[1]

$e, \pi, \chi \cdots \alpha$?

LUCIAN M. IONESCU

Abstract. Feynman amplitudes are periods, and also coefficients of the QED partition function with a formal deformation parameter the fine structure constant α. Moreover, this truly fundamental mathematical constant is the ratio of magnetic (fluxon) vs. electric charge, as well as the grading of the decay lifetimes telling apart weak from strong "interactions".

On the other (Mathematical) hand e is the "inverse" of π, another deformation parameter (no ordinary period), as Euler's famous identity $exp(2\pi i) = 1$ suggests.

In a recent work, Atiyah related α and the Todd function. But Todd classes are inverses of Chern classes, suggesting further "clues" to look for conceptual relationships between these mathematical constants, in an attempt to catch a a Platonic and Exceptional Universe by the TOE.

Contents

[1]Dedicated to the memory of Sir Michael Atiyah.

Date: February 1, 2019.

1. Introduction

Feynman integrals are scattering amplitudes from the physics side, and algebraic integrals, called periods on the mathematical side [1, 2, 3, 4]. There are exact methods [5], which at least at tree-level obtain these periods as integrals on the moduli space [22], analogs of Veneziano-Virasoro amplitudes.

The class of periods extends algebraic numbers, and are naturally graded by the degree of π, as a signature of the real numbers $R = Q_\infty$ [7] (see Gauss-Bonnet vs. Poincare-Hopf Theorems), the completion of Q at the prime at infinity in the "wrong" direction, making it an amorphous / no grading number system [1]

In what follows the articles adopts a comparison approach, pondering about the various Math-Physics aspects involved, theories well developed and tested, as well as modern mathematical models used, in connection with the above four important "numbers" (they in fact are trademarks of notable theories, having them at their core), from the point of view of theory of periods, renormalization (Riemann-Hilbert Problem) and TQFTs from subfactors, as a few examples.

[1] In a Quantum Universe, the continuum is no friend of mine ... and luckily not needed.

Overall, the article aims to extract pertinent information from the recent "telegraphic communications" of Atiyah [8][1], while placing them in the context of the new paradigm of Quantum Physics: *Quantum Information Dynamics* and the *Qubit Model*.

What we are looking for here, is sensing "mental resonances" that please the researcher, invitingly, to search and research deeper and in detail [2]. If the digressions seam out of place to the reader (memos to one self), please skip ahead ...

It is comforting to find out ([9], p.1) that all great mathematicians, e.g. Weil [10], valued " dim analogies" since they "... are essential to research and discovery"[3]

These are of course "sugestions" for investigations, and not definite claims (sort of "food for thought"); and to apologize one more time for being imprecise as a rule, the author's opinion is that we need more such brainstorming "craizy" claims[4], than comforting numerology coincidences leading nowhere conceptually. And if what is claimed helps, it is a step forward; if the reader understands it's wrong, it is another step forward; being silent about the difficult problems, yet crucial for our progress, fearing we might be "not even right", is counter-productive and takes us nowhere [5]...

2. Periods everywhere!

That is in mathematics and Physics (and chemistry, and astronomy, and in your Google calendar[6] ... yes, everywhere!).

2.1. e and π: inverse periods?

Moreover $2\pi i$ is the period of the fundamental class of $C^\times = C - \{0\}$ over the basic homology cycle S^1 (see [11], p.63; [12], p.67):

$$P = 2\pi i = \int_S^1 dz/z, \qquad (Fundamental\ Period: \ Log\ \text{"}value\text{"}).$$

It is represented as a product of other more elementary periods 2π, as an Archimedian measure of the circumference of the unit circle [7], and the algebraic number i, whether as a rotation matrix $\begin{bmatrix}0 & -1/1 & 0\end{bmatrix}$ (representation theory), or via the standard formal approach $\hat{x} \in Z[x]/(x^2+1)$, reflecting the historical development of number systems (See the Annex "Is π wrong?" for additional comments on π).

[2]Looking for final answers? "Lasciate ogni speranza, voi ch'entrate ..." the "right" questions are targeted here!

[3]The converse may not hold!.

[4]Are these "crazy enough to eventually work"!?

[5]Play to win, not not to loose!

[6]From finite and algebraic to pro-finite and transcendental

[7]Conform Archimedes computation, it is in the analytic closure of the maximal cyclotomic extension, hence in Q^{Ab}; moreover in the *constructible numbers subfield* of Gauss-Wantzel extensions, involving only Fermat primes! ($F_{-\infty} = 2$ [13], p.22; TB-continued elsewhere ...)

Now *log* as a path integral (Integral Calculus) is more fundamental in the theory of periods, then the power series approach (Differential Calculus), exhibiting $e^z = \sum z^n/n!$ as an eigenfunction of d/dz [8]

While $\log \alpha$ can be represented as a period corresponding to the divisor $D = 1, \alpha$ [12], p.67), its definition via integration is also used in Calculus I courses for defining the natural logarithm, and hence its natural basis e.

Indeed, building the Riemann surface to represent the path integral (multi-valued function) as the inverse of a branched covering map, yields the exponential (Picard approach for solving DE, like the above eigenfunction problem: $d/dz f(z) = f(z)$):

$$exp : (C, +) \to (C^\times, \cdot).$$

Then it is not surprising that exponentiating this 1x1 matrix $[P]$ of the period isomorphism of this Riemann surface yields 1 [9]:

$$Euler's\ relation: \qquad exp(\int_S^1 dz/z) = 1.$$

So, we *could* say that $e = Exp(1)$ is the "inverse period" of $P = 2\pi i$, in the sense that "it is determined by the period isomorphism in the path integral framework", and not multiplicatively, as a real number, in the framework of number systems "a la" Cauchy [10].

2.2. α **as a grading.** What we call "alpha" is context depending, with a good amount of correlation ... The basis for various studies and interpretations, whether it's numerology [14], conceptual [15] or philosophical [16], is provided by the following *definition*:

$$\alpha = e^2/hc,$$

and then the argumentation may proceed in quite many different directions: honestly, we believe we don't have a real "clue" ... or do we [8]!

Initially the *fine structure constant* was introduced by Sommerfeld, Pauli's advisor [17], in the process of computing the fine structure of the spectral lines of the Hydrogen [18].

It rightfully became Pauli's obsession, as Feynman later agreed as its explanation is the most important problem in Physics [11].

In the QED ($U(1)$-gauge theory) partition function α plays the role of a grading. When comparing with experimental amplitudes (compare with power series representations of meromorphic functions), it acquires a "magnitude" $\alpha \approx 1/137$.

[8] It is also the generating function of the groupoid of Sets: $\sum_{[n] \in Spec(Sets)} 1/|Aut([n])z^n$ [19].

[9] ... except for the expert, who definitely knows better who to explain this!

[10] Will you!? Note: p-adic numbers are pro-algebraic: Deformation Theory [20], it's not a "Cauchy story" ... and there $2\pi i = 1$!

[11] ... and will be solved only when the "other" most importamnt problem, the elementary particles *mass spectrum* will be solved.

2.3. α **as a running constant.** One objection for being fundamental is that it is a "running constant" [15]; but this really depends on the renormalization approach to QED, or the other (renormalizable) Gauge field Theories within the Standard Model.

By now we know there are other ways around to compute scattering amplitudes [5], related to the so called associahedron [21, 22] etc.

Alternatively, the Motonen-Olive duality [23, 24] in the 1970s, following the work [25], led to S-duality between QFTs which relates weak and strong couplings, providing exact methods for developing the theory (see more recent work by Witten a.a. [26] [12]).

But there is a deeper conceptual level of understanding it, since computationally speaking α seems to play a similar role to the residue of the pole $z = 1$ for an $L-function$, and again, recall the BCFW recursion method using an analog method for computing MHV amplitudes [5].

2.4. α **as an exact value.** In [27] p.4, the author rightfully treats the fine structure constant as a "physics constant", asking (implicitly) what it means to be exact.

Recall that QED can compute the anomalous moment of the electron in *terms of the QED coupling constant* α_{QED}, and compering with the experimental value to asses a value for α. It is consistent with a measurement via quantum Hall effect, with of course a theoretical model for that.

Wigner and physicists in general, were astounded by the "unreasonable effectiveness of mathematics" to explain reality; but this was at "phylosophical level" ...

Recently this "effectiveness" starts to aquire a more "literal" value: "Is really an Informational System", "The Universe is a Quantum Computer" etc.

In contrast, as a fact, the new 2019 redefinition of the *International System of Units* [28] adopts *exact values* for the Planck constant, speed of light and electric charge.

$$h = 6.62607015 \times 10^{34}\ kg \cdot m^2 \cdot s^{-1},\ c = 299792458\ m \cdot s^1,\ e = 1.60217663410^{19}\ A \cdot s.$$

This of course is meant to fix the macro units we use in terms of the fundamental ones, but shows a tendency to view the above "constannts" as indeed mathematical; what we measure is in fact what a "meter" is, what a "second" is etc.

This point of view implies that the relation between the above basic constants h, c, e is consistent with the overall "culture" regarding the fine structure constant: it is a running constant in renormalizable QFTs, but "one day", when we'll figure all out (about this particular, yet important aspect), would turn out to be a mathematical constant!

2.5. α **and Hodge structure.** The magnetic charge usually refers to Dirac's monopole, as an analog of the electric charge. yet magnetism (yes, a "relativistic effect"),

[12]Note the tight connection with geometric Langlands programme, hence with periods motives via Weil's "turntable" and Grothendieck work on anabelian geometry [29], p. 1010-04.

is "3D" theory (think magnetic vector potential) [13], and the correct concept is the *fluxon*, defined by a line integral (closed period).

In terms of quantum Hall effect, the fine structure constant can be "split" as follows:

$$g_M = h/e, g_E = e/c, \quad g_E/g_m = e^2/hc = \alpha.$$

Since electric and magnetic permitivities ϵ and μ are a simplified version of the full duality tensor (E, B, H, D etc.), i.e. a Hodge duality, the "other" deformation parameter $1/c = \sqrt{\epsilon_0 \cdot \mu}$ [14], plays the role of an *external manifestation* of the ratio between "electric" and "magnetic" aspects of the full $SU(2)$-theory (Quantum Information Dynamics).

