

SunQM-3s11: Using {N,n} QM's probability density 3D map to build a complete Solar system with time-dependent orbital movement (semi-QM deduction)

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Abstract

Combining the results from the previous SunQM series papers, a (high resolution) 3D probability density map has been constructed and it is able to describe the whole Solar system with time-dependent orbital movement. It is the Eigen description of our Solar system using Schrodinger equation's solution. In it, the Eigen n' values of all planet have been calculated. These Eigen n' values give both the orbital r and the surface r information for each planet. The result revealed that for all planets, their Eigen n' values in all three dimensions are equal ($n'_r = n'_\theta = n'_\phi = n'$). For example, for a planet at orbit $\{1,5//6\}$ (in the Solar $\{N,n//6\}$ QM structure), if it has Eigen $n' = n^*q^w = 5*6^{11} = 1.81E+9$ in each of $r\theta\phi$ -3D dimension, then it will have an orbital $r = 1.57E+11$ m, and surface $r = 7.89E+6$ m. This is very close to Earth's orbital $r = 1.49E+11$ m and surface $r = 6.38E+6$ m. For Asteroid belt and the cold-KBO, their Eigen $n'(s)$ in the r - and θ -dimension are equal ($n'_r = n'_\theta$). For example, Asteroid belt's Eigen $n' = 48$ in both r - and θ -dimension. In the current paper, although the r - and θ -dimension's probability densities were deduced with the full-QM, the ϕ -dimension's probability density was deduced with only semi-QM (for the purpose to follow the traditional QM's rule). A full-QM deduced ϕ -dimension's probability density will be given in SunQM-4s1 (where some traditional QM's rules are modified). Because $\{N,n//6\}$ QM structure covers from quark at $\{-17,1//6\}$ to the Virgo super cluster at $\{10,1//6\}$ with good consistency, and Schrodinger equation/solution can accurately describe the Solar system from $\{-2,1//6\}$ to $\{5,1//6\}$, we believe that either the whole universe or a single quark can be described by Schrodinger equation and solution. Several (lower resolution) 3D probability density maps (also based on Schrodinger equation's solution) for the whole Solar system have also been successfully built. A summary of the major results from the phase-1 of the SunQM series studies has been listed.

Introduction

The SunQM series papers ^{[1]~[15]} have shown that the formation of Solar system (as well as each planet) was governed by its $\{N,n\}$ QM. In papers SunQM-3s6, SunQM-3s7, and SunQM-3s8, it has been shown that the formation of planet's and star's (radial) internal structure is governed by the planet's or star's radial QM. In papers SunQM-3s3 and -3s9, it has been shown that the surface mass (atmosphere) movement of Sun, Jupiter, Saturn, and Earth, etc., is governed by Star's (or planet's) $\theta\phi$ -2D dimension QM. In paper SunQM-3s4 and -3s10, it has been shown that the formation of either ring structures of a planet, or the belt structures in Solar system, is also governed by the $\{N,n\}$ QM (the nLL effect). In current paper, we want to use $\{N,n\}$ QM and Schrodinger equation's solution to build a 3D probability density map for a complete Solar system with time-dependent orbital movement. Note: for $\{N,n\}$ QM nomenclature as well as the general notes for $\{N,n\}$ QM model, please see SunQM-1 section VII. Note: Microsoft Excel's number format is often used in this paper, for example: $x^2 = x^2$, $3.4E+12 = 3.4*10^{12}$, $5.6E-9 = 5.6*10^{-9}$. Note: The reading sequence for SunQM series papers is: SunQM-1, 1s1, 1s2, 1s3, 2, 3, 3s1, 3s2, 3s6, 3s7, 3s8, 3s3, 3s9, 3s4, 3s10, and 3s11. Note: for all SunQM series papers, reader should check "SunQM-4s5: Updates and Q/A for SunQM series papers" for the most recent updates and corrections.

I. To build a (time-independent) 3D probability density $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ for the current Sun from the center to the surface at {0,2//6}

In the Solar {N,n} QM theory, the whole Solar system is primarily governed by a single super large three dimensional $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ QM probability density structure which covers the Sun, all 8 planets, Asteroid and Kuiper belts, 4 undiscovered planets, and Oort cloud. Here we name this super large $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ probability density function as Sun's primary (or master) $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$. For this primary $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$, we use Sun core{0,1//6} as n=1, so it matches the Sun {N,n//q} QM structure naturally. Another reason for using {0,1//6} as n=1 is that in paper SunQM-3s10, Asteroid belt at {1,8} = {0,48//6} can be perfectly described by |48,47,47> Eigen QM state alone, and also Kuiper belt at {2,6} = {0,192//6} can be perfectly described by |192,191,m> Eigen QM state alone. This strongly suggests that using {0,1//6} as n=1 is the right choice for Sun's primary $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$. As we defined before, |nLL> means |n,l,m> with l = n-1, and m = n-1 (see SunQM-3s1). Under Sun core's total n=1, planets/belts' n(s) are calculated according to the {N,n//6} QM model and listed in column 10 of Table 1 (see also in SunQM-1 Table 3 column 7).

