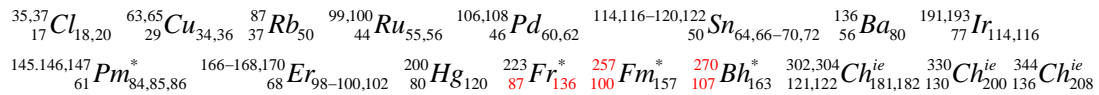
$$\alpha_{1-43-Wallis} = \frac{13 \cdot 17}{4 \cdot 43 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \dots \frac{602 \cdot 4 \cdot (8 \cdot 19 - 1)}{603 \cdot 2 \cdot 7 \cdot 43 + 1}\right)} \cdot 112 + \frac{1}{18 \cdot (2 \cdot 9 \cdot 11 + 1)} - \frac{41 \cdot (2 \cdot 11 \cdot 17 - 1)}{2 \cdot 10^{11}}$$

$$= 1/137.035999037435$$



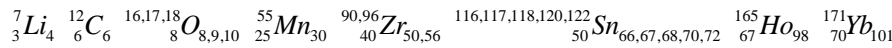
$$\alpha_{1-50-Wallis} = \frac{257}{4 \cdot 50 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \dots \frac{542}{543 \cdot 2 \cdot (270 + 1)}\right)} \cdot 112 + \frac{1}{3 \cdot 29 \cdot 61 + \frac{7 \cdot 11}{107}}$$

$$= 1/137.035999037435$$



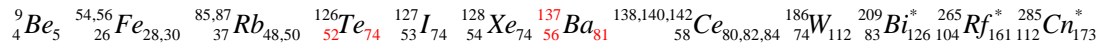
$$\alpha_{1-59-Wallis} = \frac{3 \cdot 101}{4 \cdot 59 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \dots \frac{7926}{7927 \cdot 2 \cdot 3 \cdot (10 \cdot 11 \cdot 12 + 1)}\right)} \cdot 112 + \frac{1}{40 \cdot 67 \cdot 233 + \frac{16}{25}}$$

$$= 1/137.035999037435$$



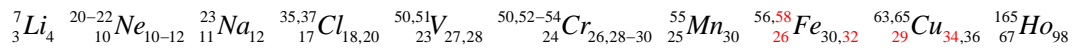
$$\alpha_{1-81-Wallis} = \frac{4 \cdot 26}{81 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \dots \frac{4814}{4815 \cdot 2 \cdot 29 \cdot 83 + 1}\right)} \cdot 112 + \frac{1}{8 \cdot 27 \cdot 37 \cdot (4 \cdot 53 - 1)}$$

$$= 1/137.035999037435$$



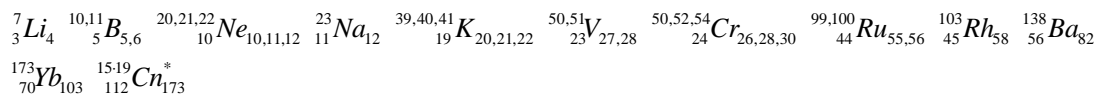
$$\alpha_{1-96-Wallis} = \frac{17 \cdot 29}{4 \cdot 96 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \dots \frac{17418}{17419 \cdot 6 \cdot (4 \cdot 6 \cdot 11^2 - 1)}\right)} \cdot 112 + \frac{1}{25 \cdot 10^{10}}$$

$$= 1/137.035999037435$$



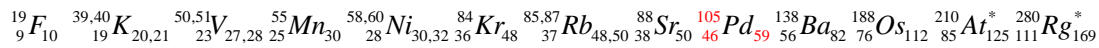
$$\alpha_{1-103-Wallis} = \frac{23^2}{4 \cdot 103 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \dots \frac{3930}{3931 \cdot 2 \cdot 15 \cdot (12 \cdot 11 - 1)}\right)} \cdot 112 + \frac{1}{23 \cdot (4 \cdot (8 \cdot 7 \cdot 19 + 1) + 1)} \cdot 4 \cdot 10^{11}$$

$$= 1/137.035999037435$$

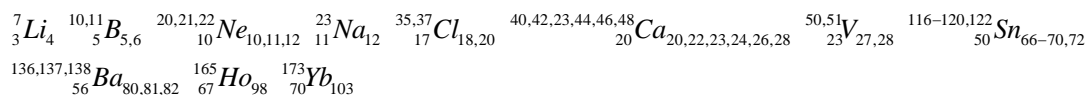


$$\alpha_{1-133-Wallis} = \frac{36 \cdot 19 - 1}{4 \cdot 7 \cdot 19 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \dots \frac{37170 \cdot 4 \cdot (4 \cdot 23 \cdot 101 + 1)}{37171 \cdot 2 \cdot 9 \cdot 5 \cdot 7 \cdot 59 + 1}\right)} \cdot 112 + \frac{1}{25 \cdot 10^{11}} \cdot 12 \cdot 37 \cdot (420 + 1)$$

$$= 1/137.035999037435$$

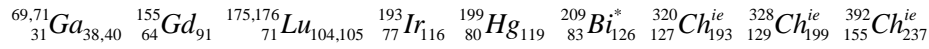


$$\alpha_{1-140-Wallis} = \frac{6^2 \cdot 20 - 1}{4 \cdot 140 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \dots \frac{5768 \cdot 10 \cdot (2 \cdot 17^2 - 1)}{5769 \cdot 56 \cdot 103 + 1}\right)} \cdot 112 + \frac{1}{5 \cdot 10^{11}} \cdot 11 \cdot 23 \cdot 67 = 1/137.035999037435$$



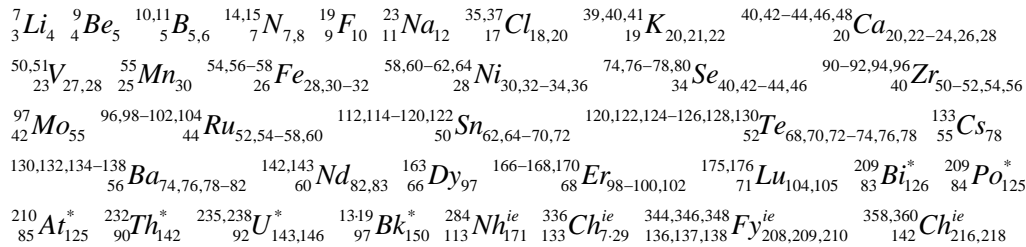
$$\alpha_{1-155\text{-Wallis}} = \frac{199}{5 \cdot 31 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{11964}{11965} \frac{2 \cdot 31 \cdot 193}{12 \cdot (12 \cdot 83 + 1) + 1}\right)} \frac{1}{112 + \frac{1}{8 \cdot 71 \cdot (10 \cdot 13 \cdot 29 - 1)}}$$

$$= 1/137.035999037435$$



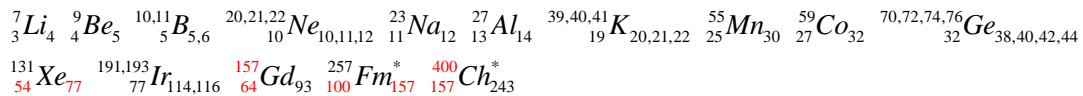
$$\alpha_{1-170\text{-Wallis}} = \frac{9 \cdot 97}{4 \cdot 10 \cdot 17 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{103324}{103325} \frac{6 \cdot 17 \cdot (4 \cdot 11 \cdot 23 + 1)}{2 \cdot 26 \cdot (4 \cdot 7 \cdot 71 - 1) + 1}\right)} \frac{1}{112 + \frac{7 \cdot 19 \cdot 83}{25 \times 10^{11}}}$$

$$= 1/137.035999037435$$

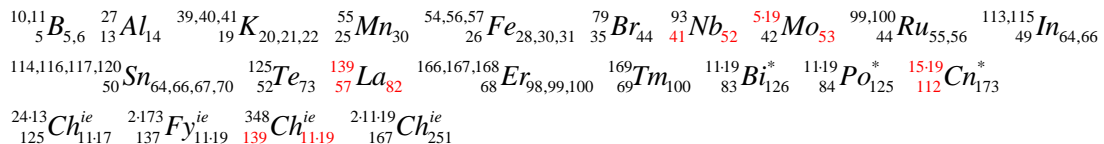


$$\alpha_{2-10\text{-Wallis}} = \frac{4 \cdot 10 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{312}{313} \frac{2 \cdot 157}{2 \cdot 12 \cdot 13 + 1}\right)}{7 \cdot 11} \frac{1}{112 - \frac{1}{2 \cdot 27^2} + \frac{32 \cdot 19 \cdot (4 \cdot (4 \cdot 7 \cdot 11 - 1) + 1)}{25 \cdot 10^{11}}}$$

$$= 1/137.035999111818$$

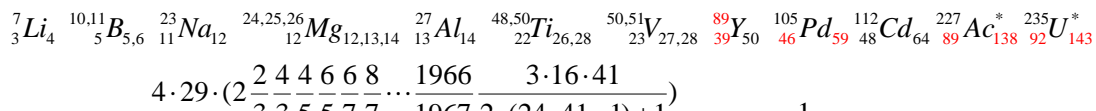


$$\alpha_{2-13\text{-Wallis}} = \frac{13 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{834}{835} \frac{4 \cdot 11 \cdot 19}{6 \cdot 139 + 1}\right)}{25} \frac{1}{112 - \frac{1}{5 \cdot 41 \cdot 49 - \frac{53}{11 \cdot 19}}} = 1/137.035999111818$$



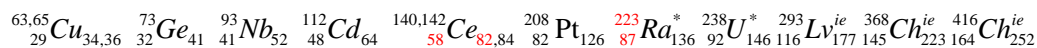
$$\alpha_{2-23\text{-Wallis}} = \frac{4 \cdot 23 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{482}{483} \frac{4 \cdot 11^2}{2 \cdot (12 \cdot 20 + 1) + 1}\right)}{3 \cdot 59} \frac{1}{112 - \frac{1}{16 \cdot 3 \cdot (4 \cdot 7 \cdot 11 - 1) + \frac{3 \cdot 89}{2 \cdot 11 \cdot 13}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-29\text{-Wallis}} = \frac{4 \cdot 29 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{1966}{1967} \frac{3 \cdot 16 \cdot 41}{2 \cdot (24 \cdot 41 - 1) + 1}\right)}{223} \frac{1}{112 - \frac{1}{4 \cdot 41 \cdot 239 + \frac{7}{16}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-33-Wallis} = \frac{4 \cdot 33 \cdot \left(2 \frac{2 \ 4 \ 4 \ 6 \ 6 \ 8 \ \dots \ 414 \ 32 \cdot 13}{3 \ 3 \ 5 \ 5 \ 7 \ 7 \ \dots \ 415 \ 2 \cdot 9 \cdot 23 + 1}\right)}{2 \cdot 127} \frac{1}{112 - \frac{1}{24 \cdot 127 - \frac{139}{16 \cdot 13}}} = 1/137.035999111818$$

$$\alpha_{2-36-Wallis} = \frac{4 \cdot 36 \cdot \left(2 \frac{2 \ 4 \ 4 \ 6 \ 6 \ 8 \ \dots \ 570 \ 4 \cdot 11 \cdot 13}{3 \ 3 \ 5 \ 5 \ 7 \ 7 \ \dots \ 571 \ 2 \cdot 15 \cdot 19 + 1}\right)}{2 \cdot 138 + 1} \frac{1}{112 - \frac{1}{10 \cdot 15 \cdot 23 - \frac{3 \cdot 11 \cdot 17}{5 \cdot 13}}} = 1/137.035999111818$$

$$\alpha_{2-125-Wallis} = \frac{5 \cdot 10^2 \cdot \left(2 \frac{2 \ 4 \ 4 \ 6 \ 6 \ 8 \ \dots \ 12880 \ 6 \cdot 19 \cdot 113}{3 \ 3 \ 5 \ 5 \ 7 \ 7 \ \dots \ 12881 \ 20 \cdot 23 \cdot 28 + 1}\right)}{31^2} \frac{1}{112 - \frac{1}{32 \cdot 239 \cdot 281}} = 1/137.035999111818$$

$$\alpha_{2-253-Wallis} = \frac{4 \cdot 11 \cdot 23 \cdot \left(2 \frac{2 \ 4 \ 4 \ 6 \ 6 \ 8 \ \dots \ 84550 \ 24 \cdot 13 \cdot 271}{3 \ 3 \ 5 \ 5 \ 7 \ 7 \ \dots \ 84551 \ 2 \cdot 19 \cdot 25 \cdot 89 + 1}\right)}{5 \cdot (4 \cdot 97 + 1)} \frac{1}{112 - \frac{27 \cdot 19 \cdot 61}{20 \times 10^{11}}} = 1/137.035999111818$$

$$\alpha_{2-269-Wallis} = \frac{269 \cdot \left(2 \frac{2 \ 4 \ 4 \ 6 \ 6 \ 8 \ \dots \ 124926 \ 2^{11} \cdot 61}{3 \ 3 \ 5 \ 5 \ 7 \ 7 \ \dots \ 124927 \ 2 \cdot 3 \cdot 47 \cdot (4 \cdot 3 \cdot 37 - 1) + 1}\right)}{11 \cdot 47} \frac{1}{112 - \frac{9 \cdot 19 \cdot 37}{10^{12}}} = 1/137.035999111818$$

