# Some Problems About The Source Of Mass In The Electroweak Theory 

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#### Abstract

We review the electroweak theory to find out some noteworthy issues. In this theory, the Higgs mechanism makes the gauge bosons obtain their mass. If the vacuum states of the Higgs fields are the sources of mass for the massive gauge bosons $W$ and $Z$ even electron $e^{-}$, then this lowest energy $v$ of the Higgs field must be smaller than the Higgs boson of 125 GeV even smaller than the electron's rest mass of 0.511 MeV . It shall be like the zero-point energy of a linearly harmonic oscillator and those massive gauge bosons consist of many such lowest-energy quanta. However, substituting the weak coupling constant $g=0.77$ into the mass equation of the $W$ boson, it directly gives $v$ equal to 208 GeV much heavier than the Higgs boson and the similar results have been revealed in some textbooks about twenty years ago. Furthermore, the scalar Higgs boson is a charge-zero $(q=0)$ and spin-zero $(S=0)$ massive particle so the vacuum states of the Higgs fields have the same characteristics if they were treated as the lowest-energy quanta. However, the massive gauge bosons $W$ and $Z$ are all spin-1 $(S=1)$ particles and moreover, $W$ bosons are charged. Therefore, how to constitute those massive gauges bosons from the vacuum states of the Higgs fields becomes a questionable concept. On the other hand, due to the local gauge invariance, all mass terms have to be removed for fermions and the Yukawa coupling can provide their mass through the Higgs mechanism. It is also a similar problem that the fermion like electron is a spin-1/2 ( $S=1 / 2$ ) massive particle and how to constitute the mass of electron from the vacuum states like the spin-zero Higgs bosons is another serious problem. Those considerations cause seriously ponder whether the Yukawa coupling is the way to provide the mass of fermion? Especially, the electron-positron pair production from two photons directly tells us that the mass of electron and positron is much easier from the photon fields through the coupling above 1.02 MeV . Even for the scalar Higgs boson $H^{0}$, it can come from different parent particles.


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## I. Introduction

Quantum field theory (QFD) combines quantum mechanics, special relativity, and classical field theory to form concepts and tools for describing the characteristics of the high-energy particles [1-5]. In the early 1950s, based on the success of quantum
electrodynamics (QED), QFT was believed by many theorists that it could eventually describe and explain all microscopically physical phenomena, not just the interactions between electrons, positrons, and photons. However, the renormalization process cannot be used universally. All infinite values from the perturbation calculations in QED can be removed by redefining a few physical quantities but this method doesn't fit to many theories. In 1954, Chen-Ning Yang and Robert Mills generalized the local symmetry of QED to build the non-Abelian gauge theory, or the so-called Yang-Mills theory [1,2]. In 1961, Sheldon Glashow tried to combine the weak interaction with the electromagnetic interaction and established a theory [1,2], but it lacked the mechanism of spontaneous symmetry breaking. The Higgs mechanism was proposed by Peter Higgs in 1964 [1,2]. Its spontaneous-symmetry-breaking mechanism shows that the gauge symmetry in Yang-Mills theory can be broken. In 1967, Steven Weinberger and Abdus Salam built the unified theory of the weak and electromagnetic interactions based on the Yang-Mills field theory, and introduced the Higgs mechanism into Glashow's electroweak theory [1,2]. Thus, the electroweak theory was obtained, the form as we see nowadays. Weinberger further proposed that the mass of quarks and leptons can also be obtained from the vacuum states of the Higgs scalar fields. Therefore, the Higgs mechanism is widely believed to explain the mass sources of particles, including $W$ and $Z$ bosons, and fermions [1-5].

However, according to the success of QED, the electron-positron pair production from two photons seems to directly tell us that the mass of electron and positron is from the photon fields through the coupling between the electron field and the photon field $[1,2]$. The mass-energy equivalence also reveals the relation between energy and mass $[1,2,6]$. If the mass of electron comes from the vacuum states of the Higgs fields, then the electron-positron pair production and its inverse process cause our confusion. Therefore, we try to discuss the source of mass in the electroweak theory from different viewpoints in this paper.

## II. Review Of The Gauge Theory And Standard Model

In this section, we briefly review the gauge theory and the minimal standard model in which $\hbar=c=1$ are used in the most places. $\hbar$ is the reduced Planck's constant and $c$ is the speed of light in free space. Theoretically speaking, the gauge fields can proceed the gauge transformation by gauge groups. The Lagrangian is invariant under the gauge transformation. $\mathrm{SU}(3)$ is the gauge group for the strong interaction, and the electroweak interaction is described by the $\mathrm{SU}(2) \times \mathrm{U}(1)$ group. What is so-called the standard model is described by the $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ group [1-5]. In the $\mathrm{U}(1)$ Higgs mechanism, first considering the Klein-Gordon Lagrangian [3,5]

$$
\begin{equation*}
L=\left(\partial_{\alpha} \Phi\right)^{*}\left(\partial_{\alpha} \Phi\right)+V\left(\Phi^{*} \Phi\right), \tag{1}
\end{equation*}
$$

where $\Phi$ is the complex Higgs field and the mass term is removed due to the gauge invariance. $\mathrm{U}(1)$ gauge transformation involves phase transformation in which the transformation group is Abelian. $\Phi$ satisfies the gauge phase transformation [3-5]

$$
\begin{equation*}
\Phi \rightarrow \Phi^{\prime}=e^{-i \Lambda} \Phi \tag{2}
\end{equation*}
$$

where $\Lambda$ is the phase. The gauge theory requires Lagrangian having global gauge invariance after gauge transformation when $\Lambda$ is a constant. The prove is as follows

$$
\begin{equation*}
L \rightarrow L^{\prime}=\left[\partial_{\mu}\left(e^{-i \Lambda} \Phi\right)\right]^{*}\left[\partial^{\mu}\left(e^{-i \Lambda} \Phi\right)\right]+V\left(e^{i \Lambda} \Phi e^{-i \Lambda} \Phi\right)=L . \tag{3}
\end{equation*}
$$