Table 1. Calculation of a planet's Eigen n' (in both r- and θ -dimension), orbital angular velocity ω (group- ω and phase- ω), ϕ position (day-0 and day-60).

NASA's data of planets						assigned N, n, period factor		set total n=1 at Sun core calc model n, rn, vn		Determine planet r-dimensional n' & w						Determine planet θ -dimensional n' & w				(N,n) calculated ω				initial $\phi =$							
unit	mass	radius	orbital velocity	orbital period	planet's body-r	N	n	period	total n from Sun core	$v_n = \sqrt{GM/r_n}$	$n' = \ln[0.1] / \ln(1+b/r_n)$	$w = \log(n'/n)$	round up w	$n'_r = n^*q^*w$	$\theta = \text{at } n^*n^*q^*w$	$r_1 = r_n / (n^*q^*w)$	$r_1 \text{ at } \{N,1//6\}$	$w(\theta) = n^*q^*w$	$n'_\theta = 0.01^*(1/2n')$	$\theta' = \arccos[0.01^*(1/2n')]$	phase $\omega_{p,ph} = v_n / r_n$	group $\omega = v_n / r_n$	period $T_s = 2\pi / (2\omega)$	day-0	day-60	day-0	day-60				
kg	m	m/s	days	m				m	m/s					m	m				arc	m	arc/s	arc/s	day	degree	arc	degree	ddegree				
Sun core	1.74E+08					0	1	6	1	1.74E+08																					
SUN	1.99E+30	6.96E+08				0	2	6	2	6.96E+08																					
(0,3) corona						0	3	6	3	1.57E+09																					
(0,4) corona						0	4	6	4	2.78E+09																					
(0,5) corona						0	5	6	5	4.35E+09																					
(0,6) corona end						0	6	6	6	6.26E+09																					
(1,2)						1	2	6	12	2.50E+10																					
Mercury	3.3E+23	5.79E+10	47400	88	2.44E+06	1	3	6	18	5.64E+10	48533	2.46E+09	11.45	11	1.09E+09	2.59E+06	4.76E-08	{-10,1//6}	11	1.09E+09	0.99999999788414	6.50E-05	3.67E+06	4.31E-07	8.61E-07	84	62	1.08	318	256	
Venus	4.87E+24	1.08E+11	35000	224.7	6.05E+06	1	4	6	24	1.00E+11	36400	1.26E+09	10.92	11	1.45E+09	6.91E+06	4.76E-08	{-10,1//6}	11	1.45E+09	0.99999999813311	5.63E-05	5.64E+06	1.82E-07	3.63E-07	200	180	3.14	288	108	
Earth	5.97E+24	1.49E+11	29800	365.2	6.38E+06	1	5	6	30	1.57E+11	29120	2.77E+09	11.24	11	1.81E+09	7.89E+06	4.76E-08	{-10,1//6}	11	1.81E+09	0.999999998730648	5.04E-05	7.89E+06	9.30E-08	1.86E-07	391	0	0.00	55	55	
Mars	6.42E+23	2.28E+11	24100	687	3.40E+06	1	6	6	36	2.25E+11	24266	2.03E+10	12.25	12	1.31E+10	5.18E+06	1.32E-09	{-11,1//6}	12	1.31E+10	0.999999999823701	1.88E-05	4.23E+06	5.38E-08	1.08E-07	676	191	3.33	223	32	
Asteroid belt	2.92E+21	4.02E+11				1	8	6	48	4.01E+11	18200																				
Jupiter	1.90E+27	7.78E+11	13100	4331	7.15E+07	2	2	5.33	64.0	7.12E+11	13658	4.56E+08	10.67	11	7.26E+08	6.94E+07	1.35E-06	{-9,1//6}	11	7.26E+08	0.999999996826621	7.97E-05	5.67E+07	9.60E-09	1.92E-08	3788	304	5.31	310	6	
Saturn	5.68E+26	1.43E+12	9700	10747	6.03E+07	2	3	5.33	95.9	1.60E+12	9106	3.25E+09	11.54	12	6.53E+09	5.21E+07	3.75E-08	{-10,1//6}	12	6.53E+09	0.99999999647402	2.66E-05	4.25E+07	2.84E-09	5.69E-09	1.28E+04	329	5.74	331	2	
Uranus	8.68E+25	2.97E+12	6800	30589	2.56E+07	2	4	5.33	127.9	2.85E+12	6829	5.71E+10	12.98	13	5.22E+10	3.27E+07	1.04E-09	{-11,1//6}	13	5.22E+10	0.9999999995925	9.39E-06	2.67E+07	1.20E-09	2.40E-09	3.03E+04	73	1.27	74	1	
Neptune	1.02E+26	4.51E+12	5400	59800	2.48E+07	2	5	5.33	159.9	4.45E+12	5463	1.48E+11	13.39	13	6.53E+10	3.73E+07	1.04E-09	{-11,1//6}	13	6.53E+10	0.99999999964740	8.40E-06	3.73E+07	6.14E-10	1.23E-09	5.92E+04	30	0.52	30	0	
Kuiper belt	1.46E+22	5.91E+12				2	6	5.33	191.9	6.40E+12	4553																				
(3,2)	7.12E+25					3	2	6	383.8	2.56E+13	2276	6.35E+12	16.00	16	5.64E+12	2.31E+07	8.05E-13	{-13,1//6}	16	5.64E+12	0.999999999999992	9.03E-07	2.31E+07	4.44E-11	8.89E-11	8.18E+05					
(3,3)	3.99E+25					3	3	6	575.6	5.76E+13	1518	4.73E+13	16.89	17	5.08E+13	2.12E+07	2.24E-14	{-14,1//6}	17	5.08E+13	0.999999999999995	3.01E-07	1.73E+07	1.32E-11	2.63E-11	2.76E+06					
(3,4)	2.75E+25					3	4	6	767.5	1.02E+14	1138	1.92E+14	17.51	18	4.06E+14	1.33E+07	6.21E-16	{-15,1//6}	18	4.06E+14	0.999999999999994	1.06E-07	1.09E+07	5.55E-12	1.11E-11	6.55E+06					
(3,5)	1.98E+25					3	5	6	959.4	1.60E+14	911	5.67E+14	18.00	18	5.08E+14	1.53E+07	6.21E-16	{-15,1//6}	18	5.08E+14	0.999999999999995	9.54E-08	1.53E+07	2.84E-12	5.69E-12	1.28E+07					

