Fermat's Last Theorem
Analysis in 6 understandable forms

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Abstract

The Pythagorean theorem is perhaps the best known theorem in the vast world of mathematics. A simple relation of square numbers, which encapsulates all the glory of mathematical science, is also justifiably the most popular yet sublime theorem in mathematical science. The starting point was Diophantus' 20 th problem (Book VI of Diophantus' Arithmetica), which for Fermat is for $\mathrm{n}=4$ *and consists in the question whether there are right triangles whose sides can be measured as integers and whose surface can be square. This problem was solved negatively by Fermat in the 17 th century, who used the wonderful method (ipse dixit Fermat) of infinite descent. The difficulty of solving Fermat's equation was first circumvented by Willes and R. Taylor in late 1994 ([1],[2],[3],[4]) and published in Taylor and Willes (1995) and Willes (1995). We present the proof of Fermat's last theorem and other accompanying theorems in 6 different independent ways. For each of the methods we consider, we use the Pythagorean theorem as a basic principle and also the fact that the proof of the first degree Pythagorean triad is absolutely elementary and useful. The proof of Fermat's last theorem marks the end of a mathematical era; however, the urgent need for a more educational proof seems to be necessary for undergraduates and students in general. Euler's method and Willes' proof is still a method that does not exclude other equivalent methods. The principle, of course, is the Pythagorean theorem and the Pythagorean triads, which form the basis of all proofs and are also the main way of proving the Pythagorean theorem in an understandable way. Other forms of proofs we will do will show the dependence of the variables on each other. For a proof of Fermat's theorem without the dependence of the variables cannot be correct and will therefore give undefined and inconclusive results.*

Part I. Pythagorean triples

I.1. Theorem 1 (Pythagorean triples 1st degree)

Let $\mathrm{P_1}$ be the set of Pythagorean triples and defined as $\mathrm{P_1}=\{(\mathrm{x,y,z})\mid \mathrm{a,b,c,x,y,z}\in \mathrm{Z}-\{0\}$ and $\mathrm{a}\cdot\mathrm{x}+\mathrm{b}\cdot\mathrm{y}=\mathrm{c}\cdot\mathrm{z}\}$. Let $\mathrm{G_1}$ be the set defined as: $\mathrm{G_1}=\{(\mathrm{x}=\mathrm{k}\cdot(\mathrm{c}\cdot\lambda-\mathrm{b}),\mathrm{y}=\mathrm{k}\cdot(\mathrm{a}-\mathrm{c}),\mathrm{z}=\mathrm{k}\cdot(\mathrm{a}\cdot\lambda-\mathrm{b})),(\mathrm{x}=\mathrm{k}\cdot(\mathrm{b}-\mathrm{c}),\ \mathrm{y}=\mathrm{k}\cdot(\mathrm{c}\cdot\lambda-\mathrm{a}),\mathrm{z}=\mathrm{k}\cdot(\mathrm{b}\cdot\lambda-\mathrm{a})),(\mathrm{x}=\mathrm{k}\cdot(\mathrm{c}+\mathrm{b}\cdot\lambda),\mathrm{y}=\mathrm{k}\cdot(\mathrm{c}-\mathrm{a}\cdot\lambda),\mathrm{z}=\mathrm{k}\cdot(\alpha+\mathrm{b}))\mid \mathrm{k},\lambda\in\mathrm{Z}^{+}\}$. We need to prove that the sets $\mathrm{P_1}=\mathrm{G_1}$.

Proof.

Given a triad (a,b,c) such that $abc\neq 0$ and are these positive integers, if we divide by $y\neq 0$, we get according to the set $\mathrm{P_1}$ then apply $\mathrm{a}\cdot(\mathrm{x/y})+\mathrm{b}=\mathrm{c}\cdot(\mathrm{z/y})$ and we call $\mathrm{X}=\mathrm{x/y}$ and $\mathrm{Z}=\mathrm{z/y}$. We declare now the sets:

$$\mathrm{F_1}=\{(\mathrm{X,Z})\}\in \mathrm{Q}^2-\{0\}\mid a\cdot\mathrm{X}+\mathrm{b}=\mathrm{Z}\cdot\mathrm{c},\ \text{where } \mathrm{a,b,c}\in\mathrm{Z}-\{0\},\ \text{and where } \mathrm{X,Z}\in\mathrm{Q}-\{0\}\}$$

and

$$\mathrm{S}_1=\left\{(\mathrm{X},\mathrm{Z})\in\mathrm{Q}^2-\{0\}\mid \mathrm{X}=\mathrm{m}-\lambda\wedge\mathrm{Z}=\mathrm{m},\ \text{where } \mathrm{m},\lambda\in\mathrm{Q}-\{0\}\right\}$$

The set $F_1\cap S_1$ has 3 points as a function of parameters m,λ and we have solutions for the corresponding final equations,

$$F_1\cap S_1=\left\langle\begin{array}{c} a\cdot(m-\lambda)+b=m\cdot c\Leftrightarrow m=\frac{a\cdot\lambda-b}{a-c}, a-c\neq 0\\ m-\lambda=\frac{c\cdot\lambda-b}{a-c}, a-c\neq 0, y=k\cdot(a-c), k\in Z^+\\ x=\frac{c\cdot\lambda-b}{a-c}\cdot y\wedge z=\frac{c\cdot\lambda-b}{a-c}\cdot y, a-c\neq 0\\ x=(c\cdot\lambda-b)\cdot k, y=k\cdot(a-c), z=k\cdot(a\cdot\lambda-b), k\in Z^+, a-c\neq 0\end{array}\right\rangle$$

Therefore

$$F_1\cap S_1=\langle x=(c\cdot\lambda-b)\cdot k, y=k\cdot(a-c), z=k\cdot(a\cdot\lambda-b), k\in Z^+, a-c\neq 0\rangle\ (I)$$

Dividing respectively by $\mathrm{x}\neq 0$ we get the set and the relations we call $\mathrm{Y}=\mathrm{y}/\mathrm{x}$ and $\mathrm{Z}=\mathrm{z}/\mathrm{x}$

$$\mathrm{F}_2=\left\{(\mathrm{Y},\mathrm{Z})\in\mathrm{Q}^2-\{0\}\mid \mathrm{a}+b\cdot(\mathrm{y}/\mathrm{x})=c\cdot(\mathrm{z}/x),\ \text{where } \mathrm{a},\mathrm{b},\mathrm{c}\in\mathrm{Z}-\{0\},\ \text{and where } \mathrm{Y},\mathrm{Z}\in\mathrm{Q}-\{0\}\right\}$$

and

$$\mathrm{S}_2=\left\{(\mathrm{Y},\mathrm{Z})\in\mathrm{Q}^2-\{0\}\mid \mathrm{Y}=\mathrm{m}-\lambda\wedge\mathrm{Z}=\mathrm{m},\ \text{where } \mathrm{m},\lambda\in\mathrm{Q}-\{0\}\right\}$$

Then as the type (I) we get the result

$$F_2\cap S_2=\langle x=(b-c)\cdot k, y=k\cdot(c\cdot\lambda-a), z=k\cdot(b\cdot\lambda-a), k\in Z^+, b-c\neq 0\rangle\ \text{(II)}$$

and finally dividing by $\mathrm{z}\neq 0$ similarly as before we call $\mathrm{X}=\mathrm{x}/\mathrm{z}$ and $\mathrm{Y}=\mathrm{y}/\mathrm{z}$

$$\mathrm{F}_3=\left\{(\mathrm{X},\mathrm{Y})\in\mathrm{Q}^2-\{0\}\mid \mathrm{a}\cdot(\mathrm{x}/\mathrm{z})+b\cdot(\mathrm{y}/\mathrm{z})=c,\ \text{where } \mathrm{a},\mathrm{b},\mathrm{c}\in\mathrm{Z}-\{0\},\ \text{and where } \mathrm{X},\mathrm{Y}\in\mathrm{Q}-\{0\}\right\}$$

and

$$\mathrm{S}_3=\left\{(\mathrm{X},\mathrm{Y})\in\mathrm{Q}^2-\{0\}\mid \mathrm{X}=\mathrm{m}-\lambda\wedge\mathrm{Y}=\mathrm{m},\ \text{where } \mathrm{m},\lambda\in\mathrm{Q}-\{0\}\right\}.$$

$$F_3\cap S_3=\langle x=(c+b\cdot\lambda)\cdot k, y=k\cdot(c-a\cdot\lambda), z=k\cdot(a+b), k\in Z^+, a+b\neq 0\}\ \text{(III)}$$

As a complement we can state that the parameter λ can be equal with $\lambda=\mathrm{p}/\mathrm{q}$, where p and q relatively primes. Therefore $\mathrm{P}_1=\mathrm{G}_1$ and the proof is complete.

I.2. Theorem 2 (Pythagorean triples 2nd degree).

Let P_2 be the set of Pythagorean triples and defined as $\mathrm{P}_2=\left\{(\mathrm{a},\mathrm{b},\mathrm{c})\mid \mathrm{a},\mathrm{b},\mathrm{c}\in\mathrm{N} \text{ and } \mathrm{a}^2+\mathrm{b}^2=\mathrm{c}^2\right\}$. Let G_2 be the set defined as: $\mathrm{G}_2=\left\{\left(\mathrm{k}\left(\mathrm{q}^2-\mathrm{p}^2\right),2\mathrm{kpq},\mathrm{k}\left(\mathrm{p}^2+\mathrm{q}^2\right)\right),\left(2\mathrm{kpq},\mathrm{k}\left(\mathrm{q}^2-\mathrm{p}^2\right),\mathrm{k}\left(\mathrm{p}^2+\mathrm{q}^2\right)\right)\mid \mathrm{k},\mathrm{p},\mathrm{q}\in\mathrm{N}^+,\right.$ $\left.\mathrm{p}\leq q,\ p \text{ and } q \text{ relatively primes}\right\}$. We need to prove that the sets $\mathrm{P}_2=\mathrm{G}_2$.

Proof.

Given a Pythagorean triad (a,b,c) such that $abc \neq 0$ and (a, b, c) are positive integers, if we divide by b^2 we get according to the set P_2 that $(\mathrm{a/b})^2 + 1 = (\mathrm{c/b})^2$, with $(\mathrm{c/b}) > 1$. We declare now the sets:

$$\mathrm{F} = \left\{(\mathrm{x}, \mathrm{y}) \in \mathrm{Q}_+^2 \mid \mathrm{x}^2 + 1 = \mathrm{y}^2, \mathrm{x} = \mathrm{a/b} \wedge \mathrm{y} = \mathrm{c/b},\ \text{where } \mathrm{a}, \mathrm{b}, \mathrm{c} \in \mathrm{Z}^+\right\}$$

and

$$\mathrm{S} = \left\{(\mathrm{x}, \mathrm{y}) \in \mathrm{Q}_+^2 \mid \mathrm{x} = \mathrm{m} - \mathrm{r} \wedge \mathrm{y} = \mathrm{m},\ \text{where } \mathrm{m}, \mathrm{r} \in \mathrm{Q}^+\right\}.$$

The set $F \cap S$ has two pairs points as a function of parameters m, r and we have solutions for the corresponding final equations as follow,

$$\begin{cases} (m-r)^2 + 1 = m^2 \Leftrightarrow m = \frac{r^2+1}{2 \cdot r}, r \neq 0,\ \text{where } m, r \in Q^+ \quad (1) \\ (m-r)^2 = 0 \Leftrightarrow m = r \wedge r = 1 \quad (2) \end{cases}$$

But we get from (1)

i) If $r = \frac{p}{q}, \{p, q$ prime numbers, $p < q\}$ we have $m = \frac{p^2+q^2}{2 \cdot p \cdot q}$ and $c = y \cdot b$ ie $c = m \cdot b = \frac{p^2+q^2}{2 \cdot p \cdot q} \cdot b$ therefore $b = 2 \cdot p \cdot q \cdot k$ (3) and final $c = \left(p^2 + q^2\right) \cdot k$ (4)

ii) If $a = (m - r) \cdot b = \frac{q^2-p^2}{2 \cdot p \cdot q} \cdot b = \left(q^2 - p^2\right) \cdot k$ (5)

Therefore the solutions is:
$a = (q^2 - p^2) \cdot k, b = 2 \cdot p \cdot q \cdot k, c = (p^2 + q^2) \cdot k$

With cyclic alternation of relations (3), (4) because b can become c and vice versa. So as a final solution we have the set

$\mathrm{G}_2 = \left\{\left(\mathrm{k}\left(\mathrm{q}^2 - \mathrm{p}^2\right), 2\mathrm{kpq}, \mathrm{k}\left(\mathrm{p}^2 + \mathrm{q}^2\right)\right), \left(2\mathrm{kpq}, \mathrm{k}\left(\mathrm{q}^2 - \mathrm{p}^2\right), \mathrm{k}\left(\mathrm{p}^2 + \mathrm{q}^2\right)\right) \mid \mathrm{k}, \mathrm{p}, \mathrm{q} \in \mathrm{N}_+, \mathrm{q} \in \mathrm{N}^*, \mathrm{p} \leq \mathrm{q}, \mathrm{p}\right.$ and q relatively primes$\}$ (6). Therefore $\mathrm{P}_2 = \mathrm{G}_2$ and the proof is complete.