Supplement 12: Comparison of two kinds of formulas of the speed of light

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-7} \alpha_{2-13}}} = \frac{5}{3} \sqrt{\frac{7 (2\pi)_{112}}{13 (2\pi)_{278}} \left(112^2 - \frac{1}{30^2 \cdot 5} + \frac{1}{60^2 \cdot 15} - \frac{1}{120^2 \cdot 15 \cdot 29}\right)}$$

=137.035999074627 (Refer to Page 12)

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-7} \alpha_{2-13}}} = \frac{5}{3} \sqrt{\frac{7 (2\pi)_{112}}{13 (2\pi)_{278}} \left(112^2 - \frac{1}{9 \cdot 5 \cdot 109 + \frac{1}{4}}\right)} = 137.035999074627$$

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-7-Wallis} \alpha_{2-13-Wallis}}} = \frac{5}{3} \sqrt{\frac{7 \left(\frac{\pi}{2}\right)_{169}}{13 \left(\frac{\pi}{2}\right)_{3139}} \left(112^2 + \frac{1}{15} - \frac{1}{4 \cdot 71} + \frac{1}{6 \cdot (48 \cdot (12 \cdot 29 - 1) + 1)}\right)}$$

=137.035999074627

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-29}\alpha_{2-29}}} = \sqrt{\frac{223(2\pi)_{3\cdot 107}}{149(2\pi)_{5\cdot 131}} \left(112^2 + \frac{1}{6\cdot 23} - \frac{1}{5\cdot 67\cdot 100 + \frac{17}{25}}\right)} = 137.035999074627$$

^{50,51}V_{23 27,28} ^{136,138}Ba_{56 80,82} ¹⁶⁵Ho_{67 98} ²²³Fr_{87 136} ²²⁷Ac_{89 138} ²⁷⁰Bh_{107 163} ^{344,346,12-29}Fy_{136,137,138 208,209,210} ³³²Ch_{131 3-67} ¹⁶⁻²³Ch_{145 223} ³⁷⁶Ch_{149 227}*

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-29-Wallis}\alpha_{2-29-Wallis}}} = \sqrt{\frac{223\left(\frac{\pi}{2}\right)_{2\cdot(2\cdot 1^2-1)}}{149\left(\frac{\pi}{2}\right)_{24\cdot 41-1}} \left(112^2 + \frac{1}{6\cdot 23} - \frac{1}{14\cdot 17\cdot 137 + \frac{29}{5\cdot 17}}\right)}$$

= 137.035999074627

^{10,11}B_{5 5,6} ^{12,13}C_{6 6,7} ^{14,15}N_{7 7,8} ^{28,29}Si_{14 14,15} ^{35,37}Cl_{17 18,20} ^{50,51}V_{23 27,28} ^{63,65}Cu_{29 34,36} ⁸⁰Se_{34 46} ^{102,104-106,108,110}Pd_{46 56,58-60,62,64}
^{136,137,138}Ba_{46 80,81,82} ¹⁴⁹Sm_{62 87} ¹⁶⁹Tm_{69 100} ²¹⁰Po_{85 125} ²²³Fr_{87 136} ²²⁷Ac_{89 138} ^{344,346,12-29}Fy_{136,137,138 208,209,210} ¹⁶⁻²³Ch_{145=5-29 223} ³⁷⁶Ch_{149 227}*

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-36}\alpha_{2-36}}} = \sqrt{\frac{277(2\pi)_{4\cdot 59}}{5\cdot 37(2\pi)_{10\cdot 19}} \left(112^2 - \frac{1}{29} + \frac{1}{6\cdot 41\cdot 47 - \frac{3\cdot 29}{139}}\right)} = 137.035999074627$$

¹⁹F_{9 10} ^{39,40,41}K_{19 20,21,22} ^{63,65}Cu_{29 34,36} ⁸²Kr_{36 46} ^{85,87}Rb_{37 48,50} ¹³⁸Ba_{56 82} ¹³⁹La_{57 82} ³⁻⁴⁷Pr_{59 2-41} ^{184,186}W_{74 110,112} ²²³Fr_{87 136} ¹²⁻²⁹Ch_{139 209}*

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-36-Wallis}\alpha_{2-36-Wallis}}} = \sqrt{\frac{277\left(\frac{\pi}{2}\right)_{6\cdot 59}}{5\cdot 37\left(\frac{\pi}{2}\right)_{15\cdot 19}} \left(112^2 - \frac{1}{36} + \frac{1}{4\cdot 43\cdot 61 - \frac{9}{43}}\right)} = 137.035999074627$$

⁹Be_{4 5} ¹⁹F_{9 10} ³⁹K_{19 20} ^{84,86}Kr_{36 48,50} ^{85,87}Rb_{37 48,50} ^{2-49,99}Tc_{43 55,56} ¹⁰⁵Pd_{46 59} ^{145,3-49}Pm_{61 84,86} ²⁷⁷Hs_{108 132} ⁶⁻³⁷Rn_{86 136} ¹⁵⁻¹⁹Cn_{112 173}
²⁹³Lv_{116 3-59} ⁶⁻⁴⁹Ts_{117=9-13 3-59} ¹⁶⁻¹⁹Ch_{122 14-13}^{ie}

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-81}\alpha_{2-125}}} = \frac{9\cdot 31}{40} \sqrt{\frac{2(2\pi)_{15\cdot 107}}{5\cdot 13(2\pi)_{53\cdot 81}} \left(112^2 + \frac{1}{2\cdot 17\cdot 53 - \frac{29}{49}}\right)} = 137.035999074627$$

¹⁹F_{9 10} ²⁷Al_{13 14} ³¹P_{15 16} ^{35,37}Cl_{17 18,20} ⁵³Cr_{24 29} ^{54,56,57}Fe_{26 28,30,31} ^{63,65}Cu_{29 34,36} ^{68,71}Ga_{31 38,40} ^{74,76,78,80}Se_{34 40,42,44,46} ⁸⁷Sr_{38 49} ⁹³Nb_{41 52}
^{90-92,94,96}Zr_{40 50-52,54,56} ⁹⁵Mo_{42 53} ^{103,105}In_{49 64,66} ¹³⁰Te_{52 78} ¹²⁷I_{53 74} ³⁻⁵³Tb_{65 94} ²⁰⁰Hg_{80 120} ²⁰⁵Tl_{81 124} ²³⁷Np_{93 144} ²⁶⁹Sg_{106 163} ²⁷⁰Bh_{107 163}*

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-81-Wallis}\alpha_{2-125-Wallis}}} = \frac{9\cdot 31}{40} \sqrt{\frac{2\left(\frac{\pi}{2}\right)_{29\cdot 83}}{5\cdot 13\left(\frac{\pi}{2}\right)_{10\cdot 23\cdot 28}} \left(112^2 + \frac{1}{10\cdot 16\cdot 19\cdot 23}\right)} = 137.035999074627$$

¹⁹F_{9 10} ²⁷Al_{13 14} ^{28,29,30}Si_{14 14,15,16} ³¹P_{15 16} ^{39,40}K_{19 20,21} ^{50,51}V_{23 27,28} ^{54,56,57,32}Fe_{26 28,30,31,32} ^{58,60,62,64}Ni_{28 30,32,34,36} ^{63,65}Cu_{29 34,36}
^{68,71}Ga_{31 38,40} ^{90,92,96}Zr_{40 50,52,56} ⁹³Nb_{41 52} ¹³⁶⁻¹³⁸Ba_{56 80-82} ¹⁴³Nd_{60 83} ²⁰⁰Hg_{80 120} ²⁰⁹Bi_{83 126} ²³⁷Np_{93 144} ¹⁵⁻¹⁹Cn_{112 173} ¹⁴⁻²⁹Ch_{160 246}^{ie}

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-170}\alpha_{2-253}}} = \frac{5}{3} \sqrt{\left(\frac{8\cdot 17}{11\cdot 23} + \frac{2\cdot 17}{11\cdot 23\cdot 97}\right) \frac{(2\pi)_{2\cdot 25\cdot 13\cdot 53}}{(2\pi)_{2\cdot 17\cdot(36\cdot 23+1)}} \left(112^2 - \frac{18\cdot 97+1}{2\cdot 10^9}\right)}$$

= 137.035999074627

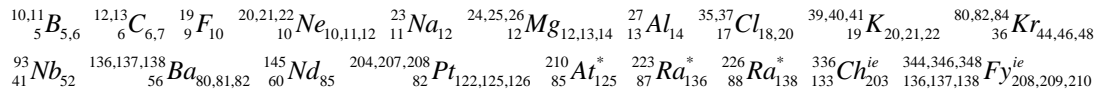
²³Na_{11 12} ^{35,37}Cl_{17 18,20} ^{36,38,40}Ar_{18 18,20,22} ^{50,51}V_{23 27,28} ⁵⁵Mn_{25 30} ^{78,80}Se_{34 44,46} ⁹⁷Mo_{42 55} ¹⁶³Dy_{66 97} ^{136,137,138}Ba_{56 80,81,82} ¹³⁻¹⁹Bk_{97 150}*

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-170-Wallis}\alpha_{2-253-Wallis}}} = \frac{5}{3} \sqrt{\left(\frac{8\cdot 17}{11\cdot 23} + \frac{2\cdot 17}{11\cdot 23\cdot 97}\right) \frac{\left(\frac{\pi}{2}\right)_{26\cdot(4\cdot 7\cdot 71-1)}}{\left(\frac{\pi}{2}\right)_{19\cdot 25\cdot 89}} \left(112^2 - \frac{19\cdot(4\cdot 83-1)}{5\cdot 10^9}\right)}$$

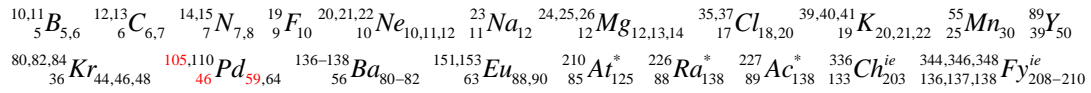
= 137.035999074627

²³Na_{11 12} ^{35,37}Cl_{17 18,20} ^{39,40,41}K_{19 20,21,22} ^{50,51}V_{23 27,28} ⁵⁵Mn_{25 30} ^{56,57}Fe_{26 30,31} ^{78,80}Se_{34 44,46} ⁹⁷Mo_{42 55} ¹⁶³Dy_{66 97} ^{136,137,138}Ba_{56 80,81,82}
¹⁴²Nd_{60 82} ^{175,176}Lu_{71 104,105} ²⁰⁹Bi_{83 126} ²²⁷Ac_{89 138} ²³²Th_{90 142} ¹³⁻¹⁹Bk_{97 150} ²⁸⁵Cn_{112 173} ⁴⁻⁷¹Nh_{113 9-19} ^{344,346,348}Fy_{136,137,138 208,209,210} ⁶⁻⁷¹Ch_{169 257}*

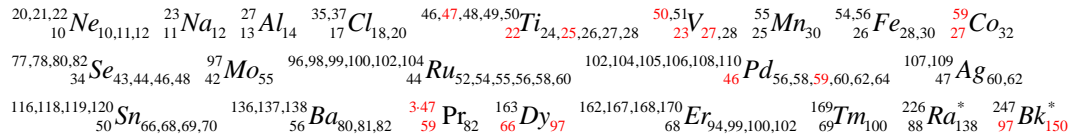
$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-133}\alpha_{2-253}}} = \sqrt{\frac{5 \cdot 17 - 10}{\frac{11}{36} - \frac{11 \cdot 23}{7 \cdot 7 \cdot 19}} \frac{(2\pi)_{13 \cdot (8 \cdot 7 \cdot 17 + 1)}}{(2\pi)_{2 \cdot 17 \cdot (36 \cdot 23 + 1)}} (112^2 - \frac{23 \cdot (12 \cdot 41 - 1)}{10^{10}})} = 137.035999074627$$



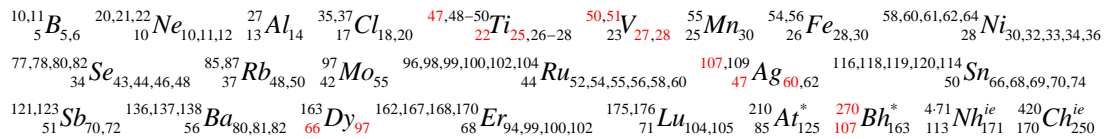
$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-133}\alpha_{2-253}}} = \sqrt{\frac{5 \cdot 17 - 10}{\frac{11}{36} - \frac{11 \cdot 23}{7 \cdot 7 \cdot 19}} \frac{(\frac{\pi}{2})_{9 \cdot 5 \cdot 7 \cdot 59}}{(\frac{\pi}{2})_{19 \cdot 25 \cdot 89}} (112^2 + \frac{19 \cdot (8 \cdot 9 \cdot 11^2 + 1)}{25 \cdot 10^9})} = 137.035999074627$$



$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-170}\alpha_{2-269}}} = \frac{2}{3} \sqrt{\frac{11 \cdot 47 \cdot 170}{97 \cdot (270 - 1)} \frac{(2\pi)_{2 \cdot 25 \cdot 13 \cdot 53}}{(2\pi)_{2 \cdot 59 \cdot (6 \cdot 59 - 1)}} (112^2 + \frac{23}{2 \cdot 10^9})} = 137.035999074627$$



$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-170-Wallis}\alpha_{2-269-Wallis}}} = \frac{2}{3} \sqrt{\frac{11 \cdot 47 \cdot 170}{97 \cdot (270 - 1)} \frac{(\frac{\pi}{2})_{26 \cdot (4 \cdot 7 \cdot 71 - 1)}}{(\frac{\pi}{2})_{3 \cdot 47 \cdot (4 \cdot 3 \cdot 37 - 1)}} (112^2 - \frac{107}{5 \cdot 10^8})} = 137.035999074627$$



Supplement 13: Construct formulas of the fine-structure constant with

Gregory-Leibniz formula of $\pi/4$ instead of 2π -e formula

Gregory-Leibniz Formula of $\pi / 4$:

$$\text{Traditional format: } \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$$

$$\text{Natural group format: } \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$$

$$\left(\frac{\pi}{4}\right)_k = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{k} = 1 + \sum_{n=1}^k \frac{(-1)^n}{2n+1}$$

$$\text{Comparable and similar to } 2\pi - e \text{ formula: } (2\pi)_k = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}$$

So, Leibniz Formula of $\pi / 4$ could be used to construct formulas of α .