When $\Lambda$ is a variable, in order to satisfy this requirement, the partial derivative $\partial_{\alpha}$ has to change to the covariant derivative $D_{\alpha}$, so we have [1-5]

$$
\begin{equation*}
D_{\mu} \equiv \partial_{\mu}+i q A_{\mu} \tag{4}
\end{equation*}
$$

where $A_{\alpha}$ is the gauge vector field and its local gauge transformation is [1-5]

$$
\begin{equation*}
A_{\mu} \rightarrow A_{\mu}^{\prime}+\partial_{\mu} \Lambda \tag{5}
\end{equation*}
$$

Hence, the Lagrangian possesses invariance shown as follows

$$
\begin{align*}
L \rightarrow L^{\prime}= & {\left[\left(\partial_{\mu}+i q A_{\mu}^{\prime}\right)\left(e^{i q \Lambda} \Phi\right)\right]^{*}\left[\left(\partial^{\mu}+i q A^{\mu^{\prime}}\right)\left(e^{i q \Lambda} \Phi\right)\right]+V\left(e^{-i q \Lambda} \Phi^{*} e^{i q \Lambda} \Phi\right) } \\
= & {\left[\left(\partial_{\mu}+i q A_{\mu}-i q \partial_{\mu} \Lambda\right)\left(e^{i q \Lambda} \Phi\right)\right]^{*}\left[\left(\partial^{\mu}+i q A^{\mu}-i q \partial^{\mu} \Lambda\right)\left(e^{i q \Lambda} \Phi\right)\right] } \\
& \quad+V\left(\Phi^{*} \Phi\right) \\
= & \left(D_{\mu} \Phi\right)^{*}\left(D^{\mu} \Phi\right)+V\left(\Phi^{*} \Phi\right)=L . \tag{6}
\end{align*}
$$

Furthermore, the local gauge invariance also requires additional Lagrangian describing this free-propagation gauge vector field, which is [3-5]

$$
\begin{equation*}
L_{p}=-\frac{1}{4} F_{\alpha \beta} F^{\alpha \beta}+\frac{1}{2} m^{2} A_{\alpha} A^{\alpha}, \tag{7}
\end{equation*}
$$

where [1-6]

$$
\begin{equation*}
F^{\alpha \beta}=\partial^{\alpha} A^{\beta}-\partial^{\beta} A^{\alpha} . \tag{8}
\end{equation*}
$$

In order to satisfy the local gauge invariance, the $m$-term has to be removed as mentioned previously. The new Lagrangian satisfying the local gauge invariance is [15]

$$
\begin{equation*}
L=\left(D_{\mu} \Phi\right)^{*}\left(D^{\mu} \Phi\right)-\frac{1}{4} F_{\alpha \beta} F^{\alpha \beta}-V\left(\Phi^{*} \Phi\right) . \tag{9}
\end{equation*}
$$

It means that all gauge bosons describing here are zero mass which are correct for the photon in the electromagnetic interaction and the gluon in the strong interaction but not for $W^{+}, W^{-}$, and $Z$ bosons. Therefore, the Higgs mechanism is used on a larger
symmetrical group to solve this unsatisfied problem.
Next, the $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge transformation is reviewed in the electroweak theory. In the section, we mainly follow the contents in Ref. 3. In the first generation of the lepton section, it consists of a left-handed doublet of fermion [1-5]

$$
\begin{equation*}
\Psi_{L}=\binom{\Psi_{v_{L}}}{\Psi_{e_{L}}}=\frac{1}{2}\left(1-\gamma^{5}\right)\binom{\Psi_{e_{v}}}{\Psi_{e}}, \tag{10}
\end{equation*}
$$

and a right-handed Fermi singlet [1-5]

$$
\begin{equation*}
\Psi_{R}=\frac{1}{2}\left(1+\gamma^{5}\right) \Psi_{e} . \tag{11}
\end{equation*}
$$

The Lagrangian also consists of a doublet of complex scalars, three $A_{\mu}^{i}$ gauge bosons related to the $\mathrm{SU}(2)$ symmetry, and one $B_{\mu}$ gauge boson related to the $\mathrm{U}(1)$ symmetry. The lepton and scalar doublets follow the transformation [3-5]

$$
\begin{equation*}
\Psi \rightarrow \Psi^{\prime}=\underbrace{e^{i g \frac{1}{2} \sigma_{i} \alpha_{i}}}_{U_{2}} \underbrace{e^{i g^{\prime} \frac{1}{2} \beta \mathbf{Y}}}_{U_{1}} \Psi \tag{12}
\end{equation*}
$$

where $i$ runs from 1 to $3, g$ and $g^{\prime}$ are respectively two coupling constants for the $\mathrm{SU}(2)$ and $\mathrm{U}(1)$ gauge groups, $\alpha_{i}$ and $\beta$ are independent rotation angles, $\sigma_{i}$ are Pauli matrices, and $\mathbf{Y}$ is the hypercharge operator [3-5]. Un-similar to the doublet, the gauge transformation for this singlet is

$$
\begin{equation*}
\Psi_{R}^{\prime}=e^{-i \frac{1}{2} g^{\prime} \beta \mathbf{Y}} \Psi_{R}=U_{1} \Psi_{R} . \tag{13}
\end{equation*}
$$

The gauge transformations for the gauge fields are [3-5]

$$
\begin{equation*}
A_{\mu}^{\prime i}=A_{\mu}^{i}+\partial_{\mu} \alpha_{i}-g \epsilon_{i j k} A_{\mu}^{j} \alpha_{k}, \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
B_{\mu}^{\prime}=B_{\mu}+\mathbf{Y} \partial_{\mu} \beta \tag{15}
\end{equation*}
$$