The set G_2 gives the total solution of the Pythagorean equation. But the landmark point for further consideration of Fermat's equation are these relations proved because they are directly related to whatever method we engage and arrive at a general proof.

These proofs are elementary not only as a tool for proving Fermat but also for proving another more generalized conjecture of Beal's. A conjecture which requires Fermat's last theorem to hold . The proofs briefly given here are documented both by the Pythagorean triads and by the correctness of the existence of integer solutions and variables. Beal's conjecture gives us a beautiful and generalized proof of Fermat's theorem.In the 2 theorems it is based on, it is proved in general for each exponent that there can be no solution to a generalized Fermat's equation if all exponents are greater than 2.A very important and essential result.

Part II. Proof Fermat's Last Theorem

Method I.

I.1. Theorem 3 (Basic theorem of Proof).

Let $\mathrm{P_n}$ be the set of Fermat triples and defined as:

$$\mathrm{P_n} = \left\{(\mathrm{a,b,c}) \mid \mathrm{a,b,c,}\ \ \mathrm{n} > 2 \in \mathrm{N}^+ \ \text{ and an } \ \mathrm{a^n + b^n = c^n, abc} \neq 0\right\}$$

Let $\mathrm{G_n}$ be the set defined as:

$$\mathrm{G_n} = \{((\mathrm{a}=0,\mathrm{c}=\mathrm{b} \text{ or } \mathrm{b}=0,\mathrm{c}=\mathrm{a} \text{ or } \mathrm{c}=0,\mathrm{a}=-\mathrm{b} \text{ or } \mathrm{a}=\mathrm{b}=\mathrm{c}=0) \mid \mathrm{n}=2\mathrm{k}+1,$$

$$(\mathrm{a}=\mathrm{b}=\mathrm{c}=0 \text{ or } \mathrm{a}=0,\mathrm{c}=\pm\mathrm{b} \text{ or } \mathrm{b}=0,\mathrm{c}=\pm\mathrm{a}) \mid \mathrm{n}=2\mathrm{k},\mathrm{k}>1) \mid \mathrm{k} \in \mathrm{N}^+\}.$$

We need to prove that the sets $\mathrm{P_n} \neq \mathrm{G_n}$ and also $\mathrm{P_n} = \emptyset$.

Proof.

We have 2 sets $\mathrm{P_n}$ and $\mathrm{G_n}$ of solutions that we need to prove are not equal and G_n is the complete set unconstrained, as we will prove of the diophantine Fermat equation. The basis of the method for the proof is the relations proved by theorems 1 & 2 of the Pythagorean triples. We start with the very basic equivalence:

$$\mathrm{a^n + b^n = c^n} \Leftrightarrow (\mathrm{a/b})^\mathrm{n} + 1 = (\mathrm{c/b})^\mathrm{n} \Leftrightarrow (\mathrm{c/b})^\mathrm{n} - (\mathrm{a/b})^\mathrm{n} = 1 \Leftrightarrow \left((\mathrm{c/b})^{\mathrm{n}/2}\right)^2 - \left((\mathrm{a/b})^{\mathrm{n}/2}\right)^2 = 1,$$

$$\mathrm{n} > 2, \ \text{ where } \mathrm{a/b, c/b} \in \mathrm{Q}^+, \mathrm{abc} \neq 0 \ (\mathrm{M1.1})$$

We declare now the sets:

$$\mathrm{F_n} = \left\{(\mathrm{a/b,c/b}) \in \mathrm{Q}_+^2 \middle| \left((\mathrm{c/b})^{\mathrm{n}/2}\right)^2 - \left((\mathrm{a/b})^{\mathrm{n}/2}\right)^2 = 1, \mathrm{n} > 2, \mathrm{a,b,c} \in \mathrm{N}^+ \right\},$$

$$\mathrm{S_n} = \left\{(\mathrm{a/b,c/b}) \in \mathrm{Q}_+^2 \mid \mathrm{m} - \lambda = \mathrm{a/b} \wedge \mathrm{m} = \mathrm{c/b}, \mathrm{m}, \lambda \in \mathrm{Q}^+\right\}$$

From this point on, **initially we solve the system freely** without constraints for variables (a,b,c), i.e if apply $(\mathrm{a,b,c}) \mid \mathrm{a,b,c} \in \mathrm{N}^+, \mathrm{m}, \lambda \in \mathrm{Q}^+$. This is because, as we will see below, the equations themselves result in at least one zero value for some variable. The following applies to the quadratic difference system:

The set $\mathrm{F_n} \cap \mathrm{S_n}$ leads to 2 categories of solutions let's look at it in detail:

$$\begin{gathered}
\left\{\left(m^{n/2}\right)^2-\left((m-\lambda)^{n/2}\right)^2=1\right\} \Leftrightarrow \\
\left\{\begin{array}{l}
\{m^{n/2}-(m-\lambda)^{n/2}=\frac{1}{t} \\
\\
m^{n/2}+(m-\lambda)^{n/2}=t
\end{array}\right\} \Leftrightarrow\left\{\begin{array}{l}
m^{n/2}=\frac{t^2+1}{2\cdot t} \\
\\
(m-\lambda)^{n/2}=\frac{t^2-1}{2\cdot t}
\end{array}\right\} \Leftrightarrow \\
\left\{\begin{array}{l}
m=\left(\frac{t^2+1}{2\cdot t}\right)^{2/n} \\
\\
(m-\lambda)=\left(\frac{t^2-1}{2\cdot t}\right)^{2/n}
\end{array}\right\} \Leftrightarrow\left\{\begin{array}{l}
m=\left(\frac{t^2+1}{2\cdot t}\right)^{2/n} \\
\\
\lambda=\left(\frac{t^2+1}{2\cdot t}\right)^{2/n}-\left(\frac{t^2-1}{2\cdot t}\right)^{2/n}
\end{array}\right\}, t \in Q^{+}
\end{gathered} \tag{M1.2}$$

Let us further assume that $\mathrm{t}=\mathrm{p/q}$ where $\mathrm{p,q} \in \mathrm{N}^{+}, p>q, \mathrm{p}$ and q relatively primes if we substitute the value of t, in relation (M1.2) then we get:

$$\left\{\begin{array}{l}
m=\left(\frac{p^2+q^2}{2\cdot p\cdot q}\right)^{2/n} \\
\\
\lambda=\left(\frac{p^2+q^2}{2\cdot p\cdot q}\right)^{2/n}-\left(\frac{p^2-q^2}{2\cdot p\cdot q}\right)^{2/n}
\end{array}\right\}, p, q \in Z_{>0}, p \text{ and } q \text{ relatively primes} \tag{M1.3}$$

We come to the **most crucial point** where we have to determine whether m and λ belong to Q^{+} or not, because by definition they must belong to Q^{+}. Because as it is in the form of relation M1.3 it is difficult to infer and therefore we will use a correlation trick. To this end, we make the following assumptions:

We define the relationships and we define as $\sigma=m$ and $\epsilon=m-\lambda$ then apply:

If where $\sigma, \varepsilon \in Q_{>0}, p$ and q relatively primes, $p, q \in Z_{>0}$

$$\left\{\begin{array}{l}
\sigma^{\mathrm{n}/2}=\left(\frac{\mathrm{p}^2+\mathrm{q}^2}{2\cdot\mathrm{p}\cdot\mathrm{q}}\right) \\
\\
\varepsilon^{\mathrm{n}/2}=\left(\frac{\mathrm{p}^2-\mathrm{q}^2}{2\cdot\mathrm{p}\cdot\mathrm{q}}\right)
\end{array}\right\} \Leftrightarrow\left\{\begin{array}{l}
\frac{\mathrm{p}}{\mathrm{q}}=\left(\sigma^{\mathrm{n}/2}+\varepsilon^{\mathrm{n}/2}\right) \\
\\
\frac{\mathrm{q}}{\mathrm{p}}=\left(\sigma^{\mathrm{n}/2}-\varepsilon^{\mathrm{n}/2}\right)
\end{array}\right\} \Leftrightarrow\left(\sigma^{\mathrm{n}/2}+\varepsilon^{\mathrm{n}/2}\right)\left(\sigma^{\mathrm{n}/2}-\varepsilon^{\mathrm{n}/2}\right)=1 \tag{M1.4}$$

We now distinguish 2 cases:

I) $p \neq q, \quad \left(\sigma^{n/2}+\varepsilon^{n/2}\right)\cdot\left(\sigma^{n/2}-\varepsilon^{n/2}\right)=1$

This case is indeterminate for the σ, ε but it gives us informations in which set each one belongs. So we have the relations:

$$\left\{\begin{array}{l}
\sigma=\left(\frac{p^2+q^2}{2\cdot p\cdot q}\right)^{2/n} \\
\\
\varepsilon=\left(\frac{p^2-q^2}{2\cdot p\cdot q}\right)^{2/n}
\end{array}\right\} \text{ where } \sigma, \varepsilon \in Q_{>0}, p \text{ and } q \text{ relatively primes, } p, q \in Z_{>0} \tag{M1.5}$$

If we divide ε and σ we get (M1.5):

$$\left\{\begin{array}{l} \dfrac{\varepsilon}{\sigma}=\left(\dfrac{p^2-q^2}{p^2+q^2}\right)^{2/n} \Leftrightarrow \\ \dfrac{\varepsilon}{\sigma}=\left(\dfrac{\left(p^2-q^2\right)\cdot\left(p^2+q^2\right)^{n/2-1}}{\left(p^2+q^2\right)^{n/2}}\right)^{2/n} \Leftrightarrow \\ \varepsilon=\dfrac{\sigma}{p^2+q^2}\sqrt[n/2]{\left(p^2-q^2\right)\cdot\left(p^2+q^2\right)^{n/2-1}} \end{array}\right\} \begin{array}{l} \text{where } \dfrac{\sigma}{p^2+q^2}\in Q_{>0}, \varepsilon\in\left(R^{+}-Q^{+}\right), \\ p \text{ and } q \text{ relatively primes, } p,q\in Z_{>0} \end{array} \tag{M1.6}$$

The last relation gives rise to the following interesting relationship

$$\left\{\varepsilon=\frac{\sigma}{p^2+q^2}\sqrt[n/2]{\left(p^2-q^2\right)\cdot\left(p^2+q^2\right)^{n/2-1}}\right\} \begin{array}{l} \text{where } \dfrac{\sigma}{p^2+q^2}\in Q_{>0}, \varepsilon\in\left(R^{+}-Q^{+}\right), \\ p \text{ and } q \text{ relatively primes, } p,q\in Z_{>0} \end{array} \tag{M1.7}$$

Which if we analyse it section by section, is interpreted as follows

$$\left\{\begin{array}{l} \dfrac{\sigma}{p^2+q^2}\in Q_{>0} \text{ and} \\ \sqrt[n/2]{\left(p^2-q^2\right)\cdot\left(p^2+q^2\right)^{n/2-1}}\in\left(R_{>0}-Q_{>0}\right) \\ \\ \text{i.e. } \varepsilon \text{ is irrational number} \end{array}\right\} \begin{array}{l} \varepsilon\in\left(R^{+}-Q^{+}\right), \\ p \text{ and } q \text{ relatively primes, } p,q\in Z_{>0} \end{array} \tag{M1.8}$$

For ε to be an positive rational number, must apply for the subroot (that it must be an integer) that:

$$\left\{\left(p^2-q^2\right)=\left(p^2+q^2\right)\right\}\Leftrightarrow q=0, p \text{ and } q \text{ relatively primes}, p,q\in Z_{>0} \tag{M1.9}$$

But this i.e **that q = 0** contradicts the assumption i.e **that q must not be zero**, so this case is impossible and is therefore rejected.