Note: There should be $(2\pi)_k \sim \left(\frac{\pi}{4}\right)_k \frac{3}{2} \sqrt[3]{\frac{\sqrt{5}+1}{2}}_k$.

That means $(2\pi)_k$ and $\left(\frac{\pi}{4}\right)_k \frac{3}{2} \sqrt[3]{\frac{\sqrt{5}+1}{2}}_k$ become convergent at the same speed.

The larger the k is, the better is the accuracy and the less is the error.

$$\alpha_{1-7-GL} = \frac{36}{8 \cdot 7 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 6 \cdot 36 + 1}\right)} \frac{1}{112 + \frac{1}{64 \cdot 13} - \frac{1}{128 \cdot 3 \cdot (4 \cdot 3 \cdot 17 \cdot 23 - 1)}}$$

$$= 1/137.035999037435$$

$$^{12,13}_6C, ^{27}_{13}Al, ^{35,37}_{17}Cl, ^{50,51}_{23}V, ^{62,64}_{28}Ni, ^{84}_{36}Kr, ^{112}_{48}Cd, ^{136-138}_{56}Ba, ^{155,156,160}_{64}Gd$$

$$\alpha_{1-22-GL} = \frac{113}{8 \cdot 22 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 5 \cdot 13 \cdot 23 + 1}\right)} \frac{1}{112 + \frac{1}{5 \cdot 17 \cdot (2 \cdot 3 \cdot 7 \cdot 13 - 1)} + \frac{3}{44}}$$

$$= 1/137.035999037435$$

$$\alpha_{1-29-GL} = \frac{149}{8 \cdot 29 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot (28 \cdot 11 - 1) + 1}\right)} \frac{1}{112 + \frac{1}{6 \cdot 25 \cdot 79 + \frac{28}{5 \cdot 23}}}$$

$$= 1/137.035999037435$$

$$^{47,50}_{22}Ti, ^{50,51}_{23}V, ^{79}_{35}Br, ^{135}_{56}Ba, ^{149}_{62}Sm$$

$$\alpha_{1-36-GL} = \frac{5 \cdot 37}{8 \cdot 36 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 4 \cdot 113 + 1}\right)} \frac{1}{112 + \frac{1}{3 \cdot 29 \cdot 47 - \frac{2 \cdot 47}{4 \cdot 83 - 1}}}$$

$$= 1/137.035999037435$$

$$^{24}_{12}Mg, ^{63,65}_{29}Cu, ^{83}_{36}Kr, ^{85,87}_{37}Rb, ^{107,109}_{47}Ag, ^{112,113}_{48}Cd, ^{184,186}_{74}W, ^{223}_{87}Fr^*$$

$$^{284}_{113}Nh^{ie}, ^{288}_{115}Mc^{ie}, ^{223}_{144}Ch^{ie}$$

$$\alpha_{1-43-GL} = \frac{13 \cdot 17}{8 \cdot 43 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 128 \cdot 3 + 1}\right)} \frac{1}{112 + \frac{1}{4 \cdot (64 \cdot 9 + 1) - \frac{136}{137}}}$$

$$\text{or } = \frac{13 \cdot 17}{8 \cdot 43 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 128 \cdot 3 + 1}\right)} \frac{1}{112 + \frac{1}{4 \cdot (64 \cdot 9 + 1)} - \frac{12 \cdot 11 \cdot 13}{25 \cdot 10^{11}}}$$

$$= 1/137.035999037437 \text{ or } 1/137.035999037435$$

$$^{19}_9F, ^{77}_{34}Se, ^{84}_{36}Kr, ^{112}_{48}Cd, ^{136,137}_{56}Ba, ^{99}_{43}Tc^*, ^{222}_{86}Rn^*, ^{285}_{112}Cn^*, ^{8 \cdot 43, 2 \cdot 173}_{136, 137}Fy^{ie}, ^{16 \cdot 13, 209}_{16, 13, 209}$$

$$\alpha_{1-50-GL} = \frac{257}{8 \cdot 50 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 173 + 1}\right)} \frac{1}{112 + \frac{1}{16 \cdot 11 \cdot 13 - \frac{28}{6 \cdot 31}}}$$

$$= 1/137.035999037436$$

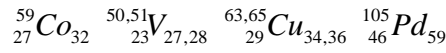
$$^{23}_{11}Na, ^{31}_{15}P, ^{47,48,50}_{22}Ti, ^{55}_{25}Mn, ^{54,56,57,58}_{26}Fe, ^{58,60}_{28}Ni, ^{69,71}_{31}Ga$$

$$^{116,119,120}_{50}Sn, ^{136,137}_{56}Ba, ^{150}_{62}Sm, ^{168}_{68}Er, ^{169}_{69}Tm, ^{173}_{70}Yb, ^{400}_{80}Hg, ^{209}_{84}Po^*$$

$$^{237}_{93}Np^*, ^{257}_{100}Fm^*, ^{285}_{112}Cn^*, ^{288}_{115}Mc^*, ^{2 \cdot 173}_{137}Fy^*, ^{400}_{157}Ch^*, ^{6 \cdot 71}_{169}Ch^*$$

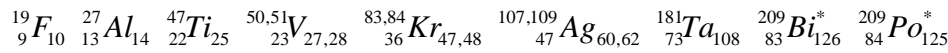
$$\alpha_{1-59-GL} = \frac{3 \cdot 101}{8 \cdot 59 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 6 \cdot 29^2 + 1}\right)} \cdot 112 + \frac{1}{27 \cdot (8 \cdot 9 \cdot (8 \cdot 3 \cdot 13 - 1) - 1) + \frac{10}{23}}$$

$$= 1/137.035999037435$$



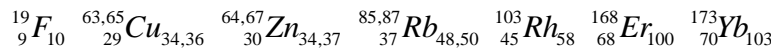
$$\alpha_{1-81-GL} = \frac{4 \cdot 13}{81 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 6 \cdot 7 \cdot 73 + 1}\right)} \cdot 112 + \frac{1}{5 \cdot 23 \cdot (36 \cdot 47 + 1) + \frac{19}{25}}$$

$$= 1/137.035999037435$$



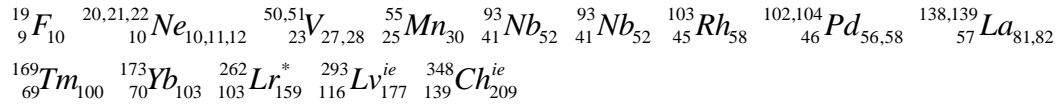
$$\alpha_{1-96-GL} = \frac{7 \cdot 29}{8 \cdot 96 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 10 \cdot (30 \cdot 37 - 1) + 1}\right)} \cdot 112 + \frac{1}{9 \cdot 103 \cdot (4 \cdot 17 \cdot 41 + 1)}$$

$$= 1/137.035999037435$$



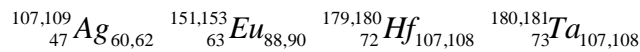
$$\alpha_{1-103-GL} = \frac{23^2}{8 \cdot 103 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 18 \cdot 139 + 1}\right)} \cdot 112 + \frac{1}{2 \cdot (10 \cdot 23 \cdot 41 + 1)} \cdot \frac{1}{25 \cdot 10^{10}}$$

$$= \frac{23^2}{8 \cdot 103 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 18 \cdot 139 + 1}\right)} \cdot 112 + \frac{1}{4 \cdot 10^{11}} = 1/137.035999037435$$



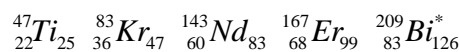
$$\alpha_{1-133-GL} = \frac{36 \cdot 19 - 1}{8 \cdot 7 \cdot 19 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 16 \cdot 3 \cdot 17 \cdot 29 + 1}\right)} \cdot 112 + \frac{1}{3 \cdot 107 \cdot 151} \cdot \frac{1}{4 \cdot 10^{11}}$$

$$= 1/137.035999037435$$

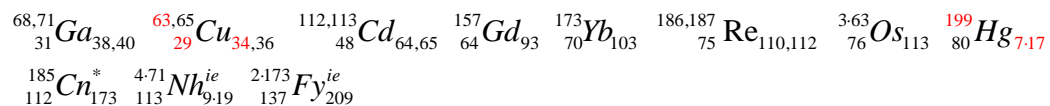


$$\alpha_{1-140-GL} = \frac{36 \cdot 20 - 1}{8 \cdot 7 \cdot 20 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 22 \cdot 167 + 1}\right)} \cdot 112 + \frac{1}{128 \cdot 19 \cdot 83 + \frac{2}{17}}$$

$$= 1/137.035999037435$$

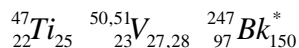


$$\alpha_{1-155-GL} = \frac{199}{10 \cdot 31 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 64 \cdot 7 \cdot 17 + 1}\right)} \cdot 112 + \frac{1}{9 \cdot 7 \cdot 29 \cdot 107} \cdot \frac{1}{25 \cdot 10^{11}} \text{ or } \frac{113 \cdot 173}{25 \cdot 10^{10}}$$



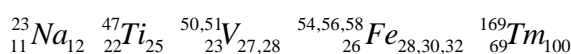
$$\alpha_{1-170-GL} = \frac{3^2 \cdot 97}{8 \cdot 170 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 6 \cdot 19 \cdot (2 \cdot 17^2 - 1) + 1}\right)} \frac{1}{112 + \frac{7 \cdot 23 \cdot 41}{25 \cdot 10^{11}}}$$

$$= 1/137.035999037435$$



$$\alpha_{2-10-GL} = \frac{8 \cdot 10 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 18 \cdot 11 + 1}\right)}{7 \cdot 11} \frac{1}{112 - \frac{1}{3 \cdot 23 \cdot (2 \cdot 13 \cdot 23 + 1)} + \frac{30}{47} \text{ or } \frac{37}{58}}$$

$$= 1/137.035999111818$$

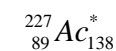


$$\alpha_{2-13-GL} = \frac{8 \cdot 13 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 28 \cdot 19 + 1}\right)}{100} \frac{1}{112 - \frac{1}{10 \cdot (10 \cdot 23 - 1) + \frac{7 \cdot (36 \cdot 11 + 1)}{100^2}}}$$

$$= 1/137.035999111818$$

$$\alpha_{2-23-GL} = \frac{8 \cdot 23 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 4 \cdot 7 \cdot 11 + 1}\right)}{3 \cdot 59} \frac{1}{112 - \frac{1}{36 \cdot 61 - \frac{89}{170}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-29-GL} = \frac{8 \cdot 29 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 4 \cdot (24 \cdot 13 + 1) + 1}\right)}{223} \frac{1}{112 - \frac{1}{25 \cdot 49 \cdot 89 + \frac{4}{11}}}$$

$$= 1/137.035999111818$$

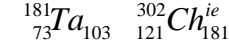
$$\alpha_{2-33-GL} = \frac{4 \cdot 33 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 24 \cdot 11 + 1}\right)}{127} \frac{1}{112 - \frac{1}{81 \cdot 25} + \frac{1}{2 \cdot 43 \cdot 61 \cdot (2 \cdot 11 \cdot 49 - 1)}}$$

$$= 1/137.035999111818$$



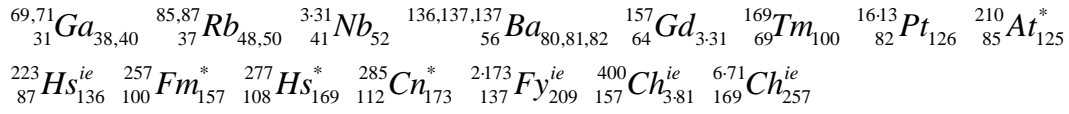
$$\alpha_{2-36-GL} = \frac{8 \cdot 36 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 181 + 1}\right)}{2 \cdot 138 + 1} \frac{1}{112 - \frac{1}{16 \cdot (2 \cdot 7 \cdot 11 \cdot 17 - 1) + \frac{6}{7}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-125-GL} = \frac{8 \cdot 125 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 8 \cdot 25 \cdot 41 + 1}\right)}{31^2} \frac{1}{112 - \frac{1}{2 \cdot 13^2 \cdot 37 \cdot 137}}$$

$$= 1/137.035999111818$$

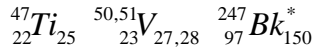


$$\alpha_{2-253-GL} = \frac{8 \cdot 11 \cdot 23 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 6 \cdot (30 \cdot 13 \cdot 23 + 1) + 1}\right)}{5 \cdot (4 \cdot 49 + 1)} \frac{1}{112 - \frac{6 \cdot 49}{25 \cdot 10^9}}$$

$$= 1/137.035999111818$$

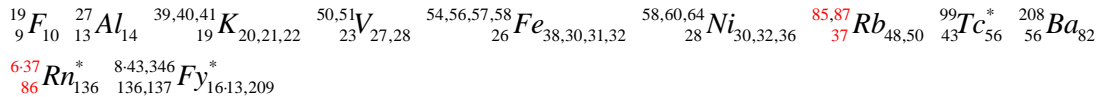
$$\alpha_{2-269-GL} = \frac{2 \cdot (270 - 1) \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 30 \cdot 11 \cdot (2 \cdot 11^2 - 1) + 1}\right)}{11 \cdot 47} \frac{1}{112 - \frac{2 \cdot 47 \cdot 97}{25 \cdot 10^{11}}}$$

$$= 1/137.035999111818$$



Supplement 14: Construct formulas of the speed of light with α_{1-GL} and α_{2-GL}