2.6. α **and S-duality.** Alternatively, as mentioned briefly above, the electric-magnetic duality understood as a symmetry between electric and magnetic charge, was generalized as Motonen-Olive duality and led to a reciprocity between the weak and strong coupling constants [24, 25, 29, 26] we now call S-duality.

It is interesting, in view of the above Hodge structure/duality discussion, to ponder on the role of the combined complex coupling constant used to develop the GNO-idea of a dual group and MO-duality [30], p.2:

$$\tau = \theta/A(S^1) + i1/fracg^2 A(S^2), \quad A(S^1) = 2\pi, A(S^2) = 4\pi,$$

when the S-duality is extended with the introduction of a "vacuum angle" θ.

To the author the above relation suggests a deeper reason for relating "electric" and "magnetic" Lie generators: the mechanics of the point particle S^0 was extended to the dynamics of a string S^1, and later to D-branes, but the only other "extended objetc" needed is the qubit $SU(2)$, with its Bloch sphere S^2 as a "closed brane", part of the Hopf fibration $S^1 \to S^3 \to S^2$, instrumental in the theory of solitons, skyrmions and of course, Quantum Computing.

Then Poincare / Hodge duality seams to be the way to relate the physics of strings and baryons.

3. Internal vs. External: what is mass?

This is relevant here, since the fine structure constant grades also the decay lifetimes of the elementary particles, whether in "strong" or "weak" interactions [31], as separated historically in the course of development of the Standard Model [32], Ch.2 (see §3.3 for additional details); hence the structure of the mass spectrum and

[13]... and we *know* that EM cannot be practically explained without extending $U(1) \to SU(2)$, as in Tesla work, torsion waves, a.k.a. scalar waves [33] etc, as the Electro-Weak Theory starts to catch up at a tentative fundamental level.

[14]Lorentz group may be associated to a deformation of the Galilei group; or c can be viewed as "central charge" of quaternions, as the infinitesimal deformation of the Lie algebra $(R^3, \times)$.

its stability theory is directly related to the fine structure constant, as "The Master Constant" e^2/hc [15].

Mechanical aspects (energy-momentum etc., space-time dependent), are referred to as "external"; quantum numbers refer to *internal* aspects (Wigner modeled them via irreducible representations). The hint towards unification is implicit in Einstein's relation $(E/c)^2 = p^2 + (m_0 c)^2$, and requires an understanding of what mass really is: a measure of the amount of structure, e.g. representation of a group of symmetry, consisting of the internal states.

How the electro-magnetism (as an $SU(2)$, or better $SL_2(C)$ theory, to include boosts etc.) are separated at "low energies", is encoded in α, as a Hodge structure and its associated theory of the period matrix ... When a resolution is used to represent the structure, in the sense of deformation theory (Kodaira etc .), it should exhibit the formal series in this deformation parameter, a mixture of h and $1/c$ ... somehow!

Note that, looking at the generalized momentum $P = mv + eA$, mass m is internal, and e external, yet they cross-couple to the corresponding flows.

Alternatively one can "split" the fine structure constant as

$$\alpha = h/e^2 \cdot c, \quad \sigma_{Hall} = \alpha \frac{1}{c},$$

where e^2/h is the quantum Hall conductivity ("internal"?), while c corresponds to the the quantum information transfer "externally" (explaining why any "classical computation" is limited in its rate of change) ...

3.1. **What about χ?** The Euler characteristic measures the "ratio" between relations and degrees of freedom in an exact sequence, i.e. when we "resolve" a structure into its generators and relations. For example, in Poincare-Hopf Theorem a vector field has sources and sinks, as *constraints* regarding how the flow may branch-out, otherwise *freely* (think a traffic problem on a network instead). In Gauss-Bonnet, intuitively the geodesics (or whatever flows) curve to close "freely", except for the topological relation of the ambient manifold.

What and how much of the above "imagery" carries-over to an $SU(2)$-quantum information flow, which when split into $U(1) \to SU(2)$ (rather considering the Hopf bundle), yields at the level of a generating function, the fine structure constant (Hodge structure with two deformation parameters), as a grading parameter? [16]

3.2. **... and Todd classes.** Now classification of vector bundles is governed by Chern classes. Dirac monopoles as instantons are controlled in this way [34].

[15] It is the 1:3 quark strength ratio, if we count the leptons as a 4-th color, following Barut [35]; R, G, B are of course the 3D frame basis of a baryon ... at least in the Qubit Model [36]

[16] The first step would be to try to make this statement precise; the second: to succeed; 3rd: conjecture; 4th: prove!

But here we advocate that magnetic charges are periods for a different type of divisor then electric charges (Gauss Theorem).

Todd classes are "inverse" to Chern classes perhaps similar to how $2\pi i$ is inverse to e for $C^\times$, or more general Riemann Surfaces (more to be said later on §4.3).

But one would need additional "room": 3D Thorston's classification of manifolds built out of trinions, or an $SU(2)$-network version of Chen Classes ... ?

3.3. **Weak or Strong interactions: what's the difference!?** Another objection for α not being "The" fundamental coupling constant is the argument that there are "other" such coupling constants, overunity in the Strong Interaction, hence "proving" that "worst things can happen" (not just energy dependence: being "variable constants").

The history of the Standard Model (see [32] Ch.2), shows clearly that physicists did not accept quarks as elementary particles, in spite of the success of the quark model in classifying *reasonably* (at that stage of overflow of new particles discovered: "there should be a fine ... " not a Nobel prize ...). Heisenberg (the "Master of us all" in physics [110]) new it does not make sense to talk about "particles" between I/O of a process, advocating the S-matrix approach: Quantum Computing, before the classical era of computing really started!

In brief, the separation of interactions as "weak" vs. "strong" comes from phenomenology of the lifetimes, where two quite distinct regions exist: 10^{-10} decays (roughly) vs. 10^{-22} resonances.

The gauge paradigm was an easy solution [17], already well developed: adapt the gauge group! Yet even for weak decays, the distances involved rule out the idea of space or time, especially after Einstein's implementation and confirmation of the philosophical lesson given by Mach, and after Heisenberg's Uncertainty relations, proving that "space-time indeed does not physically exist in Quantum Physics; it's not that " "we" can't measure them" ...

The internal quantum states *are* in fact modeled as Wigner pointed out representations in an 'internal space"; together with Weinberg's approach it is the Point Form of Quantum Mechanics (still used to this very day; usually avoiding principle bundles and monodromy etc. [34]).

Of course String Theory maps "internal" to "external", a la Kaluza-Klein, and, in view of Commutative Algebra and Sheaf Theory, or Gelfand-Neimark-Segall Theorem, one can go, *in principle*, back-and-forth between internal and external ...

Yet the new milenium's paradigm *is* Quantum Information Processing / Computing / Topologically etc., and it relies on Category Theory language (Turaev's Graphical Calculus generalizes Feynman's, and SM's quark line diagramatic approach).

[17]... at first! having to confine quarks turned harder than initially estimated! Introduction of color points again that three quarks form a qubit frame etc.

Returning to the main point to be developed elsewhere [36], the concept of "force" (implicitly appealing to energy-momentum and time-position), is adequate only for a QED gauge theory approach, to be extended as Electro-Weak Theory just because the "correct" gauge group is $U(1) \to U(2)$ (filtration; in fact it's the Hopf qubit / quaternionic bundle).

The weak decays, involving quark flavor transitions, change the "Klein geometry" of the baryons (finite "vertical gauge groups"), corresponding somehow to crystalographic groups (including binary Platonic, with their relation to exceptional Lie groups, already observed to be instrumental in TOEs as an alternative to GUts appraoch to go beyond SM).

The "strong decays" are indeed resonances of quantum networks, and can be thought of as discrete "vibrations", e.g. Wytoffian operations [37], i.e. changes of nodes, lines etc. (homological complex and its periods), in a similar way drum modes are used in Schrodinger's orbital theory.

In both cases, no "force" is adequate for their description. The final argument will be a deduction of the mass spectrum out of the corresponding group theory, once the concept of mass is well established, as hinted above.

3.4. **Alpha grading Mass and Time.** Back to the role of the "fine structure constant" $g_E/g_M \approx 137$: related to the $3:1$ dimensions ratio of Hopf qubit bundle (somehow), phenomenological it is related to a fundamental unit of mass and a grading of the time scale.

3.4.1. *The Unit of Mass: amount of structure.* The $70MeV's$, either as fraction of m_{mu} or m_e, the muon or electron mass, was advocated by several researchers: Nambu, Barut, Mac Gregor, Palazzi, Varlamov ... [38]. How mass is associated to internal symmetries remains to be understood, yet there are already several "clues" this is related to the above α (TBA).

An earlier attempt of the author involved primes (finite irreducible strings, or rather finite cyclotomic quantum phase) and Riemann zeros, as a dual spectrum [39], p.19, [13], p.6-8, 22, [40]. It recently became clear that 2D and 3D finite groups lurk underneath this idea [36, 41], as geometry of baryons.

3.4.2. *The Time Scale Grading.* Elementary particles, or rather the associated decays[18], cluster according to their lifetimes, as analyzed by MacGregor [31].

Note that lifetimes $\tau = \hbar/\Gamma$ are inverses of mass-widths Γ; for example for the *charged pion*[19], $\tau_\pi = 2.6033 \times 10^{-8}$ sec. corresponds to an energy band width of $\Gamma = 2.528 \times 10^8 eV$.

[18]Per Heisenberg's principle.

[19]Quarkonium states are a totally different "ball-game".

