

Relation of accelerations in two inertial frames in Special Relativity theory

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ABSTRACT

In special relativity theory, we discover the relation of inertial frames' accelerations. In this theory, we can understand general state of the relation of inertial frames' accelerations

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I.Introduction

In special relativity theory, we discover the relation of inertial frames' accelerations.

At first, 4-dimension-Lorentz differential transformation is

$$d\vec{x} = d\vec{x}' + \gamma\vec{v}_0 dt' - (1-\gamma)\frac{\vec{v}_0 \cdot d\vec{x}'}{v_0^2}\vec{v}_0 \quad (1)$$

$$dt = \gamma(dt' + \frac{\vec{v}_0 \cdot d\vec{x}'}{c^2}) , \quad \gamma = 1/\sqrt{1-\frac{v_0^2}{c^2}} \quad (2)$$

If we rewrite Eq(1)-(2) in the other way,

$$\frac{d\vec{x}}{d\tau} = \frac{d\vec{x}'}{d\tau} + \gamma\vec{v}_0 \frac{dt'}{d\tau} - (1-\gamma)\frac{\vec{v}_0}{v_0^2} \cdot \frac{d\vec{x}'}{d\tau}\vec{v}_0 \quad (3)$$

$$dt = \gamma(dt' + \frac{\vec{v}_0 \cdot d\vec{x}'}{c^2}) = \gamma dt'(1 + \frac{\vec{v}_0 \cdot \vec{u}'}{c^2}), \quad \vec{V} = \frac{d\vec{x}}{dt}, \vec{u} = \frac{d\vec{x}'}{dt'} \quad (4)$$

2. The relation of inertial frames' accelerations

For calculating the acceleration,

$$\begin{aligned} \vec{a} &= \frac{d}{dt} \left(\frac{d\vec{x}}{d\tau} \right) = \frac{1}{\gamma(1 + \frac{\vec{v}_0 \cdot \vec{u}'}{c^2})} \left[\frac{d}{dt'} \left(\frac{d\vec{x}'}{d\tau} \right) + \gamma\vec{v}_0 \frac{d}{dt'} \left(\frac{dt'}{d\tau} \right) - (1-\gamma)\frac{\vec{v}_0}{v_0^2} \cdot \frac{d}{dt'} \left(\frac{d\vec{x}'}{d\tau} \right)\vec{v}_0 \right] \\ &= \frac{1}{\gamma(1 + \frac{\vec{v}_0 \cdot \vec{u}'}{c^2})} \left[\vec{a}' + \gamma\vec{v}_0 \frac{d}{dt'} \left(\frac{1}{\sqrt{1-\frac{u'^2}{c^2}}} \right) - (1-\gamma)\frac{\vec{v}_0 \cdot \vec{a}'}{v_0^2}\vec{v}_0 \right] \end{aligned} \quad (5)$$

In this time, the constant acceleration a_0' is

$$a_0' = \frac{d}{dt'} \left(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}} \right) \rightarrow u = \frac{a_0' t'}{\sqrt{1 + \frac{a_0'^2 t'^2}{c^2}}} \quad (6)$$

Hence,

$$\frac{d}{dt'} \left(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \right) = \frac{d}{dt'} \left(\sqrt{1 + \frac{a_0'^2 t'^2}{c^2}} \right) = \frac{a_0'}{c^2} \frac{a_0' t'}{\sqrt{1 + \frac{a_0'^2 t'^2}{c^2}}} = \frac{a_0'}{c^2} u \quad (7)$$

$$\vec{v}_0 \cdot \vec{u} = v_0 u \cos\theta' , \quad \vec{v}_0 \cdot \vec{a}_0' = v_0 a_0' \cos\theta' \quad (8)$$