The covariant derivative now is [3-5]

$$
\begin{equation*}
\vec{D}_{\mu}=\vec{\partial}_{\mu}+i g \mathbf{A}_{\mu}+i \frac{1}{2} g^{\prime} B_{\mu} \tag{16}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathbf{A}_{\mu}=\frac{1}{2} \sigma_{i} A_{\mu}^{i} . \tag{17}
\end{equation*}
$$

The arrow " $\rightarrow$ " means the derivative acting on the right and " $\leftarrow$ " means the derivative
acting on the left. The double arrow " $\leftrightarrow$ " means the derivative acting on both sides and equal to right minus left. If we define a gauge field tensor [3-5]

$$
\begin{equation*}
\mathbf{F}_{\mu \nu}=D_{\mu} \mathbf{A}_{v}-D_{\nu} \mathbf{A}_{\mu}, \tag{18}
\end{equation*}
$$

then it satisfies gauge invariance under the gauge transformation. This gauge field tensor has similar definition which is [3-5]

$$
\begin{equation*}
\mathbf{F}_{\mu \nu}=\frac{1}{2} \sigma_{i} F_{\mu \nu}^{i} . \tag{19}
\end{equation*}
$$

It gives the electromagnetic type Largrangian [3-5]

$$
\begin{equation*}
L_{f i e l d}=-\frac{1}{4} F_{\mu \nu}^{i} F^{i \mu \nu}=-\frac{1}{2} \operatorname{tr}\left(\mathbf{F}_{\mu \nu} \mathbf{F}^{\mu v}\right) \tag{20}
\end{equation*}
$$

by using the relation

$$
\begin{equation*}
\operatorname{tr}\left(\sigma_{i} \sigma_{j}\right)=\delta_{i j} . \tag{21}
\end{equation*}
$$

Similarly, the gauge field tensor for $B_{\mu}$ associated with $\mathrm{U}(1)$ symmetry is [3-5]

$$
\begin{equation*}
B_{\mu \nu}=\partial_{\mu} B_{v}-\partial_{v} B_{\mu} . \tag{22}
\end{equation*}
$$

Therefore, the unbroken Largrangian is [3-5]

$$
\begin{equation*}
L=L_{\text {lepton }}+L_{\text {field }}+L_{\text {scalr }}+L_{\text {int }}, \tag{23}
\end{equation*}
$$

where

$$
\begin{align*}
& L_{\text {lepton }}=\bar{\psi}_{L}\left(\frac{i}{2} \gamma^{\mu} \overleftrightarrow{D}_{\mu}\right) \psi_{L}+\bar{\psi}_{R}\left(\frac{i}{2} \gamma^{\mu} \overleftrightarrow{D}_{\mu}\right) \psi_{R},  \tag{24}\\
& L_{\text {field }}=-\frac{1}{2} \operatorname{tr}\left(\mathbf{F}_{\mu \nu} \mathbf{F}^{\mu \nu}\right)-\frac{1}{4} B_{\mu \nu} B^{\mu \nu}  \tag{25}\\
& L_{\text {scalar }}=\left(D_{\mu} \Phi^{\dagger}\right)\left(D^{\mu} \Phi\right)-m^{2}|\Phi|^{2}-\lambda^{2}|\Phi|^{4}, \tag{26}
\end{align*}
$$

and

$$
\begin{equation*}
L_{i n t}=-G_{e}\left[\bar{\Psi}_{R}\left(\Phi^{\dagger} \Psi_{L}\right)+\left(\bar{\Psi}_{L} \Phi\right) \Psi_{R}\right] . \tag{27}
\end{equation*}
$$

After spontaneous symmetry breaking, the energy density has a minimum when[1-5]

$$
\begin{equation*}
|\Phi|=\frac{v}{\sqrt{2}}=\frac{1}{\sqrt{2}} \sqrt{-\frac{m^{2}}{\lambda^{2}}} \tag{28}
\end{equation*}
$$

where $v$ is determined from a minimum of the meson Hamiltonian from the linear sigma model in the long-wavelength limit [3-5], which is

$$
\begin{equation*}
H_{\Phi}=V\left(\Phi^{*} \Phi\right)=\frac{1}{2} m^{2}|\Phi|^{2}+\frac{1}{4} \lambda^{2}|\Phi|^{4} . \tag{29}
\end{equation*}
$$

This lowest-energy state $\Phi$ is called the vacuum state of the Higgs field. The fields are now constrained and they are defined by the choice of gauge. Due to the constraint, the gauge invariance is spontaneously broken. In this $\mathrm{SU}(2) \times \mathrm{U}(1)$ model, the Higgs field is a complex doublet state. Now, the doublet state is represented by [3-5]

$$
\begin{equation*}
\Phi=\frac{1}{\sqrt{2}}\binom{\varphi_{1}+i \varphi_{2}}{v+H+i \varphi_{3}} . \tag{30}
\end{equation*}
$$

Adapting the unitary gauge $\hat{\mathrm{U}}$ to transform $\Phi$ to a new state $\Phi_{0}$, in which $\varphi_{1}=\varphi_{2}=\varphi_{3}=0$, so we have [3-5]

$$
\begin{equation*}
\Phi^{\prime}=\widehat{U} \Phi=\Phi_{0}=\frac{1}{\sqrt{2}}\binom{0}{v+H} . \tag{31}
\end{equation*}
$$

