Energy Localization problem pointed to Virtual Term in First Order Deviation Equation

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Abstract

Due to his solution of Energy Localization problem in General Relativity the author finds out, that tidal forces of Black Hole can compress the falling astronaut instead of ripping him into parts. Moreover, found necessity of inclusion mathematical correction made "by hand" into first order Deviation equation. Four different methods in this paper gave the same results!

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A. On energy Localization problem

By recalling the basic need to study problems in an inertial coordinate system (tetrad) [recall the demand for an inertial tetrad in the Galilean and Einstein Postulates of Relativity: in a non-inertial tetrad would be changed laws, but latter comes in conflict with Metrology [1]], we found no problem with the local conservation of the most basic laws of physics. But others have faced major problems (cf. e.g. Refs. [2]).

The vector of rate in the local (ON) tetrad has

$$\frac{d\,B^{\hat\nu}}{d\tau} = e^{\hat\nu}_{\alpha}\,\frac{D\,B^{\alpha}}{d\tau}\,. \tag{1}$$

Thus, if $B^{\hat\nu}$ conserves in inertial tetrad, then

$$\frac{d\,B^{\hat\nu}}{d\tau} = 0\,, \qquad \frac{D\,B^{\alpha}}{d\tau} = 0\,. \tag{2}$$

But because

$$B^{\alpha} = e^{\alpha}_{\hat\nu}\,B^{\hat\nu}\,, \tag{3}$$

then the inertial tetrad is defined by

$$\frac{D\,e^{\alpha}_{\hat\nu}}{d\tau} = \frac{d\,e^{\alpha}_{\hat\nu}}{d\tau} + \Gamma^{\alpha}_{\beta\,\gamma}\,e^{\beta}_{\hat\nu}\,u^{\gamma} = 0\,. \tag{4}$$

In particular a solution of this describes the Earth axis yearly fixation on Polar Star area.

As well, this solves Energy Localization problem in General Relativity. The known formula

$$T^{\nu\,\mu}_{;\nu} = 0 \tag{5}$$

in inertial ON tetrad is the needed conservation of energy-momentum

$$T^{\hat\nu\,\hat\mu}_{,\hat\nu} = 0\,, \tag{6}$$

because in inertial ON tetrad all the Christoffel Symbols are zero

$$\Gamma^{\hat\alpha}_{\hat\nu\,\hat\mu} = 0 \tag{7}$$

due to the Strong Equivalence Principle: physics in free-moving laboratory is independent of gravity (spacetime position).

B. The model in use

A motion of long bodies in curved spacetime is a fascinating theme because the point-like particles are way too simple idealization. However, the large bodies do lose the interest of a reader, because of the tremendous number of details. Remains the golden area of study: the small object, but not a microscopic – a drop of "perfect fluid". A drop of fluid is falling along the geodesic line because the drop is small. There are waters in heaven, look: [3].

As a background example, the author considers the Schwarzschild metric of Black Hole spacetime. The velocity one finds using integral of motion $u_t = -E$, and the norm $u^\nu\, u_\nu = -1$, the non-zero components are

$$u_t = -E, \qquad u_r = -\frac{\sqrt{E^2 - 1 + (2M/r)}}{1 - (2M/r)}\,, \tag{8}$$

where $E = \sqrt{1 - (2M/r_0)}$. The M, $S^{\hat{\nu}}$, τ and r are being measured in meters: they are "geometrised". Initial velocity (when $r = r_0$) is zero $u_r = 0$.

C. Usefulness of first-order Deviation Equation

We are sure, what the complicated algorithms, often with extensive use of the Second order Deviation Equation (in its higher approximation terms) are written, e.g., [4]. However, in the present manuscript, the author presents the easily accessible way to study any spacetime of interest: first order Deviation Equation.

Please note, that unlike the known Deviation Equation, the author's First order Deviation Eq.(12) includes the property of the bundle of geodesics: a starting point $r_0 \equiv \eta$, whereas proper time along each geodesic $\tau \equiv \lambda$. The calculation with the known Deviation equation is much more complicated because it includes the second order derivatives.

I. FIRST METHOD: ALTERNATIVE TO THE KNOWN DEVIATION EQUATION

In this section, the equation of state of the fluid is zero pressure $p = 0$: the dust is a particular case of a fluid.