Hence impossible to be a Rational number and logically there will be 2 additional cases.

II)Assume t is integer, then similarly will apply $\left(\sigma^{(n/2)}-e^{(n/2)}\right)\left(\sigma^{(n/2)}+e^{(n/2)}\right)=1$.

From Theorem 4 (Method II page 8,9), for n odd or even, it is proved that valid $\lambda=1$ & $m=1$ if we accept that λ is an integer. We come to relationship (M1.2 page 5) then because we have the ratio $m=\left(\left(t^2+1\right)/(2t)\right)^{(2/n)}=1\Leftrightarrow t=1$. The value of t is therefore independent of n. But when $t=1$ we will have $t=p/q=1\Leftrightarrow p=q$. There is now only one case left to consider what happens when $t=1$ and completes the proof.

III) If $\mathrm{t} = 1$ then $\mathrm{p} = \mathrm{q}$ and furthermore $\left(\sigma^{(\mathrm{n}/2)} + \mathrm{e}^{(\mathrm{n}/2)}\right)\left(\sigma^{(\mathrm{n}/2)} + \mathrm{e}^{(\mathrm{n}/2)}\right) = 1$.

From relationship (M1.4) we have

$$\left\{\begin{array}{l} \left(\sigma^{n/2} + \varepsilon^{n/2}\right) = 1 \\ \\ \left(\sigma^{n/2} - \varepsilon^{n/2}\right) = 1 \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \sigma = \pm 1, \varepsilon = 0, n = 2 \cdot k, k \in N^{+}, k > 1 \\ \\ \sigma = 1, \varepsilon = 0, n = 2 \cdot k + 1, k \in N^{+} \end{array}\right\} \tag{M1.10}$$

Aggregated results for

$$\boxed{\begin{array}{l} 1.n = 2\mathrm{k} + 1, \mathrm{k} \in \mathrm{N}^{*} \\ i).\text{if apply: } a/b = m - \lambda, c/b = m \\ \mathrm{m} = 1 \wedge \mathrm{m} - \lambda = 0 \Leftrightarrow \mathrm{a} = 0 \wedge \mathrm{c} = \mathrm{b} \\ ii).\text{if apply: } \mathrm{b/a} = \mathrm{m} - \lambda, \mathrm{c/a} = \mathrm{m} \\ \text{and } \mathrm{m} = 1 \wedge \mathrm{m} - \lambda = 0 \Leftrightarrow \mathrm{b} = 0 \wedge \mathrm{c} = \mathrm{a} \\ iii).\text{If } m - \lambda = -1 \wedge m = 0 \Leftrightarrow a = -b, c = 0 \\ iv).\text{If } \mathrm{m} - \lambda \neq 0 \wedge \mathrm{m} \neq 0 \Leftrightarrow \mathrm{a} = \mathrm{b} = \mathrm{c} = 0 \\ \\ 2.n = 2k, k \in N^{+}, k > 1 \\ i).\text{if apply: } a/b = m - \lambda, c/b = m \\ \mathrm{m} = \pm 1 \wedge \mathrm{m} - \lambda = 0 \Leftrightarrow \mathrm{a} = 0 \wedge \mathrm{c} = \pm \mathrm{b} \\ ii).\text{if apply: } \mathrm{b/a} = \mathrm{m} - \lambda, \mathrm{c/a} = \mathrm{m} \\ \text{and } m = \pm 1 \wedge m - \lambda = 0 \Leftrightarrow b = 0 \wedge c = \pm a \\ iii).\text{If } m - \lambda \neq 0 \wedge m \neq 0 \Leftrightarrow a = b = c = 0 \end{array}} \tag{M1.11}$$

From these 2 cases we can easily conclude that the set of solutions of the intersection of the sets $F_n \cap S_n$ arises the $\mathrm{G_n}$ which is:

$$\mathrm{G_n} = \{((\mathrm{a} = 0, \mathrm{c} = \mathrm{b} \text{ or } \mathrm{b} = 0, \mathrm{c} = \mathrm{a} \text{ or } \mathrm{c} = 0, \mathrm{a} = -\mathrm{b} \text{ or } \mathrm{a} = \mathrm{b} = \mathrm{c} = 0) \mid \mathrm{n} = 2\mathrm{k} + 1,$$
$$(\mathrm{a} = \mathrm{b} = \mathrm{c} = 0 \text{ or } \mathrm{a} = 0, \mathrm{c} = \pm \mathrm{b} \text{ or } \mathrm{b} = 0, \mathrm{c} = \pm \mathrm{a}) \mid \mathrm{n} = 2\mathrm{k}, \mathrm{k} > 1) \mid \mathrm{k} \in \mathrm{N}^{+}\}.$$

Finally, we proved that the solution sets $Pn \neq Gn$, since the assumptions we made must hold and we must keep the integer positive value in each variable, which is absolutely necessary. Since the results of the solution (in table (M1.11)) contradict the hypothesis because $abc \neq 0$ and since $\{(a, b, c) \mid a, b, c \in N^{+}\}$ holds for the variables. **Therefore, there is no solution** to F.L.T for $n > 2$ in N^{+} and hence $\mathrm{Pn} = \emptyset$. Of course, we accept solutions to Fermat's equation **only if** our variables take values **from the set Z**, as shown in table (M1.11).

Method II.

II.1. Theorem 4 (Basic theorem of Proof).

Let P_n be the set of Fermat triples and defined as $P_n = \{(a,b,c) \mid$ a.b.c, $n > 2 \in N*$ and an $a^n + b^n = c^n$, $abc \neq 0\}$. Let $\mathrm{G_n}$ be the set defined as:

$$\mathrm{G_n = \{((a = 0, c = b \text{ or } b = 0, c = a \text{ or } c = 0, a = -b \text{ or } a = b = c = 0) \mid n = 2k+1,}$$
$$\mathrm{(a = b = c = 0 \text{ or } a = 0, c = \pm b \text{ or } b = 0, c = \pm a) \mid n = 2k, k > 1) \mid k \in N^+\}.}$$

We need to prove that the sets $\mathrm{P_n \neq G_n}$ and also $\mathrm{P_n} = \emptyset$.

Proof

We have 2 sets P_n and G_n of solutions that we need to prove are not equal and G_n is the complete set unconstrained, as we will prove of the diophantine Fermat equation. The basis of the method for the proof is the relations proved by theorems 1&2 of the Pythagorean triples. We start with the very basic equivalence

$$\mathrm{a^n + b^n = c^n \Leftrightarrow (a/b)^n + 1 = (c/b)^n \Leftrightarrow (c/b)^n - (a/b)^n = 1, n > 2, \{a/b, c/b\} \in Q^+, abc \neq 0} \qquad \text{(M2.1)}$$

The set $\mathrm{F_0 \cap S_n}$ leads to 2 categories of solutions let's look at it in detail for n i.e. $\mathrm{n = 2r + 1, r \geq 1}$ and $\mathrm{n = 2r, r > 1, r = N^+}$

i) $\mathrm{n = 2r + 1}, r \in \mathrm{N^+}$

We declare now the sets:

$$F_n = \left\{(a/b, c/b) \in Q^{2+} \mid (c/b)^n - (a/b)^n = 1, n > 2, a, b, c \in N^+\right\},$$
$$S_n = \left\{(a/b, c/b) \in Q^{2+} \mid m - \lambda = a/b \wedge m = c/b, m, \lambda \in Q^+\right\}$$

From this point on, **initially we solve the system freely** without constraints for variables (a,b,c), i.e if apply $(a,b,c) \mid a,b,c \in N^+, m, \lambda \in Q^+$: As we will see below, the equations themselves lead to at least one zero value for some variable that we will obviously exclude. The following applies to the system:

We define the function $F(m,\lambda) = (m-\lambda)^{(2r+1)} - m^{2r+1} + 1 = 0, m, \lambda \in Q^+$ (M2.2)

To find the discriminant we need to find the first derivative and substitute it into the original function under the condition that it is >= 0.

Therefore: $(2r+1)\cdot(m-\lambda)^{2r} - (2r+1)\cdot m^{2r} = 0 \Leftrightarrow \left\{ \begin{array}{c} m = \frac{\lambda}{2} \\ \lambda = 0 \end{array} \right\}$, but because $\lambda \neq 0$ we accept only the $m = \frac{\lambda}{2}$ and with substitution in the original equation we have $F(m,\lambda) \geq 0$ which must apply into discriminant:

$$D = \left(\frac{\lambda}{2} - \lambda\right)^{2r+1} - \left(\frac{\lambda}{2}\right)^{2r+1} + 1 = 1 - 2\cdot\left(\frac{\lambda}{2}\right)^{2r+1} \geq 0 \Leftrightarrow \lambda \leq 2\cdot\left(\frac{1}{2}\right)^{1/(2r+1)}$$

Therefore $0 < \lambda < 2 \cdot \left(\frac{1}{2}\right)^{1/(2r+1)}$

But then $\lambda < 2 \cdot \left(\frac{1}{2}\right)^{1/(2r+1)}$ and for $r \to \infty$ then $\lambda \to 2$. How ever because $\lambda > 0$ it follows that the only integer value of $\lambda = 1$ and therefore the unique solution $m = 1$ will also result

ii) $\mathrm{n} = 2\mathrm{r}, \mathrm{r} > 1, \mathrm{r} \in \mathrm{N}^{+}$.

In this second case according to relation (M3.1) we declare now the sets:

$$\mathrm{F_n} = \left\{(\mathrm{a/c}, \mathrm{b/c}) \in \mathrm{Q}^2+ \mid (\mathrm{a/c})^{\mathrm{n}} + (\mathrm{b/c})^{\mathrm{n}} = 1, \mathrm{n} > 2, \mathrm{a}, \mathrm{b}, \mathrm{c} \in \mathrm{N}^{+}\right\},$$

$$\mathrm{S_n} = \left\{(\mathrm{a/c}, \mathrm{b/c}) \in \mathrm{Q}^2+ \mid \mathrm{m} - \lambda = \mathrm{a/c} \wedge \mathrm{m} = \mathrm{b/c}, \mathrm{m}, \lambda \in \mathrm{Q}^{+}\right\}$$

We define the function $F(m, \lambda) = 1 - (m - \lambda)^{(2r)} - m^{2r} = 0, m, \lambda \in Q^{+}$ (M 2.3). To find the discriminant we need to find the first derivative and substitute it into the original function under the condition that it is >= 0. Therefore: $-(2r) \cdot (m - \lambda)^{2r-1} - (2r) \cdot m^{2r-1} = 0 \Leftrightarrow \left\{\mathrm{m} = \frac{\lambda}{2}\right\}$, therefore $\mathrm{m} = \frac{\lambda}{2}$, and with substitution in the original equation $\mathrm{F}(\mathrm{m}, \lambda) \geq 0$ which must apply into discriminant:

$$\mathrm{D} = 1 - \left(\frac{\lambda}{2} - \lambda\right)^{2\mathrm{r}} - \left(\frac{\lambda}{2}\right)^{2\mathrm{r}} = 1 - 2 \cdot \left(\frac{\lambda}{2}\right)^{2\mathrm{r}} \geq 0 \Leftrightarrow \lambda \leq 2 \cdot \left(\frac{1}{2}\right)^{1/(2r)}.$$

Therefore $0 < \lambda < 2 \cdot \left(\frac{1}{2}\right)^{1/(2r)}$. But then $\lambda < 2 \cdot \left(\frac{1}{2}\right)^{1/(2r)}$ and for $r \to \infty$ then $\lambda \to 2$. However because $\lambda > 0$ it follows that the only integer value of $\lambda = 1$ and therefore the unique solution $m = 1$ will also result.

