Remarks on Birch and Swinnerton-Dyer Conjungture

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Abstract

These short remarks show deriviation of Birch and Swinnerton-Dyer conjungture. As a consequence new one resulting constant free equality of Birch and Swinnerton-Dyer conjungture proposed

We know, that Euler product in general equals to the infinitisimal sum. Threfore, for big enough N finite Euler product can be approximated as finite sum in the same way as follow:

$$\prod_{p<N} \frac{N_p}{p} = \prod_{p<N} P(p,1) = \prod_{p<N} \frac{1}{1 - \frac{1}{p}\frac{pN_p - p^2}{N_p}} \approx \sum_{n<N} \frac{1}{n}\frac{nN_n - n^2}{N_n} \tag{1}$$

where N_p is the number of points modulo p for a large number of primes p on elliptic curves whose rank r was known and N_n is the number of points modulo n for a large number of positive integers n on elliptic curves for the same rank r. On the other hand, the finite sum can be approximated as integral by using Euler–Maclaurin formula [1]:

$$\sum_{n<N} \frac{1}{n}\frac{nN_n - n^2}{N_n} \approx \int_{x=1}^{N} \frac{N_x - x}{N_x} dx \tag{2}$$

Let's deside, that $N_x = x/\left(1 - C\frac{r(\log x)^{r-1}}{x}\right)$ dependance is valid[1]. After inserting this expresion into the integral we obtain:

$$\int_{x=1}^{N} \frac{N_x - x}{N_x} dx = C\int_{x=1}^{N} \frac{r(\log x)^{r-1}}{x} dx = C(\log N)^r \tag{3}$$

Finally, expanding N_x near 0 and taking just first term, we can write for big enough N:

$$\prod_{p<N} \frac{N_p}{p} \approx \frac{N_N \log N}{r}$$

References

[1] Abramowitz, Milton; Stegun, Irene A., eds. (1972). *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables.* New York: Dover Publications. ISBN 978-0-486-61272-0. tenth printing., pp. 16, 806, 886

[1] It can be proved by numeric experiment