According to this transformation, the other fields become [3]

$$
\begin{gather*}
\psi_{L}^{\prime}=\hat{\mathrm{U}} \psi_{L},  \tag{32}\\
\psi_{R}^{\prime}=\hat{\mathrm{U}} \psi_{R},  \tag{33}\\
\mathbf{A}_{\mu}^{\prime}=\hat{\mathrm{U}} \mathbf{A}_{\mu} \hat{\mathrm{U}}^{\dagger}+\frac{i}{g}\left(\partial_{\mu} \hat{\mathrm{U}}\right) \hat{\mathrm{U}}^{\dagger}, \tag{34}
\end{gather*}
$$

and

$$
\begin{equation*}
B_{\mu}^{\prime}=B_{\mu}+\frac{i}{g^{\prime}}\left(\partial_{\mu} \hat{\mathrm{U}}\right) \hat{\mathrm{U}}^{\dagger} . \tag{35}
\end{equation*}
$$

However, the new Lagrangian after the gauge transformation by using above new fields is no longer gauge invariance because the gauge is broken by replacing $\Phi$ with $\Phi_{0}$. But there is still another gauge transformation keeping the Lagrangian gauge invariance. The following local gauge transformation,

$$
\begin{equation*}
U_{Q}=e^{-i e f\left(\frac{1}{2} \tau_{3}+\frac{1}{2} \mathbf{Y}\right)} \tag{36}
\end{equation*}
$$

can keep the gauge invariance where $f$ is a function of coordinates. The new gauge fields $\left(A_{\mu}, Z_{\mu}\right)$ has a connection with the original fields $\left(A_{\mu}^{3}, B_{\mu}\right)$ through this relation [1-5]

$$
\binom{Z_{\mu}}{A_{\mu}}=\left(\begin{array}{rr}
\cos \theta_{W} & -\sin \theta_{W}  \tag{37}\\
\sin \theta_{w} & \cos \theta_{w}
\end{array}\right)\binom{B_{\mu}}{A_{\mu}^{3}} .
$$

This relation also gives [1-5]

$$
\begin{equation*}
e=\frac{g g^{\prime}}{\sqrt{g^{2}+g^{\prime 2}}}=g \sin \theta_{w}=g^{\prime} \cos \theta_{W}=\sqrt{2} g_{W} \sin 2 \theta_{W} \tag{38}
\end{equation*}
$$

where $\theta_{W}$ is the missing angle and $g_{W}$ is a convenient parameter. At the same time, the charged vector boson fields are defined as [3-5]

$$
\begin{equation*}
W_{\mu}^{+}=\frac{1}{\sqrt{2}}\left(A_{\mu}^{1}-i A_{\mu}^{2}\right) \tag{39}
\end{equation*}
$$

and

$$
\begin{equation*}
W_{\mu}^{-}=\frac{1}{\sqrt{2}}\left(A_{\mu}^{1}+i A_{\mu}^{2}\right) \tag{40}
\end{equation*}
$$

Due to such transformation $U_{Q}$, the mass terms of the bosons $W$ and $Z$ are defined as [3-5]

$$
\begin{equation*}
M_{W}^{2}=\frac{g^{2}}{4} v^{2} \tag{41}
\end{equation*}
$$

and

$$
\begin{equation*}
M_{z}^{2}=\frac{g^{2}}{4 \cos ^{2} \theta_{W}} v^{2} \tag{42}
\end{equation*}
$$

Then the new Lagrangian after this local gauge transformation becomes several parts [3-5]

$$
\begin{align*}
L_{\text {kinetic }}^{E W}=\bar{\Psi}_{e} & {\left[\frac{i}{2}\left(\vec{\partial}_{\mu}-\grave{\partial}_{\mu}\right) \gamma^{\mu}-m_{e}\right] \Psi_{e}-\frac{1}{4} F_{\mu \nu} F^{\mu v}-\frac{1}{4} Z_{\mu \nu} Z^{\mu \nu}-\frac{1}{2} W_{\mu \nu}^{+} W^{-\mu \nu} } \\
& +\frac{1}{2} m_{Z}^{2} Z_{\mu} Z^{\mu}+m_{W}^{2} W_{\mu}^{+} W^{-\mu}+\frac{1}{2}\left(\partial_{\mu} H\right)\left(\partial^{\mu} H\right)-\frac{1}{2} m_{H}^{2} H^{2} \tag{43}
\end{align*}
$$

$L_{N}^{E W}=\bar{\Psi}_{e}\left[e \gamma^{\mu} A_{\mu}+\sqrt{2} g_{W} \gamma^{\mu}\left(-2 \sin ^{2} \theta_{W}+\frac{1-\gamma^{5}}{2}\right) Z_{\mu}\right] \Psi_{e}$
$=e J_{E M}^{\mu} A_{\mu}+\frac{g}{\cos \theta_{W}}\left(J_{Z}^{\mu}-2 \sin ^{2} \theta_{W} J_{E M}^{\mu}\right) Z_{\mu}$,
$L_{C}^{E W}=-\frac{g}{\sqrt{2}}\left[\bar{\Psi}_{e} \gamma^{\mu}\left(\frac{1-\gamma^{5}}{2}\right) \Psi_{\nu} W_{\mu}^{+}+\right.$h.c. $]$,
$L_{e_{\nu}}^{E W}=\bar{\Psi}_{e_{\nu}} \gamma^{\mu}\left[\frac{i}{2}\left(\vec{\partial}_{\mu}-\bar{\partial}_{\mu}\right)-\sqrt{2} g_{W} Z_{\mu}\right] \Psi_{e_{\nu}}$,
$L_{H}^{E W}=-\frac{g m_{H}^{2}}{4 M_{W}} H^{3}-\frac{g^{2} m_{H}^{2}}{32 M_{W}^{2}} H^{4}$,