This analysis is obtained for integer λ. If $\lambda \in Q$, then we use the result of Theorem 3 , as a lemma, in particular, it follows from (M1.19) that if $\mathrm{p} \neq \mathrm{q}$ then this is impossible and therefore $\lambda = \mathrm{p/q} = 1$. Consequently for $\mathrm{n} > 2$ for values of $\boldsymbol{\lambda = 1}$ **as only integer and** $\boldsymbol{m = 1}$. These values for λ, m lead to the unique solution of the set:

$$\mathrm{G_n} = \{((\mathrm{a} = 0, \mathrm{c} = \mathrm{b} \text{ or } \mathrm{b} = 0, \mathrm{c} = \mathrm{a} \text{ or } \mathrm{c} = 0, \mathrm{a} = -\mathrm{b} \text{ or } \mathrm{a} = \mathrm{b} = \mathrm{c} = 0) \mid \mathrm{n} = 2\mathrm{k} + 1,$$

$$(\mathrm{a} = \mathrm{b} = \mathrm{c} = 0 \text{ or } \mathrm{a} = 0, \mathrm{c} = \pm\mathrm{b} \text{ or } \mathrm{b} = 0, \mathrm{c} = \pm\mathrm{a}) \mid \mathrm{n} = 2\mathrm{k}, \mathrm{k} > 1) \mid \mathrm{k} \in \mathrm{N}^{+}\}.$$

But according to the original hypothesis that $abc \neq 0$, implies that there can be no solution.

The only therefore integer value is $(\lambda, m) = (1, 1)$ and therefore as we proved again $\mathrm{Pn} \neq \mathrm{Gn}$ and $\mathrm{Pn} = \emptyset$ since the assumptions we made must also hold must keep the integer positive value in each variable, which is absolutely necessary.

Method III.

Proof of FLT by maximum of discriminant using Frey's elliptic curves.

III.1. Theorem 5. (Basic theorem of Proof).

In 1955, Taniyama noted that it was plausible that the $\mathrm{N_p}$ attached to a given elliptic curve always arise in a simple way from a modular form (in modern terminology, that the elliptic curve is modular). In 1985 Frey observed that this did not appear to be true for the elliptic curve attached to a nontrivial solution of the Fermat equation an $a^p + b^p = c^p, p > 2$. His observation prompted Serre to revisit some old conjectures implying this, and Ribet proved enough of his conjectures to deduce that Frey's observation is correct: the elliptic curve attached to a nontrivial solution of the Fermat equation is not modular. Finally, in 1994 Wiles (assisted by Taylor) proved that every **elliptic curve in a large class is modular**, there by proving Fermat's Last Theorem. It was Gerhard Frey [7] who completely transformed FLT into a problem about elliptic curves. In essence, Frey said this: if I have a solution $a^n + b^n = c^n$ to the Fermat equation for some exponent $n > 2$, then I'll use it to construct the following elliptic curve:

$$E : y^2 = x\left(x - a^n\right)\left(x + b^n\right) = g(x) \tag{M3.1}$$

Now if f is a polynomial of degree k and if $\mathrm{r}_1, \mathrm{r}_2, \ldots \mathrm{r}_k$ are all of its roots, then the discriminant Δ(f) of f is defined by

$$\Delta(f) = \prod_{1<i<j\leq k} \left(\mathrm{r}_i - r_j\right)^2 \tag{M3.2}$$

If f is monic with integer coefficients, it turns out that Δ(f) is an integer. The three roots of the polynomial g(x) on the right-hand side of the Frey curve are $0, a^n$ and $-b^n$ using the fact that $a^n - (-b^n) = a^n + b^n = c^n$ and a little algebra, we find that $\Delta(g) = (abc)^{2n}$. Frey said that an elliptic curve with such a discriminant must be really strange. In particular, such a curve cannot possibly be what is called modular (never mind what that means). Now here's a thought, said he; what if you could manage to prove two things: first, that a large class of elliptic curves is modular, and second, that the Frey curve is always a member of that class of curves? Why, you'd have a contradiction-from which you could conclude that there is no such curve. That is, there is no such solution to the Fermat equation ... that there is no counterexample to Fermat's Last Theorem ... and so Fermat's Last Theorem is true. We will try to give another proof using the well-known theory of classical analysis using the discriminant more understandable and faster. The steps we follow are in order as:

i). Since we have accepted as correct the relevant theory for Frey's elliptic curves equation (M3.1) will apply $\mathrm{y}^2 = \mathrm{x}\left(\mathrm{x} - \mathrm{a}^{\mathrm{n}}\right)\left(\mathrm{x} + \mathrm{b}^{\mathrm{n}}\right)$ and if we differentiate it with respect to x we get the relations analytically:

$$\left\langle \begin{array}{l} 2y\dfrac{dy}{dx} = \dfrac{d}{dx}\left\{x\left[x^2 + \left(b^n - a^n\right)x - (ab)^n\right]\right\} = \dfrac{d}{dx}\left\{x^3 + \left(b^n - a^n\right)x^2 - (ab)^n x\right\} \\ 2y\dfrac{dy}{dx} = 3x^2 + \left(b^n - a^n\right)2x - (ab)^n,\ y\dfrac{dy}{dx} = \dfrac{3}{2}x^2 + \left(b^n - a^n\right)x - \dfrac{(ab)^n}{2} \\ \dfrac{dy}{dx} = \dfrac{\frac{3}{2}x^2 + \left(b^n - a^n\right)x - \frac{(ab)^n}{2}}{\sqrt{x^3 + \left(b^n - a^n\right)x^2 - (ab)^n x}} = 0 \end{array} \right\rangle \tag{M3.3}$$

It must therefore be true that the numerator is equal to zero i.e.

$$\left\langle \begin{array}{l} x^2 + \dfrac{2}{3}\left(b^n - a^n\right)x - \dfrac{(ab)^n}{3} = 0, \left(x + \dfrac{1}{3}\left(b^n - a^n\right)\right)^2 - \dfrac{\left(b^n - a^n\right)^2}{3^2} - \dfrac{(ab)^n}{3} = 0 \\ x = -\dfrac{1}{3}\left(b^n - a^n\right) \pm \dfrac{1}{3}\sqrt{\left(b^n - a^n\right)^2 + 3(ab)^n} \\ x = \dfrac{-\left(b^n - a^n\right) \pm \sqrt{\left(a^n + b^n\right)^2 - (ab)^n}}{3} \\ x = \dfrac{-\left(b^n - a^n\right) \pm \sqrt{\left(a^n + b^n - (ab)^{n/2}\right)\left(a^n + b^n + (ab)^{n/2}\right)}}{3} = \dfrac{-\left(b^n - a^n\right) \pm \sqrt{\Delta}}{3} \\ \Delta = \text{ is the discriminant and } b > a \end{array} \right\rangle \tag{M3.4}$$

Because $\left(a^n + b^n + (ab)^{n/2}\right) > 0$ and this after $(a, b, c) \in N^+$, it follows that the representation $\left(a^n + b^n - (ab)^{n/2}\right) \geq 0$ (M3.5). But apply $a^n + b^n = c^n (M3.6)$ we will get $(a \cdot b) = c^2 (M3.7)$. Finally, from relations (M3.5, M3.6, M3.7) it will follow that $(\mathrm{a^n} + \mathrm{b^n} - \mathrm{c}^n) \geq 0$ (M3.8). From relationships (M3.6 and M3.8) the equation results

$$\left\langle \begin{array}{l} \text{But from (M3.8) apply only " = ", therefore we have:} \\ \mathrm{a^n} + \left(\dfrac{\mathrm{c}^2}{\mathrm{a}}\right)^{\mathrm{n}} - \mathrm{c^n} = 0,\ \ (\mathrm{a^n})^2 - \mathrm{a^n c^n} + (\mathrm{c^n})^2 = 0 \\ \left(\mathrm{a^n} - \dfrac{\mathrm{c^n}}{2}\right)^2 + (\mathrm{c^n})^2 - \dfrac{1}{4}(\mathrm{c^n})^2 = 0, \left(\mathrm{a^n} - \dfrac{\mathrm{c^n}}{2}\right)^2 + \dfrac{3}{4}(\mathrm{c^n})^2 = 0 \\ \mathrm{a} = \mathrm{c}\left[\dfrac{(1 \pm \mathrm{i}\sqrt{3})}{2}\right]^{1/\mathrm{n}},\ \ \mathrm{i} = \sqrt{-1} \end{array} \right\rangle \tag{M3.9}$$

That is, there is a complex number for a related to c or b related to c respectively. So we do not find an integer relationship between the variables as has been proven. According to relation (M3.9) it follows that in relation (M3.4) the **Discriminant $\boldsymbol{\Delta = 0}$**, another very basic conclusion, which leaves out as we see the variable $\mathrm{c} \cdot ([8], [9])$. Our penultimate goal is to calculate x with respect to our new discoveries and the final goal is to calculate y. From relation (M3.4) it follows that

$$\left\{ \begin{array}{l} x = \dfrac{-(b^n - a^n) \pm \sqrt{\Delta}}{3} = \dfrac{-(b^n - a^n)}{3} \\ \Delta = 0,\ \text{the discriminant} \end{array} \right\} \tag{M3.10}$$

Finally, we have for the calculation of y the relationships

$$\left\langle \begin{array}{l} b > a \\ y = \dfrac{\sqrt{(b^n - a^n)(b^n + 2a^n)(a^n + 2b^n)}}{3^{3/2}} \text{ must subroot of } y > 0 \\ \dfrac{d^2y}{dx^2} = \dfrac{3x^4 + 4(b^n - a^n)x^3 - 6(ab)^n x^2 - (ab)^{2n}}{4\{x(x - a^n)(x + b^n)\}^{3/2}} \\ \left.\dfrac{d^2y}{dx^2}\right|_{y'=0} = -\dfrac{3^{9/2}}{4}\dfrac{\left[\frac{(b^n - a^n)^2}{3} + (ab)^n\right]^2}{[(b^n - a^n)(b^n + 2a^n)(a^n + 2b^n)]^{3/2}} \langle 0 \end{array} \right\rangle \tag{M3.11}$$

So there is a maximum at this point but in fact we cannot accept its existence because there is no positive integer so that D = 0 is satisfied. This is what Frey has stated as the forbidden point of existence. In general we consider 2 cases in relation to $y^2 = x(x - a^n)(x + b^n)$:

A. y = 0. In this case there are 3 categories anaphorically with a,b,c.

A_1 : If $x = 0$ then apply $a \cdot b = 0 \Leftrightarrow a = 0 \vee b = 0$ which is rejected because $a \cdot b \cdot c \neq 0$

A_2 : If $x = a^n$ then apply $c = 0$, but is rejected because $\mathrm{a \cdot b \cdot c} \neq 0$

A_3 : If $x = -b^n$ then apply $c = 0$, but is rejected because $\mathrm{a \cdot b \cdot c} \neq 0$

B. $\mathbf{y \neq 0}$ and $\mathbf{y \in N^{+}}$, in this case there are 2 categories anaphorically with $\mathbf{a, b, c}$.