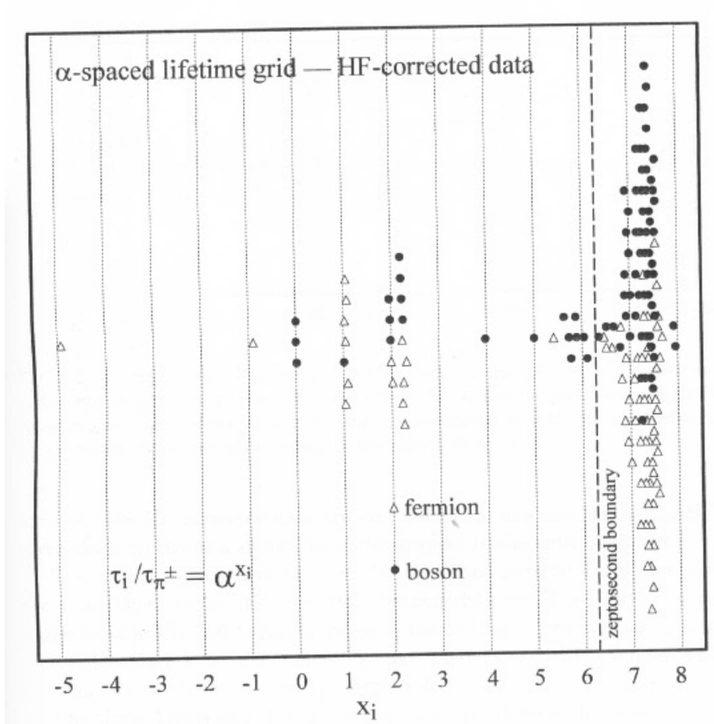

FIGURE 1. α-quantization of lifetime of particle (Fig.2.6.2 loc. cit.)

In $U(1)$-electronics at low energies "RLC-Theory", this corresponds to the quality factor of the corresponding resonant circuit; a parallel here as "infotronics" [42] is of conceptual value for understanding the "internal theory" (again, "forces" are not adequate at this point; rather it is Harmonic Analysis).

Fig. 1 reproduces here Fig. 2.6.2 [31], showing the α-grading of lifetime τ, relative to the muon's lifetime taken as the base time-unit: $\xi = \tau/\tau_\pi$. As explained in loc. cit. *"The α-quantization of the 36 threshold-states particles to the left of the zeptosecond boundary"* (weak interaction, not the "strong" sector), *"... extends over 11 powers of α, or 23 orders of magnitude, with no "rogue" particle lifetime in evidence."*.

The fact that the pions can be used as "ground level" (mass and lifetime) is an indication of the existence of a "tensor form $\otimes$" of the Balmer Law for the spectrum of Hydrogen: $E(n \to m) = Rydberg(1/n - 1/m)$ [111], where the "quark flavor" transitions correspond to qubit Klein geometries (and their $\Gamma \to SU_2$ and SU_3 finite subgroups representations theory), in a similar way the crude theory of principal quantum number n labeling electronic shells enter in the above formula.

The weak decays lifetimes $\alpha^x, x = -1..6$ probably correspond to the genus of a Riemann surface blow-up of the Klein singularity of the corresponding finite subgroup [112], with a grading similar to the loop order of Feynman diagrams in QED.

As mentioned upfront in the introduction, these are of course "sugestions" for investigations, and not definite claims (sort of "food for thought").

3.5. **What about Gravity!?** ... and of course "Gravity" is *not* fundamental, and it can be derived from the Electro-Weak Theory (fractional complex charges), if an expert would take the hints from [43] seriously.

4. Todd map and more chirilic leters: Ж ("Zhe") and Ч ("Che")

In [8] Atiyah is making use of the Todd map $T = t_-^{-1} \circ t_+$ (p.5) and Hirzebruch formalism (p.3)[20], involving the Functional Analysis of von Neuman algebras, within the framework of renormalization.

In view of Connes and Kreimer work on renormalization [44], renormalization is "just" a Riemann-Hilbert Problem, solved via *Birkchoff's decomposition* of the type apparent in the definition of Todd map. The docomposition is infact based on a graded Hopf algebra structure, as a solution of a convolution equation (Mautrer-cartan?), using of course the anti-pode (inverting Green's kernel?).

The grading of Connes-Kreimer Hopf algebra of rooted tree is essential, guarantying the existence of an antipode the moment a bialgebra structure is present (and hence one can write the basic equation of deformation quantization: Maurer-cartan Equation; see [45, 46]).

Remark 4.1. It is interesting that the prime spectrum $Spec(Z)$ has a POSet structure corresponding to the Hopf algebra of rooted trees (foundational structure for hierarchy in any theory), while being dual to the "other" primordial God given structure: the zeros of the Riemann Zeta Function ζ, or better, the poles of the *fermionic* Zeta Function $\zeta_- = DT(\mu) = 1/\zeta(s)$, the Dirichlet (Discrete Melin) transform of the Mobius function [40, 47, 48].

Now von-Neumann algebras (factors) are instrumental in building TQFTs [49], a categorical version of Feynman's diagramatic approach, via a graphical calculus (see Turaev's work; and Reshetikhin's).

Thus the technical discours revolves in the same circle of *ideas* ... Is it "la meme Jeanette autremaen ... categorifier"!?

4.1. **On Hirzebruch formalism.** Here we just "puzzle" upon some similarities, for a lack of expertise, and the time to acquire it [21]

The use by Hirzebruch of "formal algebraic process of multiplicative sequences" is reminiscent of formal groups in the spirit of Tate, Liubin etc., which are a generalization of Lie Theory as an early stage of Deformation Theory.

[20] Which are *not* the author's comfort zone ...

[21] We must publish or perish, right? So, not too "idle" philosophical research for now: we must retire first!

The whole"adelic theory" is in fact part of the Deformation Theory of finite fields (p-adic numbers) [20], in the context of the Algebraic Quantum Group of Rational Numbers Q [50, 51].

Since (loc. cit.) "Todd exponential, whose generating function is the Bernoulli function ..." places this in the context of a the inverse of a discrete derivative $De^x = (e^x - e^0)/x$, which can lead in so many directions: Riemann branching covers, algebraic ramification theory, Drinfeld' associator and ZK-equations etc.

4.2. **Чas the Todd map inverse of π.** And now, why the Todd inverse of π, denoted by Ж$=T(\pi)$, would be a renormalized version, hence a possible series in *alpha*, or even the fine structure itself? (Think, maybe, of Teichmuler digits as an example of re-coordinatizing formal deformations).

The Todd map $T = t_+ \circ t_-$ is defined as a change of coordinates (loc. cit. §3.4), reminiscent of a gluing of two open charts, the two embeddings of the center $Z(A)$ of the hyperfactor $A = \otimes_N M_2(C)$ [22]. It also appears as a Birkhoff decomposition as in the process of renormalization (Riemann-Hilbert Problem; the grading via the Hopf algebra makes gives a direct solution via the unique antipode, once the bialgebra structure is specified).

But while π, or rather the fundamental period $2\pi i = \int_S^1 dz/z$, has a clear algebraic-geometric conceptual meaning in the context of the "Hodge structure" $Z[i] \to C$ (or better embedding into CP^1, to view the residue as extracting a finite part of the real integral on the "real equator" gluing the two half-spheres of the Riemann sphere), the author can't say the same about $T(\pi)$: what is the analogous "extraction" / residue, a "non-commutative period" (algebraic cohomological duality pairing in the contex of hyperfatcors; the quantum dimension is a trace of the identity I_A ...)?

Claiming Ж$=T(\pi)$ as a renormalized value of the Euclidean circle half circumference (as explained above, on the analysis side, not topological), suggests such a transfer from CP^1 to some modular tensor category (see [49, 52]).

On the other hand, one could look at a different Math-habitat for such a non-commutative fundamental period, analog of π: could be an adelic Veneziano amplitude:

$$Jacobi\ sum: J(c, c') = g(c)g(c')/g(cc'), \quad g(c) = gauss\ sum,$$

as p-adic analogues of Euler's Gamma and Beta integrals. These are directly related to periods on moduli spaces and the associahedron [22].

Remark 4.2. Note "en passant", the "correct" zeta function can be introduced via Melin transform of the Gaussian distribution and Fourier eigenfunction $e^{-x^2/2!}$ (see Garrett [53]), which exhibits the $\sqrt(\pi) = \Gamma(1/2)$ together with $\zeta = DT(1)$.

[22]Habitat: braided tensor category $SU(2) - Mod$.

The actual calculation of Жand Чwith its heavy use of base 2 (F_2), makes one think of some amazing coincidences involving Bernoulli numbers and regular primes, with counting binary digits ... maybe even with Galois group ring elements like Stickelberger's, instrumental in the study of Gauss sums and their arguments:

$$g(\xi_{\mathcal{P}})^n = \mathcal{P}^\theta, \quad \theta = \sum_{Z/p^\times} a\sigma_a^{-1}.$$

4.2.1. *Euler's constant.* On the other hand the use of Euler's γ constant to define Ч$=T(\pi)$ as the Todd inverse of the "Euclidean coupling constant" (loc. cit. p.2), and "Abel integral formula difference of logarithms" (continuous and discrete), cycles back to the same larger context of adelic mathematics and ramification (algebraic vs. geometric) theory ...

Finally, the equality

$$\frac{T(\gamma)}{\gamma} = \frac{T(\pi}{\pi} = \cdots \frac{dg}{g}$$

exhibits the pattern of a *gauge term* $g^{-1}dq$ (see e.g. [57], p.7):

$$A \mapsto g^{-1}Ag + g^{-1}dg, \quad g : M \to G,$$

in perfect "resonant harmony" with the other voices of this polymath composition of Sir Atiyah ... The toy example is the 1-form dz/z and the associated period $frm{-}e\pi i$.

Now Feynman Path Integrals in a gauge theory (Chern-Simons Theory, e.g. loc. cit.) can be computed via *equivariant localization* (Duistermaat-Heckman principle [54]), as a sum of indexes (residues) at finite fixed points of a vector field: it's "just" an application of the equivariant integration formula of Atiyah-Bott (1984) [55] for the stationary phase approximation. Then, presented in this way, are these still genuine periods, or a pro-algebraic generalization? And in what sense the Todd map gives such a renormalized ("extracting the finite") value of a classical period (via some deformation quantization?)? And what is this $T(\pi)/\pi$ "in general", for other periods? Partial "clues" will be collected in §5.

4.2.2. *... and Chern classes.* But if a traditional cohomological pairing of d and $\int$ is reasonably clear to understandt, what is the "other" non-linear operator besides T? Maybe the *Chern classes*? From [113] we learn that "Todd class acts like a reciprocal of a Chern class, or stands in relation to it as a conormal bundle does to a normal bundle.