$$
\begin{align*}
& L_{H V}^{E W}=g M_{W} W_{\mu}^{+} W^{-\mu}\left(H+\frac{g}{4 M_{W}} H^{2}\right)+\frac{1}{2} \frac{g M_{Z}}{\cos \theta_{W}} Z_{\mu} Z^{\mu}\left(H+\frac{g}{4 M_{Z} \cos \theta_{W}} H^{2}\right),  \tag{48}\\
& L_{3 g}^{E W}=i W^{+\mu} W^{-v}\left[e F_{\mu \nu}+g \cos \theta_{W} Z_{\mu \nu}\right]+i\left(e A^{v}+g \cos \theta_{W} Z^{v}\right)\left[W_{\mu \nu}^{+} W^{-\mu}-\right. \\
& \left.W_{\mu \nu}^{-} W^{+\mu}\right] \text {, }  \tag{49}\\
& L_{4 g}^{E W}=\frac{1}{2} g^{2}\left[W_{\mu}^{+} W^{+\mu} W_{v}^{-} W^{-v}-W_{\mu}^{+} W_{v}^{-} W^{+\nu} W_{\mu}^{-}\right] \\
& -g^{2} \cos ^{2} \theta_{W}\left[W_{\mu}^{+} W^{-\mu} Z_{v} Z^{v}-W_{\mu}^{+} W_{v}^{-} Z^{v} Z^{\mu}\right] \\
& -e g \cos \theta_{W}\left[2 W_{\mu}^{+} W^{-\mu} A_{v} Z^{v}-W_{\mu}^{+} W^{-v}\left(A^{v} Z^{\mu}+A^{\mu} Z^{v}\right)\right] \\
& +e^{2}\left[W_{\mu}^{+} W^{-\mu} A_{v} A^{v}\right. \\
& \left.+W_{\mu}^{+} W_{v}^{-} A^{\mu} A^{\nu}\right], \tag{50}
\end{align*}
$$

and the Yukawa coupling [3-5]

$$
\begin{equation*}
L_{\text {Yukawa }}^{E W}=-\frac{g}{2} \frac{m_{e}}{M_{W}} \bar{\Psi}_{e} \Psi_{e} H . \tag{51}
\end{equation*}
$$

$m_{H}$ in $L_{H}^{E W}$ is the mass of the Higgs boson, and the electromagnetic current and neutral weak current are respectively [3-5]

$$
\begin{equation*}
J_{E W}^{\mu}=\bar{\Psi}_{e} \gamma^{\mu} \Psi_{e} \tag{52}
\end{equation*}
$$

And [3-5]

$$
\begin{equation*}
J_{Z}^{\mu}=I_{Z}^{e} \bar{\Psi}_{e}\left(\frac{1-\gamma^{5}}{2}\right) \Psi_{e} \tag{53}
\end{equation*}
$$

where $I_{Z}^{e}=1 / 2$ is the weak isospin of electron. The Yukawa coupling also gives the relation [3]

$$
\begin{equation*}
m_{e}=\frac{G_{e} v}{\sqrt{2}} . \tag{54}
\end{equation*}
$$

Because the gauge theory doesn't permit fermions possessing mass, their mass is thought to come from the vacuum states of the Higgs fields due to the Yukawa coupling. However, such results exist some basic problems pointed out in the next section.

## III. Some Problems About The Source of Mass In The Electroweak Theory

When we study the classical electrodynamics, the Lorentz force clearly tells us that a particle of charge $q$ like electron can be accelerated or decelerated by the electromagnetic fields. The rate of change of energy equation in covariant form is [6]

$$
\begin{equation*}
\frac{d p^{\alpha}}{d \tau}=m \frac{d U^{\alpha}}{d \tau}=\frac{q}{c} F^{\alpha \beta} U_{\beta} \tag{55}
\end{equation*}
$$

where $U^{\alpha}$ is the four velocity, $p^{\alpha}$ is the four-vector momentum, $\tau$ is the proper time, and $F^{\alpha \beta}$ is the second-rank antisymmetric field-strength tensor defined previously. The equations of motion in the ( $\vec{x}, t$ ) coordinates are [6]

$$
\begin{equation*}
\frac{d \vec{p}}{d t}=q\left[\vec{E}+\frac{\vec{u}}{c} \times \vec{B}\right] \tag{56}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d E}{d t}=q \vec{u} \cdot \vec{E} . \tag{57}
\end{equation*}
$$

According to the special relativity, the mass-energy equivalence explicitly reveals that the charged particles like electrons increase their total energy as well as their relativistic mass when their speeds raise [6]. The famous energy-mass equivalent equation is

$$
\begin{equation*}
E=\gamma m_{e} c^{2}, \tag{58}
\end{equation*}
$$

where $\gamma$ is the Lorentz factor and $\gamma m_{e}$ is the relativistic mass, reveals how the total energy varies with the velocity of a particle. Therefore, classical electrodynamics clearly tells us that the electromagnetic fields ( $\vec{E}, \vec{B}$ ) and the corresponding scalar and vector potentials $(\Phi, \vec{A})$ have something to do with the mass of electrons.

Furthermore, in quantum electrodynamics (QED), when we consider the electropositron pair annihilation as shown in Fig. 1(a) or the electron-positron pair production as shown in Fig. 1(b), both results directly tell us that the relation between electron field $\psi_{e}$ and photon field $A^{\mu}$. The missing photon directly becomes to an electron and a positron in which the process is clearly described in QED. If electron and positron come from the vacuum states of the Higgs fields, not from the incident photon, then the disappeared photon also means the missing energy which directly violate the massenergy equivalence. On the other hand, the successful searches of Higgs bosons at ATLS and CMS [7], one of the rare processes about a neutral Higgs boson decaying to two photons is [2,7-9]

$$
\begin{equation*}
H^{0} \rightarrow \gamma+\gamma \tag{59}
\end{equation*}
$$

where $H^{0}$ is the neutral Higgs boson and the Feynman diagram associated with the parts of vertices $[2,8,9]$ as shown in Fig. 1(c). In those vertices, $H^{+}$and $H^{-}$are charged Higgs bosons. This process tells us the interaction between the Higgs field and photon field. The average mass of $H^{0}$ is fit to $125.10 \pm 0.14 \mathrm{GeV} / c^{2}$ [7] and the process in Fig. 1(c) directly transfers the mass energy of $H^{0}$ into two photon energy which just proves the mass-energy equivalence.

