Doppler effect and the law of conservation of energy-momentum.

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Abstract: Using the law of conservation of energy-momentum and the postulate of the constancy of the speed of light, the dependences for the length of the electromagnetic wave with red and blue shift when the Doppler effect are derived. Moreover, using this approach, general formulas for the gravitational shift (red and blue) are derived, and the Compton effect is explained from these positions. It is also shown that for any redshift, galaxies will always have a speed lower than the speed of light in a vacuum. Taking the speed of light in vacuum, as the escape velocity for the visible part of the Universe, the mass of the Universe is calculated.

Keywords: The Doppler effect, the law of conservation of energy-momentum, red and blue shift, gravitational shift, the speed of galaxies, the mass of the Universe.

INTRODUCTION.

The Doppler effect is always there when there is a movement of the source of waves relative to the receiver of these waves [1, 2]. An environment should be taken into account only if waves move in it, for example, sound waves in air, or in water. But, for the movement of electromagnetic radiation, the medium is not needed, therefore, the analysis of the Doppler effect during the movement of photons is an analysis of the Doppler effect in a "pure form". We will see the very essence of the effect. Moreover, this "essence" is also the cause of the gravitational shift (both red and blue). This "essence" will explain the Compton effect (direct and reverse), and will make it possible to calculate the mass of the visible part of the Universe. Just amazing! It turns out that everything has one reason, and this reason is A. Einstein's STR and the law of conservation of energy.

But, before the analysis, we recall the Doppler effect. So, if we have a wave receiver and the source of these waves moves towards it, then the receiver will register a shorter wavelength than the source emitted (this is a blueshift, since the wavelength has decreased). If the wave source moves away from the receiver, the receiver will register a longer wavelength than the source emitted (this is a redshift, as the wavelength has increased). To derive general formulas for the Doppler effect (and gravitational shift), we need to take into account the energy-momentum conservation law [3 – 6] and the fact that photons move with speed light in a vacuum (naturally, we will consider the "pure" case in a vacuum). This is sufficient for a rigorous derivation of the formulas of the above effects. So, imagine the conclusion.

RESULTS AND DISCUSSION.

Let the photon source (reference frame 1) move from the photon receiver (reference frame 2) at a speed v. Consider the case when a photon with a frequency ν1 is emitted by a source. The photon energy will be:

$$E = h*\nu 1$$

The mass of the photon will be equal to:

$$E = h*\nu 1 = m0*c\hat{}2$$

$$m0 = h*\nu 1/c\hat{}2$$

We especially note that we can consider the photon mass m0 as an analog of the rest mass for elementary particles. But, when a photon is emitted by a source (which moves away from the receiver), we must also take into account the momentum that the photon will lose due to the movement of the source. This is obvious as the source moves from the receiver. The magnitude of this impulse is equal to:

$$p = m0*v$$

where

m0 - is the "rest mass of the photon",

v – is the speed of the photon source with respect to the receiver.

If the photon were a classical body (billiard ball, etc.), then, according to the law of conservation of momentum, such a photon-billiard ball would simply reduce its speed (more precisely, would receive an impulse in accordance with the law of conservation of momentum). But, a real photon cannot change its speed (neither increase nor decrease). Considering the fact that during emission the photon loses a part of the pulse (relative to the receiver), in accordance with the law of conservation of energy, the photon must reduce its energy. The photon simply has no choice, since it cannot reduce speed, and its energy-momentum has decreased. Therefore, the photon frequency decreases. Then the photon energy will be equal to:

$$E = h*\nu 2$$

where ν2 < ν1, since the photon source moves from the receiver, that is, the pulse reduces the photon energy.

Recall the relativistic law of conservation of energy-momentum.

$$E\hat{}2 = (m0\hat{}2*c\hat{}2) + p\hat{}2*c\hat{}2$$

Since the photon loses energy-momentum, we can strictly write:

$$(h*\nu 2)\hat{}2 = (h*\nu 1)\hat{}2 - (m0*v)\hat{}2 * c\hat{}2$$

where ν1 – photon frequency when emitted by a source,

ν2 — photon frequency upon registration by the receiver, v — is the speed of the photon source with respect to the receiver (movement from the receiver, redshift).

From this equation it is obvious that m0 is the "rest mass of the photon", and this is true and strict. A decrease in the frequency of the photon occurs because the photon cannot reduce its speed. For convenience, the equation can be rewritten for the wavelength, given the following relationships:

$$\nu 1 * \lambda 1 = c$$

$$\nu 2 * \lambda 2 = c$$

$$m0 = h*\nu 1/c^2$$

We get the ratio

$$\nu 2^2 = \nu 1^2 - \nu 1^2 * v^2/c^2$$

Therefore, we can write the relativistic law of conservation of energy-momentum for wavelengths.

$$1/(\lambda 2)^2 = 1/(\lambda 1)^2 - 1/(\lambda 1)^2 * v^2/c^2$$

Making elementary contractions, we obtain the equation:

$$\lambda 1^2 * c^2 = \lambda 2^2 * c^2 - \lambda 2^2 * v^2$$

Where do we get

$$\lambda 1^2 = \lambda 2^2 (1 - v^2/c^2)$$

$$\lambda 2^2 = \lambda 1^2 /(1 - v^2/c^2)$$

Finally, we obtain the expression for λ2.

$$\lambda 2 = \lambda 1/(1 — v^2/c^2)^{0.5}$$

In fact, we got a relativistic increase in the wavelength when the source moves from the receiver. As we know, the linear dimensions of objects decrease as they move toward the observer (relativistic effects of STR). Note that this equation is for redshift when $\lambda 2 > \lambda 1$.