The derivation of Deviation Equation [5], pages 58, 291 shall be made more clear, because the starting from the bundle of trajectories $x^\alpha = x^\alpha(\lambda, \eta)$ and definition of a tangent to the geodesic line $u^\alpha = \partial x^\alpha/\partial\lambda$ one came to the wrong assertion $\mathrm{grad}\, u^\alpha \equiv \partial_u\, u^\alpha \neq 0$. But here is

$$\mathrm{grad}\, u^\alpha := \frac{\partial u^\alpha}{\partial x^\nu} = \frac{\partial}{\partial x^\nu}\Big(\frac{\partial x^\alpha}{\partial \lambda}\Big) = \frac{\partial}{\partial \lambda}\Big(\frac{\partial x^\alpha}{\partial x^\nu}\Big) = \frac{\partial}{\partial \lambda}\,\delta^\alpha_\nu = 0\,. \tag{9}$$

One shall rewrite the official derivation using the alternative denotations $U^\alpha(\{x^\nu\};\,\lambda,\eta) = U^\alpha(\{x^\nu(\lambda,\eta)\};\,\lambda,\eta) = u^\alpha(\lambda,\eta)$ with

$$U^\alpha_{,\nu} \equiv \frac{\partial U^\alpha(x^0,x^1,x^2,x^3)}{\partial x^\nu} \neq 0. \tag{10}$$

Because mathematically speaking

$$\frac{\partial^2\, x^\alpha}{\partial\eta\,\partial\lambda} = \frac{\partial^2\, x^\alpha}{\partial\lambda\,\partial\eta}\,, \tag{11}$$

then obviously holds [8]

$$\frac{\Delta\eta\,\partial n^\alpha}{\partial\lambda} = \Delta\eta\,\frac{\partial u^\alpha}{\partial\eta}\,, \tag{12}$$

where $n^\alpha = \partial x^\alpha/\partial\eta$ and $\Delta\eta = \mathrm{const}$. With $n^\alpha = n^{\hat u}\, e_{\hat u}{}^\alpha$, where $n^{\hat u}$ is the projection of the vector n^α on the free-falling ON reference frame with $e^{\hat q}_\alpha e^{\hat u\,\alpha} = \eta^{\hat q\,\hat u} = \mathrm{diag}(-1,1,1,1)$ it turns into

$$\frac{d\,S^{\hat u}}{d\lambda}\, e_{\hat u}{}^\alpha = \Delta\eta\,\frac{\partial u^\alpha}{\partial\eta} - S^{\hat u}\,\frac{\partial e_{\hat u}{}^\alpha}{\partial\lambda}\,. \tag{13}$$

with $S^{\hat u} := \Delta\eta\, n^{\hat u}$.

Now, because we have realized the necessity of the Eqs.(9), (10), holds

$$\frac{\partial u^\alpha}{\partial\eta} \equiv U^\alpha_{,\nu}\,\frac{\partial x^\nu}{\partial\eta} + \psi\,\frac{\partial\, U^\alpha}{\partial\eta}\,, \tag{14}$$

where in the absence of virtual correction would be $\psi = 1$ and

$$\frac{\partial x^\nu}{\partial\eta} = n^\nu = S^{\hat u}\, e^\nu_{\hat u}/\Delta\eta\,. \tag{15}$$

In case of Schwarzschild metric with proper time $\tau \equiv \lambda$ we come to

$$M\, S^{\hat 1} + \frac{d\,S^{\hat 1}}{d\tau}\, r\,\sqrt{r^2\,(E^2-1) + 2\,M\,r} - \psi\, r^2 = 0\,, \tag{16}$$

and latter τ-derivative (note, that $r = r(\tau)$) results in

$$\frac{d^2\,S^{\hat 1}}{d\tau^2} = \frac{2\,M}{r^3}\,S^{\hat 1}\,. \tag{17}$$

But Eq.(16) is clearly unphysical, because the equations must allow $S^{\hat 1} = dS^{\hat 1}/d\tau = 0$ case. Therefore, the choice $\psi = 0$ is welcomed. That choice does not contradict the experiments, so it is like the calibration freedom in Electrodynamics: it is remarkable, that Eq.(17), which directly relates to detectable stresses in material, is ψ-independent. However, the Hilbert's problem has the negative answer: mathematics based on arithmetic axioms (mathematics is in basis of physics) appears to be not self-consistent.

Because this solution has fixed $S^{\hat{0}} = \frac{d\,S^{\hat{0}}}{d\tau} = 0$, then the $S^{\hat{1}}$ can be recognized as the distance between the dust-particles (as well as the Strong Equivalence principle stays [6], what the same time shall be in the locality of the observer, namely $S^{\hat{0}} = 0$).