Note: all planets data is obtained from NASA's Planetary Fact Sheet at: <http://nssdc.gsfc.nasa.gov/planetary/factsheet/>, Sun's data is from: <https://en.wikipedia.org/wiki/Sun>. According to wiki "Asteroid belt", Asteroid belt's mass = 4% of Moon mass = 0.04 * 7.3E+22 = 2.92E+21 kg. Note: Kuiper belt's mass was assumed to be 5x of Earth's mass here, in comparison with wiki "Kuiper belt" 's ~0.1x of measured, or ~30x of modeled (of Earth's mass). Note: 1.99E+30 kg is for the whole Sun (including Sun core). Note: In column 12, the modeled orbit v_n is calculated by using classical physics $F = ma = mv_n^2 / r_n$, $F = GMm / r_n^2$, $mv_n^2 / r_n = GMm / r_n^2$, $r_n v_n^2 = GM$, $v_n = \sqrt{GM / r_n}$. Note: for the undiscovered {3,n=2..5} planets, the estimated mass in column 2 is copied from Table 3b in SunQM-1s1, and the estimated surface-r in column 6 is copied from Table 2 in SunQM-3s6. Note: in column 15, the integer w is round-up (or down) according to column 17's calculated b value which should close to planet's surface-r (at column 6). Note: From column 18's r_1 value, check SunQM-1s2 Table 1 to find the corresponding {N,1}, and fill to column 19. Notice that {1,n=3..6}'s r_1 fits to the Hot-G r track in SunQM-1s2 Table 1, and {2,n=3..6}'s and {3,n=3..6}'s r_1 fits to the Cold-G r track in SunQM-1s2 Table 1.