(a)

(b)

(c)

Figure 1. (a) The electron-positron pair production describing in QED. (b) Two-photon production by the electron-positron pair annihilation also describing in QED. (c) The neutral Higgs boson decays to two photons including the parts of vertices [8,9]. The forbidden interaction between the photon field and the Higgs field.

Next, we have to check the physical properties between electron and the vacuum state of the Higgs field. As we know, the Higgs boson is the scalar boson predicted in the SM, which is charge-zero $(q=0)$ and spin-zero $(S=0)$ particle [1,5, 7-9]. The lowest energy state of the Higgs field shall have the same characteristics. When we look at the equation describing the relation between $m_{e}$ and $v$, it causes our curiosity about how the lowest Higgs bosons constitute a charged fermion like electron? It makes us wonder whether the following constitution is possible?

$$
\begin{equation*}
\sum v(q=0, S=0) \cdots \cdots \cdots \cdots m_{e}\left(q=-\cdots,-\cdots=\frac{1}{2}\right) \tag{60}
\end{equation*}
$$

Furthermore, the finding of the Higgs bosons seems to give the neutral and charged gauge bosons $W^{+}, W$, and $Z^{0}$ a reasonable explanation of their mass source. However, all those gauge bosons have $S=1$, and it also causes our curiosity about how the spin- 0 Higgs bosons can constitute the spin-1 gauge bosons? It also makes us wonder whether the following constitution is possible?

If the massive particles obtain their mass from the vacuum states of the Higgs fields, their physical properties shall also determine such as the charge and spin at the same time. However, the vacuum states of the Higgs bosons have no ability for determine these physical quantities. Especially, the lifetime of electron approaches to infinite and the mass of electron is very stable. The vacuum states of the Higgs fields are more likely the energy fluctuation in space. In the Higgs mechanism, the vacuum state of the Higgs field is the lowest energy state that is similar to the zero-point energy of the harmonic oscillator. The eigen-energy of the linearly harmonic oscillator is [3]

$$
\begin{equation*}
E_{n}=\left(n+\frac{1}{2}\right) \hbar \omega \tag{62}
\end{equation*}
$$

where $n=0,1,2, \ldots$ and $\omega$ is the oscillating frequency. If the mass of electron comes from the vacuum states of the Higgs fields, then it shall constitute some lowest-energy quanta $E_{\text {lowest }}$ which is

$$
\begin{equation*}
m_{e} c^{2}=N E_{\text {lowest }} \tag{63}
\end{equation*}
$$

where $N$ is an integer. Therefore, we can limit the maximum of the lowest energy by considering this value less than the lightest elementary particle so we have

$$
\begin{equation*}
E_{\text {lowest }}<m_{e} c^{2}=0.511 \mathrm{MeV} . \tag{64}
\end{equation*}
$$

Then we might ask that is it possible to exist inner structure inside electron like quarks and gluons consisting of proton and neutron? According to this, the mass of the gauge bosons $W$ and $Z$ shall be integer times $E_{\text {lowest }}$ and their mass ratios shall be fractional, which are

$$
\begin{equation*}
\frac{M_{W}}{M_{z}}=\cos \theta_{W} \tag{65}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{m_{e}}{M_{W}}=\frac{\sqrt{2} G_{e}}{g} \tag{66}
\end{equation*}
$$

Furthermore, it requires the conditions

$$
\begin{equation*}
\frac{g}{2}>1 \& \frac{G_{e}}{\sqrt{2}}>1 . \tag{67}
\end{equation*}
$$

However, the $g$ value is 0.66 reported in Ref. 1 and 0.77 calculated in Ref. 3. If we substitute the latter into the mass equation of $W$ boson, it gives

$$
\begin{equation*}
v=\frac{2 M_{W}}{g}=208.31 \mathrm{GeV}! \tag{68}
\end{equation*}
$$

The mass of $W$ boson is 80.2 GeV used here [1-5]. The other values are reported 248 GeV [10] and 250 GeV [11]. It is even heavier than the Higgs boson. It is an unbelievable value for the prediction in the electroweak theory! What so called the lowest energy is larger than the excited energy, the Higgs boson. This energy is not a reference energy because we really treat the Higgs boson with energy 125 GeV . If it is a reference energy and all particles are higher than it, then we have to face the problems describing in Eqs. (60) and (61). The lowest energy of the Higgs field higher than the Higgs boson is very an unreasonable result and it makes the initial assumption failed.

Even in the possible process transferring from a photon to the $Z$ boson as shown in Fig. 2, it seems that the electron-positron pair have something to do with this process but it still has some problems. First, this is a higher-order process which means it hard to happen. Second, once this process dominates the final source of the electron-positron pair largely from the $Z$ gauge boson, it means that the necessary correction in QED even the Maxwell's equations in classical electrodynamics. However, the experiments to verify the predictions in QED have proven the highly accurate calculations in QED like the gyromagnetic ratio [3]. Therefore, even the Feynman diagram in Fig. 2 is reasonable, it only contributes very slight part in the electron-positron pair production.

When two photons are directly produced from the electron-positron annihilation, then the Higgs fields will not be the intermediate state as shown in Fig. 1(b). As mentioned previously, the vacuum state of the Higgs field seems to more likely provide energy fluctuation due to the Heisenberg's uncertainty principle if its energy is very low. It is almost impossible to provide a stable source of mass for electron with infinite lifetime. $W$ and $Z$ gauge bosons, exist in a very short time, not long-term stable particles, so their mass coming from the vacuum states of the Higgs fields seem to be reasonable without mentioning the spin problem. It is because the characteristic of the vacuum state provides energy fluctuation and the energy gathering in a tiny space within a very short time becomes possible.