The Todd class plays a fundamental role in generalizing the classical Riemann-Roch theorem to higher dimensions, in the Hirzebruch-Riemann-Roch theorem and the Grothendieck-Hirzebruch-Riemann-Roch theorem."

4.3. **Todd and Chern Classes: "inverse" to one another ...** But these "mental resonances" are too volatile (short lived, perhaps), to be reliable in chasing α ...

The analysis of how e is the inverse of the "periods coupling constant" π (uniformizer), in the sense of a larger theory (Riemann branching covers), while reciprocal in the analytic realm (EulerвЃЄs formula in "char 0"), justifies perhaps to look at how similarly, the Todd and Chern classes (Principal and associated bundles realm of gauge theories) are *dual* with "global period" Ξ the Euler-Poincare characteristic in the "discrete" (graded) case of formal groups (Lie exponential vs. Riemann Surface, e.g. log and exp : $(C, +) -> (C*, .)$ etc.).

While searching for further connections, one may think of the algebraic realm of number fields (and Hirzebruch Q algebras), in parallel with the p-adic version of the theory (Weyl Conjectures, where the Riemann Hypothesis was proved), since such "numbers" are in fact deformations of finite fields (from the trivial formal series $F_p[x]$ to Z_p, together with their unique Galois extensions) in the "right" direction of the carry-over 2-cocycle; think of finite fields as tangent bundles / Lie objects, and Teichmuller-Witt vectors as 1-parameter formal groups (dynamics).

Then, what are Todd-Chern classes here? What convolution algebras are we talking about?

5. Other lessons from QFT, CFT, TQFT and beyond

QED relates the anomalous magnetic moment of the electron $g - 2$ (Lande g-factor) and a perturbation series of Feynman integrals in a formal parameter α_{QED} as a coupling constant, identified as Somerfeld's *fine structure constant*. Otherwise it cannot explain it, hence predict its value; it is a parameter in the SM too.

The (T)QFT framework, via fields or diagrams, or *generalized cobordisms* (CFT, [56] etc.) allows to define a partition function of the theory, whose coefficients are periods [11, 57, 2, 3] [23], which plays a similar role to the (toy model of a) partition function of the category of finite sets $e^x = \sum_{[n]} z^n/|Aut([n])|$. More generally see [58], p.8 (τ is a graph in a given class, e.g. Feynman diagram):

$$Path\ Integral: \ \frac{\int_{\mathcal{P}} e^{\lambda^{-1}S(\phi)} D\phi}{\int_{\mathcal{P}} e^{\lambda^{-1}S_0(\phi)} D\phi} = \sum_{\gamma \in \Gamma} \frac{\lambda^{-\chi(\tau)}}{Aut\ \tau} w(\tau), \quad \lambda : Planck's\ constant.$$

The weights $w(\gamma)$ are periods as for example in Kontsevich's formula for the coefficients of the Formality Theorem [45] (for some category of admissible graphs and propagators), determined by the action functional [24].

Now in deformation quantization (loc. cit.), which is another approach to quantization, in the context of Chern-Simons Theory (e.g. [57] Z_k, p.7-8), similar "players"

[23] See the period quasi-isomorphism of operads [11], p.20.

[24] These are a sort of poly-logarithms, defining multidimensional analogs of e, perhaps ...

to center of the Hirzebruch's hyperfinite factor $A(Q)$, Todd map $T = t_-^{-1} \circ t^+$, center $C \to A(Q)$, Todd exponential with generating function the *Bernoulli function* $Q(x) = x/(1 - e^{-x})$ [8] occur in connection with the *center of the UEA* $U(g)$ of the gauge Lie algebra g (better a $U(1) \to U(2)$ analysis, perhaps), via the Duflo-Kirillov isomorphism I_{DK} [11], p.43., with its decomposition:

$$I_{alg}^{-1} \circ I_T = I_{PBW} \circ I_{strange},$$

where $I_{strange}$ is an invariant operator on $Sym(g)$ associated to "... a power series on g at zero, reminiscent of the square root of the Todd class":

$$I_{strange} : \ \gamma \mapsto exp(\sum B_{2k}/4k(2k)! \, Trace(ad(\gamma)^{2k})$$

and the r.h.s. is $det(q(ad(\gamma))$, with $q(x)^{-2}$ equivalent to the above Bernoulli function, ... and also related to the "Tomita-Takesaki flow of weights for *von Neumann factors*" (whatever that may mean! author's emphasis) ...

There are too many "coincidences" here, in the two apparently different presentations, not to deserve a closer look for a deeper "motive" [25] ...

Geniuses like Grothendieck and other contemporaries, could see "... all the incarnations of a given object" (or may be Meta-Pattern, like ADE-Correspondence, or Ramification Theory in Alg. NT and Complex Analysis etc.) "through their various ephemeral cloak." [59], p.373.

Is this the case too, of a "meta-pattern" in a non-commutative analog of the π and e "reciprocity"? One of course would immediately think of motives and (perhaps) Grothendieck-Teichmuller group [26]

6. Chern-Weil Homomorphism and Todd classes

The "true" extension of theories from EM to EW via the gauge group *extension* $U(1) = Z(SU(2))$ as the center of $G = SU(2)$ (see footnote 9), and with Chern-Weil homomorphism for the corresponding Lie algebra $g = su(2)$ and Chern / Todd classes of associated Chern-Simons Theory,

$$Alpha \ side : \quad CW : C[g]^G \to H^\bullet_{deRham}(M) : \ Periods \ side,$$

[25] [8] is perhaps in "telegraphic mode", half-dismissed by "journalists" and reporters, but [45] is "rock solid": to build on it (but how?) ... and hard to understand! (for the present author)

[26] There is perhaps a collective subconscious insight that the absolute / *Cosmic Galois Group* is at work here ... Pauli got help from Jung to decipher his actual dreams [16], in desperate" chase of his dream; if a Mersenne search can do so much with a network of computers, imagine a network of Mathematicians and Physicists brainstorming in a quantum random search to maximize the relational value of these concepts!

seams to be the correct framework for the details "lifting" (the veil of the) analog of Euler-Hamilton formula from [8]:

$$U(1)\ side: \quad e^{2\pi i} = 1 \quad \cdots \to \quad [e^{Ж}]^{4\pi\alpha} = 1 \quad SU(2)\ side.$$

Here $4\pi\alpha$ is the 3D-analog of the fundamental 2D-period $2\pi i$, and the Chern character:

$$c_t(E) = det(I - t\frac{\Omega}{2\pi i}) = e^{-Tr(\frac{t}{2\pi i}\Omega)}, \quad \leftrightarrow [e^{Ж}]^w = 1, \quad Ж = Tr(2\pi i \Omega^{-1}), \quad w \leftrightarrow \alpha \quad (?)$$

of a principal bundle with curvature $\Omega = dA + A \wedge A$, as the partition function of the theory (compare with gauge theory partition function, e.g. QED, with its coupling constant α; or $S(e \xrightarrow{\gamma} e)$ probability amplitude?).

Now α^{-1}, is claimed to be a candidate for $T(\pi)$ [8], §2.5, p.4. This is to be compared with the *reciprocal* to the Todd exponential/character, since the classes are reciprocal (see [113] for sheaf version):

$$\sum_{i=1}^{dim(M)} dim_{\mathbf{C}}(H^i_{deRham}(E)) = \chi(M) = \int Ch^\bullet(E) \wedge Td^\bullet(TM),$$

where the Todd class $Td(E) = \prod Q(\alpha_i)$ is defined via Bernoulli function $Q(x) = x/(1 - e^{-x}$ and Chern roots α_i (loc. cit.).

Is this perhaps a discrete (adelic) algebraic-geometry analog of building a Riemann surface covering map via path integration (the e and π "**C**-story")? The presence of roots, Bernoulli numbers (Galois groups and covers: algebraic and analytic ramification theory), is perhaps a mark we are on the right trail: Hirzebruch-Riemann-Roch Theorem (Tate: adelic version? ... a tale of primes and zeros?) ...

A connection with the $U(g) - mod$ theory (abstract algebraic / category theory framework / TQFTs), would perhaps clarify the role of the center (Harish-Chandra isomorphism) and its connection with the Todd map $T = t_-^{-1} \circ t_+$.

Now in what sense $Ж = T(\pi)$ is a "renormalized" real number version of π [8], §4, p.4, is unclear to the author (as expected), but let's remember that "most good numbers" are at least "functions" [58], if not trademarks for entire theories! (like 2π or e).

An intermediary (small) step between the two formulas above is via de Moivre formula for quaternions [60]:

$$||(x, y, z)|| = 1, \ e^{2\pi(x\sigma_x + y\sigma_y + z\sigma_z)} = 1 \quad SU(2)\ Pauli\ matrices \sigma_x, \sigma_y, \sigma_z,$$

where $\vec{r} = (x, y, z)$ is a pure unit quaternion, determining a "90° rotation" via conjugation ($SU(2) - >> SO(3)$ 2:1 covering map), similarly to the complex analog i in 2D.

It is related to the Lorentz group (or Mobius transformations and spinors, if needed [5]) via the *hermitean model*[61]:

$$\begin{bmatrix} z - ct & x + iy \\ x - iy & z + ct \end{bmatrix}.$$

This can also be related with the Standard Model via the separation of rotations and boots (reductive group setup):

$$0 \to SU(2) \to SL_2(C) \to m \to 0 \leftrightarrow SU(2)^L \oplus SU_2^R.$$

7. Todd Map and Adelic Deformation Quantization

In [8] ӁК$=T(\pi)$ is defined via the Todd map, which "maps Euler's formula to the Euler-Hamilton formula". Let us try to "decode" this ...

$e^{\text{ӁК}w} = 1$ is not the quaternionic version $e^{2\pi\vec{r}} = 1$, as explained above using de Moivre formula, but spelling out the definitions:

$$t_+(w) = it_-(w), i \in \mathbf{C}, \quad t_+(\text{ӁК}) = \pi t_-(\text{ӁК})$$

looks like a Birkhoff factorization in a quantum group, i.e. a deformation of a (hopefully graded) Hopf algebra. So let us review what hyperfactors are, then taking the p-adic numbers as a well understood example of deformation of algebraic numbers $Z_p = (F_p[x], \star)$ (as well as their field extensions) [20], use the Birkhoff decomposition type in the context of Baker-Campbell-Hausdorf series, the "mother of all deformation quantizations".