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-7-GL} \alpha_{2-13-GL}}} = \frac{5}{3} \sqrt{\frac{7 \left(\frac{\pi}{4}\right)^{6 \cdot 36}}{13 \left(\frac{\pi}{4}\right)^{4 \cdot 7 \cdot 19}} \left(112^2 + \frac{1}{9} - \frac{1}{23 \cdot 43 + \frac{37}{3 \cdot 43}}\right)} = 137.035999074627$$



$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-29-GL} \alpha_{2-29-GL}}} = \sqrt{\frac{223 \left(\frac{\pi}{4}\right)^{2 \cdot (28 \cdot 11 - 1)}}{149 \left(\frac{\pi}{4}\right)^{4 \cdot (24 \cdot 13 + 1)}} \left(112^2 + \frac{1}{2 \cdot 59} - \frac{1}{17 \cdot (14 \cdot 83 + 1) + \frac{4}{11}}\right)}$$

$$= 137.035999074627$$

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-36-GL}\alpha_{2-36-GL}}} = \sqrt{\frac{277 \left(\frac{\pi}{4}\right)^{4 \cdot 113}}{5 \cdot 37 \left(\frac{\pi}{4}\right)^{2 \cdot 181}} \left(112^2 + \frac{1}{40} - \frac{1}{2 \cdot 7 \cdot 11 \cdot 23 + \frac{9}{37}}\right)}$$

= 137.035999074627

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-81-GL}\alpha_{2-125-GL}}} = \frac{9 \cdot 31}{40} \sqrt{\frac{2 \left(\frac{\pi}{4}\right)^{6 \cdot 7 \cdot 73}}{5 \cdot 13 \left(\frac{\pi}{4}\right)^{8 \cdot 25 \cdot 41}} \left(112^2 + \frac{1}{37 \cdot 53 + \frac{19}{2 \cdot 43}}\right)}$$

= 137.035999074627

¹²⁷₅₃I₇₄

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-170-GL}\alpha_{2-253-GL}}} = \frac{5}{3} \sqrt{\left(\frac{8 \cdot 17}{11 \cdot 23} + \frac{2 \cdot 17}{11 \cdot 23 \cdot 97}\right) \frac{\left(\frac{\pi}{4}\right)^{6 \cdot 19 \cdot (2 \cdot 17^2 - 1)}}{\left(\frac{\pi}{4}\right)^{6 \cdot (30 \cdot 13 \cdot 23 + 1)}} \left(112^2 - \frac{19 \cdot 43}{8 \cdot 10^8}\right)}$$

= 137.035999074627

²³₁₁Na₁₂ ^{35,37}₁₇Cl_{18,20} ^{39,40,41}₁₉K_{20,21,22} ^{40,42,43,44,46,48}₂₀Ca_{20,22,23,24,26,28} ^{50,51}₂₃V_{27,28} ⁵⁵₂₅Mn₃₀ ^{56,57}₂₆Fe_{30,31}
^{64,66,68}₃₀Zn_{34,36,38} ^{76,77,78,80}₃₄Se_{42,43,44,46} ⁹⁷₄₂Mo₅₅ ^{97,98,99}₄₃Tc_{54,55,56} ¹⁶³₆₆Dy₉₇ ^{136,137,138}₅₆Ba_{80,81,82} ²²²₈₆Ac₁₃₆
¹³⁻¹⁹₉₇Bk₁₅₀ ¹⁵⁻¹⁹₁₁₂Cn₁₇₃ ^{8-43,346,348}_{136,137,138}Fy_{208,209,210}^{ie}

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-170-GL}\alpha_{2-269-GL}}} = \frac{2}{3} \sqrt{\frac{11 \cdot 47 \cdot 170}{97 \cdot (270 - 1)} \frac{\left(\frac{\pi}{4}\right)^{6 \cdot 19 \cdot (2 \cdot 17^2 - 1)}}{\left(\frac{\pi}{4}\right)^{2 \cdot 30 \cdot 11 \cdot (2 \cdot 11^2 - 1)}} \left(112^2 - \frac{23 \cdot 49}{10^{10}}\right)}$$

= 137.035999074627

²³₁₁Na₁₂ ^{35,37}₁₇Cl_{18,20} ^{39,40,41}₁₉K_{20,21,22} ^{40,42,43,44,46,48}₂₀Ca_{20,22,23,24,26,28} ^{46,47,48,49,50}₂₂Ti_{24,25,26,27,28} ^{50,51}₂₃V_{27,28}
⁵⁵₂₅Mn₃₀ ^{64,66,68}₃₀Zn_{34,36,38} ^{76,77,78,80}₃₄Se_{42,43,44,46,48} ^{80,82,83,84,86}₃₆Kr_{44,46,47,48,50} ^{86,88}₃₇Rb_{48,50} ^{84,86,87,88}₃₈Sr_{46,48,49,50}
⁹⁷₄₂Mo₅₅ ^{97,98,99}₄₃Tc_{54,55,56} ^{107,109}₄₇Ag_{60,62} ^{113,115}₄₉In_{64,66} ¹⁶³₆₆Dy₉₇ ²⁰⁹₈₃Bi₁₂₆ ²⁰⁹₈₄Po₁₂₅ ²¹⁰₈₅At₁₂₅ ²²²₈₆Rn₁₃₆
²²³₈₇Fr₁₃₆ ²²⁶₈₈Ra₁₃₈ ¹³⁻¹⁹₉₇Bk₁₅₀ ¹⁵⁻¹⁹₁₁₂Cn₁₇₃ ^{344,346,348}_{136,137,138}Fy_{208,209,210}^{ie}

Supplement 15: DNA-Protein model of formulas of α and nuclides

$$\text{DNA: } \alpha_1 = \alpha_{1-7} = \frac{6^2}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{113}{112}\right)^{225}} \frac{1}{112 + \frac{1}{75^2}}} = 1/137.035999037435$$

Gene Factors: 3 4 5 6 7 9 12 18 25 36 44(7·2 π \approx 44) 75 112 113 225

Direct Derived Factors: 30 35 42 88 150 226;

7 14 28 56 224; 42 82-84 126 166-168; 68 69 136 137 138.

Emerging Factors: 11 13 19 48 71 173 187 209 *et al.*

Protein: ^{12,13}₆C_{6,7} ⁹₄Be₅ ^{10,11}₅B_{5,6} ^{12,13}₆C_{6,7} ^{14,15}₇N_{7,8} ¹⁹₉F₁₀ ²⁴⁻²⁶₁₂Mg_{8,12-14} ²⁸⁻³⁰₁₄Si₁₄₋₁₆ ⁵⁵₂₅Mn₃₀ ^{54,56}₂₆Fe_{28,30}
^{58,60,61,64}₂₈Ni_{30,32,33,36} ⁷⁵₃₃As₄₂ ^{82,83,84}₃₆Kr_{46,47,48} ^{96,98}₄₂Mo_{54,56} ⁸⁸₃₈Sr₅₀ ¹⁰⁰₄₄Ru₅₆ ^{112,113}₄₈Cd_{64,65} ¹²⁵₅₂Te₇₃ ¹²⁹₅₄Xe₇₅
¹³⁶⁻¹³⁸₅₆Ba₈₀₋₈₂ ^{142-146,148,150}₆₀Nd_{82-86,88,90} ¹⁵⁰₆₂Sm₈₈ ^{175,176}₇₁Lu_{104,105} ^{185,187}₇₅Re_{110,112} ^{188,189}₇₆Os_{112,113} ^{207,208}₈₂Pt_{125,126} ²⁰⁹₈₃Bi₁₂₆^{*}
²⁰⁹₈₄Po₁₂₅ ²¹⁰₈₅At₁₂₅ ²²²₈₆Rn₁₃₆ ²²³₈₇Fr₁₃₆ ²²⁶₈₈Ra₁₃₈ ²²⁷₈₉Ac₁₃₈ ²⁸⁵₁₁₂Cn₁₇₃ ⁴⁻⁷¹₁₁₃Nh₁₇₁^{ie} ^{312,314}_{125,126}Ch_{187,188}^{ie} ²¹⁷³₁₃₇Fy₂₀₉^{ie} ^{366,372}_{144,147}Ch_{222,225}^{ie}

$$\text{DNA: } \alpha_2 = \alpha_{2-13} = \frac{13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{9 \cdot 31}{2 \cdot 139}\right)^{557}}}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} = 1/137.035999111818$$

Gene Factors: 2 3 5 9 10 13 29 31 64 82(13·2π ≈ 82) 100 112 139

Direct Derived Factors: 26 39 52 104 169, 50 200, 58 87 116, 32 96 128 192, 93;

7 14 28 56 224; 42 82-84 126 166-168; 68 69 136 137 138.

Emerging Factors: 19 38 57 71 76 113 114 157 173 187 208 209 210 *et al.*

Protein: ${}^7_3\text{Li}$ ${}^{19}_9\text{F}$ ${}^{27}_{13}\text{Al}$ ${}^{29}_{14}\text{Si}$ ${}^{54,56,57,58}_{26}\text{Fe}$ ${}^{58}_{28}\text{Ni}$ ${}^{63,65}_{29}\text{Cu}$ ${}^{69,71}_{31}\text{Ga}$ ${}^{74}_{32}\text{Ge}$ ${}^{89}_{39}\text{Y}$
 ${}^{93}_{41}\text{Nb}$ ${}^{96}_{42}\text{Mo}$ ${}^{100}_{44}\text{Ru}$ ${}^{112,116}_{48}\text{Cd}$ ${}^{116}_{50}\text{Sn}$ ${}^{128}_{52}\text{Te}$ ${}^{136,137,138}_{56}\text{Ba}$ ${}^{139}_{57}\text{La}$ ${}^{136,138,140,142}_{58}\text{Ce}$ ${}^{149}_{62}\text{Sm}$ ${}^{156,157,160}_{64}\text{Gd}$ ${}^{162}_{66}\text{Dy}$ ${}^{166,167,168}_{68}\text{Er}$ ${}^{169}_{69}\text{Tm}$ ${}^{175}_{71}\text{Lu}$ ${}^{187}_{75}\text{Re}$ ${}^{188-190,192}_{76}\text{Os}$
 ${}^{200}_{80}\text{Hg}$ ${}^{208}_{82}\text{Pb}$ ${}^{209}_{83}\text{Bi}^*$ ${}^{209}_{84}\text{Po}^*$ ${}^{210}_{85}\text{At}^*$ ${}^{223}_{87}\text{Fr}^*$ ${}^{237}_{93}\text{Np}^*$ ${}^{247}_{96}\text{Cm}^*$ ${}^{257}_{100}\text{Fm}^*$ ${}^{15-19}_{112}\text{Cn}^*$ ${}^{4-71}_{113}\text{Nh}^{ie}$
 ${}^{310,312}_{124,125}\text{Ch}^{ie}$ ${}^{318}_{126}\text{Ch}^{ie}$ ${}^{330}_{130}\text{Ch}^{ie}$ ${}^{344,346,12-29}_{136,137,138}\text{Fy}^{ie}$ ${}^{12-29}_{139}\text{Ch}^{ie}$ ${}^{400}_{157}\text{Ch}^{ie}$ ${}^{6-71}_{169}\text{Ch}^{ie}$

Supplement 16: Some formulas of α₂

$$\alpha_{2-45} = \frac{5 \cdot 9 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{30 \cdot 36}{13 \cdot 83}\right)^{17 \cdot 127}}}{2 \cdot 173} \frac{1}{112 - \frac{1}{2 \cdot (32 \cdot 13 \cdot 97 - 1) - \frac{15}{16}}}$$