Figure 2. The possible Feynman diagram for two photons through the electron bubble and $Z$ boson intermediates to become to an electron-positron pair.

## IV. The New Explanation About The Source Of Mass

According to the above discussions, first thing is to re-explain the Yukawa coupling from the Lagrangian. Because of the local gauge invariance, all the mass terms in original Lagrangian have to be removed. At the moment, the Yukawa coupling is added to make fermion have mass through the Higgs mechanism which has been shown in the previous section. However, the spin- $1 / 2$ massive particle shall not constitute by the spin-0 bosons as mentioned before, so the Yukawa coupling in the interaction part of the Lagrangian is no longer in charge of the source of the fermion mass. Then we may ask what the meaning for the coupling between the electron and Higgs fields is? The fundamental vertex of this coupling is shown in Fig. 3(a). When we consider the twoelectron scattering event by exchanging a virtual Higgs boson [2], the Feynman diagram is shown in Fig. 3(b). This coupling of the Higgs boson to electron doesn't mean the
electron mass coming from the vacuum states of the Higgs fields even the mass equation has something to do with $v$. We have discussed that $v$ shall be smaller than the electron mass, 0.511 MeV , but the calculation shows it much heavier than the Higgs boson. However, some report [2,7,12] shows a channel that a Higgs boson finally decays to four leptons

$$
\begin{equation*}
H^{0} \rightarrow Z+Z^{*} \rightarrow 2 l^{-}+2 l^{+}(l=\text { electron, muon }) \tag{69}
\end{equation*}
$$

if this neutral scalar particle is really the Higgs boson. The corresponding Feynman diagram is shown in Fig. 3 (c) where the lepton is electron. The fraction of the experimental results for such process which the final products are four electrons is [7]

$$
\begin{equation*}
\frac{\Gamma_{i}}{\Gamma}<1.9 \times 10^{-3} . \tag{70}
\end{equation*}
$$

The fraction is very small that means such process is a rare process. Even the Higgs boson can decay to two or four leptons, this thing takes place very rarely. If the mass term in Eq. (51) is changed from $M_{W}$ to $M_{z}$,

$$
\begin{equation*}
L_{\text {Yukawa }}^{E W}=-\sqrt{2} g_{W} \frac{m_{e}}{M_{Z}} \bar{\Psi}_{e} \Psi_{e} H, \tag{71}
\end{equation*}
$$

then it would make something more reasonable because the neutral gauge bosons $Z^{0}$ take part in the process in Eq. (69) or Fig. 3 (c). In Addition, the ratio of $m_{e}$ to $M_{Z}$ makes the quantity $v$ disappear. However, this decay shows four electrons originating from the scalar Higgs bosons $H^{0}$, not the vacuum states of the Higgs fields, so the vacuum states of the Higgs fields are not the source of electrons in this case.

In fact, the Higgs boson can be generated from several processes and some of the Feynman diagrams are drawn in Fig. 4. The annihilation of the electron-positron pair can generate the neutral gauge boson $Z^{0}$ and reveals a Higgs boson $H^{0}$ as shown in Fig. 4(a), which is the reaction

$$
\begin{equation*}
e^{-}+e^{+} \rightarrow Z^{0} \rightarrow Z^{0}+H^{0} \tag{72}
\end{equation*}
$$

It is also a way to generate a Higgs boson $H^{0}$ through the interaction between $u$ and $d$ quarks as shown in Fig. 4(b). Both processes produce the Higgs bosons through the weak interaction. The third way to produce Higgs bosons at ATLAS is through the collisions of ultra-high energy protons at 7 and 8 TeV [12,13] which are described in Eqs. (59) and (69), respectively. Eq. (59) is the event happened through gluon fusion in $p p$ collisions as shown in Fig. 4(c) where $p$ is proton, $g$ is gluon, $t$ and $\bar{t}$ are top and anti-top quarks, and $b$ and $\bar{b}$ are bottom and anti-bottom quarks. Those figures reveal that even the scalar Higgs boson $H^{0}$, it can come from different parent particles
involving different interactions, so the source of mass is not fixed for $H^{0}$.


Figure 3. (a) The fundamental vertex of direct coupling between the electron and Higgs boson. (b) The Feynman diagram of the interaction between two electrons through the intermediate Higgs boson. (c) The Feynman diagram for the process decaying from the Higgs field to the final four leptons such as electrons [7].


Figure 4. (a) The Feynman diagram for the production of the Higgs boson from the $Z$ gauge boson generated from the annihilation of the electron-positron pair [2]. (b) The Feynman diagram for the production of the Higgs boson from two quarks through the weak interaction [2]. (c) The corresponding Feynman diagram in the biphoton decay from the Higgs boson producing by gluon fusion in two-proton collisions [13] where $p$ is proton, $g$ is gluon, $t$ and $\bar{t}$ are top and anti-top quarks, and $b$ and $\bar{b}$ are bottom and anti-bottom quarks.

Next, we think about a reasonable term in the Lagrangian to make sure that the mass of electrons as well as the muon $\mu$ and pion $\pi$ comes from the photon fields, that is,

$$
\begin{equation*}
L_{\text {pair production }}^{E W}=e \bar{\psi}_{e} \gamma^{\mu} \psi_{e} A_{\mu}=e J_{E M}^{\mu} A_{\mu} \tag{73}
\end{equation*}
$$

which is the electromagnetic current in the electroweak theory and has been already
mentioned in QED. Due to this term, the total scattering cross section can be calculated in QED. The coupling between the electron field and photon field also directly reveals the energy transfer from electron and positron to two photons and vice versa as shown in Figs. 1(a) and (b). It also the obvious verification of the mass-energy equivalence. These two cases don't need the Higgs fields to participate in, and mass and energy can transfer to each other as the Lorentz force performs in the classical electrodynamics.