But recall that p-adic numbers are deformations [20], and occur as weights in the context of hyperfinite factors [8] §4 ...

7.1. **Hyperfinite factors, Birkhoff decomposition and Chern-Weil Homomorphism.**

Type II_1-factors [62] are like the von Neumann *group algebra* of a countable discrete group, e.g. $SL(n, Z)$ (and its congruence subgroups, towards Chevalley groups and ADE-classification), have the right properties to implement Quantum Mechanics of states and transitions, as in the context of $C^* - algebras$. But these group algebra structure is better "upgraded" to Hopf algebras, leading to the framework of Quantum Groups and associated modular categories (what's "wrong" with finite "vertical gauge groups"!), used in CFT, TQFT, VOAs modern approach to quantization.

Type II_1 factors have finite trace, playing the role of quantum dimension in the modern frameworks of quantum groups and their category of representations. Jones tower, as the closure of a union of subfactors of this type, exhibits the Temperley-Leib relations, a fundamental structure in recoupling theory and Topological Quantum Computing.

"... the von Neumann group algebra of the infinite symmetric group of all the permutations of a countable set that fix all but a finite number of elements give the (unique) *hyperfinite type II_1 factor*." [62].

But by now it is clear that we need braided categories (categorical framework corresponding to the punctured Riemann surfaces models in CFT), to account for Feynman amplitudes as periods (see [7]).

Remark 7.1. The need for more general braided categories, not necessarily symmetric, explains the role of the cosmic Galois group, embeded in the Grothendieck-Teichmuller group which acts on the braided categories [63], p.3, as studied by Drinfeld in the 1990s [64] (see also [114], explaining its role in the framework of operads).

The main physics role of braidings, with its 3-strings braiding relations, is to implement the Galois action of $SU(3)$-color on the $SU(2)-Mod$ (Qubit Model). Quantization yielding mass as a quantum invariant, amounts to considering their finite subgroups.

The Birkhoff factorization/decomposition plays the role of a non-commutative Cauchy Residues Theorem, and may be thought as the "mathematically precise content of renormalization" (tip of the iceberg?). The prototype can be derived from Baker-Capbell-Hausdorf formula [89](universal deformation quantization), in the presence of a Rota-Baxter operator ("gluing" the two halves). This is a framework similar to Yang-Baxter Equation satisfied by the R-matrix, providing a braiding (Yangians and Drinfeld associator etc.) compatible with a duality (essence of Hopf algebras and associated categories of representations: a model for creation and annihilation operators in QFT).

Now in this quantum context (geometric or categorical), we have to deal with *equivariant periods*, not just de Rham period isomorphism, to compute amplitudes, that is *equivariant de Rham cohomology* (geometric framework of gauge field theory); and, "probably", the *algebraic equivariant de Rham period isomorphism* is somehow directly related to the Chern-Weil homomorphism in the Chern-Weil theory computing invariants of principle bundles (compare with Witten's Z_k invariants in Chern-Simons Theory [57]).

How to translate from the Hirzebruch and von Newmann language to the Chern-Simons gauge theory, and then to modern multiple-zeta values [65, 66, 67, 22], looks for the present author like a "champolionic task" ...

Recall that the main goal is to find the "right" modular category, so that the Turaev's Graphical Calculus (Topological Quantum Computing) for vacuum-to-vacuum amplitude (particion function) would define the "fine structure constant" as a formal parameter, and together with the categorical duality (interpreted as external/internal: g_E/g_M), would yield the "ratio" $\approx 1/137$.

7.2. **From Topologic and Analytic to Algebraic ... and beyond!** Then, "One" (can't say "we" at this stage!) would "just" go from Abelian Algebraic Number Theory (cyclotomic numbers as group rings of finite subgroups of $U(1)$ and Complex Analysis as $U(1)$-equivariant functions) to *Anabelian Number Theory*, or equivalently Algebraic

Geometry of Moduli spaces ([2], remembering that Riemann surfaces correspond to number fields via Belyi's Theorem; the non-commutative side of Galois Theory [68]), right into Grothendieck's dessin d'enfant [69], period isomorphisms and motives ... and of course aiming to understand the Cosmic Galois group ...

Now this is deeply related to finite subgroups of $SU(2)$ (binary point groups / Chevalley groups and Galois theory, of number fields or Belyi surfaces of Fuchsian triangular groups) and another mysterious "meta-pattern", the ADE-correspondence (Klein singlarities, blow-up Riemann surfaces, McKay correspondence, Coxeter-Weyl and Lie, VOAs and all that jazz) ...

8. Additional comments on Atiyah's article regarding α

The said news [70] we will just record a few additional comments, and close, to take more time to think and ask around, since, unfortunately, direct feedback on these thoughts are no longer an option[27]...

8.1. **Weights and Bernoulli.** The involvement of p-adic numbers suggests the discrete groups could be $SL_2(F_q)$, together with their deformations (when passing to $F(G)$, or better $U(g)$) $F_q -> Z_p[\theta]$ (p-adic extensions).

The weights are related to the "quantum dimensions", and a candidate of a partition function to be investigated in this context is Verlinde's formula.

The multiplicative sequences "tool", e.g. Todd polynomials, associated to formal groups (Tate, Liubin, etc. ... Deformation Theory), seam a "algebraic topologic" analog to Witt vectors in "finite characteristic" (extension to adeles just places the investigation in the "correct" setup of "graded Algebraic Quantum Groups", e.g. van Daele [50]).

Now the prominent role of π and Euler-Mascheroni constant γ seams to steam out of their relation to Riemann Zeta Function values:

$$\zeta(1) = \sum 1/n \text{``} = \text{''} \prod_{p \in Spec(Z)} (1 - \frac{1}{p}), \quad \zeta(2) = \pi^2/3!.$$

Hence working with 2 as a grading in the computation of α, or equivalently as claimed, with another prime p, places the main formula [8], (8.11), p.13:

$$Ж = \lim_{n,j \to \infty} B^n_{k(j)}/2^{2n}$$

in the realm of adelic mathematics (as it should: "The Ultimate Physics Theory is Number Theory" [71]). But how is this related to the Hirzebruch context is not familiar to the author (Tate: Riemann-Mangoldt exact formula as "just" Adelic Riemann-Roch Theorem!?).

[27]Access to Sir Atiyah's manuscripts, would be an invaluable help ... (any Research Journal!?).

Still, the basis 2 is special, since quadratic extension towers build constructible numbers (Gauss-Wantzel Theorem[28] (loc. cit. p.8), for which the Local-to-Global Principle holds), with π a profinite limit, and possibly $1/\alpha$"="ӁК$= T(\pi)$ too.

Indeed, citing from [8], §9.1, p14: "The arithmetic version of the algebraic structure in [6] built from the group of order 1" (finite field with one element? as a philosophy relating permutations to their linearized version, representation theory, as in passing from the modular arithmetic to cyclotomic numbers, perheps ...) "... is an infinite tower of extensions of $Q(\rho)$ leading to ӁК.".

On the other hand using only cyclotomic numbers, even the maximal Abelian extension Q_{Ab} places the computation outside of the *Anabelic Geometry*, and within the context of finite (quantum phases) subgroups of $U(1)$ (quantum phases), rather then in the context of binary point groups $E_6..E_8$ of TOEs, and non-abelian Galois groups, which are related to the Feynman amplitudes (integrals on moduli spaces [22]) ... and MZVs, Feynman integrals as periods etc. fall within the *homotopical analog of de Rham theory*, via Chen integrals (associahedron).

8.2. **On the computation of α.** Regarding the actual computation, based on the definition of Ч[8] Eq. (7.1):

$$2\text{Ч} = \lim_{n\to\infty} \sum_{j=1}^{j=n} 2^{-j} (1 - \int_{1/j}^{j} \log_2 x dx,$$

which together with Eq. (1.1) $\text{Ч}/\gamma = \text{ӁК}/\pi$, should yield $\alpha = 1/\text{ӁК}$, was checked in a direct-verbatim way by others [117, 116], without providing a confirmation at this stage.

Recasting the equation in terms of the fundamental period $2\pi i$ (residue), Abel's summation formula [72] relating $\log(x) = \int_1^x dx/x$ and $Log(x) = \sum_1^n 1/j$ (in view of Mascheroni constant), with applications to the representation of the Riemann Zeta Function (because of the Prime Number Theorem like summation), together with the Cauchy principal part of the improper integral (residue), could perhaps provide a further clue for understanding it conceptually, irrespective of a numerical error or "mising" factor [29].

The other computation based on Bernoulli / Todd polynomials Eq. (8.11), and its relation to the above computation, is unfortunately not clear to the author at this time.

8.3. **On "Further Comments".**

[28]The role of Fermat primes in conection with quark flavors, as being the simplest primes was stated in [13], p.22, together with a conjectural formula for the fine structure constant.

[29]... or "missing" exponent! see Chebysheff function and prime counting in terms of logarithms [73]), since the fine structure constant *has* to be related to the prime spectrum (finite strings) and Riemann zeros (some "mass spectrum") somehow ...

8.3.1. *The four historical interactions.* The role of the four division algebras as underlying (somehow) the four fundamental interactions is a theme quite often present in the modern articles on "beyond the SM" models (GUts and TOEs).

But as explained earlier in connection with mass and lifetimes of elementary particles (measured in energy units), these are "separate" interactions due to the historical development of the SM. In fact, mass and lifetimes have a natural quantum unit of energy which is a *power of the fine structure constant*: 70 $MeV \approx 1/\alpha m_e$ for mass and $\tau = \alpha^k \tau_{\pi^\pm}, k = ... -1, -2, 0, 1, 2, 3...$ for lifetimes (see [31]; Fig. 1, p.9 in this article). The "weak" vs. "strong" correspond to two ranges: $k = -1..6$ for weak decays which, in the author's Qubit Model correspond to a change of symmetry group of the qubit frame modeling baryons (flavor changes), while the "strong interaction" is just a resonant phenomenon (think band filters and resonant circuits), with range $k = 7, 8$. The $U(1) \to U(2) \to U(3)$ from loc. cit. §9.4 has a different interpretation in the context of the Qubit Model.