Amazingly, the radial size of the body shrinks despite the positive acceleration of deviation:

$$f = \frac{d^2\,S^{\hat{1}}}{d\tau^2} > 0\,, \qquad \frac{d\,S^{\hat{1}}}{d\tau} < 0\,, \tag{18}$$

where the $\psi = 0$ was taken. The author gives the following explanation to it. The deviation forces (f) are not forces at all. Why? The Strong Equivalence principle stays clear: the Physics of the small laboratory is not affected by outside curvature of spacetime. So, to introduce alien force into such oasis is conceptually wrong.

II. SECOND METHOD: THE KNOWN DEVIATION EQUATION AGREES

In this section pressure $p = 0$.

The following holds for any value of the constant ψ. Is expected, that in (inertial) tetrad

$$\frac{d^n\,h^{\hat{\nu}}}{d\tau^n} = e^{\hat{\nu}}_{\alpha}\,\frac{D^n\,h^{\alpha}}{d\tau^n}\,, \tag{19}$$

where

$$h^{\hat{\nu}} = e^{\hat{\nu}}_{\mu}\,h^{\mu}\,, \quad h^{\alpha} = e^{\alpha}_{\hat{\nu}}\,h^{\hat{\nu}}\,, \tag{20}$$

For any tensor h^{ν} and any n. P.S. the rank of a tensor can be any. Then the inertial tetrad is defined by

$$\frac{D\,e^{\alpha}_{\hat{\nu}}}{d\tau} = \frac{d\,e^{\alpha}_{\hat{\nu}}}{d\tau} + \Gamma^{\alpha}_{\beta\,\gamma}\,e^{\beta}_{\hat{\nu}}\,u^{\gamma} = 0\,. \tag{21}$$

The known is [5]

$$\frac{D^2\,n^{\alpha}}{d\tau^2} = -R^{\alpha}_{\mu\,\rho\,\nu}\,u^{\mu}\,u^{\nu}\,n^{\rho}\,. \tag{22}$$

Thus, when fixed $S^{\hat{0}} = 0$, holds the radial

$$\frac{d^2\,S^{\hat{1}}}{d\tau^2} = -e^{\hat{1}\,\alpha}\,R_{\alpha\,\mu\,\rho\,\nu}\,u^{\mu}\,u^{\nu}\,(e^{\rho}_{\hat{1}}\,S^{\hat{1}})\,, \tag{23}$$

which in case of Schwarzschild metric gives the

$$\frac{d^2 S^{\hat{1}}}{d\tau^2} = \frac{2\,M}{r^3}\,S^{\hat{1}}\,, \tag{24}$$

which exactly matches the Eq.(17).

III. THIRD METHOD: DENSITY FROM ENERGY-MOMENTUM

In this section non-zero pressure $p \neq 0$ is allowed.

Is known (from [5], pages 226–227, see Appendix B), that the rate of compression of a perfect fluid behaves as

$$\frac{d\rho}{d\tau} = -(\rho + p(\rho))\,u^{\mu}_{;\mu} \tag{25}$$

Here the $u^{\mu}_{;\mu}$ is the tensor of the zero rank – the scalar because the derivative in the 4-divergence is the covariant (one, which uses the Christoffel symbols). Case of the viscous fluid is in Appendix A.

If you insert the velocity u^{ν} into the divergence, you get to know, that $u^{\mu}_{;\mu} \sim 1/u^r \to -\infty$. For Schwarzschild Black Hole holds

$$D := u^{\mu}_{;\mu} = M\,\frac{4\,r - 3\,r_0}{\sqrt{2\,M\,r_0\,r^3\,(r_0 - r)}} \tag{26}$$

With the zero at $r = 3r_0/4$ as the start of the compression. At initial moment (i.e. $r = r_0$) the $D > 0$ and infinite (behaves like $1/\sqrt{r_0 - r}$), the density of the drop drops, but the integration is finite $\int (d\rho/d\tau)\,d\rho < \infty$. Then at $r = 3\,r_0/4$ the $D < 0$ and the drop begins to shrink. Notably, this happens at infinite distance from the Black Hole, if the r_0 is infinite. This effect does not fit into the intuition, where the gravity deviation forces are trying to rip apart the “falling astronaut body”. Such an unexpected result hardly can be found in Newton’s age, even while we still have a weak field at $r = (3/4)r_0 \gg 2M$. The deadly ripping $D \gg 0$ never begins, however at $r = 0$ the $D < 0$ and is infinite. At this moment the D behaves like $-1/r^{3/2}$, which integral is diverging at the curvature singularity $r = 0$.

IV. FOURTH METHOD: GEOMETRIC DENSITY CHANGE

In this section pressure $p = 0$.