In principle it applies to that $y = \sqrt{\dfrac{(c^n - 2a^n)\cdot(c^n + a^n)\cdot(2c^n - a^n)}{3^3}}$ if we change b to a.

From 2 relationship $\left(b \cdot a = c^2 \;\&\; a^n + b^n = c^n\right)$ implies $a^n = c^n\left(\frac{1}{2}(1 \pm i\sqrt{3})\right)(a, b, c) \in N^+$.

So we have:

$B_1)\, a^n = c^n\left(\frac{1}{2}(1 - i\sqrt{3})\right)$

With replacement we have $y = \sqrt{\dfrac{(c^n - 2a^n)\cdot(c^n + a^n)\cdot(2c^n - a^n)}{3^3}} = \sqrt{\dfrac{ic^{3n}}{3^{1/2}}}$.

If i replace with $c^n = 3^{1/2} \cdot i \cdot k^{2r}, k, r \in N^+$ then $y = k^{3r}$, i.e Integer positive.

$B_2) \cdot a^n = c^n\left(\frac{1}{2}(1 + \mathrm{i}\sqrt{3})\right)$

With replacement we have $\mathrm{y} = \sqrt{\dfrac{(c^n - 2a^n)\cdot(c^n + a^n)\cdot(2c^n - a^n)}{3^3}} = \sqrt{-\dfrac{ic^{3n}}{3^{1/2}}}$

If i replace with $c^n = -3^{1/2} \cdot i \cdot k^{2r}, k, r \in N^+$ then $y = k^{3r}$, i.e Integer positive.

For Frey's curve with the original formula is forbidden to exist & cannot be drawn with Fermat's conditions under the resulting conditions.As we can see in these 2 cases, in order to have y an integer, we need $(a, b, c) \in C - R$. Therefore these 2 cases **A, B rejected** for the reasons explained and furthermore we have complex variable values and there **is not solution** for the Fermat equation $\mathrm{a^n + b^h = c^n}$ for some exponent $\mathrm{n} > 2$, in integers, with **use of Frey's elliptic curves.**

Method IV.

Method using the generalises Fermat equation

IV.1 Theorem 6[11]

Any equation form $x^p + y^q = z^w$ with positive integers x, y, z, p, q, w where $p, q, w > 1$, is transformed into a final Diophantine equation with GCD$(x, y, z) = 1$ then and only then, when at least one exponent equals 2. This equation will belong to a class of equations with exponents that be consistent with the criteria $\sigma(p, q, r) > 1, \sigma(p, q, r) = 1$ or $\sigma(p, q, r) < 1$ with a limited number equations, in accordance with chapter 4.

Proof

The number of the forms of $x^q + y^p = z^w, x, y, z, p, q, w \in Z^+ \wedge \{q >= 2, p >= 2, w >= 2\}$ after simplifying the terms of the $GCD[x, y, z]$, Lemma 1, Lemma 2 limited to 6. Depending on the ascending order of exponents $\{p, q, w\}$ of original Diophantine equation $x^p + y^q = z^w, x, y, z, p, q, w \in Z^+ \wedge \{q >= 2, p >= 2, w >= 2\}$ and after simplifying the terms with the number $\varepsilon = GCD[x, y, z]$, we receive a total of 6 cases where any stemming detail has as follows

1. $\lambda^p \cdot \varepsilon^{p-q} + \mu^q = \varepsilon^{w-q} \cdot \sigma^w, w > p > q \in Z$
2. $\lambda^p + \varepsilon^{q-p} \mu^q = \varepsilon^{w-p} \cdot \sigma^w, w > q > p \in Z$
3. $\lambda^p \cdot \varepsilon^{p-q} + \mu^q = \varepsilon^{w-q} \cdot \sigma^w, p > w > q \in Z$
4. $\lambda^p \cdot \varepsilon^{p-w} + \varepsilon^{q-w} \cdot \mu^q = \sigma^w, p > q > w \in Z$
5. $\lambda^p \cdot \varepsilon^{p-w} + \varepsilon^{q-w} \mu^q = \sigma^w, q > p > w \in Z$
6. $\lambda^p + \varepsilon^{q-p} \mu^q = \varepsilon^{w-p} \cdot \sigma^w, q > w > p \in Z$

But these exhibitors must comply with the Fermat-Catalan criteria, but here we will analyse them in general terms, distinguishing 3 general cases:

if we accept that p,q and w are fixed positive integers and that these exponents must satisfy the criteria of chapter 4[10,12], and after first accepting $p, q, w >= 2$, we will prove that at least one exponent equals 2 using these criteria alone. So according to this logic the following 3 cases will apply:

Case 1$^{\mathbf{rd}}$

$0 < 1/p + 1/q + 1/w < 1$

In order to we calculate the exhibitors present in the open interval $(0, 1)$ solve the inequality as z and we get

$$1/w < 1 - \frac{p+q}{p \cdot q} \Rightarrow w > \frac{p \cdot q}{p(q-1) - q}$$

The inequality has integer solutions which arise only in accordance with the 3 equations:

$$\begin{aligned} &(1).\ p \cdot (q-1) - q = 1 \\ &(2).\ q = \varphi \cdot (p \cdot (q-1) - q) \\ &(3).\ p = \varepsilon \cdot (p \cdot (q-1) - q) \\ &\ \ \varepsilon, \varphi \in Z \end{aligned}$$

1.From the first equation it follows that

$p \cdot (q-1) = q + 1 \Rightarrow p = \frac{1+q}{q-1} = 1 + \frac{2}{q-1}$ which implies 2 prerequisites:

i) $q - 1 = 1 \Rightarrow q = 2 \wedge p = 3$

ii) $q - 1 = 2 \Rightarrow q = 3 \wedge p = 2$

because should the $(q-1)$ must divide 2

And for 2 exhibitor cases we get $\boxed{w > 6 \Rightarrow w \geq 7}$

Therefore Thus arise the two triads $\boxed{p = 3, q = 2, w \geq 7}$ and $\boxed{p = 2, q = 3, w \geq 7}$

2. Similarly from the second equation $\boldsymbol{q = \phi \cdot (p \cdot (q-1) - q)}$ we get:

$$q = \phi \cdot p \cdot (q-1) - q \cdot \varphi \Rightarrow p = \frac{q \cdot (1+\phi)}{\phi \cdot (q-1)}$$

$i)\phi(q-1)=1 \Rightarrow \phi=\dfrac{1}{q-1}=1 \wedge q-1=1 \Rightarrow q=2$

$p=\dfrac{q\cdot(1+\phi)}{\phi\cdot(q-1)}=\dfrac{2\cdot 2}{1}=4$

$w>\dfrac{p\cdot q}{p(q-1)-q}=\dfrac{4\cdot 2}{4\cdot 1-2}=4, w\geq 5$

Hence the triad

$\boxed{p=4, q=2, w\geq 5}$

$ii)q=\sigma(q-1)\wedge(1+\phi)=\lambda\cdot\phi$

$a)\phi=\dfrac{1}{\lambda-1}\Rightarrow \lambda-1=1\Rightarrow(\lambda=2\wedge\phi=1)$

$\beta)q\cdot(\sigma-1)=\sigma\Rightarrow q=\dfrac{\sigma}{\sigma-1}=1+\dfrac{1}{\sigma-1}==2\wedge\sigma-1=1\Rightarrow(\sigma=2\wedge q=2)$

$p=\dfrac{q\cdot(1+\phi)}{\phi\cdot(q-1)}=\dfrac{2\cdot 2}{1\cdot 1}=4, w>\dfrac{p\cdot q}{p(q-1)-q}=\dfrac{4\cdot 2}{4\cdot(2-1)-2}=4, w\geq 5$

Therefore resulting triad

$\boxed{|p=4, q=2, w\geq 5|}$

$iii)q=\sigma\cdot\phi\wedge(1+\phi)=\lambda\cdot(y-1)$

$a)\lambda=\dfrac{1+\phi}{y-1}=\dfrac{1}{q-1}+\dfrac{\phi}{q-1}\wedge q-1=1\Rightarrow(q=2\wedge\lambda=3)$

$\sigma\cdot\phi=2\Rightarrow(\sigma=1\wedge\phi=2),(\sigma=2\wedge\phi=1)$

$q=2\wedge\phi=1\Rightarrow p=\dfrac{q\cdot(1+\phi)}{\phi\cdot(q-1)}=\dfrac{2}{1}\dfrac{2}{1}=4, w>\dfrac{p\cdot q}{p(q-1)-q}=\dfrac{4\cdot 2}{4\cdot 1-2}=4$

$q=2\wedge\phi=2\Rightarrow p=\dfrac{q\cdot(1+\phi)}{\phi\cdot(q-1)}=\dfrac{2}{2}\dfrac{3}{1}=3, w>\dfrac{p\cdot q}{p(q-1)-q}=\dfrac{3\cdot 2}{3\cdot 1-2}=6$

Thus arise the two triads

$\boxed{p=4, q=2, w\geq 5}$ and $\boxed{p=3, q=2, w\geq 7}$

3.Similarly from equation $p=\varepsilon\cdot(p\cdot(q-1)-q)$ take that:

$p=\varepsilon\cdot p\cdot(q-1)-q\cdot\varepsilon\Rightarrow q=\dfrac{p\cdot(1+\varepsilon)}{\varepsilon\cdot(p-1)}$

$i)\varepsilon(p-1)=1 \Rightarrow \varepsilon=\dfrac{1}{p-1}=1 \wedge p-1=1 \Rightarrow p=2$

$q=\dfrac{p\cdot(1+\varepsilon)}{\varepsilon\cdot(p-1)}=\dfrac{2\cdot 2}{1}=4$

$w>\dfrac{p\cdot q}{p(q-1)-q}=\dfrac{4\cdot 2}{2\cdot 3-4}=4, w\geq 5$

Therefore shows the triad

$\boxed{q=4, p=2, w\geq 5}$

$ii)p=\varepsilon(p-1)\wedge(1+\varepsilon)=\lambda\cdot\varepsilon$

$a)\varepsilon=\dfrac{1}{\lambda-1} \Rightarrow \lambda-1=1 \Rightarrow (\lambda=2\wedge\varepsilon=1)$

$b)p\cdot(\varepsilon-1)=\varepsilon \Rightarrow p=\dfrac{\varepsilon}{\varepsilon-1}=1+\dfrac{1}{\varepsilon-1}==2\wedge\varepsilon-1=1 \Rightarrow (\varepsilon=2\wedge p=2)$

$q=\dfrac{p\cdot(1+\varepsilon)}{\varepsilon\cdot(p-1)}=\dfrac{2\cdot 2}{1\cdot 1}=4, w>\dfrac{p\cdot q}{p(q-1)-q}=\dfrac{4\cdot 2}{2\cdot(4-1)-4}=4, w\geq 5$

Hence the triad

$\boxed{p=2, q=4, w\geq 5}$

$iii)p=\varepsilon\cdot\phi\wedge(1+\varepsilon)=\lambda\cdot(p-1)$

$a)\lambda=\dfrac{1+\varepsilon}{p-1}=\dfrac{1}{p-1}+\dfrac{\varepsilon}{p-1}\wedge p-1=1 \Rightarrow (p=2)$

$\varepsilon\cdot\phi=2 \Rightarrow (\varepsilon=1\wedge\phi=2), (\varepsilon=2\wedge\phi=1)$

$p=2\wedge\phi=1 \Rightarrow q=\dfrac{p\cdot(1+\phi)}{\phi\cdot(p-1)}=\dfrac{2}{1}\dfrac{2}{1}=4, w>\dfrac{p\cdot q}{p(q-1)-q}=\dfrac{4\cdot 2}{2\cdot 3-4}=4$

$p=2\wedge\phi=2 \Rightarrow q=\dfrac{p\cdot(1+\phi)}{\phi\cdot(p-1)}=\dfrac{2}{2}\dfrac{3}{1}=3, w>\dfrac{p\cdot q}{p(q-1)-q}=\dfrac{3\cdot 2}{2\cdot 1-2}=6$