= 1/137.035999111818

${}^{10,11}_5\text{B}$ ${}^{19}_9\text{F}$ ${}^{27}_{13}\text{Al}$ ${}^{31}_{15}\text{P}$ ${}^{35,37}_{17}\text{Cl}$ ${}^{54,56,57,58}_{26}\text{Fe}$ ${}^{64,66,68}_{30}\text{Zn}$ ${}^{83,84}_{36}\text{Kr}$ ${}^{103}_{45}\text{Rh}$
 ${}^{126,128}_{52}\text{Te}$ ${}^{163}_{66}\text{Dy}$ ${}^{173}_{70}\text{Yb}$ ${}^{209}_{83}\text{Bi}^*$ ${}^{209}_{84}\text{Po}^*$ ${}^{247}_{97}\text{Bk}^*$ ${}^{285}_{112}\text{Cn}^*$ ${}^{344,2-137}_{136,137}\text{Fy}^{ie}$ ${}^{318-320}_{127}\text{Ch}^{ie}$

$$\alpha_{2-45\text{-Wallis}} = \frac{2 \cdot 5 \cdot 9 \cdot \left(2 \frac{2}{3} \frac{4}{5} \frac{6}{7} \frac{8}{7} \dots \frac{3234}{3235} \frac{4 \cdot (8 \cdot 101 + 1)}{2 \cdot 3 \cdot 49 \cdot 11 + 1}\right)}{173} \frac{1}{112 - \frac{1}{12 \cdot 37 \cdot (2 \cdot 126 - 1) - \frac{25}{31}}}$$

= 1/137.035999111818

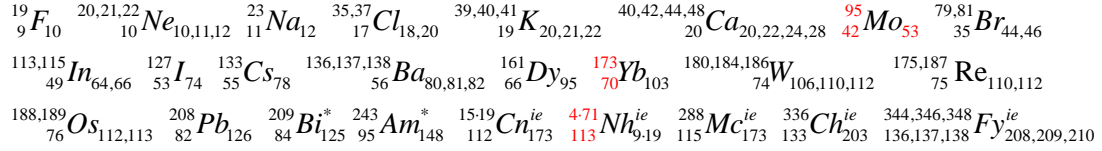
${}^{10,11}_5\text{B}$ ${}^{19}_9\text{F}$ ${}^{23}_{11}\text{Na}$ ${}^{55}_{25}\text{Mn}$ ${}^{69,71}_{31}\text{Ga}$ ${}^{85,87}_{37}\text{Rb}$ ${}^{101}_{44}\text{Ru}$ ${}^{103}_{45}\text{Rh}$ ${}^{113,115}_{49}\text{In}$
 ${}^{171,173}_{70}\text{Yb}$ ${}^{209}_{83}\text{Bi}^*$ ${}^{209}_{84}\text{Po}^*$ ${}^{285}_{112}\text{Cn}^*$ ${}^{288}_{115}\text{Mc}^{ie}$ ${}^{2-137}_{137}\text{Fy}^{ie}$

$$\alpha_{2-45\text{-GL}} = \frac{4 \cdot 5 \cdot 9 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot (24 \cdot 43 - 1) + 1}\right)}{173} \frac{1}{112 - \frac{1}{25 \cdot (22 \cdot 109 + 1) + \frac{31}{59}}}$$

= 1/137.035999111818

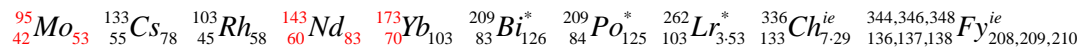
${}^{10,11}_5\text{B}$ ${}^{19}_9\text{F}$ ${}^{47}_{22}\text{Ti}$ ${}^{68,71}_{31}\text{Ga}$ ${}^{99}_{43}\text{Tc}^*$ ${}^{103}_{45}\text{Rh}$ ${}^{107,109}_{47}\text{Ag}$ ${}^{118,120}_{50}\text{Sn}$ ${}^{3-47}_{59}\text{Pr}$
 ${}^{148,150}_{62}\text{Sm}$ ${}^{173}_{70}\text{Yb}$ ${}^{183}_{74}\text{W}$ ${}^{222}_{86}\text{Lr}^*$ ${}^{276}_{109}\text{Mt}^*$ ${}^{285}_{112}\text{Cn}^*$ ${}^{288}_{115}\text{Mc}^{ie}$ ${}^{294}_{118}\text{Og}^{ie}$ ${}^{8-43,2-173}_{136,137}\text{Fy}^{ie}$

$$\alpha_{2-173} = \frac{173 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{8 \cdot 17 \cdot 59}{71 \cdot 113}\right)^{9 \cdot (2 \cdot 9^2 \cdot 11 + 1)}}}{10 \cdot 7 \cdot 19} \cdot \frac{1}{112 - \frac{53 \cdot (42 \cdot 41 - 1)}{4 \cdot 10^{11}}} = 1/137.035999111818$$



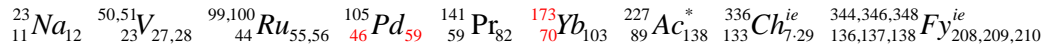
$$\alpha_{2-173-Wallis} = \frac{2 \cdot 173 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{24068}{24069} \frac{10 \cdot 29 \cdot 83}{2 \cdot 22 \cdot (42 \cdot 13 + 1) + 1}\right)}{5 \cdot 7 \cdot 19} \cdot \frac{1}{112 - \frac{3 \cdot 53 \cdot (20 \cdot 27 + 1)}{25 \cdot 10^{11}}} = 1/137.035999111818$$

$$= 1/137.035999111818$$

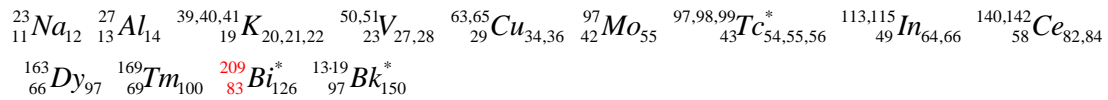


$$\alpha_{2-173-GL} = \frac{4 \cdot 173 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 12 \cdot (4 \cdot 11 \cdot 29 + 1) + 1}\right)}{5 \cdot 7 \cdot 19} \cdot \frac{1}{112 - \frac{1}{4 \cdot 7 \cdot 23 \cdot 59 \cdot 89}} = 1/137.035999111818$$

$$= 1/137.035999111818$$

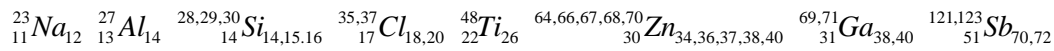


$$\alpha_{2-49} = \frac{49 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{16 \cdot 13}{9 \cdot 23}\right)^{5 \cdot 83}}}{13 \cdot 29} \cdot \frac{1}{112 - \frac{1}{7 \cdot (6 \cdot 11^2 + 1) + \frac{11 \cdot 19}{3 \cdot 97}}} = 1/137.035999111818$$



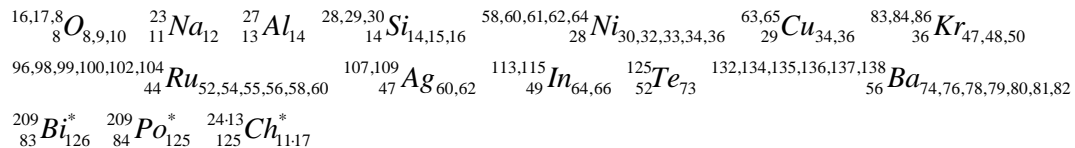
$$\alpha_{2-49-Wallis} = \frac{4 \cdot 49 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{622}{623} \frac{48 \cdot 13}{2 \cdot (10 \cdot 31 + 1) + 1}\right)}{13 \cdot 29} \cdot \frac{1}{112 - \frac{1}{10 \cdot (30 \cdot 17 - 1) + \frac{2 \cdot 7 \cdot 11}{300}}} = 1/137.035999111818$$

$$= 1/137.035999111818$$



$$\alpha_{2-49-GL} = \frac{8 \cdot 49 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 36 \cdot 11 + 1}\right)}{13 \cdot 29} \cdot \frac{1}{112 - \frac{1}{11^2 \cdot 47 - \frac{36}{125}}} = 1/137.035999111818$$

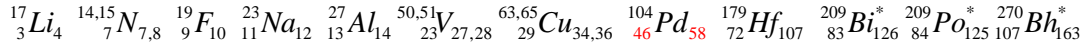
$$= 1/137.035999111818$$



Supplement 17: Some formulas of the speed of light

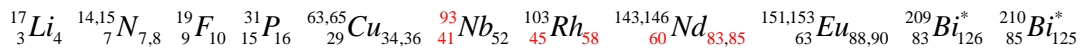
$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-81}\alpha_{2-49}}} = \frac{9}{4 \cdot 7} \sqrt{\frac{29 (2\pi)_{15 \cdot 107}}{2 (2\pi)_{9 \cdot 23}} \left(112^2 - \frac{1}{2 \cdot 23} + \frac{1}{7 \cdot (2 \cdot 11 \cdot 19 + 1)} - \frac{13}{7 \cdot 11}\right)}$$

$$= 137.035999074627$$



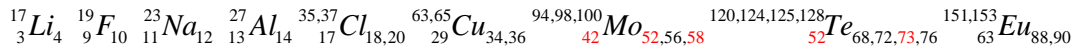
$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-81}\text{-Wallis}\alpha_{2-49}\text{-Wallis}}} = \frac{9}{4 \cdot 7} \sqrt{\frac{29 \left(\frac{\pi}{2}\right)_{29 \cdot 83}}{2 \left(\frac{\pi}{2}\right)_{10 \cdot 31+1}} \left(112^2 - \frac{1}{9 \cdot 5} + \frac{1}{5 \cdot 17 \cdot 41 + \frac{11}{60}}\right)}$$

$$= 137.035999074627$$



$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-81}\text{-GL}\alpha_{2-49}\text{-GL}}} = \frac{9}{4 \cdot 7} \sqrt{\frac{29 \left(\frac{\pi}{4}\right)_{6 \cdot 7 \cdot 73}}{2 \left(\frac{\pi}{4}\right)_{36 \cdot 11}} \left(112^2 - \frac{1}{4 \cdot 13} + \frac{1}{9 \cdot 7 \cdot 11 \cdot 13 + \frac{17}{18}}\right)}$$

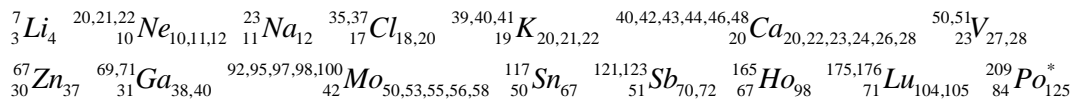
$$= 137.035999074627$$



Supplement 18: Other formulas of α_2

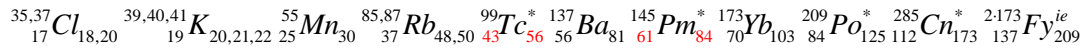
$$\alpha_{2-42} = \frac{2 \cdot 3 \cdot 7 \cdot e^2 \cdot \frac{e^2}{\left(\frac{2}{1}\right)^3} \cdot \frac{e^2}{\left(\frac{3}{2}\right)^5} \cdots \frac{e^2}{\left(\frac{42 \cdot 11}{20 \cdot 23 + 1}\right)^{13 \cdot 71}}}{17 \cdot 19} \frac{1}{112 - \frac{1}{12 \cdot (10 \cdot 11 \cdot 67 - 1) + \frac{5}{12}}}$$

$$= 1/137.035999111818$$



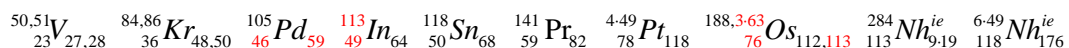
$$\alpha_{2-42}\text{-Wallis} = \frac{8 \cdot 3 \cdot 7 \cdot \left(2 \frac{2}{3} \frac{4}{5} \frac{4}{5} \frac{6}{7} \frac{6}{7} \frac{8}{7} \cdots \frac{1384}{1385} \frac{2 \cdot 9 \cdot 7 \cdot 11}{2 \cdot 4 \cdot 173 + 1}\right)}{17 \cdot 19} \frac{1}{112 - \frac{1}{2 \cdot 25 \cdot 29 \cdot 61 - \frac{37}{3 \cdot 43}}}$$

$$= 1/137.035999111818$$



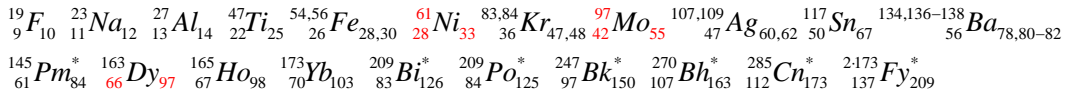
$$\alpha_{2-42}\text{-Wallis} = \frac{16 \cdot 3 \cdot 7 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot 3^2 \cdot 7^2 + 1}\right)}{17 \cdot 19} \frac{1}{112 - \frac{1}{4 \cdot (2 \cdot 23 \cdot 113 - 1) + \frac{10}{59}}}$$

$$= 1/137.035999111818$$



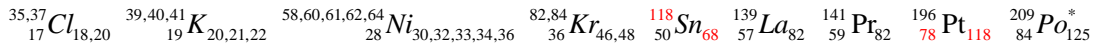
$$\alpha_{2-61} = \frac{61 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{9 \cdot 173}{4 \cdot (4 \cdot 97 + 1)}\right)^{11 \cdot (6 \cdot 47 + 1)}}}{7 \cdot 67} \frac{1}{112 - \frac{1}{2 \cdot 6 \cdot 13 \cdot (8 \cdot 7 \cdot 11 \cdot 13 + 1)}}$$