... and α is *not* "only related to electro-magnetism.", unless we use the term in the extended sense of Electro-Weak Theory (updated in the context of the Qubit Model), as anticipated by various other authors claiming that EM is an $SO(3)$ or $SU(2)$ theory (e.g. Tesla, torsion fields etc.).

Regarding the current "study of α" (loc. cit. §9.10) in the light of latest experimental data, we should be aware of the *fact* that the "theoretical lense" through which we "look" at the data is biassed by a mandatory historical tren (Gauge Field Theory), followed due to its successes, but also because the formalism was already in place: "One more range of phenomena to model? No problem: fit it in the GFT box!!" [30]

8.3.2. *Energy and Information: what's the difference anyways!?* In §9.7 the role of entropy as determining energy, and conversly, was remarkably mentioned, but it was limited to theory "true in model but not in physical world". This just reflects the limitations of current physical models, not the physical world.

Indeed, Einsteins mass-energy relation is the tip of the iceberg, that mass is a measure of the amount of internal states and possible transitions ("geometry jumps"; think Quantum Turing Machine), hence of internal symmetry, as started to be investigated by the present author before, on "philosophical grounds" (conceptuology and formulae-logy [74]).

8.3.3. *Renormalize e or π?* The "renormalization" of π or e, in fact related as reciprocal, is a Deformation Theory business. The bicharacter z^s of the Dirichlet series, Riemann included, is the larger framework, involving both Fourier and Melin transform. Now deform this in the adelic context of the Algebraic Quantum Group Q ...

[30] We know how the grant funding system works ... and yes, we need continuity in developments, stability etc., until a paradigm shift happens.

It is interetsting that Atiyah also finally thinks of α as a ratio $e/Ж$, possibly related to an electric vs magnetic ratio of coupling constants, corresponding to the dichotomy external/internal. Since "external" (e.g. Mechanics viewpoint) is associated with "classical" (description/computing/logic etc.), while "internal" is associated to "quantum" (discrete/superpositions/symmetry), it could be such a ratio, characteristic 0, ungraded / continuum Mathematics models, vs. characteristic p, graded / discrete Mathematics models based on p-adic numbers (see e.g. [48, 75]), which are deformations of finite fields [20].

Euler discoverd how π is ubiquitous in commutative mathematics (loc. cit. §9.9), which we interprete as the theory of cyclotomic numbers (Abelian Algebraic Number Theory; quantum phase), as the 2D-finite geometry (apposed to Complex Analysis and full $U(1)$ quantum phase).

It is expected that α (hence Ж) to be ubiquitous in non-commutative mathematics (§9.9), of 3D-finite subgroups of $SO(3)$ or $SU(2)$ (and beyond: vertical gauge subgroups of $SU(N)$). Rather replacing $n!$ by $p(n)$, let us remember the role of quantum integers $[n]_q$, with q formal or a root of unity ...

8.4. **Understanding the electron ... and proton!** In Atiyah's article there are so many "seeds" of deeper clues regarding not only *alpha*, but Math and Physics in general (especially at a second reading!), that it is worth "decoding" and translating the von Neumann-Hirzebruch (Atiyah-Singer Index Theory) in the modern fremework of deformation theory, adeles and Categorical Models (Quantum groups, Frobenius, Lefschets, periods and motives etc.) [31].

Einstein is known to have many "big catchy claims", one of which refers to electrons "I would just like to understand the electron."; but he also implicitely said the same about the photon ... But we can't understand "external" independently of "internal": we have to understand the proton too, *at the same time*!

In other words, the electron-proton is the unit, and phisicists like Barrutt understood that the electron plays the role of a forth "color". Moreover, fermions and bosons are aspects unified by the network approach to quantum physics (super-symmetry is just a formal attempt for unification); but this is another story ...

8.4.1. *The Rainbow Bridge: Physics and Mathematics.* The pot of gold is, perhaps, what mass is (and Riemann zeros?).

A classical bridge was envisioned by Manin and explained by Atiyah [9], not to mention many others working at its construction (too many to list).

It needs being developed to the point where we see that "reality is mathematical", close to the Platonic view [41], finally explaining Wigner's "unreasonable effectiveness of mathematics", who roughly started the whole approach (particles as irreducible representations).

[31]The Standard Model is also "ready to be Categorified", e.g. Topological Quantum Computing, Qubit Model etc.; but waiting for that "green light" called funding ...

So, the "quantum bridge" aluded to by Atiyah, from QFT to Laglands, is being developed, but it is more of a loom of theories, due to various "schools", developers and independent studies ... [32]

The human limitations are by far not approached yet, in Science; working in isolation is the culprit, specialization and "upgrading" (unifying the gauge groups in GUTs is not major progress: just "simplification" and reorganizing data). Indeed "we need the collective wisdom of all branches of science." (loc. cit. §10) [33].

In conclusion, [8] is a wonderful source of ideas, to be read at least twice!

Unfortunately the access to the listed bibliography is more difficult than expected ... at least for now ... [34] Atiyah's mathematical-physics final swan dance may turn out comparable with Ramanujan's notebooks, but on the other side of the spectrum: conceptually loaded, to be deciphered, and then proved right ...

9. Conclusions

There is a fuzzy "feeling" that e, π on one hand, and χ, α are related in a meaningful way ...

... and that really is really mathematical, thus one should be able to prove $\alpha = e^2/hc$, once the theory would be able to explain what mass really is.

If The Universe is really quantum, it is locally finite, and quatum phase is cyclotomic, while qubits have Galois groups as "vertical gauge groups" (which must also be "horizontal").

Thus a study of how quark flavor can be modeled as Platonic TOY Geometry (notation from [76]), as $SU(3)$-bounded Galois groups, and dynamics governed by the corresponding exceptional Lie algebras, full circle to VOAs would be a place to start: *The Beauty and The Monster: an ADE-Classification Story*! [35]

Ultimately, there should be "Finite String Theory" ("Integral Conformal Field Theory"): the parallel between the Algebraic Geometry in the non-graded case of complex numbers and that of graded case of $\bar{Q}_p$ is well know to go quite deep.

Once the role of $\alpha = g_E/g_M$ as a Hodge structure *related concept* (to play safe!) is clarified, the goals to achieve in this "Graal Quest" should become apparent.

An speaking of "String Theory", upgrading Mechanics from point particles to 1D-strings, why stop there!? The "vibrating think" ("brane"), is S^3, the qubit, a central concept in ... everything: instantons (Hopf bundle), gauge theory ($SU(2)$ is The group; $SU(3)$ just changes qubit frames: baryons), so:

[32]Diversity is good, but a clear formulated new paradigm is needed: not axiomatic, but rather as an *Expert System*.

[33]... "and beyond", what Science chooses *not* to study openly, yet!

[34]Please share, if you have some of Atiyah's unpublished recent papers; a memorial website, maybe?

[35]Why ADE only? because the Universe is constructible ;)

$$S^0\ Mechanics \to S^1\ String\ Theory \to S^3\ Quantum\ Computing!$$

The octonions should model morphisms / channels of quantum information (mesons). Mathematically are a natural step beyond quaternions, but physically they have a different role.

In conclusion, we have partition functions of gauge groups (Lie type in QFT or quantum groups in TQFT) with periods as coefficients and a formal parameter (generating functions), associated to certain generalized cobordism categories.

In view of the "extension" $EM \to Eletro-Weak$ theories $U(1) \cong Z(SU(2)) \to SU(2)$[36], the "fine structure constant", or better $4\pi\alpha$ [37] as a mathematically computable number, could be in fact a *ratio*:

$$Math:\ \alpha_{U(1)}/\alpha_{SU(2)} = \cdots ?$$

because in fact, from a physical point of view, it is an internal quantum structure ("mass from symmetries") to external change behaviour ("conserved forces") ratio [77][38]:

$$Physics: 4\pi\alpha = e^2 \frac{1}{hc\epsilon_0} = e^2 \frac{\mu_0 c}{h} = \frac{g_E}{g_M} = \frac{Z_0}{R_0},$$

$$Z_0^2 = \frac{\mu_0}{\epsilon_0}, \quad R_0 = 1/G_0 = e^2/h,$$

where the last step is an "RLC-analog" in quantum computing. The *conductance quantum* G_0 through QI-channels, e.g. transit time of an electron[39]), can be derived from Heisenberg's uncertainty principle as $(eV)/(e/I) \approx h$ [40].

So, finally, what *is* the fine structure constant!? I don't know, yet, but, "yippee-ki-yay", we will not give up ... [78]

(to be continued [41] ...)

[36]Not the SM's tilted product $U(1) \otimes SU(2)^L \otimes SU(2)^R \otimes \cdots$ ...

[37]From Gauss-Bonnet / Descartes angular defect, to Chern-Weyl homomorphism and Chern-Todd classes ...

[38]Noether Theorems in the context of external/internal degrees of freedom, modeling classical information/quantum information Master-Slave Paradigm.

[39]Dual to the transition time of a photon between two electrons, related to the transition probability (Fermi Golden Rule), and hence to the square of the fine structure constant: Feynman, QED [115].

[40]The role of Heisenberg uncertainty principle is that of a cut-off of when "external space-time based concepts are no longer adequate for modeling internal aspects, exhibiting quantum behaviour and properties.

[41]The baton of the challenge has been passed successfully, Dr. Hooft!

10. Acknowledgements

The author would like to thank for the opportunity to research the topic of periods at IHES, and the patience of the professors listening to his opinion during the talk.

Comments regarding the above ideas will be much appreciated ...

Some additional references consulted by the author but not cited in this article, were nevertheless left part of the bibliography, for the reader to consult, if needed.

Annex - Is π "wrong"!?

Some tangential comments were triggered by [79] and [80]. I hope they might help.

Math is an "ecosystem", concepts support one another, and some go way out of their natural habitat ... π is not "wrong", but a "running constant" of geometry for areas $A(r)/r^2$, at zero curvature, while 2π is the length analog. Both reflect a measure theory approach, as an $L - norm$, while the oriented (Riemann integral/de Rham) version is the period $2\pi i$, which incorporates the "curvature (Lie) generator" i.