Thus arise the two triads

$\boxed{q=4, p=2, w\geq 5}$ and $\boxed{q=3, p=2, w\geq 7}$

Total we have 12 cases for exhibitors and cyclically we will have

(i)

$$\boxed{\begin{array}{l} p=3,q=2,w\geq 7\wedge p=2,q=3,w\geq 7\\ w=3,p=2,q\geq 7\wedge w=2,p=3,q\geq 7\\ w=3,q=2,p\geq 7\wedge w=2,q=3,p\geq 7\\ q=4,p=2,w\geq 5\wedge q=2,p=4,w\geq 5\\ w=4,p=2,q\geq 5\wedge w=2,p=4,q\geq 5\\ w=4,q=2,p\geq 5\wedge w=2,q=4,p\geq 5 \end{array}}$$

Which in relation to equations take the form

(ii)

$$\boxed{\begin{array}{l} x^3+y^2=z^w,w\geq 7\\ x^2+y^3=z^w,w\geq 7\\ x^2+y^q=z^3,q\geq 7\\ x^3+y^q=z^2,q\geq 7\\ x^p+y^2=z^3,p\geq 7\\ x^p+y^3=z^2,p\geq 7 \end{array}} \qquad \boxed{\begin{array}{l} x^2+y^q=z^4,q\geq 5\\ x^4+y^q=z^2,q\geq 5\\ x^2+y^4=z^w,w\geq 5\\ x^4+y^2=z^w,w\geq 5\\ x^p+y^2=z^4,p\geq 5\\ x^p+y^4=z^2,p\geq 5 \end{array}}$$

Characteristics mention the work of Jamel Ghanouchi "A new approach of Fermat-Catalan conjecture" that achieves the same result.

The generalized Fermat conjecture (Darmon and Granville, 1995; Darmon, 1997), also known as the Tijdeman-Zagier conjecture and as the Beal conjecture (Beukers, 2012), is concerned with the case χ <1. It states that the only non-trivial primitive solutions to $x^q + y^p = z^w$ with $\sigma(\mathrm{p,g,r}) < 1$ are

$$2^5+7^2=3^4,\quad 7^3+13^2=2^9,\quad 2^7+17^3=71^2,\quad 3^5+11^4=122^2,$$
$$17^7+76271^3=21063928^2, 1414^3+2213459^2=65^7, 9262^3+15312283^2=113^7,$$
$$43^8+96222^3=30042907^2 \text{ and } 33^8+1549034^2=15613^3.$$

The generalized Fermat conjecture has been documented for many signatures (p, q, r), including many infinite families of signatures, starting with Fermat's last theorem (p, p, p) by Wiles (1995). The remaining cases are reported in Chapter 4[10].

Case 2rd. $1/p + 1/q + 1/w = 1$

i) From case 1 shows that overall we have 12 cases for exhibitors and and we roundly take:

(i)		(ii)
$p=3,q=2,w\geq 7 \wedge p=2,q=3,w\geq 7$		$p=3,q=2,w>6 \wedge p=2,q=3,w>6$
$w=3,p=2,q\geq 7 \wedge w=2,p=3,q\geq 7$		$w=3,p=2,q>6 \wedge w=2,p=3,q>6$
$w=3,q=2,p\geq 7 \wedge w=2,q=3,p\geq 7$	$\Leftrightarrow$	$w=3,q=2,p>6 \wedge w=2,q=3,p>6$
$q=4,p=2,w\geq 5 \wedge q=2,p=4,w\geq 5$		$q=4,p=2,w>4 \wedge q=2,p=4,w>4$
$w=4,p=2,q\geq 5 \wedge w=2,p=4,q\geq 5$		$w=4,p=2,q>4 \wedge w=2,p=4,q>4$
$w=4,q=2,p\geq 5 \wedge w=2,q=4,p\geq 5$		$w=4,q=2,p>4 \wedge w=2,q=4,p>4$

But the inequality (ii), for example, $p = 3, q = 2, w > 6$ as well as the inequality $q = 4, p = 2, w > 4$ which is characteristic of the group of exhibitors according to the criterion $0 < 1/p + 1/q + 1/w < 1$, so for the exponent group to have equality, 12 relations will apply cyclically as follows:

(iii)

$p=3,q=2,w=6 \wedge p=2,q=3,w=6$
$w=3,p=2,q=6 \wedge w=2,p=3,q=6$
$w=3,q=2,p=6 \wedge w=2,q=3,p=6$
$q=4,p=2,w=4 \wedge q=2,p=4,w=4$
$w=4,p=2,q=4 \wedge w=2,p=4,q=4$
$w=4,q=2,p=4 \wedge w=2,q=4,p=4$

ii) Pending from only the case $3/p = 1 \Rightarrow p = 3$ which implies $p = q = w = 3$. But this case according to the proof of Fermat's theorem does not accept solutions with exponents greater than 2.

Case 3$^{\text{rd}}$. $1/p + 1/q + 1/w > 1$

Originally accept that $p >= 2, q >= 2$ and $w >= 2$. We examine three cases:

i) $p = q = w = 2$ which is true

ii) $p = q = 2 \Rightarrow w > 2$ which is true we cyclically for the other exhibitors that $p = w = 2 \Rightarrow q > 2$ and $q = w = 2 => p > 2$.

iii) For all other cases will apply in accordance with the relation (iii) the second case, because now would force the inequality < 6, i.e total of 12 relations for all exhibitors.

$p=3,q=2,\{2<=w<=5\} \wedge p=2,q=3,\{2<=w<=5\}$
$w=3,p=2,\{2<=q<=5\} \wedge w=2,p=3,\{2<=q<=5\}$
$w=3,q=2,\{2<=p<=5\} \wedge w=2,q=3,\{2<=p<=5\}$
$q=4,p=2,\{2<=w<=3\} \wedge q=2,p=4,\{2<=w<=3\}$
$w=4,p=2,\{2<=q<=3\} \wedge w=2,p=4,\{2<=q<=3\}$
$w=4,q=2,\{2<=p<=3\} \wedge w=2,q=4,\{2<=p<=3\}$

(iv)

IV.2 Theorem 7[11,12]

The equation $x^p + y^q = z^w$ with positive integers x, y, z and extra $(p, q, w >= 2)$ and p, q and w are fixed positive integers is solved if and only if apply the conditions of Theorem 5, $(1, 2, 3)$ cases for exponents p, q, w with extra $(x, y, z) = 1$, and at least one of them equal 2. Therefore Beal's Conjecture is true with the above conditions, because accepts that there is no solution under the condition that all values of the exponents greater of 2.

Proof

For the equation $x^p + y^q = z^w$ with positive integers $x, y, z, (p, q, w >= 2)$ demonstrated that solved if and only if apply the conditions of Theorem 5 (i, ii, iii) for the exponents p, q, w with extra $(x, y, z) = 1$, so we have analytical

i) $1/p + 1/q + 1/w < 1$

According to Theorem 5, and 1 case, there is a solution to obtain values for the group of exhibitors $\{p, q, w\}$ as follows:

$$\boxed{\begin{array}{l} p = 3, q = 2, w \geq 7 \wedge p = 2, q = 3, w \geq 7 \\ w = 3, p = 2, q \geq 7 \wedge w = 2, p = 3, q \geq 7 \\ w = 3, q = 2, p \geq 7 \wedge w = 2, q = 3, p \geq 7 \\ q = 4, p = 2, w \geq 5 \wedge q = 2, p = 4, w \geq 5 \\ w = 4, p = 2, q \geq 5 \wedge w = 2, p = 4, q \geq 5 \\ w = 4, q = 2, p \geq 5 \wedge w = 2, q = 4, p \geq 5 \end{array}}$$

Which clearly shows that $p = 2$ or $q = 2$ or $w = 2$. Therefore least one exponent $= 2$.

ii) $1/p + 1/q + 1/w = 1$

It happens the second case, Theorem 5, for exist solution will arrive at values for the group of exhibitors $\{p, q,$ w$\}$ as follows:

$$\boxed{\begin{array}{l} p = 3, q = 2, w = 6 \wedge p = 2, q = 3, w = 6 \\ w = 3, p = 2, q = 6 \wedge w = 2, p = 3, q = 6 \\ w = 3, q = 2, p = 6 \wedge w = 2, q = 3, p = 6 \\ q = 4, p = 2, w = 4 \wedge q = 2, p = 4, w = 4 \\ w = 4, p = 2, q = 4 \wedge w = 2, p = 4, q = 4 \\ w = 4, q = 2, p = 4 \wedge w = 2, q = 4, p = 4 \end{array}}$$

Which also seems that $p = 2$ or $q = 2$ or $w = 2$. Therefore least one exponent equal 2.

iii) $1/p + 1/q + 1/w > 1$

For the third case, the Theorem 5, to obtain a solution we will arrive at values for the group of exhibitors

$\{p, q, w\}$ as follows:

$p = 3, q = 2, \{2 <= w <= 5\} \wedge p = 2, q = 3, \{2 <= w <= 5\}$
$w = 3, p = 2, \{2 <= q <= 5\} \wedge w = 2, p = 3, \{2 <= q <= 5\}$
$w = 3, q = 2, \{2 <= p <= 5\} \wedge w = 2, q = 3, \{2 <= p <= 5\}$
$q = 4, p = 2, \{2 <= w <= 3\} \wedge q = 2, p = 4, \{2 <= w <= 3\}$
$w = 4, p = 2, \{2 <= q <= 3\} \wedge w = 2, p = 4, \{2 <= q <= 3\}$
$w = 4, q = 2, \{2 <= p <= 3\} \wedge w = 2, q = 4, \{2 <= p <= 3\}$

in which at least appear that one of the $p = 2$ or $q = 2$ or $w = 2$.

Therefore at least one exponent equals 2 to have a solution and hence play Beal's Conjecture is true, because it recognizes that there is no solution if all values of the exponents greater 2.

IV.3.Theorem 8. (F.L.T) For any integer $n > 2$, the equation $\mathrm{x}^n + \mathrm{y}^n = \mathrm{z}^n$ has no positive integer solutions

An equation of the form $x^a + y^b = z^c$ (Beals') to have a solution, according to theorems $\{6, 7\}$, must have at least one exponent equal to 2. And since in Fermat's last theorem we have $a = b = c = n$, it follows directly that the only solution that Fermat's equation $\mathrm{x}^n + \mathrm{y}^n = \mathrm{z}^n$ can have is when $\mathrm{n} = 2$. So for $\mathrm{n} > 2$ there is no solution.

Part III. 2 Solutions of F.L.T. by simulation method

Method V.

V.1. Theorem 9.(Trigonometric simulation of Fermat's equation - Pythagorean equation).[5]

Let P_n be the set of Fermat triples and defined as:

$$P_n = \{(a, b, c) \mid a, b, c, n > 2 \in N \text{ and } a^n + b^n = c^n, abc \neq 0\}.$$

Let $\mathrm{G_n}$ set of simulation be the set defined as:

$\mathrm{G}_{nT} = \{$ If $\mathrm{a^n/c^n + b^n/c^n = 1, a^n/c^n = sin^n(x) \wedge b^n/c^n = cos^n(x)}, \mid \mathrm{a, b, c} \in \mathrm{N}^+$, $\mathrm{sin^n(x)} < \mathrm{sin^2(x)} < 1, \mathrm{cos^n(x)} < \mathrm{cos^2(x)} < 1$. The solutions are $((\sin(x) = 1, \cos(x) = 0)$ or $(\sin(x) = 0, \cos(x) = 1)) \mid n = 2k + 1, k \in N^+, ((\sin(x) = \pm 1, \cos(x) = 0$ or $(\sin(x) = 0, \cos(x) = \pm 1) \mid n = 2k, k > 1, k \in N^+\}$. We need to prove that the ts $P_n \neq G_{nT}$ and also $P_n = \emptyset$.