## V. What Problems We Have To Face In The Electroweak Theory

In the gauge theory, the mass terms have to be removed because of the requirement of the gauge invariance. Hence, introducing the Higgs mechanism through the spontaneous symmetry breaking on the vacuum states seems to open a door for the losing term in the Lagrangian. However, this model assumes the vacuum state to be the lowest-energy state and the Higgs boson to be the excited state. How large the vacuum state is doesn't appear in the gauge theory, it can only be deduced through some parameters like the mass of the $W$ gauge boson and the weak coupling constant $g$ in Eq. (41). Through the experimental data, the energy of the vacuum state $v$ is higher than 200 GeV even twice the Higgs boson! This is the first problem that we have to face. Can we avoid it by using the form in Eq. (66)? In that, the relation between the mass of electron $m_{e}$ and the vacuum state of Higgs field $v$ is subtly replaced with $g_{w}\left(m_{e} / M_{W}\right)$. Actually, the vacuum state of the Higgs boson has its physical role in the Higgs mechanism. Such deal mathematically avoids the existence of $v$ but loses some physical meaning. The roles of the spontaneous symmetry breaking are just the vacuum states of the Higgs fields. Without them, the spontaneous symmetry breaking loses meaning and many particles cannot obtain mass through the Higgs mechanism in the electroweak theory as well as in the standard model. Therefore, it is not a right way to avoid the existence of the vacuum states in this theory because these states are the basis of this theory.

Next, the thing we have to consider is whether the scalar particle at 125 GeV is the Higgs boson or not? If it were true, then it is at the energy lower than the vacuum state of the Higgs field. If it were not true, then we have to continue to find the right Higgs boson at higher energy more than 250 GeV at lease. The second problem is whether the vacuum state of the Higgs field and the Higgs boson more than 250 GeV are reasonable or not?

If the vacuum state of the Higgs field is very low, it would be at the level as we know about the vacuum state. Then the third problem is whether the scalar and uncharged vacuum states of the Higgs fields can be the mass sources of the spin-1 gauge bosons and spin-1/2 fermions or not? This question has been appeared in Eqs. (60) and (61)
and it involves how to introduce the source of mass in the gauge theory like the Yukawa coupling for leptons. In Fig. 4, we see some channels to produce the scalar gauge boson $H^{0}$ and in Fig. 1(a) we see an easier way to produce the electron-positron pair through two photons only 1.02 MeV . The vacuum states of the Higgs fields as the source of mass from seem not to be the reasons for the above cases.

## VI. Conclusions

In the electroweak theory, the Higgs mechanism makes the gauge bosons obtain their mass. We review this theory and find out some noteworthy issues. First, the lowest energy $v$ at the vacuum state of the Higgs bosons provide the sources of mass re the sources of mass for the massive gauge bosons $W$ and $Z$ even electron $e^{-}$. Their mass can have relations with $v$ accompanying with a coupling constant. According to the mass equation, substituting the weak coupling constant $g=0.77$ into the $W$ boson, it gives 208 GeV much heavier than the Higgs boson, 125 GeV . This is a very confused value because this lowest energy of the Higgs field must be smaller than the Higgs boson of 125 GeV even smaller than the electron's rest mass of 0.511 MeV . It shall be like the zero-point energy of a linearly harmonic oscillator and those massive gauge bosons consist of many such lowest-energy quanta. If the lowest-energy state is higher than the excited state, the Higgs boson of 125 GeV , then the basis of the concept about the source of mass loses its rightness.

Second, the scalar Higgs boson is a massive particle with $q=0$ and $S=0$ so the vacuum states of the Higgs fields have the same characteristics if they were treated as the lowestenergy quanta. However, the massive gauge bosons $W$ and $Z$ are all particles with $S=1$ and especially, $W$ bosons are charged. Therefore, how to constitute those massive gauges bosons from the vacuum states of the Higgs fields becomes a questionable concept. The local gauge invariance requires the mass term to be necessarily removed for all fermions and the Yukawa coupling can provide their mass through the Higgs mechanism. The similar problem is that the fermion like electron is a spin- $1 / 2$ massive particle and how to constitute the mass of electron from the vacuum states of the Higgs fields is another serious problem. Those considerations cause us think about whether the Yukawa coupling is the way to provide the mass of fermion?

Third, the electron-positron pair production from two photons directly tells us that the mass of electron and positron is from the photon fields through the coupling. According to the success of QED, the process describing in Fig. 2 is very few so it is also an obvious case that the mass of electron is reasonable from the photon field, not the vacuum states of the Higgs fields. Hence, the production of the positron-electron pair clearly indicates that the electron and positron come from the photon field. In the electroweak theory, $W$ and $Z$ gauge fields also interact with the photon fields. It also
shows that gauge bosons are not only generated from Higgs fields. Even if the existence of Higgs boson is proved, it seems to not prove that the mass gauge bosons in the electroweak action are completely generated from the vacuum states of the Higgs fields!

In summary, the lowest energy of the Higgs field higher than the Higgs boson makes the initial assumption failed in the electroweak theory when the Higgs mechanism is adopted for explaining the source of mass in the gauge theories. By definition, the vacuum states of the Higgs fields are more likely the energy perturbation in space due to the Heisenberg's uncertainty principle. These states shall correspond to very small energy, not as high as more than 200 GeV . It is not an appropriate way to provide a stable source of mass for electron with infinite lifetime. The production of the positronelectron pair clearly reveals that the electron and positron come from the photon field. The Yukawa coupling needs to be re-explained because it is not in charge of the source of the mass electron. Even for the scalar Higgs boson $H^{0}$, it can come from different parent particles.

## Acknowledgement

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