2π is connected to e in the algebraic differential forms setup, as explained at the beginning of the article, or introducing e first via Differential Calculus, with e^x as eigenfunction of d/dx (natural shift basis $x^n/n!$, or graph differential for bamboos), or the Integral / convolution algebra point of view via real Fourier transform (duality), and the Gaussian eigenfunction $f(x) = e^{-x^2/2!}$.

The volume of the sphere relative its radius (a conformal geometry constant):

$$Vol(S^n)/r^n = n \cdot \frac{\pi^{n/2}}{(n/2)!},$$

or in terms of some other "Gamma function", e.g. using the Mellin transform of the Gauss distribution $\Gamma_f(s)$ [53], p.5, to account for $\pi^{n/2}$ factor via a hidden 2π rescaling of the real axis.

Remark 10.1. The debate could continue relating Haar measure of $(R_+, \cdot, dx/x)$ and that of the unit circle via a Cayley transform, with the arithmetic counting measure and the natural quotient $Z \to R \to S^1$ ...

References

[1] M. Kontsevich and D. Zagier, Periods, 2001, http://www.maths.ed.ac.uk/~aar/papers/kontzagi.pdf
[2] F. Brown, On periods of some Feynman integrals, https://arxiv.org/abs/0910.0114, 2009
[3] Oliver Schnetz, Quantum periods: a census of ϕ^4-periods, https://arxiv.org/abs/0801.2856, 2008.
[4] L. M. ionescu, "Periods, Feynman Integrals and Jacobi sums", http://my.ilstu.edu/~lmiones/ 2017.
[5] R. Britto, F. Cachazo, B. Feng, and E. Witten, вЂЁDirect proof of tree-level recursion relation in Yang-Mills theory,вЂ№ Phys.Rev.Lett. 94 (2005) 181602, arXiv:hep-th/0501052
[6] H. Fresse, Homotopy of operads and Grothendieck-Teichmuleer groups, Mathematical Surveys and Monographs Volume: 217; 2017.

[7] L. M. Ionescu, "Periods: Variations sur un theme de Kontsevich", IHES Feb. 6, 2018, http://my.ilstu.edu/ lmiones/
[8] M. Atiyah, The fine structure constant, blog.sciencenet.cn/home.php?mod=attachment&id=256716
[9] Yuri Manin, New dimensions in geometry, Proceedings of the meeting held by the Max-Planck-Institut fΓёr Mathematik, Bonn June 15вЃY22, 1984, Editors Friedrich HirzebruchJoachim SchwermerSilke Suter, p. 59-101; https://people.mpim-bonn.mpg.de/zagier/files/doi/10.1007/BFb0084581/chapter04.pdf
[10] A. Weil, De la metaphysique aus mathematique, http://dream.inf.ed.ac.uk/projects/coinvent/weil60.pdf
[11] M. Kontsevich,Operads and motives in deformation quantisation. Letters in Mathematical Physics 48 (1999), pp. 35вЃY72. https://arxiv.org/pdf/math/9904055.pdf
[12] B. Friedrich, Periods and Algebraic deRham Cohomology, https://home.mathematik.uni-freiburg.de/arithgeom/preprints/friedrich-periods.pdf
[13] L. M. Ionescu, Remarks on physics as number theory, 19th Natural Philosophy Alliance Proceedings, Vol. 9, p. 232-244. http://empslocal.ex.ac.uk/people/staff/mrwatkin/zeta/ionescu2011.pdf
[14] G. Dattoli, The fine structure constant and numerical alchemy, ENE, 2010. https://arxiv.org/abs/1009.1711
[15] Sean Carroll, Atiyah and the fine-structure constant, www.preposterousuniverse.com/blog/2018/09/25/atiyah-and-the-fine-structure-constant/
[16] P. Varlaki and I. J. Rudas, Twin concept of fine structure constant as the "Self Number-Archetype" in perspective of the Pauli-Jung correspondence, Acta Polytechnica Hungarica, Vol. 6, No.2, 2009.
[17] M. A. Sherbon, Michael Sherbon. Wolfgang Pauli and the Fine-Structure Constant. Journal of Science, World Science Publisher, 2012, 2 (3), pp.148-154 <10.2139/ssrn.1934553>. <hal-01304518>
[18] Wikipedia, Relativistic orbit, https://en.wikipedia.org/wiki/Old_quantum_theory#Relativistic_orbit
[19] J. Baez, Groupoidification, http://math.ucr.edu/home/baez/groupoidification.pdf
[20] L. M. Ionescu, On p-adic Frobenius lifts and p-adic periods, from a Deformation Theory viewpoint, https://arxiv.org/abs/1801.07570
[21] Nima Arkani-Hamed, Jacob L. Bourjaily, Freddy Cachazo, Alexander B. Goncharov, Alexander Postnikov, Jaroslav Trnka, Scattering Amplitudes and the Positive Grassmannian, https://arxiv.org/abs/1212.5605
[22] F. Brown, "Multiple zeta values and periods of moduli spaces $M_{0,n}$", 2006, https://arxiv.org/abs/math/0606419
[23] C. Montonen and G. Olive, http://cds.cern.ch/record/422232/files/CM-P00061677.pdf
[24] Wikipedia, Motonen-Olive duality, https://en.wikipedia.org/wiki/Montonen%E2%80%93Olive_duality.
[25] P. Goddard, J. Nuyts and D. Olive, Gauge theories and magnetic charge, Nuclear Physics B, Volume 125, Issue 1, 4 July 1977, Pages 1-28
[26] Anton Kapustin and Edward Witten, Electric-Magnetic Duality And The Geometric Langlands Program, https://arxiv.org/pdf/hep-th/0604151.pdf
[27] John D. Cook, "Riemann hypothesis, the fine structure constant and the Todd function", 2018, https://www.johndcook.com/blog/2018/09/24/riemann-hypothesis-the-fine-structure-constant-and-the-todd-function/
[28] Wikipedia, 2019 redefinition of SI base units, https://en.wikipedia.org/wiki/2019_redefinition_of_SI_base_units#Redefinition

[29] E. Frenkel, Gauge theory and Langlands duality, 0906.2747v1, 2009.
[30] N. Dorey, C. Fraser and T. J. Hollowood, "S-duality in N=4 supersymmetric gauge theories with arbitrary gauge groups, hep-th/9605069v1, 1996.
[31] M. H. Mac Gregor, The power of alpha, World Scientific Pub Co Inc (March 14, 2007).
[32] D. Griffith, Introduction to elementary particle physics, Wiley-VCH; 2nd edition (October 13, 2008).
[33] K. Meyl, Scalar waves: Experimental kit, https://www.meyl.eu/go/index.php?dir=50_Experimental&page=1&sublevel=2
[34] Naber, Topology, Geometry and Gauge Fields, J. P. Naber, Applied Mathematical Sciences (Book 141), Springer; 2nd ed. 2011 edition (April 6, 2011).
[35] A. O. Barut, Stable Particles as Building Blocks of Matter, Surveys High Energ. Phys 1, 113, 1980.
[36] L. M. Ionescu, The Qubit Model: a Platonic and exceptional quantum theory, E678-TOE.pdf, Nov. 2018.
[37] Wikipedia: Coxeter-Dynkin-Diagrams
[38] L. M. Ionescu, On mass formulas and MZVs, work in progress; TBA 2019.
[39] L. M. Ionescu, On the arrow of time, Theoretical Physics, Vol. 2, No. 3, September 2017. https://arxiv.org/abs/0708.4180
[40] L. M. Ionescu, On prime numbers and Riemann zeros, Journal of Advances in Applied Mathematics, Vol. 2, No. 4, October 2017 https://dx.doi.org/10.22606/jaam.2017.24002, http://oaji.net/articles/2017/3749-1509417677.pdf
[41] L. M. Ionescu, The Platonic universe, ISU Alg. Seminar presentation, 2018.
[42] L. M. Ionescu, Infotronics: Theory and Experiment, http://my.ilstu.edu/ lmiones/ResearchProjects_2010.htm
[43] L. M. Ionescu, "Quantum Gravity: an essay", Physics Essays, Volume 30: Pages 308-313, 2017; https://physicsessays.org/browse-journal-2/product/1591-15-lucian-m-ionescu-quantum-relativity-an-essay.html
[44] A. Connes and D. Kreimer, Renormalization in quantum field theory and the Riemann-Hilbert problem I, https://arxiv.org/abs/hep-th/9912092
[45] M. Kontsevich, Deformation quantization I, https://arxiv.org/abs/q-alg/9709040
[46] L. M. Ionescu, Cohomology of Feynman graphs and perturbative quantum field theory, https://arxiv.org/abs/math/0506142
[47] L. M. Ionescu, A natural partial order on the set of prime numbers, Notes on Number Theory and Discrete Mathematics, Vol. 21, 2015, No. 1, 1вЃY9; https://arxiv.org/pdf/1407.6659.pdf
[48] L. M. Ionescu: Finite String Theory Project, http://my.ilstu.edu/ lmiones/ 2016;
[49] Vijay Kodiyalam and V. S. Sunder, Topological Quantum Field Theories from Subfactors, 1st Edition, Chapman & Hall/CRC Research Notes in Mathematics Series, 2000.
[50] A. Van Daele, An algebraic framework for group duality, Advances in Mathematics, Volume 140, Issue 2, 25 December 1998, Pages 323-366.
[51] L. M. Ionescu, The Algebraic Quantum Group of Rationals, http://my.ilstu.edu/ lmiones/, Articles and Presentations, ISU Alg. Sem., 2011.
[52] V. Turaev, Quantum Invariants of Knots and 3-Manifolds, De Gruyter Studies in Mathematics, Vol. 18, 2nd ed. 2010.
[53] P. Garrett, Riemann's explicite/exact formula, 2013, http://www-users.math.umn.edu/ garrett/m/mfms/notes_2013-14/02_Riemann_and_zeta.pdf
[54] Atiyah, Michael Francis; Bott, Raoul (1984), "The moment map and equivariant cohomology", Topology. an International Journal of Mathematics, 23 (1): 1вЃY28,