From this, it is easy to obtain the redshift Δλ, since $\lambda 2= \lambda 1 + \Delta\lambda$.

$$\lambda 1 + \Delta\lambda = \lambda 1/(1 - v^2/c^2)^{0.5}$$

$$\Delta\lambda = \lambda 1 * (1/(1 — v^2/c^2)^{0.5} — 1)$$

This clearly shows that when the photons source moves at a speed close to the speed of light, the red shift will simply increase (to infinity). That is, we will never see galaxies that move at a speed greater than the speed of light. Moreover, it is easy to show that if the galaxy moved at the speed of light, then it could no longer emit photons. Such a galaxy turns into an analogue of a black hole, and it can no longer emit photons. This is fundamental and very easy to show. To do this, we write the equation.

$$(h*\nu 2)^2 = (h*\nu 1)^2 - (m0*v)^2 * c^2$$

where ν1 – is the photon frequency when emitted by a source,

$\nu 2$ — is the photon frequency upon registration by the receiver,

v — is the speed of the photon source with respect to the receiver (movement from the receiver, redshift).

If v = c, then the right side of the equation vanishes (remember that E = h*ν1 = m0*c^2):

$$(h*\nu 2)^2 = (h*\nu 1)^2 - (m0*v)^2 * c^2 = 0$$

that is, ν2 = 0, that is, a photon cannot radiate fundamentally.

Naturally, at high speeds beyond the speed of light, the galaxy will also not be able to radiate, for the above reason. Therefore, the tales that distant galaxies (for some fantastic reasons) can move at a speed greater than the speed of light must be forgotten. Claims that objects having a rest mass other than zero can move at speeds greater than the speed of light in a vacuum refer to fiction, not science. Just need to carefully read A. Einstein's STR.

Considering the fact that at redshift we obtained the relativistic equation for

$$\lambda 2 = \lambda 1/(1 — v^2/c^2)^{0.5}$$

By analogy, we can immediately write the equation for the wavelength at a blueshift. Naturally, this will be a relativistic decrease in the wavelength:

$$\lambda 2 = \lambda 1 * (1 — v^2/c^2)^{0.5}$$

where λ1 – is the wavelength when emitted by the source,

λ2 - wavelength upon registration by the receiver,

v is the speed of the photon source with respect to the receiver (movement toward the receiver, blueshift).

Note that the presented derivation of the formula for redshift was done to understand the essence of the process (we will need this when moving to the gravitational shift). In the general case, these formulas (for redshift and blueshifts) are displayed in several lines, using the Einstein STR and the energy-momentum conservation law.

For example, consider a blueshift. We write the corresponding equation.

$$(h*\nu 2)^2 = (h*\nu 1)^2 + p^2 * c^2$$

where ν1 – is the photon frequency when emitted by a source,

ν2 is the photon frequency upon registration by the receiver,

v is the speed of the photon source in relation to the receiver (movement to the receiver, blueshift),

p = m0*v, p — momentum.

With a blueshift, we have ν2 > ν1.

Therefore, in accordance with the law of conservation of energy-momentum, we can write the following equations.

$$h\nu_1 = m_0 c^2$$

$$h\nu_2 = m_0 c^2 / (1 - v^2/c^2)^{0.5}$$

Where do we get the equation:

$$h\nu_2 = h\nu_1 / (1 - v^2/c^2)^{0.5}$$

$$\nu_2 = \nu_1 / (1 - v^2/c^2)^{0.5}$$

Given that $\nu = c / \lambda$, it is easy to go to wavelengths.

$$\lambda_2 = \lambda_1 \cdot (1 - v^2/c^2)^{0.5}$$

That is, we got a relativistic decrease in wavelength, completely analogous to a relativistic decrease in linear dimensions.

From here it is also easy to obtain the value of the blueshift $\Delta\lambda$, since $\lambda_2 = \lambda_1 - \Delta\lambda$.

$$\lambda_1 - \Delta\lambda = \lambda_1 (1 - v^2/c^2)^{0.5}$$

$$\Delta\lambda = \lambda_1 \cdot (1 - (1 - v^2/c^2)^{0.5})$$

It is clearly seen that for $v = c$, we obtain the maximum blueshift for the inverse Compton effect ($\Delta\lambda = \lambda_1$). Since it is the electrons that transmit the energy-momentum to the photons, therefore, the energy of the photons increases (that is, the blueshift). The usual Compton effect can be considered as a redshift, since photons transfer energy-momentum to electrons, and therefore the photon energy decreases.