$$= 1/137.035999111818$$



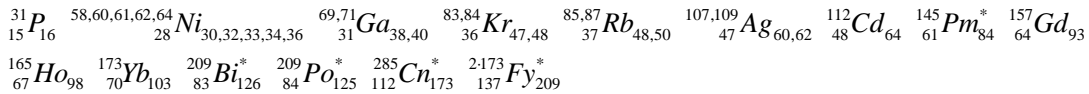
$$\alpha_{2-61-Wallis} = \frac{4 \cdot 61 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{5670}{4671} \frac{8 \cdot (12 \cdot 59 + 1)}{2 \cdot 10 \cdot (36 \cdot 13 - 1) + 1}\right)}{7 \cdot 67} \frac{1}{112 - \frac{1}{2 \cdot 97 \cdot (10 \cdot 9 \cdot 17 + 1) - \frac{19}{28}}}$$

$$= 1/137.035999111818$$



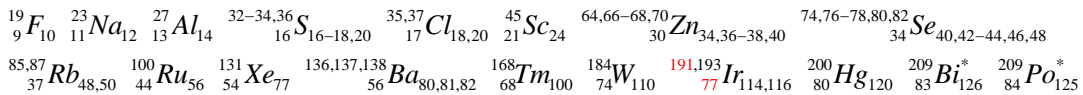
$$\alpha_{2-61-GL} = \frac{8 \cdot 61 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot (16 \cdot 3 \cdot 31 - 1) + 1}\right)}{7 \cdot 67} \frac{1}{112 - \frac{1}{4 \cdot 3 \cdot 5 \cdot (2 \cdot 7 \cdot 173 + 1) - \frac{37}{62}}}$$

$$= 1/137.035999111818$$



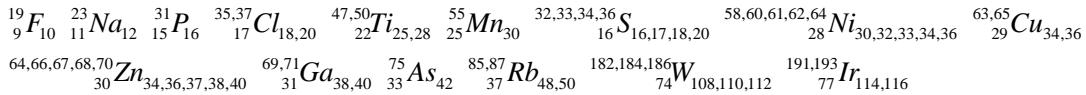
$$\alpha_{2-77} = \frac{7 \cdot 11 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{11 \cdot 191}{3 \cdot 7 \cdot 100}\right)^{4201}}}{16 \cdot 37} \frac{1}{112 - \frac{1}{5 \cdot (4 \cdot 9 \cdot 7 \cdot 13 \cdot 17 - 1) + \frac{5}{16}}}$$

$$= 1/137.035999111818$$



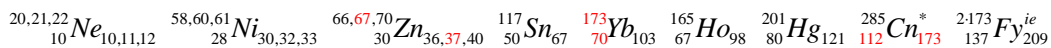
$$\alpha_{2-77-Wallis} = \frac{7 \cdot 11 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{6300}{6301} \frac{2 \cdot 16 \cdot (2 \cdot 9 \cdot 11 - 1)}{2 \cdot 2 \cdot 9 \cdot 25 \cdot 7 + 1}\right)}{4 \cdot 37} \frac{1}{112 - \frac{1}{2 \cdot 15 \cdot 17 \cdot 29 \cdot 31 - \frac{5}{6}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-77-GL} = \frac{7 \cdot 11 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 10 \cdot (6 \cdot 67 - 1) + 1}\right)}{2 \cdot 37} \frac{1}{112 - \frac{1}{28 \cdot (10 \cdot 28 \cdot (2 \cdot 173 + 1) - 1)}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-93} = \frac{93 \cdot e^2 \cdot \frac{e^2}{\left(\frac{2}{1}\right)^3} \cdot \frac{e^2}{\left(\frac{3}{2}\right)^5} \cdots \frac{e^2}{\left(\frac{12 \cdot 227}{7 \cdot (4 \cdot 97 + 1)}\right)^{13(10 \cdot 42 - 1)}}}{5 \cdot 11 \cdot 13} \cdot \frac{1}{112 - \frac{1}{10 \cdot 12 \cdot (8 \cdot 9 \cdot 7 \cdot 13 + 1) + \frac{7}{12}}} = 1/137.035999111818$$

$$\alpha_{2-93-Wallis} = \frac{4 \cdot 93 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \cdots \frac{8170 \cdot 36 \cdot 227}{8171 \cdot 2 \cdot 5 \cdot 19 \cdot 43 + 1}\right)}{5 \cdot 11 \cdot 13} \cdot \frac{1}{112 - \frac{1}{45 \cdot (30 \cdot 11 \cdot 53 + 1) + \frac{4}{13}}} = 1/137.035999111818$$

$$\alpha_{2-93-GL} = \frac{8 \cdot 93 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot 9 \cdot 17^2 + 1}\right)}{5 \cdot 11 \cdot 13} \cdot \frac{1}{112 - \frac{1}{60 \cdot (30 \cdot 11 \cdot 23 + 1) + \frac{14}{33}}} = 1/137.035999111818$$

$$\alpha_{2-109} = \frac{109 \cdot e^2 \cdot \frac{e^2}{\left(\frac{2}{1}\right)^3} \cdot \frac{e^2}{\left(\frac{3}{2}\right)^5} \cdots \frac{e^2}{\left(\frac{9 \cdot (3 \cdot 128 - 1)}{2 \cdot (42 \cdot 41 + 1)}\right)^{61 \cdot 113}}}{2 \cdot (2 \cdot 11 \cdot 19 + 1)} \cdot \frac{1}{112 - \frac{1}{3 \cdot 47 \cdot (4 \cdot 7 \cdot 13^2 + 1) + \frac{3}{4}}} = 1/137.035999111818$$

$$\alpha_{2-109-Wallis} = \frac{2 \cdot 109 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \cdots \frac{10338 \cdot 2 \cdot 10 \cdot 11 \cdot 47}{10339 \cdot 2 \cdot 3 \cdot (42 \cdot 41 + 1) + 1}\right)}{2 \cdot 11 \cdot 19 + 1} \cdot \frac{1}{112 - \frac{1}{49 \cdot 67 \cdot 313}} = 1/137.035999111818$$

$$\alpha_{2-109-GL} = \frac{4 \cdot 109 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 6 \cdot (8 \cdot 137 + 1) + 1}\right)}{2 \cdot 11 \cdot 19 + 1} \cdot \frac{1}{112 - \frac{1}{49 \cdot 97 \cdot 151 - \frac{5}{13}}} = 1/137.035999111818$$

Note: $8 \cdot 137 + 1 = 18 \cdot 61 - 1$

$$\alpha_{2-141} = \frac{3 \cdot 47 \cdot e^2 \cdot \frac{e^2}{\left(\frac{2}{1}\right)^3} \cdot \frac{e^2}{\left(\frac{3}{2}\right)^5} \cdots \frac{e^2}{\left(\frac{22 \cdot (2 \cdot 11^2 - 1)}{9 \cdot 19 \cdot 31}\right)^{23(42 \cdot 11 - 1)}}}{4 \cdot (16 \cdot 17 - 1)} \cdot \frac{1}{112 - \frac{1}{18 \cdot 17 \cdot (6 \cdot 11 \cdot 137 - 1)}} = 1/137.035999111818$$

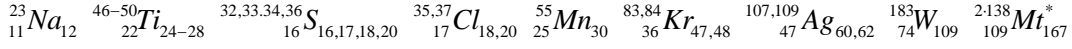
$$\alpha_{2-141-Wallis} = \frac{3 \cdot 47 \cdot \left(2 \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} \cdots \frac{15904 \cdot 6 \cdot 11 \cdot (2 \cdot 11^2 - 1)}{15905 \cdot 2 \cdot 16 \cdot 7 \cdot 71 + 1}\right)}{16 \cdot 17 - 1} \cdot \frac{1}{112 - \delta_2} = 1/137.035999111818$$

$$\delta_2 = \frac{71 \cdot (10 \cdot 11 \cdot 47 + 1)}{10^{12}} \approx \frac{1}{7 \cdot 43 \cdot (13 \cdot 24 \cdot 29 + 1)} \approx \frac{1}{125 \cdot (20 \cdot 109 - 1)} \approx \frac{1}{8 \cdot 23 \cdot 113 \cdot (12 \cdot 11 - 1)}$$

$${}^{23}_{11}\text{Na}_{12} \quad {}^{27}_{13}\text{Al}_{14} \quad {}^{63,65}_{29}\text{Cu}_{34,36} \quad {}^{71}_{31}\text{Ga}_{40} \quad {}^{98,99}_{43}\text{Tc}_{55,56} \quad {}^{107,109}_{47}\text{Ag}_{60,62} \quad {}^{136}_{56}\text{Ba}_{80} \quad {}^{142}_{60}\text{Nd}_{82} \quad {}^{175,176}_{71}\text{Lu}_{104,105} \quad {}^{209}_{84}\text{Po}_{125} \quad {}^{223}_{87}\text{Fr}_{136} \quad {}^{4-71}_{113}\text{Nh}_{171}^{ie}$$

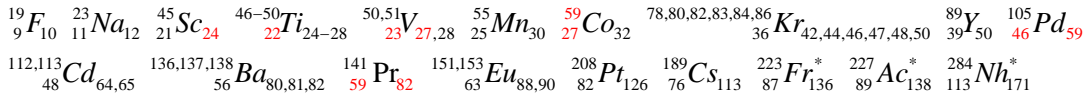
$$\alpha_{2-141-GL} = \frac{2 \cdot 3 \cdot 47 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 4 \cdot (10 \cdot 11 \cdot 23 + 1)}\right)}{16 \cdot 17 - 1} \cdot \frac{1}{112 - \frac{4 \cdot 109 \cdot (2 \cdot 3 \cdot 47 - 1)}{25 \cdot 10^{11}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-189} = \frac{27 \cdot 7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{12 \cdot (36 \cdot 23 - 1) \text{ or } (14 \cdot 59 + 1)}{2 \cdot 11^2 \cdot 41 + 1}\right)^{89 \cdot 223}}}{12 \cdot 11^2 + 1} \cdot \frac{1}{112 - \frac{2 \cdot 3 \cdot 7 \cdot 113}{25 \cdot 10^{11}}}$$

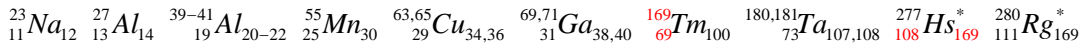
$$= 1/137.035999111818$$



$$\alpha_{2-189-Wallis} = \frac{4 \cdot 27 \cdot 7 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{29770}{19771} \frac{36 \cdot (36 \cdot 23 - 1) \text{ or } (14 \cdot 59 + 1)}{2 \cdot 5 \cdot 13 \cdot (12 \cdot 19 + 1)}\right)}{12 \cdot 11^2 + 1} \cdot \frac{1}{112 - \delta_2}$$

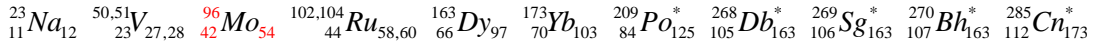
$$= 1/137.035999111818$$

$$\delta_2 = \frac{13^2 \cdot 29 \cdot 31}{25 \cdot 10^{11}} \approx \frac{9 \cdot 25 \cdot 37 \cdot 73}{4 \cdot 10^{11}}$$