Please note, what the azimuthal size of the dust cloud does shrink as $1/r$ while approaching the curvature singularity. This azimuthal contraction increases the density of the dust cloud as $1/r^2$, because the geometry shows $\rho \sim 1/(S^{\hat{1}}\, r^2)$. Then

$$\frac{d\rho}{d\tau} = \frac{d}{d\tau}\left(\frac{K}{S^{\hat{1}}(\tau)\, r^2(\tau)}\right). \tag{27}$$

Here the $K = \text{const}$.

From the Eqs.(13),(14),(27) with $M = 1/2$ has appeared

$$\frac{d\rho}{d\tau} = \rho\, \frac{3\, r_0 - 4\, r}{2\, \sqrt{r_0\, r^3\, (r_0 - r)}} + \psi\, \rho\, F(r)\,, \tag{28}$$

where $F(r)$ is certain function. That is exactly the Eqs.(25),(26) for $\psi = 0$ and $p(\rho) = 0$.

V. CONCLUSION

The points in this paper are proven now by four alternative approaches. Therefore, they are true and must be published. For a future paper one can notice, that point $r = 0$, where $\rho \to \infty$, becomes the point $r = r_m > 0$ for more general Black Hole metrics, as example: Kerr metric. [7] But there is no curvature singularity at $r = r_m$. The introduction of virtual matter heals latter inconsistency. Dark Matter is a class of virtual matter. Dark Energy is a class of Dark Matter.

VI. APPENDIXES

A. Appendix A: Navier-Stokes fluid singularity

In this section non-zero pressure $p \neq 0$ is allowed.

The energy-momentum tensor of viscose fluid was taken [5], and density rate appeared in

$$-\frac{d\rho}{d\tau} - (\rho + p)\, D + Z\, D^2 + \eta\, H = 0\,, \tag{29}$$

where $H := u^\alpha_{;\mu}\, u^\mu_{;\alpha}$ and $Z = \zeta - (2\eta/3)$. One can show, what $H = (u^\alpha\, u^\mu_{;\alpha})_{;\mu} - d\,D/d\tau \approx -d\,D/d\tau$, because a drop of fluid is moving along gedesic, because drop is small, therefore $u^\alpha\, u^\mu_{;\alpha} \approx 0$. So, looking at the perfect fluid solution, one could conclude, that the $H > 0$ in the vicinity of the central singularity. With this, if the $Z > 0$, then the catastrophic compression is inevitable. If the $Z < 0$ the density ρ still behaves as the $\exp|D|$ (looking at the perfect fluid case). Then, the D^2 term cannot cancel the compression. The main contribution to density is then

$$\rho \sim \exp\Big(\int \big(- D + \frac{Z\,D^2}{\rho} \big)\, d\tau \Big)\,. \tag{30}$$

In the case of the finite sum under integral, should be $\rho \to |Z\,D|$. However, the density diverges now, because it is known, what $D \to -\infty$.

B. Appendix B: density rate for perfect fluid

Consider a drop of "perfect fluid" falling into Black Hole. Because the drop is small, the velocity of every part of it is the velocity of the fall. The equation of matter is $T^{\mu\nu}_{;\nu} = 0$, thus $u_\mu\, T^{\mu\nu}_{;\nu} = 0$, where

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p\, g^{\mu\nu}\,. \tag{31}$$

Thus,

$$-(\rho + p)_{,\,\nu}\, u^\nu - (\rho + p)\, u^\nu_{;\,\nu} + (\rho + p)\, u^\nu\, u^\mu_{;\,\nu} u_\mu + p_{,\,\nu}\, u^\nu = 0\,, \tag{32}$$

where $u^\mu_{;\,\nu} u_\mu = 0$, because $(u^\mu u_\mu)_{;\,\nu} = (-1)_{;\,\nu} = 0$. We have $u^\nu = dx^\nu/d\tau$, then

$$-\frac{d(\rho + p)}{d\tau} - (\rho + p)\, u^\nu_{;\,\nu} + \frac{dp}{d\tau} = 0\,. \tag{33}$$

This has no solution, unless the fluid is compressible. Let the equation of state is $p = p(\rho)$, then

$$\frac{d\rho}{d\tau} = -(\rho + p(\rho))\, u^\nu_{;\,\nu}\,. \tag{34}$$

Now the rate (and sign) of density change depends on the $D := u^\nu_{;\,\nu}$.

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[1] I have invented the following definition of Nature: it is what the Standard Instruments do measure, and Instruments are what measure the Nature. To measure correctly the Instruments must be seen as invariants of Metrology, the unchangeables: any places, times, and universes in multiverse which have alien laws or different fundamental constants are not physical. Because the Instruments in those places would be changed.

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