Proof.

Using a similar procedure as Theorem 3, we will prove Theorem 4 under the conditions we assumed for Fermat's equation to hold. If the solutions are identical then the solutions are equivalent and the simulation is true with respect to the solution sets. As we have mentioned we can equate the equation and $a^n + b^n = c^n(5.1)$ with the trigonometric equation $\sin^n(x) + \cos^n(x) = 1(5.2)$ and $a^n/c^n = \sin^n(x) \wedge b^n/c^n = \cos^n(x), \mid a, b, c \in N + (5.3)$. The proof passes though 2 parts to prove that it does not apply for power for even positive numbers integers greater than 2, i.e. $n > 2$ and the proof is divided into 2 parts:

V.2. Part A.

The equivalent Diophantine trigonometric equation $\sin^{2k+1}(x) + \cos^{2k+1}(x) = 1(5.4)$ has no solutions with $\sin(x) \neq 0$ and $\cos(x) \neq 0$ for $k \in N^+$.

Proof.

Let's assume that x is a solution of equation (4.4). We can easily (because $0 \leq \cos(x) \leq 1$ & $0 \leq \sin(x) \leq 1$ find that:

$$\cos^{2k+1}(x) \leq \cos^2(x) \& \sin^{2k+1}(x) \leq \sin^2(x) \tag{5.5}$$

if in at least one of the relations (4.5), the inequality applies then if we add in parts we will have

$$\sin^{2k+1}(x) + \cos^{2k+1}(x) \leq 1 \tag{5.6}$$

Therefore the trigonometric solution of (i1) will result from the group

$$s = \left\langle \begin{array}{c} \cos^{2k+1}(x) = \cos^2(x) \\ \sin^{2k+1}(x) = \sin^2(x) \end{array} \right\rangle \Rightarrow \left\langle \begin{array}{c} \cos^2(x)\left(\cos^{2k-1}(x) - 1\right) = 0 \\ \sin^2(x)\left(\sin^{2k-1}(x) - 1\right) = 0 \end{array} \right\rangle \Rightarrow \left\langle \begin{array}{c} \cos(x) = 0 \vee \cos(x) = 1 \\ \sin(x) = 0 \vee \sin(x) = 1 \end{array} \right\rangle \Rightarrow \tag{5.7}$$

The **system $<$ s $>$** leads to the solutions

$(t \in Z, x = 2\pi t) \| \left(\{t \in Z, \left(x = \frac{\pi}{2} + 2\pi t\right)\right)$ (5.8)

This is the only solution of the system and we will get the results.

Great results

$$\boxed{\begin{array}{l} 1. \operatorname{Sin}(x) = 1, \cos(x) = 0 \Rightarrow b = 0 \text{ and } c = a \\ 2. \operatorname{Sin}(x) = 0, \cos(x) = 1 \Rightarrow a = 0 \text{ and } c = b \end{array}} \tag{5.9}$$

V.3. Part B.

The equivalent Diophantine trigonometric equation $\sin^{2k}(x) + \cos^{2k}(x) = 1$ (5.10) has no solutions with $\sin(x) \neq 0$ and $\cos(x) \neq 0$ for $k \in N^+, k > 1$.

Proof.

For the same reasons as before we assume that x is a solution of equation (5.9 & 5.10). If we Apply the restrictions $0 \leq \cos(x) \leq 1$ & $0 \leq \sin(x) \leq 1$) we find that:

$$\cos^{2k}(x) \leq \cos^2(x) \& \sin^{2k}(x) \leq \sin^2(x) \tag{5.11}$$

The trigonometric solution of (5.10) as clustered system will take the form:

$$s' = \left\langle \begin{array}{c} \cos^{2k}(x) = \cos^2(x) \\ \sin^{2k}(x) = \sin^2(x) \end{array} \right\rangle \Rightarrow \left\langle \begin{array}{c} \cos^2(x)\left(\cos^{2k-2}(x) - 1\right) = 0 \\ \sin^2(x)\left(\sin^{2k-2}(x) - 1\right) = 0 \end{array} \right\rangle \Rightarrow \left\langle \begin{array}{c} \cos(x) = 0 \vee \cos(x) = \pm 1 \\ \sin(x) = 0 \vee \sin^2(x) = 1 \end{array} \right\rangle \Rightarrow$$

$$\Rightarrow \left\langle \begin{array}{c} \cos(x) = 0 \vee \cos(x) = \pm 1 \\ \sin(x) = 0 \vee \sin(x) = \pm 1 \end{array} \right\rangle \quad (5.12)$$

The $\mathbf{< s' >}$ **system** results in the solutions.

1. $(t \in Z, x = 2\pi t) \| \left(t \in Z, \left(x = -\frac{\pi}{2} + 2\pi t, x = \frac{\pi}{2} + 2\pi t\right)\right)$ (5.13)

2. $(t \in Z, x = 2\pi t + \pi) \| \left(t \in Z, \left(x = -\frac{\pi}{2} + 2\pi t, x = \frac{\pi}{2} + 2\pi t\right)\right)$ (5.14)

The only system solutions will be

Great results

$$\boxed{\begin{array}{l} 1. \sin(x) = 1, \cos(x) = 0 \Rightarrow b = 0 \text{ and } c = a \\ 2. \sin(x) = -1, \cos(x) = 0 \Rightarrow b = 0 \text{ and } c = -a \\ 3. \sin(x) = 0, \cos(x) = 1 \Rightarrow c = 0 \text{ and } b = a \\ 4. \sin(x) = 0, \cos(x) = -1 \Rightarrow c = 0 \text{ and } b = -a \end{array}} \quad (5.15)$$

From these 2 parts we can easily conclude that as set of solutions arises the $\mathrm{G_{nT}}$ which is:

$\mathrm{G_{nT}} = \{((\mathrm{a} = 0, \mathrm{c} = \mathrm{b} \text{ or } \mathrm{b} = 0, \mathrm{c} = \mathrm{a}) \mid \mathrm{n} = 2\mathrm{k}{+}1, (\mathrm{a} = 0, \mathrm{c} = \pm\mathrm{b} \text{ or } \mathrm{b} = 0, \mathrm{c} = \pm\mathrm{a}) \mid \mathrm{n} = 2\mathrm{k}, k > 1) \mid \mathrm{k} \in \mathrm{N}^+\}$

We finally proved that the sets of solutions $P_n \neq G_{nT}$, because the results of the solution in (Tables (5.9 & 5.15)) contradicts the hypothesis since $abc \neq 0$ and since for the variables apply $\{(a, b, c) \mid a, b, c \in N^+$. As we observe the proofs of **Theorems 3 and 9 are equivalent according to the results**. Also according to trigonometry. in Theorem 9, we do not accept that the terms $\sin(x)$ and $\cos(x)$ are simultaneously zero, which is known to be excluded trigonometrically. Summarizing we can accept that both forms of proof belong to the same **Method I**.

Method VI.

VI.1. Theorem 10. (Basic theorem of Proof).[6]

Let P_n be the set of Fermat triples and defined as $P_n = \{(x, y, z) \mid x, y, z, n > 2 \in N^+$ and an $x^n + y_n^n = z^n, xyz \neq 0\}$. Let $\mathrm{G_n}$ be the set defined as:

$$\mathrm{G_n} = \{((\mathrm{G_1} \mid \mathrm{n} = 2\mathrm{k} + 1) \;\text{ and }\; (\mathrm{G_2} \mid \mathrm{n} = 2\mathrm{k}, k > 1)) \mid \mathrm{k} \in \mathrm{N}^+\}$$

We need to prove that the sets $\mathrm{P_n} \neq \mathrm{G_n}$ and also $\mathrm{P_n} = \emptyset$.

VI.2. Part A.

The Diophantine equation $x^{2k} + y^{2k} = z^{2k}$ has no solution to the positive integers for $k > 1$, $k \in \mathrm{N}^+$.

Proof.

We bring the original equation $x^n + y^n = z^n$ and we put $n = 2 \cdot k$ where $k \in \mathrm{N}^+$and then $x^{2k} + y^{2k} = z^{2k}$ (1) which comes into the form $(x/y)^{2k} + 1 = (z/y)^{2k}$ after we divide by y, since $y \neq 0$. A basic effort to solve the equation can be done with one replacement of the original variables which is done:

If we call $(z/y)^k = m$ (2) and $(x/y)^k = m - l$ (3) then from (1) $\Rightarrow -2\mathrm{ml} + \mathrm{l}^2 + 1 = 0 \Rightarrow \mathrm{m} = \frac{\mathrm{l}^2+1}{2l}, m, l \in Q+$(4)

$\& m - l = \frac{1-l^2}{2l}, m, l \in Q+$ (5)

I. From the relation (2) we get ...

$$(z/y)^k = m => z^k = \frac{\left(1+l^2\right)}{2l} y^k \tag{6}$$

But then for the variable y we will have a relation of form $y = \left(1+l^2\right)^f \cdot \left(1-l^2\right)^t \cdot (2l)^s \cdot g$ (7), where $g = w \cdot q^2, l = p/q$ (8) where (p, q) relatively primes and $(w, p, q, f, s, t) \in Z^+$ Combining relations (6, 7, 8) we get the final relation,

$$z^k = \left(\frac{q^2+p^2}{(2p) \cdot q}\right) \left(\frac{q^2+p^2}{q^2}\right)^{f \cdot k} \cdot \left(\frac{q^2-p^2}{q^2}\right)^{t.k} \cdot ((2p)/q)^{k \cdot s} \cdot w^k \cdot q^{2k} \tag{9}$$

if we order each and every one term and equalize them i.e Powers with the Power of z (that is factorization) will have,

$$z^k = \left(q^2+p^2\right)^{f \cdot k+1} \cdot \left(q^2-p^2\right)^{t \cdot k} \cdot (2 \cdot p)^{k \cdot s-1} w^k \cdot q^{2k-2f \cdot k-2t \cdot k-k \cdot s-1} \tag{10}$$

From relation (10) by comparing the powers for all terms we will have the system,

$$\begin{aligned} &f \cdot k + 1 = t_1 \cdot k \\ &k \cdot s - 1 = t_2 \cdot k \\ &2 \cdot k - s \cdot k - 2 \cdot t \cdot k - 2 \cdot f \cdot k - 1 = t_3 \cdot k \end{aligned} \tag{11}$$

The solve of this system is,

$$\boxed{\begin{aligned} &\text{If } t, t_2 \in Z \wedge \\ &((a \in Z \wedge f = a \wedge t_1 = -1 + a \wedge k = -1) \vee (a \in Z \wedge f = a \wedge t_1 = 1 + a \wedge k = 1)) \wedge \\ &\wedge (t_3 = 2 - 2t - 2t_1 - t_2 \wedge s = -f + t_1 + t_2) \end{aligned}} \tag{12}$$

The solutions of the system (12) as we see are analytically

1. Because $n = 2k$ and $k = -1$ which means that $n = -2$ which is rejected because must $n > 0$
2. Because $n = 2k$ and $k = 1$ which means that $n = 2$ in this case the solution is known.

Therefore **the only solution that is accepted is n = 2.**

II. Also from the relation (3) we get ...

$$(x/y)^k = m - l => x^k = \frac{(1-l^2)}{2l} y^k \tag{13}$$

Similar to the variable y, we will have a form relation $y = (1+l^2)^f \cdot (1-l^2)^t \cdot (2l)^s \cdot g$ (14), where $g = w \cdot q^2, l = p/q$ (15) where (p.g) relatively primes and $(w, p, q, f, s, t) \in Z^+$.