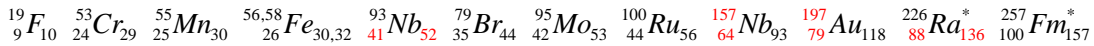
$$\alpha_{2-189-GL} = \frac{8 \cdot 27 \cdot 7 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 8 \cdot 23 \cdot 103 + 1}\right)}{12 \cdot 11^2 + 1} \cdot \frac{1}{112 - \frac{16 \cdot (6 \cdot 7 \cdot 11^2 - 1)}{25 \cdot 10^{11}} \text{ or } \frac{3 \cdot 7 \cdot 19 \cdot 163}{2 \cdot 10^{12}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-205} = \frac{5 \cdot 41 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{79 \cdot 157}{2 \cdot 9 \cdot 13 \cdot 53}\right)^{5 \cdot 11^2 \cdot 41}}}{8 \cdot 197} \cdot \frac{1}{112 - \frac{2 \cdot (4 \cdot 11 \cdot (16 \cdot 17 - 1) - 1)}{25 \cdot 10^{11}}}$$

$$= 1/137.035999111818$$



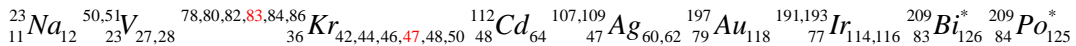
$$\alpha_{2-205-Wallis} = \frac{5 \cdot 41 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{37208}{37209} \frac{10 \cdot 61^2}{2 \cdot 4 \cdot (6 \cdot 25 \cdot 31 + 1)}\right)}{2 \cdot 197} \cdot \frac{1}{112 - \frac{4 \cdot 3 \cdot 5 \cdot 37^2 + 1}{2 \cdot 10^{12}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-205-GL} = \frac{5 \cdot 41 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 8 \cdot 9 \cdot 7 \cdot 47 + 1}\right)}{197} \cdot \frac{1}{112 - \frac{3 \cdot 41 \cdot (16 \cdot 7 \cdot 11 + 1)}{2 \cdot 10^{12}} \text{ or } \frac{23 \cdot 83 \cdot 197}{5 \cdot 10^{12}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-221} = \frac{13 \cdot 17 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{6 \cdot 11 \cdot (2 \cdot 7 \cdot 17 + 1) - 1}{4 \cdot (8 \cdot 17 \cdot 29 - 1)}\right)^{9.5 \cdot (7 \cdot 100 + 1)}}}{17 \cdot 100 - 1} \cdot \frac{1}{112 - \frac{3 \cdot 7 \cdot 139}{4 \cdot 10^{11}}} = 1/137.035999111818$$

$$\alpha_{2-221\text{-Wallis}} = \frac{4 \cdot 13 \cdot 17 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{47318}{47319} \frac{10 \cdot 4 \cdot 7 \cdot 13^2}{2 \cdot 59 \cdot 401 + 1}\right)}{17 \cdot 100 - 1} \cdot \frac{1}{112 - \frac{19^2 \cdot 103}{25 \cdot 10^{11}}} = 1/137.035999111818$$

$$\alpha_{2-221\text{-GL}} = \frac{8 \cdot 13 \cdot 17 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 4 \cdot 17 \cdot (2 \cdot 13 \cdot 17 + 1) + 1}\right)}{17 \cdot 100 - 1} \cdot \frac{1}{112 - \frac{7 \cdot 23 \cdot (6 \cdot 7 \cdot 10 + 1)}{25 \cdot 10^{11}}} = 1/137.035999111818$$

$$\alpha_{2-234} = \frac{2 \cdot 9 \cdot 13 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{2 \cdot 41 \cdot 47 - 1}{36 \cdot 107}\right)^{5 \cdot 23 \cdot 67}}}{7 \cdot 257} \cdot \frac{1}{112 - \frac{1}{7 \cdot 29 \cdot (2^9 - 1) + \frac{11}{14}}} = 1/137.035999111818$$

$$\alpha_{2-234\text{-Wallis}} = \frac{8 \cdot 9 \cdot 13 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \dots \frac{11556}{11557} \frac{2 \cdot (20 \cdot 17^2 - 1)}{2 \cdot 2 \cdot 27 \cdot 107 + 1}\right)}{7 \cdot 257} \cdot \frac{1}{112 - \frac{1}{5 \cdot (12 \cdot 47 \cdot (4 \cdot 163 + 1) + 1)}} = 1/137.035999111818$$

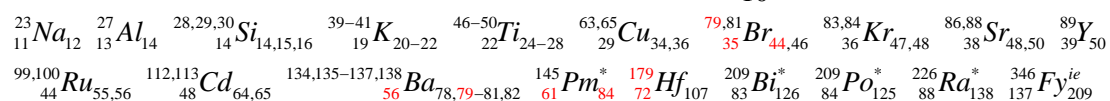
$$\alpha_{2-234\text{-GL}} = \frac{16 \cdot 9 \cdot 13 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 13 \cdot (4 \cdot 71 - 1) + 1}\right)}{7 \cdot 257} \cdot \frac{1}{112 - \frac{1}{5 \cdot 11^2 \cdot (13 \cdot 100 - 1) + \frac{14}{19} \text{ or } \frac{17}{23}}} = 1/137.035999111818$$

$$\alpha_{2-237} = \frac{3 \cdot 79 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{20 \cdot (24 \cdot 43 - 1)}{2 \cdot 13^2 \cdot 61 + 1}\right)^{11 \cdot 23 \cdot 163}}}{2 \cdot (10 \cdot 7 \cdot 13 + 1)} \cdot \frac{1}{112 - \frac{163 \cdot (4 \cdot 79 + 1)}{2 \cdot 10^{12}}} = 1/137.035999111818$$

$$\alpha_{2-237\text{-Wallis}} = \frac{6 \cdot 79 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{61858}{61859} \frac{60 \cdot (24 \cdot 43 - 1)}{2 \cdot 157 \cdot 197 + 1}\right)}{10 \cdot 7 \cdot 13 + 1} \cdot \frac{1}{112 - \frac{18 \cdot (8 \cdot (12 \cdot 41 - 1) + 1)}{25 \cdot 10^{11}}} = 1/137.035999111818$$

$$\alpha_{2-237\text{-GL}} = \frac{6 \cdot 79 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 13 \cdot (4 \cdot 71 - 1) + 1}\right)}{7 \cdot 257} \cdot \frac{1}{112 - \frac{1}{5 \cdot 11^2 \cdot (13 \cdot 100 - 1) + \frac{14}{19} \text{ or } \frac{17}{23}}} = 1/137.035999111818$$

$$\alpha_{2-237-GL} = \frac{12 \cdot 79 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 20 \cdot 11 \cdot 179 + 1}\right)}{10 \cdot 7 \cdot 13 + 1 \text{ or } 48 \cdot 19 - 1} \cdot \frac{1}{112 - \frac{3 \cdot 61 \cdot 137}{10^{12}}} = 1/137.035999111818$$



22. Discussion and Conclusion

Regarding the fine-structure constant, Richard Feynman said: “is it related to π or perhaps to the base of natural logarithms?”⁴ Our answer is that it relate to 2π -e, 2π , $\pi/2$ and $\pi/4$ formulas. He also deduced that the maximum element should be the 137th element Fynmanium (Fy) based on the analyses of the electron line velocity of his ideal hydrogen-like atoms. Our answer is that the natural end of the elements is the 112th element Copernicium (Cn^{*}), but the elements could have some ideal extensions, and above all, the fine-structure constant does relate to elements.

So, based on the analyses of ideal and real natural maximum element, Chen’s Chirality and Poetry Model of Atomic Nuclei⁷ and 2π -e formula^{6,7,8}, we deduced two series of Chen’s formulas of the fine-structure constant which gave two values $\alpha_1=1/137.035999037435$ and $\alpha_2=1/137.035999111818$. The factors in the formulas are much coincident to nucleon numbers of some nuclides, this means the formulas should be correct (too many coincidences mean too few possibilities to be wrong, or too many coincidences imply science). And we indicate the reason of $\alpha \approx 1/137.036$ is that it’s almost the equal ratio factor between 112 and 168 (more precisely 168-1/3) which are the key stable numbers (magic numbers) in Chen’s Chirality and Poetry Model of Atomic Nuclei⁷.

With Chen’s formulas of the fine-structure constant, we predicted the nucleon numbers of all 119th to 170th ideal extended elements; we theoretically or mathematically calculated the speed of light in atomic units, i.e., $c_{au}=1/\alpha_c=1/(\alpha_1\alpha_2)^{1/2}=137.035999074627$; we deduced a concise Schrödinger-Chen equation of hydrogen atom which included α_1/α_2 factor; in analogy to α and its formulas, α_p (the second fine-structure constant) and its formulas were hypothesized, and the proton charge radius r_p was supposed to be 0.833027203 fm; in the end we discovered that the approximate rational numbers of 2π marvelously and directly related to nuclides. Based on these, a mathematic shell model of elements was established and a picture

of elements and ideal extended elements was depicted.

In their relations to nuclides, 2π formulas can only be certain approximate rational numbers and 2π -e formulas in Chen's formulas of the fine-structure constant can only take certain k values. So we also believe the two values of the fine-structure constant should be rational numbers with definite digits rather than irrational numbers with infinite digits, and actually the fine-structure constant is transformed to nucleon numbers of 136, 137 and 138 in the world of nuclides.

In a recent paper¹¹, physicist Nicolas Gisin commented that in 1920s there once was a debate between mathematicians David Hilbert and Luitzen Egbertus Jan Brouwer. Hilbert was promoting formalized mathematics, in which every real number with its infinite series of digits is a completed individual object. On the other side the Luitzen Egbertus Jan Brouwer was defending the view that each point on the line should be represented as a never-ending process that develops in time, a view known as intuitionistic mathematics. Hilbert and his supporters clearly won the debate. In consequence, formalized mathematics has been adopted as the language of physics. In the end of his paper, Nicolas Gisin said: "Physics can be as successful if built on intuitionistic mathematics, even if this breaks its marriage to determinism. Contrary to usual expectations, I bet that the next physical theory will not be even more abstract than quantum field theory, but might well be closer to human experience."

In this paper we adopted mathematical language like intuitionistic mathematics, but we go ahead even more. The formulas of 2π , 2π -e and the fine-structure constant consist of integer factors and relate to nucleon numbers of nuclides, and hence correlate with each others. So in this paper we may use super-intuitionistic mathematics or decoding methodology with features of multi-correlations of integer factors or rational numbers which relate to nucleon numbers of nuclides, and it seems it is the real language in the world of nuclides. As we know an atomic nucleus is a N-body system and chaos should be its real state, so it seems N-body chaos returns to integers. In overall, Leopold Kronecker's famous saying "God made the integers, all else is the work of man" should be correct in the world of nuclides or even in other fields of the real world. It seems an irrational number can only be a rational number to play roles in the real world.

"God is a pure mathematician!" declared British astronomer Sir James

Jeans(1877-1946). The physical Universe does seem to be organized around elegant mathematical relationships³. The fine-structure constant may be the most important number in physics. As it is dimensionless, it could be called the proportional ruler of the nature or the bridge of mathematics and physics. And we have successfully given reasonable and precise formulas of it. In some sense, we explain the bridge between mathematics and physics, or we may realize the unification of mathematics and physics. It seems we prove the saying “God is a pure mathematician”. At least, it seems that good mathematics means good physics, and some pure mathematical numbers do have scientific meanings.

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Acknowledgements

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Appendix I: Research History

Items	Page	Discover/Create	Revise/Supplement
2π -e formula	3	2013/4-12	
Formulas related to 2π -e formula	4	2013/4-12	
Preliminary applications of 2π -e formula and its related formulas	6	2013/4-12	
Chen’s Periodic Table of Elements and Natural Group Theory	6	2014-2017/12	
Chirality and Poetry Model of Atomic Nuclei	6	2017/12-2018/3	
Chen’s theory of the fine-structure constant	6	2018/4-6	2018/7-2020/1
Original Inspiration for Formulas of the Fine-structure Constant	6	2018/4/12	
Logical deduction of Chen’s formulas of the	6 7	2018/4/12-24	

fine-structure constant			
α_1 (α_{1-7})	67	2018/4/12	2018/4/20 (+1/75 ²)
α_2 (α_{2-13})	7	2018/4/24	2018/9/18-20 (280→278 <i>et al.</i>)
Calculation tables and diagrams of α	8	2018/4/12-24	2018/9/18-20
$\alpha_{1-(3/2)}$	9	2019/4/25	
$\alpha_{2-(3/2)}$	9	2019/4/25	
α_c^2	10	2018/6/8-9, 9/18-19; 2019/4/17-19	
$1/\alpha_c^2$	10	2019/12/14	
112/137≈137/168 <i>et al.</i>	11	2018/4-6	
^{136,137,138} Fy _{208,209,210}	11	2019/12-2020/1	
^{125,126} Ch, ¹⁴⁴⁻¹⁴⁹ Ch, ^{153,154} Ch, ^{155,156} Ch, ¹⁵⁷ Ch, ¹⁶³ Ch, ¹⁶⁴⁻¹⁶⁸ Ch, ¹⁶⁹ Ch	11	2019/12-2020/1	
c_{au} formulas	12	2019/12/16	2020/1/5-8, 2/24, 3/29
The special 29 and 75 factors	12-15	2019/4/22-24	
α_1/α_2 in Schrödinger Equation of Hydrogen Atom	16	2018/4-6	2019/12/13
α_1/α_2	16	2019/8/28-29	
The two kinds of general formulas of α	17	2019/6/27	2019/7/2-3
Parameters and results of α_{1-m}	18	2019/7/2	
α_{1-1}	19	2019/7/2	
α_{1-2}	19	2019/6/26	
α_{1-3}	19	2019/5/26	2019/6/26
α_{1-4}	19	2019/6/27	
α_{1-5}	19	2019/6/27	
α_{1-6}	19	2019/6/27	
α_{1-7} (α_1)	19	2018/4/12	2018/4/20 (+1/75 ²)
α_{1-9}	20	2019/6/28	
α_{1-11}	20	2019/6/29	
α_{1-13}	20	2019/6/29	
α_{1-16}	20	2019/6/29-30	
α_{1-17}	20	2019/6/30	2019/10/29
α_{1-19}	20	2019/7/1	
α_{1-20}	20	2018/4-6	2019/6/26
α_{1-22}	21	2019/5/25	2019/12/12
α_{1-23}	21	2019/7/4	
α_{1-25}	21	2019/7/4	
α_{1-27}	21	2019/5/25-26	
α_{1-29}	21	2019/5/25	
α_{1-31}	21	2019/7/4	
α_{1-32}	21	2019/7/5	