Therefore, Eq(5) is

$$\begin{aligned} \vec{a} &= \frac{1}{\gamma(1 + \frac{\vec{v}_0 \cdot \vec{u}}{c^2})} [\vec{a}_0' + \gamma \vec{v}_0 \frac{a_0'}{c^2} u - (1-\gamma) \frac{\vec{v}_0 \cdot \vec{a}_0'}{v_0^2} \vec{v}_0] \\ &= \frac{1}{\gamma(1 + \frac{v_0 u \cos \theta'}{c^2})} [\vec{a}_0' + \gamma \vec{v}_0 \frac{a_0'}{c^2} u - (1-\gamma) \frac{v_0 a_0' \cos \theta'}{v_0^2} \vec{v}_0] \end{aligned} \quad (9)$$

If we calculate Eq(9),

$$\begin{aligned} a^2 &= \vec{a} \cdot \vec{a} = \frac{1}{\gamma^2(1 + \frac{v_0 u \cos \theta'}{c^2})^2} [a_0'^2 + \gamma^2 v_0^2 \frac{a_0'^2}{c^4} u^2 + (1-\gamma)^2 a_0'^2 \cos^2 \theta' \\ &\quad + 2a_0'^2 \gamma v_0 \cos \theta' \frac{u}{c^2} - 2\gamma v_0 \frac{a_0'}{c^2} u (1-\gamma) \cos \theta' - 2(1-\gamma) a_0'^2 \cos^2 \theta'] \\ &= \frac{1}{(1 + \frac{v_0 u \cos \theta'}{c^2})^2} a_0'^2 [\frac{1}{\gamma^2} + (\frac{v_0^2 u^2}{c^4} + 2 \frac{v_0 u}{c^2} \cos \theta' + \frac{v_0^2}{c^2} \cos^2 \theta')] \end{aligned} \quad (10)$$

Finally, the relation of inertial frames' accelerations is

$$a = \frac{\sqrt{(1 - v_0^2/c^2) \left[\frac{v_0^2 u^2}{c^4} + \frac{v_0 u}{c^2} \cos \theta' + \frac{v_0^2}{c^2} \cos^2 \theta' \right]}}{(1 + \frac{v_0 u \cos \theta'}{c^2})} a_0' \quad (11)$$

In special case, if $\theta' = 0$ in Eq(11), [1]

$$a = a_0' = \frac{d}{dt'} \left(\frac{u}{\sqrt{1 - \frac{u^2}{c^2}}} \right) = a_0 = \frac{d}{dt} \left(\frac{V}{\sqrt{1 - \frac{V^2}{c^2}}} \right), \quad \vec{V} = \frac{d\vec{x}}{dt}, \vec{u} = \frac{d\vec{x}'}{dt'} \quad (12)$$

Eq(11) is the general state of Eq(12).

According to the relativity principle, if we use $-v_0, V, \theta$ instead of v_0, u, θ' in Eq(11), we can obtain

the next relation of inertial frames' accelerations.

$$a' = \frac{\sqrt{(1 - v_0^2/c^2) \left[\frac{v_0^2 V^2}{c^4} - \frac{v_0 V}{c^2} \cos \theta + \frac{v_0^2}{c^2} \cos^2 \theta \right]}}{(1 - \frac{v_0 V \cos \theta}{c^2})} a_0 \quad (13)$$

In this time, the constant acceleration a_0 is

$$a_0 = \frac{d}{dt} \left(\frac{V}{\sqrt{1 - \frac{V^2}{c^2}}} \right), \quad \vec{v}_0 \cdot \vec{V} = v_0 V \cos \theta, \quad \vec{v}_0 \cdot \vec{a}_0 = v_0 a_0 \cos \theta \quad (14)$$

Additional special case, if $\theta = 0$ in Eq(13), [1]

$$a = a_0 = \frac{d}{dt} \left(\frac{V}{\sqrt{1 - \frac{V^2}{c^2}}} \right) = a_0' = \frac{d}{dt'} \left(\frac{u}{\sqrt{1 - \frac{u^2}{c^2}}} \right), \quad \vec{v} = \frac{d\vec{x}}{dt}, \quad \vec{u} = \frac{d\vec{x}'}{dt'} \quad (15)$$

3. Conclusion

We can understand general state of the relation of inertial frames' accelerations in this theory.

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