Similarly, we can consider the redshift. We write the corresponding equation.

$$(h\nu_2)^2 = (h\nu_1)^2 + p^2 c^2$$

where ν_1 – photon frequency when emitted by a source,

ν_2 is the photon frequency upon registration by the receiver,

v is the speed of the photon source in relation to the receiver (movement from the receiver, redshift),

$p = m_0 v$, p — momentum.

At redshift, we have $\nu_2 < \nu_1$.

Therefore, in accordance with the law of conservation of energy-momentum, we can write the following equations.

$$h\nu_2 = m_0 c^2$$

$$h\nu_1 = m_0 c^2 / (1 - v^2/c^2)^{0.5}$$

Where do we get the equation:

$$h\nu_1 = h\nu_2 / (1 - v^2/c^2)^{0.5}$$

$$\nu_2 = \nu_1 \cdot (1 - v^2/c^2)^{0.5}$$

Given that $\nu = c / \lambda$, it is easy to go to wavelengths.

$$\lambda_2 = \lambda_1 / (1 - v^2/c^2)^{0.5}$$

Using the data of the formula (Doppler effect) for the red and blueshift, it is easy to pass to the gravitational shift [7 - 11]. So, for redshift we have the formula:

$$\lambda 2 = \lambda 1 / (1 — v^2/c^2)^{0.5}$$

As we have already analyzed, the photon loses part of the energy-momentum, and therefore its wavelength increases. But, if we consider the radiation of a photon, for example from a star, then the photon must also overcome the attraction to this star. Therefore, such a photon will also lose part of the energy in accordance with the law of conservation of energy-momentum. The classical body (m0) must overcome the escape velocity (v) to overcome the attraction to the star. Here it is our favorite impulse that a photon should lose!

$$p = m0*v$$

Just for a photon we will take into account the "rest mass of a photon":

$$m0 = h*\nu 1/c^2$$

where ν1 — is the photon frequency during radiation.

Naturally, the momentum that the photon will lose when overcoming the attraction to the star will be equal to:

$$p = m0*v$$

where $m0 = h*\nu 1/c^2$,

v — is the escape velocity for a given star (or other space object).

Therefore, if we record the photon wavelength under ideal conditions (the gravitational potential is zero), then we will record an increase in the photon wavelength. The wavelength of the photon that has overcome the attraction of the star will be equal to:

$$\lambda 2 = \lambda 1 / (1 — v^2/c^2)^{0.5}$$

where λ1 — is the primary wavelength of the photon (photon wavelength upon emission),

λ2 - photon wavelength recorded by the observer (gravitational potential is zero),

v is the escape velocity for the star (or other space object).

If we will register this photon under conditions when the gravitational potential is not equal to zero, for example on planet Earth. That with this photon, with a wavelength of λ2, a blueshift will occur, in accordance with the gravitational potential of the planet. Then, when registering this photon, we get a wavelength λ3 equal to:

$$\lambda 3 = \lambda 2 * (1 — v2^2/c^2)^{0.5}$$

where v2 — is the escape velocity for the planet.

Note that the escape velocity is the velocity that must be given to the body at a certain distance from the star (planet, etc.) in order to bring this body out of the gravitational influence of the star [12]. Recall that the escape velocity for an object of mass M, at a distance r, from its center of mass, is equal to:

$$v^2 = 2*G*M/r$$

$$v = (2*G*M/r)^{0.5}$$

where G – is the gravitational constant.

Therefore, if you wish, from the above formulas with a escape velocity, you can easily go to the classical formulas of gravitational shift, with gravitational potentials at points. But, it seems to us that working with escape velocity is more visual and enjoyable, all the more it makes it possible to elementary calculate the mass of the visible Universe.

Consider the visible part of Universes with a radius h, mass M. Then, the escape velocity for such a Universe, at a distance h, will be equal to:

$$v^2 = 2*G*M / h$$

Naturally, for the visible part of the Universes, the escape velocity should be as large as possible. That is, numerically the escape velocity for the visible Universe will be equal to the speed of light in vacuum. Therefore, we can easily calculate the mass of the visible part of the Universes of radius h.

$$M = h*c^2 / (2*G)$$

Assuming h to be 13.8 billion light-years, we get the Universe mass equal to $8.825*10^{52}$ kilograms, which is in good agreement with modern estimates (10^{53} kg.) [13, 14].

$$M = h*c^2 / (2*G) = 1.308*10^{26} * 3*10^8 * 3*10^8 /(2 * 6.67*10^{(-11)})$$

$$M = 8.825 * 10^{52} \text{ kg.}$$

Note that when calculating the radius of the visible part of the Universe (h), we did not take into account the expansion of the Universe (the Big Bang theory), since the Big Bang theory contradicts A. Einstein's STR [15].

CONCLUSION.

Thus, it is shown that the reason for the change in the electromagnetic wavelength when the Doppler effect is a change in the photon momentum, which, according to the law of energy-momentum conservation, leads to a change in the photon energy (i.e., to a change in wavelength). A change in the photon momentum is also the cause of the gravitational shift (red and blue) and the Compton effect. This change in the photon momentum occurs due to the effects of STR A. Einstein. The mass of the visible part of the Universe is also calculated.

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