By combining relations $(13, 14, 15)$ we get the relation,

$$x^k = \left(\frac{q^2 - p^2}{(2p) \cdot q}\right) \left(\frac{q^2 + p^2}{q^2}\right)^{f \cdot k} \cdot \left(\frac{q^2 - p^2}{q^2}\right)^{t \cdot k} \cdot ((2p)/q)^{k \cdot s} \cdot w^k \cdot q^{2k} \tag{16}$$

Doing factorization we come to form,

$$x^k = (q^2 + p^2)^{f \cdot k} \cdot (q^2 - p^2)^{t \cdot k + 1} \cdot (2 \cdot p)^{k \cdot s - 1} w^k \cdot q^{2k - 2f \cdot k - 2 \cdot t \cdot k - k \cdot s - 1} \tag{17}$$

From the relationship (17) comparing to the desirable powers for all terms we will have the system,

$$\begin{aligned} &\mathrm{t \cdot k + 1 = t_1 \cdot k} \\ &\mathrm{k \cdot s - 1 = t_2 \cdot k} \\ &\mathrm{2 \cdot k - s \cdot k - 2 \cdot t \cdot k - 2f \cdot k - 1 = t_3 \cdot k} \end{aligned} \tag{18}$$

The solve of this system is,

$$\boxed{\begin{aligned} &\mathrm{If\ f, t_2 \in Z \wedge} \\ &\mathrm{((a \in Z \wedge t = a \wedge t_1 = -1 + a \wedge k = -1) \vee (a \in Z \wedge t = a \wedge t_1 = 1 + a \wedge k = 1)) \wedge} \\ &\mathrm{\wedge (t_3 = 2 - 2f - 2t_1 - t_2 \wedge s = -t + t_1 + t_2)} \end{aligned}} \tag{19}$$

The specific solutions of the system(19) are two, as we see, are analytically.

1. Because $n = 2k$ and $k = -1$ which means that $n = -2$ which is rejected because must $n > 0$
2. Because $n = 2k$ and $k = 1$ which means that $n = 2$ in this case the solution is known.

Therefore **the only solution that is accepted is n = 2.**

VI.3. Part B.

The Diophantine equation $x^{2k+1} + y^{2k+1} = z^{2k+1}$ has no solution to the positive integers for $\mathbf{k \in N^+}$.

Proof.

We start from the original equation $x^n + y^n = z^n$ and we put $n = 2 \cdot k + 1$ where $k \in \mathrm{Z}^+$and then $x^{2k+1} + y^{2k+1} = z^{2k+1}$ (1*) which becomes at the form $(x/y)^{2k+1} + 1 = (z/y)^{2k+1}$ after we divide by y, since $y \neq 0$.

The effort to solve the equation can be done by replacing the primary variables as follows:

If we call $(\mathrm{z/y})^{2(\mathrm{k}+1/2)} = \mathrm{m}^2$ (2*) and $(\mathrm{x/y})^{2(\mathrm{k}+1/2)} = (\mathrm{m}-\mathrm{l})^2$ (3*) then from (1*) =>

$$-2ml + l^2 + 1 = 0 \Rightarrow m = \frac{l^2+1}{2l} \, (4^*) \, \& \, \mathrm{m} - \mathrm{l} = \frac{1-\mathrm{l}^2}{2 \,|}, \mathrm{m}, \mathrm{l} \in \mathrm{Q} + (5^*) \, .$$

I. From the relation (2*) we get...

$$\begin{aligned} &(z/y)^{2(k+1/2)} = m^2 \Rightarrow z^{2(k+1/2)} = \left(\frac{(1+l^2)}{2l}\right)^2 y^{2(k+1/2)} \Rightarrow \\ &\Rightarrow z^{(k+1/2)} = \left(\frac{(1+l^2)}{2l}\right) y^{(k+1/2)} \end{aligned} \tag{6*}$$

But then for the variable y we will have a relation of form, $y = \left(1+l^2\right)^f \cdot \left(1-l^2\right)^t \cdot (2l)^s \cdot g$ (7*) where $g = w \cdot q^2, l = p/q$ (8*) where (p, q) relatively primes and $(w, p, q, f, s, t) \in Z^+$ Combining relations $(6^*, 7^*, 8^*)$ we get the final relation,

$$z^{k+1/2} = \left(\frac{q^2+p^2}{(2p)\cdot q}\right) \left(\frac{q^2+p^2}{q^2}\right)^{f(k+1/2)} \cdot \left(\frac{q^2-p^2}{q^2}\right)^{t(k+1/2)} \cdot ((2p)/q)^{(k+1/2)\cdot s} \cdot w^{k+1/2} \cdot q^{2(k+1/2)} \tag{9*}$$

if we order each and every one term and equalize them Powers with the Power of z (that is factorization) will have,

$$z^{k+1/2} = \left(q^2+p^2\right)^{1+f(k+1/2)} \cdot \left(q^2-p^2\right)^{t\cdot(k+1/2)} \cdot (2\cdot p)^{-1+(k+1/2)\cdot 5} w^{k+1/2} \cdot q^{2(k+1/2)-2f(k+1/2)-2\cdot t(k+1/2)-(k+1/2)\cdot s-1} \tag{10*}$$

From the relationship (10*) comparing to the desirable powers for all terms we will have the system,

$$\begin{aligned} &\mathrm{f} \cdot (\mathrm{k}+1/2) + 1 = \mathrm{t}_1 \cdot (\mathrm{k}+1/2) \\ &(\mathrm{k}+1/2) \cdot \mathrm{s} - 1 = \mathrm{t}_2 \cdot (\mathrm{k}+1/2) \\ &2 \cdot (\mathrm{k}+1/2) - \mathrm{s} \cdot (\mathrm{k}+1/2) - 2\mathrm{t} \cdot (\mathrm{k}+1/2) - 2\mathrm{f} \cdot (\mathrm{k}+1/2) - 1 = \mathrm{t}_3 \cdot (\mathrm{k}+1/2) \end{aligned} \tag{11*}$$

We get the solve of the last system,

$$\boxed{\begin{aligned} &\text{If } t, t_2 \in Z \wedge \\ &((a \in Z \wedge f = a \wedge t_1 = -2 + a \wedge k = 0) \vee (a \in Z \wedge f = a \wedge t_1 = 2 + a \wedge k = -1)) \wedge \\ &\wedge (t_3 = 2 - 4f - 2t + 2t_1 - t_2 \wedge s = f - t_1 + t_2) \end{aligned}} \tag{12*}$$

The solutions of the system (12*) are analytical

1. Because n = 2k + 1 and k = −1 which means that n = −1 which is rejected because must n > 0
2. Because n = 2k + 1 and k = 0 which means that n = 1 in this case the solution is known.

Therefore **the only solution that is accepted is n = 1.**

II. Also from the relation (3*) we get

$$\begin{aligned}&(x/y)^{2k+1} = (m-l)^2 \Rightarrow x^{2(k+1/2)} = \left(\frac{(1-I^2)}{2l}\right)^2 y^{2(k+1/2)} \Rightarrow \\ &\Rightarrow x^{k+1/2} = \left(\frac{(1-I^2)}{2l}\right) y^{k+1/2}\end{aligned} \tag{13*}$$

Similar to the variable y, we will have a form relation $y = \left(1+l^2\right)^f \cdot \left(1-l^2\right)^t \cdot (2l)^s \cdot g$ (14*), where $g = w \cdot q^2, l = p/q$ (15*) where $(p,q))$ relatively primes and $(w,p,q,f,s,t) \in Z^+$ By combining relations $(13^*, 14^*, 15^*)$ we get the relation,

$$\mathrm{x}^{\mathrm{k}+1/2} = \left(\frac{q^2-\mathrm{p}^2}{(2\mathrm{p})\cdot\mathrm{q}}\right) \cdot \left(\frac{q^2+\mathrm{p}^2}{q^2}\right)^{f\cdot(k+1/2)} \cdot \left(\frac{q^2-p^2}{q^2}\right)^{t(k+1/2)} \cdot ((2p)/q)^{(k+1/2)\cdot s} \cdot w^{k+1/2} \cdot q^{2(k+1/2)} \tag{16*}$$

Doing factorization we come to form,

$$\mathrm{x}^{\mathrm{k}+1/2} = \left(\mathrm{q}^2+\mathrm{p}^2\right)^{\mathrm{f}(\mathrm{k}+1/2)} \cdot \left(\mathrm{q}^2-\mathrm{p}^2\right)^{1+\mathrm{t}(\mathrm{k}+1/2)} \cdot (2\cdot\mathrm{p})^{(k+1/2)s-1} \mathrm{w}^{\mathrm{k}+1/2} \cdot q^{2(k+1/2)-2\mathrm{f}(k+1/2)-2\cdot\mathrm{t}(k+1/2)-(k+1/2)\cdot s-1} \tag{17*}$$

From the relationship (17*) comparing to the desirable powers for all terms we will have the system,

$$\begin{aligned}&t\cdot(k+1/2)+1 = t_1\cdot(k+1/2) \\ &(k+1/2)\cdot s - 1 = t_2\cdot(k+1/2) \\ &2\cdot(k+1/2) - s\cdot(k+1/2) - 2t\cdot(k+1/2) - 2f\cdot(k+1/2) - 1 = t_3\cdot(k+1/2)\end{aligned} \tag{18*}$$

The solve of this last system is,

$$\boxed{\begin{aligned}&\text{If } \mathrm{f}, \mathrm{t}_2 \in \mathrm{Z} \wedge \\ &((\mathrm{a} \in \mathrm{Z} \wedge \mathrm{t} = \mathrm{a} \wedge \mathrm{t}_1 = -2+\mathrm{a} \wedge \mathrm{k} = -1) \vee (\mathrm{a} \in \mathrm{Z} \wedge \mathrm{t} = \mathrm{a} \wedge \mathrm{t}_1 = 2+\mathrm{a} \wedge \mathrm{k} = 0)) \wedge \\ &\wedge (\mathrm{t}_3 = 2 - 2\mathrm{f} - 2\mathrm{t}_1 - \mathrm{t}_2 \wedge \mathrm{s} = -\mathrm{t} + \mathrm{t}_1 + \mathrm{t}_2)\end{aligned}} \tag{19*}$$

The solutions of the system (19*) are analytically

1. Because n = 2k + 1 and k = −1 which means that n = −1 which is rejected because must n > 0
2. Because n = 2k + 1 and k = 0 which means that n = 1 in this case the solution is known.

Therefore **the only solution that is accepted is n = 1.**

If we assume as we proved on pages $1-3$ that G_1 and G_2 are the solutions for $n = 1$ and 2 of the general Fermat equation $a^n + b^n = c^n$. Therefore we have after the analysis we did: $G_n = \{((G_1 \mid n = 2k+1)$ and $(G_2 \mid n = 2k, k > 1)) \mid \mathrm{k} \in N^+\}$. This means that we proved that the sets $\mathrm{P_n} \neq \mathrm{G_n}$ and also $\mathrm{P_n} = \emptyset$ because $\mathrm{n} > 2 \in \mathrm{N}^+$ for $\mathrm{x^n} + \mathrm{y^n} = z^\mathrm{n}$, and should apply $\mathrm{xyz} \neq 0$.

Finally, after examining the two parts, it was proved that for Fermat's equation $x^n + y^n = z^n$ there is no solution in positive integers, for $\mathbf{n > 2, n \in N^+}$ and $x, y, z \neq 0$.

Epilogue

According to the methods developed, the first two methods satisfy the assumption that the solution set for the Fermat equation with $n > 2$ in positive integers is the empty set, because it turns out that at least one variable is equal to zero. Frey's 3 rd method for elliptic functions shows us that at least, one variable will necessarily be zero and therefore agrees with the hypothesis that there can be no solution. The 4th method follows from the condition that for the generalized equation $\mathrm{x^p} + \mathrm{y^q} = \mathrm{z^w}$, at least one exponent must be equal to 2 , and thus falls under the Pythagorean diophantine equation. Finally for the last 2, namely the trigonometric proof and the exponents equation method are two simulations, which otherwise prove that there is no solution, i.e. for the first one at least one variable must be equal to zero while the second one restricts the exponents to be equal to $n = 1$ or $n = 2$, which are known the solutions them, from Theorems 1 and 2.

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