α_{1-33}	22	2019/7/5	
α_{1-34}	22	2019/5/24	
α_{1-36}	22	2019/5/24	
α_{1-43}	22	2019/12/15	
α_{1-50}	22	2018/4-6	2019/6/26
α_{1-59}	22	2019/5/25	
α_{1-81}	22	2019/5/25-26	
α_{1-96}	23	2019/5/25-26	2019/12/29
α_{1-103}	23	2019/12/15	
α_{1-133}	23	2019/5/26	2019/12/29
α_{1-140}	23	2019/5/26	
α_{1-155}	23	2019/12/15	
α_{1-170}	23	2019/7/2	2019/7/9
<hr/>			
Parameters and results of α_{2-m}	24	2019/7/3	
α_{2-1}	25	2018/4-6	2019/5/26-27
α_{2-4}	25	2019/5/28	
α_{2-5}	25	2019/6/22	
α_{2-6}	25	2019/6/23	
α_{2-7}	25	2019/5/24	2020/3/5
α_{2-9}	25	2019/6/21	
α_{2-10}	26	2019/5/28	
α_{2-11}	26	2019/6/22	
$\alpha_{2-13} (\alpha_2)$	26	2018/4/24	2018/9/18-20 (280→278 <i>et al.</i>)
α_{2-15}	26	2019/6/19	
α_{2-17}	26	2019/6/24	
α_{2-18}	26	2019/6/24	
α_{2-19}	27	2019/6/24	
α_{2-23}	27	2019/6/23	
α_{2-24}	27	2019/6/23	
α_{2-25}	27	2019/6/24	
α_{2-27}	27	2019/6/21	
α_{2-29}	27	2019/6/20	
α_{2-31}	28	2019/7/6	
α_{2-32}	28	2019/7/6	
α_{2-33}	28	2019/7/6	
α_{2-36}	28	2019/6/25	
α_{2-37}	28	2019/7/7	
α_{2-38}	28	2019/7/7	
α_{2-125}	29	2019/5/25	2019/12/20
α_{2-253}	29	2019/7/3	2019/7/9
α_{2-269}	29	2019/7/7	2019/12/19

$a_0/r_e, r_e; a_0/r_p, r_p$	30	2019/12/19-23	
$\alpha_{p/1}$	30	2020/1/2	
$\alpha_{p/2}$	30 31	2020/1/2-3	
${}_{164}\text{Ch}_{252}$	31	2020/1/3	
Direct relationships between 2π and nuclides	31	2020/1/8-10	
Some correlations of formulas of 2π and α	32 33	2020/1/11-13	
Chen's Mathematic Shell Model of Nuclides	33	2020/1/12-13	
${}_{130,131}\text{Ch}_{200,201}$	20 22 26 27	2020/1/26-28	
${}_{119,120,121}\text{Ch}_{179,180,181}$	21 28 31	2020/1/28-29	2020/2/5 (add 121)
${}_{128,129}\text{Ch}_{198,197/199}$	23 26 31	2020/1/28-29	2020/1/31
${}_{139}\text{Ch}_{209}$	21 26 28	2020/1/29	
${}_{132,133}\text{Ch}_{202,203}$	21 23	2020/1/31	
${}_{169}\text{Ch}_{257}$	11 22 27	2020/1/29-30	
${}_{157}\text{Ch}_{243}$	11 20 21 22 31	2019/1/8	
${}_{158}\text{Ch}_{243}$	11 20 22 25 30 31	2019/1/31	
${}_{169}\text{Ch}_{257}$	11 22 26 27	2020/1/29-30	
${}_{134,135}\text{Ch}_{206,205}$	20	2020/1/31	
${}_{127}\text{Ch}_{191,193}$	27	2020/1/31	2020/2/1 (add 191)
${}_{150,151,152}\text{Fy}_{228,229,230}$	19 28	2020/2/2	
${}_{170}\text{Ch}_{250}$	23 26	2020/2/3	
Chen's Picture of Elements and Ideal Extended Elements	41	2018/1-3 2020/2/2-5	2020/2/12, 16, 17, 19, 22-24
Supplement 1	37	2020/2/11-12	
$2\pi=62831853/10^7$	37	2020/2/11-12	
Supplement 2	37	2020/2/12	
${}_{124}\text{Ch}_{186}$	37	2020/2/12	
Supplement 3	37	2020/2/16-17	
${}_{143}\text{Ch}_{220,221}$	36	2020/2/17	
Supplement 4	38	2020/2/18-21	
${}_{122}\text{Ch}_{182}$	22	2020/2/19	
Supplement 5	38	2020/2/21	
${}_{127}\text{Ch}_{192}$ ${}_{167}\text{Ch}_{251}$	38	2020/2/21	
Supplement 6	39	2020/2/21-22, 24-25	
${}_{140,141}\text{Ch}_{212,215}$	39	2020/2/22	
${}_{123}\text{Ch}_{183,185}$	36	2020/2/23	
${}_{159/161,160}\text{Ch}_{245,246}$	36	2020/2/23	
${}_{165}\text{Ch}_{255}$	36	2020/2/23	
${}_{162}\text{Ch}_{246}$	36	2020/2/23	
Supplement 7	40	2020/2/25-26	
Supplement 8	41	2020/2/26	

Supplement 9	41-44	2020/2/27-3/3	
Supplement 10	44	2020/3/7-8	2020/3/12-14
Supplement 11	45-48	2020/3/21-27	
α_{1-7} -Wallis	45	2020/3/21	
α_{1-22} -Wallis	45	2020/3/25	
α_{1-29} -Wallis	45	2020/3/25-26	
α_{1-36} -Wallis	45	2020/3/26	
α_{1-43} -Wallis	46	2020/3/26	
α_{1-50} -Wallis	46	2020/3/22	
α_{1-59} -Wallis	46	2020/3/26	
α_{1-81} -Wallis	46	2020/3/23	
α_{1-96} -Wallis	46	2020/3/26	
α_{1-103} -Wallis	46	2020/3/26	
α_{1-133} -Wallis	46	2020/3/25	
α_{1-140} -Wallis	46	2020/3/26	
α_{1-155} -Wallis	47	2020/3/23	
α_{1-170} -Wallis	47	2020/3/24	
α_{2-10} -Wallis	47	2020/3/26-27	
α_{2-13} -Wallis	47	2020/3/22	
α_{2-23} -Wallis	47	2020/3/26	
α_{2-29} -Wallis	47	2020/3/23	
α_{2-33} -Wallis	48	2020/3/27	
α_{2-36} -Wallis	48	2020/3/25	
α_{2-125} -Wallis	48	2020/3/23	
α_{2-253} -Wallis	48	2020/3/25	
α_{2-269} -Wallis	48	2020/3/25	
Supplement 12	48-50	2020/3/22-30	
$C_{au}=(\alpha_{1-7}\alpha_{2-13})^{-1/2}$	48	2020/3/27	
$C_{au}=(\alpha_{1-7}\text{-Wallis}\alpha_{2-13}\text{-Wallis})^{-1/2}$	48	2020/3/22	
$C_{au}=(\alpha_{1-29}\alpha_{2-29})^{-1/2}$	49	2020/3/27	
$C_{au}=(\alpha_{1-29}\text{-Wallis}\alpha_{2-29}\text{-Wallis})^{-1/2}$	49	2020/3/27	
$C_{au}=(\alpha_{1-36}\alpha_{2-36})^{-1/2}$	49	2020/3/27-28	
$C_{au}=(\alpha_{1-36}\text{-Wallis}\alpha_{2-36}\text{-Wallis})^{-1/2}$	49	2020/3/27	
$C_{au}=(\alpha_{1-81}\alpha_{2-125})^{-1/2}$	49	2020/3/23	
$C_{au}=(\alpha_{1-81}\text{-Wallis}\alpha_{2-125}\text{-Wallis})^{-1/2}$	49	2020/3/23	
$C_{au}=(\alpha_{1-70}\alpha_{2-253})^{-1/2}$	49	2020/3/28	
$C_{au}=(\alpha_{1-170}\text{-Wallis}\alpha_{2-253}\text{-Wallis})^{-1/2}$	49	2020/3/25	
$C_{au}=(\alpha_{1-170}\alpha_{2-269})^{-1/2}$	50	2020/3/29-30	
$C_{au}=(\alpha_{1-170}\text{-Wallis}\alpha_{2-269}\text{-Wallis})^{-1/2}$	50	2020/3/29-30	
$C_{au}=(\alpha_{1-133}\alpha_{2-253})^{-1/2}$	50	2020/3/30	
$C_{au}=(\alpha_{1-133}\text{-Wallis}\alpha_{2-253}\text{-Wallis})^{-1/2}$	50	2020/3/30	

Supplement 13	50-54	2020/3/31-4/4
α_{1-7} -GL	51	2020/4/1
α_{1-22} -GL	51	2020/4/1
α_{1-29} -GL	51	2020/4/1
α_{1-36} -Leibniz	51	2020/4/1
α_{1-43} -GL	51	2020/4/2
α_{1-50} -GL	51	2020/4/2
α_{1-59} -GL	52	2020/4/2
α_{1-81} -GL	52	2020/4/2
α_{1-96} -GL	52	2020/4/2
α_{1-103} -GL	52	2020/4/2
α_{1-133} -GL	52	2020/4/2
α_{1-140} -GL	52	2020/4/2
α_{1-155} -GL	52	2020/4/2
α_{1-170} -GL	53	2020/4/1
α_{2-10} -GL	53	2020/4/3
α_{2-13} -GL	53	2020/4/1
α_{2-23} -GL	53	2020/4/3
α_{2-29} -GL	53	2020/4/1
α_{2-33} -GL	53	2020/4/3-4
α_{2-36} -GL	54	2020/4/3-4
α_{2-125} -GL	54	2020/4/2
α_{2-253} -GL	54	2020/4/3
α_{2-269} -GL	54	2020/4/1
Supplement 14	54-55	2020/4/3-
$C_{au}=(\alpha_{1-7-GL}\alpha_{2-13-GL})^{-1/2}$	54	2020/4/3
$C_{au}=(\alpha_{1-29-GL}\alpha_{2-29-GL})^{-1/2}$	54	2020/4/4
$C_{au}=(\alpha_{1-36-GL}\alpha_{2-36-GL})^{-1/2}$	55	2020/4/4
$C_{au}=(\alpha_{1-81-GL}\alpha_{2-125-GL})^{-1/2}$	55	2020/4/3
$C_{au}=(\alpha_{1-170-GL}\alpha_{2-253-GL})^{-1/2}$	55	2020/4/3
$C_{au}=(\alpha_{1-170-GL}\alpha_{2-269-GL})^{-1/2}$	55	2020/4/3
Supplement 15	55-56	2020/4/3-4
Supplement 16	55	2020/4/2-7
α_{2-45}	56	2020/4/5
α_{2-45} -Wallis	56	2020/4/8
α_{2-45} -GL	56	2020/4/7-8
α_{2-173}	57	2020/4/6
α_{2-173} -Wallis	57	2020/4/7
α_{2-173} -GL	57	2020/4/7
α_{2-49}	57	2020/4/8
α_{2-49} -Wallis	57	2020/4/8

$\alpha_{2-49-GL}$	57	2020/4/8
Supplement 17	58	2020/4/8-9
$C_{au}=(\alpha_{1-81}\alpha_{2-49})^{-1/2}$	58	2020/4/8
$C_{au}=(\alpha_{1-81-Wallis}\alpha_{2-49-Wallis})^{-1/2}$	58	2020/4/9
$C_{au}=(\alpha_{1-81-GL}\alpha_{2-49-GL})^{-1/2}$	58	2020/4/9
α_{2-42}	58	2020/4/9
$\alpha_{2-42-Wallis}$	58	2020/4/11-12
$\alpha_{2-42-GL}$	58	2020/4/12
α_{2-61}	58	2020/4/9
$\alpha_{2-61-Wallic}$	59	2020/4/11
$\alpha_{2-61-GL}$	59	2020/4/11
α_{2-77}	59	2020/4/9
$\alpha_{2-77-Wallic}$	59	2020/4/12
$\alpha_{2-77-GL}$	59	2020/4/12
α_{2-93}	60	2020/4/10
$\alpha_{2-93-Wallic}$	60	2020/4/12
$\alpha_{2-93-GL}$	60	2020/4/12
α_{2-109}	60	2020/4/10
$\alpha_{2-109-Wallis}$	60	2020/4/13
$\alpha_{2-109-GL}$	60	2020/4/12-13
α_{2-141}	60	2020/4/10
$\alpha_{2-141-Wallis}$	60	2020/4/13
$\alpha_{2-141-GL}$	61	2020/4/13
α_{2-189}	61	2020/4/10
$\alpha_{2-189-Wallis}$	61	2020/4/13
$\alpha_{2-189-GL}$	61	2020/4/13
α_{2-205}	61	2020/4/10
$\alpha_{2-205-Walis}$	61	2020/4/13-14
$\alpha_{2-205-GL}$	61	2020/4/13
α_{2-221}	62	2020/4/10
$\alpha_{2-221-Wallis}$	62	2020/4/14
$\alpha_{2-221-GL}$	62	2020/4/14
α_{2-234}	62	2020/4/10
$\alpha_{2-234-Wallic}$	62	2020/4/12
$\alpha_{2-234-GL}$	62	2020/4/12
α_{2-237}	62	2020/4/10-11
$\alpha_{2-237-Wallic}$	62	2020/4/12
$\alpha_{2-237-GL}$	63	2020/4/12
Preparing this paper	1-72	2019/12/1-2020/4/17

Note: Dates were recorded according to Beijing Time; ie means ideal extended; GL means Gregory and Leibniz.