[55] Loring W. Tu, What is equivariant cohomology?, Notices of the AMS, March 2011, Vol. 58, No.3, p.423-6.
[56] L. M. Ionescu, TQFTs on generalized cobordism categories, NSF GP 2002, http://my.ilstu.edu/~lmiones/ISUP/ISU-GP.html
[57] M. Kontsevich, Feynman diagrams and low-dimensional topology, First European Congress of Mathematics Paris, July 6вЃY10, 1992 pp 97-121; see also pdf at IHES, 2006. https://www.ihes.fr/ maxim/TEXTS/Feynman%20%20diagrams%20and%20low-dimensional%20topology.pdf
[58] Yuri I. Manin, Numbers as functions, math.NT/1312.5160, 2013.
[59] P. Cartier, Alexander Grothendieck: a country known only by name, Notices of the AMS, Vol. 62, No.4, April 2015, p.373-382.
[60] E. Cho, De Moivre's formula for quaternions, https://core.ac.uk/download/pdf/82697108.pdf
[61] Wikipedia, Mobius transformation (Lorentz transformations), https://en.wikipedia.org/wiki/M%C3%B6bius_transformation#Lorentz_transformation
[62] Wikipedia: Von Neumann algebra; Hyperfinte type II factor; Chern-Weil homomorphism; Birkhoff factorization;
[63] P. Guillot, An elementary approach to dessins d'enphants and Grothendieck-Teichmuller group, 1309.1968v2, 2014.
[64] V. G. Drinfeld, On quasitriangular quasi Hopf algebra and a group that is closely connected to $Gal(\bar{Q}/Q)$, Algebra i Analiz 2 (1990), no.4, 149-181.
[65] Jianqiang ZHAO and Xia ZHOU, Witten Multiple Zeta Values Attached to $SL(4)$, TOKYO J. MATH. VOL. 34, NO. 1, 2011; ttps://projecteuclid.org/euclid.tjm/1313074447
[66] J. Zhao, Multiple Zeta Functions, *Multiple Polylogarithms and Their Special Values*, Series on Number Theory and Its Applications: Volume 12, World Scientific, 2016.
[67] T. Terasoma, Selberg integral and multiple zeta values, https://arxiv.org/pdf/math/9908045.pdf
[68] G. V. Belyi, On Galois extensions of a maximally cyclotomic field, Math USSR Izvestija Vol. 14 (1980), No.2
[69] L. Schneps, Dessins d'enphants on the Riemann sphere, The Grothendieck Theory of Dessins d'Enfant, London Math. Soc. LN 200, Cambridge Univ. Press, 1994.
[70] P. Ghosh, Mathematician Sir Michael Atiyah dies at age 89, BBC News, Jan. 14, 2019.
[71] V. Volovich, Number Theory as the Ultimate Physics Theory, http://empslocal.ex.ac.uk/people/staff/mrwatkin/zeta/volovich1.pdf
[72] Wikipedia, Abel summation formula, https://en.wikipedia.org/wiki/Abel%27s_summation_formula#Representation_of_Riemann's_zeta_function
[73] Wikipedia, Prime Number Theorem; prime counting function, https://en.wikipedia.org/wiki/Prime_number_theorem#Proof_sketch
[74] L. M. Ionescu, The search of a new unifying principle, https://arxiv.org/abs/0705.1116
[75] B. Dragovich1, A. Yu. Khrennikov2, S. V. Kozyrev3 and I. V. Volovich, On p-Adic Mathematical Physics, https://arxiv.org/pdf/0904.4205.pdf
[76] Michael Atiyah and Paul Sutcliffe, Polyhedra in physics, chemistry and geometry, https://arxiv.org/abs/math-ph/0303071
[77] Wikipedia: Fine structure, Fine-structure constant", Planck charge, Impedance of free space, Conductance quantum, Landau pole, Quantum triviality,
[78] Angela Tung, A brief history of yippee-ki-yay, https://theweek.com/articles/462065/brief-history-yippeekiyay
[79] M. Hartl, The tau manifesto, 2018, https://tauday.com/tau-manifesto
[80] B. Palais, π is wrong!, http://www.math.utah.edu/ palais/pi.pdf

[81] ICM 2018 News, A fantastic journey through Atiyah mathematical mind, http://www.icm2018.org/wp/2018/08/06/a-fantastic-journey-through-atiyahs-mathematical-mind/
[82] J. A. de Azcarraga, On Mathematics and Physics: a conversation with Sir Michael Atiyah, https://www.uv.es/ azcarrag/pdf/2018%20REF%20Conversation%20Atiyah%20English.pdf
[83] P. Woit, Atiyah talk at Santa Barbara, 2005; https://www.math.columbia.edu/~woit/wordpress/?p=285
[84] M. Kontsevich, Periods, IHES Oct. 2006, http://www.ihes.fr/~maxim/TEXTS/Periods-short.pdf.
[85] M. Kontsevich, talk / Graph polynomial conjecture, 1997
[86] Quanta Magazine, Strange numbers found in quantum particles collisions, 2016.
[87] J. M. Commelin, Periods (and why FTC is a fundamental theorem), 2015.
[88] S. Muller-Stach, What is a period, Notices AMS
[89] K. Ebrahim-Fard, Li Guo, and D. Manchon, Birkhoff type decomposition and the BCH reursion, https://arxiv.org/pdf/math-ph/0602004.pdf
[90] L. M. Ionescu, A holistic approach to quantum physics, article.aascit.org/file/pdf/9401292.pdf; see also “A note on manifolds vs. networks as mathematical models in modern physics”, May 2018; “What’s a Muon Anyways!?”, July 2018; http://my.ilstu.edu/~lmiones/ResearchProjects_2010.htm
[91] D. Zagier, Values of zeta functions and their applications, https://people.mpim-bonn.mpg.de/zagier/files/scanned/ValuesofZetaFunsAndApplications/fulltext.pdf
[92] D. J. Broadhurst, D. Kreimer, Association of multiple zeta values with positive knots via Feynman diagrams up to 9 loops, https://arxiv.org/abs/hep-th/9609128
[93] A. L. Kholodenko, “New” Veneziano amplitudes from “old” Fermat hypersurfaces, hep-th/o212189
[94] Pierre Cartier, “A mad day’s work: from Grothendieck to Connes and Kontsevich”, Bulletin of the AMS, Vol. 38, No. 4, Pages 389вЃY408. https://www.ams.org/journals/bull/2001-38-04/S0273-0979-01-00913-2/S0273-0979-01-00913-2.pdf
[95] J. Carlson, P. Griffiths, What is a period domain?, AMS Vol.55, No.11, p.1418.
[96] L. M. Ionescu and R. Sumitro, Periods and applications, https://arxiv.org/abs/1708.09277
[97] J. Earl and L. M. Ionescu, Modeling Quark Flavor, ISU UG Research poster, April 2018.
[98] Vladimir Voevodsky, A^1-Homotopy Theory, Documenta Mathematica, Extra Volume ICM 1998 I 579вЃY604, http://www.math.ias.edu/vladimir/files/A1_homotopy_ICM_1998_Berlin_published.pdf
[99] J. Ajub, Periods and the conjectures of Grothendieck and KontsevichвЃYZagier, http://user.math.uzh.ch/ayoub/PDF-Files/periods-GKZ.pdf
[100] R. Feynman, The Feynman Lectures on Physics, http://www.feynmanlectures.caltech.edu/
[101] L. M. Ionescu, A note on manifolds vs. networks as mathematical models in modern physics, 2018.
[102] L. M. Ionescu, The Feynman Legacy, math/0701069.
[103] L. M. Ionescu, “The Digital World Theory v.1: An Invitation”; “Q++ and a Non-standard Model: DWT v.2”, 2007; http://my.ilstu.edu/~lmiones/ResearchProjects_2010.htm
[104] L. M. Ionescu, “Periods, Feynman Integrals and Jacobi sums”, ISU presentation, Jan. 2017, http://my.ilstu.edu/~lmiones/.
[105] E. Wigner, “The unreasonable effectiveness of Mathematics in the Natural Sciences”, Communications in Pure and Applied Mathematics, Vol. 13, No. I, Feb. 1960, New York: John Wiley & Sons, Inc.; http://www.maths.ed.ac.uk/~v1ranick/papers/wigner.pdf

[106] Wikipedia: MZV,
https://en.wikipedia.org/wiki/Multiple_zeta_function#Two_parameters_case
[107] C. Davis and K. S. Kedlaya, Witt vectors with p-adic coefficients and Fontaine rings, https://www.math.uci.edu/~davis/padic.pdf
[108] L. M. Ionescu, From Lie Theory to Deformation Theory and Quantization, https://arxiv.org/abs/0704.2213
[109] L. M. Ionescu, Periods: from Global to Local, https://arxiv.org/abs/1806.08726
[110] W. Dunham, Euler: The Master of Us All, Dolciani Mathematical Expositions (Book 22), 1999.
[111] Wikipedia: Rydberg constant
[112] Joris van Hoboken, "Platonic solids, binary polyhedral groups, Kleinian singularities and Lie algebras of the type A, D, E.", Master Thesis, University of Amsterdam, Faculty of Mathematics, 2002.
[113] Wikipedia, Todd class, https://en.wikipedia.org/wiki/Todd_class
[114] B. Fresse, Homotopy of operads and Grothendieck-Teichmuler groups, Mathematical Surveys and Monographs, Volume: 217; 2017, https://bookstore.ams.org/surv-217/
[115] R. Feynman, QED: The Strange Theory of Light and Matter, Princeton Science Library, Princeton Press, 1988.
[116] Reddit Math user: u/swni, "Atiyah's computation of the fine structure constant (pertinent to RH preprint)",
www.reddit.com/r/math/comments/9ig4ei/atiyahs_computation_of_the_fine_structure/
[117] Arturo Quirantes, "Physics with Excel: Michael Atiyah and the fine structure constant",
http://elprofedefisica.es/2018/09/25/fisica-excel-atiyah-constante-estructura-fina/

Department of Mathematics, Illinois State University, IL 61790-4520
E-mail address: lmiones@ilstu.edu