I-a. To build a (time-independent) 3D probability density $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ for the current Sun from the center to the surface at {0,2//6}

For the current Sun ball, a detailed r-dimension description is given in SunQM-3s8 section-I. Because from Sun surface to Sun core, it belongs to a {0,1//6} orbit shell space, therefore we also can described it with n=1, l = n-1 = 0, m = -l ... +l = 0, or |1,0,0>, or by

$$r^2 * |R(1,0)|^2 * |Y(0,0)|^2 \quad \text{eq-1}$$

We know that either $r^2 * |R(1,0)|^2$ function or $|Y(0,0)|^2$ function is a perfect sphere, so the production of these two is still a perfect sphere. Because the current Sun has 100% mass occupancy up to surface- r at $\{0,2\}$, therefore the shape of our Sun follows the primary $r^2 * |R(1,0)|^2 * |Y(0,0)|^2$ and it is a perfect sphere.

In the Solar $\{N,n\}$ QM, the space inside Sun core with r less than the r of $\{0,1//6\}$ is described by $\{-1,n=1..5//6\}$ orbit shells, and the space inside the r of $\{-1,1//6\}$ is described by $\{-2,n=1..5//6\}$ orbit shells, and the space inside the r of $\{-2,1//6\}$ is described by $\{-3,n=1..5//6\}$ orbit shells, and so on so forth (see SunQM-3 Figure 3a). To use (Sun's) primary $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$, we have to set $N=0$ for Sun's whole $\{N,n//6\}$ QM structure. In the $\{N,n\}$ QM's nomenclature, we have $\{-1,n//6\} = \{0, n/(6^1)//6\}$, $\{-2,n//6\} = \{0, n/(6^2)//6\}$, and $\{-3,n//6\} = \{0, n/(6^3)//6\}$ (see SunQM-1 section-VII). So $\{-1,n=1..5//6\}$ orbit shells is re-written as $\{0,n=(1/6, 2/6, 3/6, 4/6, 5/6)//6\}$ orbit shells, with the fractional quantum number as $n = 1/6, 2/6, 3/6, 4/6$ and $5/6$. And $\{-2,n=1..5//6\}$ orbit shells is re-written as $\{0,n=(1/6^2, 2/6^2, 3/6^2, 4/6^2, 5/6^2)//6\}$ orbit shells, with the fractional quantum number as $n = 1/6^2, 2/6^2, 3/6^2, 4/6^2$ and $5/6^2$, and so on so forth.

So, the original probability function $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ is only suitable for each specific N 's $\{N,n//q\}$ QM where n is always the base n (i.e., $N=0$, or $N=2$, etc.). For a $\{N,n//q\}$ QM, **let's define that $\{N,n//q\}$ is a general form (where n is base n), and let's define $\{0,n^*q^N//q\}$ is the primary form.** Obviously, in the primary form $\{0,n^*q^N//q\}$, the quantum number is no longer base n . It is either high frequency n (if $N>0$), or sub-base n (if $N<0$). For general form $\{N,n//q\}$, we write it as $|n,l,m\rangle$. For primary form $\{0,n^*q^N//q\}$, we write the QM state as $|n^*q^N,l,m\rangle$ (in which we do NOT calculate out the value of n^*q^N). According to the rule of "all mass between r_n and r_{n+1} belongs to orbit n (see paper SunQM-3s2)", the uncalculated and calculated n^*q^N have different n and covers different r ranges. For example, for $\{1,3//6\}$, it is $|3,l,m\rangle$. For $\{0,3^*6^1//6\}$, it is $|3^*6^1,l,m\rangle$, we do NOT write it as $|18,l,m\rangle$ because they have different meaning: $|18,l,m\rangle$ QM state covers r -dimension from r of $n=18$ to $n=19$ with r_1 at $\{0,1//6\}$, while $|3^*6^1,l,m\rangle$ QM state covers r -dimension from r of $n=3$ to $n=4$ with r_1 at $\{1,1//6\}$, which equivalent to r of $n = 3^*6^1 = 18$ to $n = 4^*6^1 = 24$ with r_1 at $\{0,1//6\}$. Considering the rule of "all mass between r_n and r_{n+1} belongs to orbit n ", so $|3^*6^1,l,m\rangle$ QM state is always a linear combination of $|18,l,m\rangle, |19,l,m\rangle, |20,l,m\rangle, |21,l,m\rangle, |22,l,m\rangle,$ and $|23,l,m\rangle$ QM states. This is always true when $|3^*6^1,l,m\rangle$ QM state is at $\sim 100\%$ mass occupancy. Only when the mass occupancy $\ll 1\%$, then $|3^*6^1,l,m\rangle$ QM state may equal to $|18,l,m\rangle$ QM state.

Accordingly, we write the probability function for a primary form $\{0,n^*q^N//q\}$ QM state as

$$r^2 * |R(n^*q^N,l)|^2 * |Y(l,m)|^2 \quad \text{eq-2}$$

However, due to that if $N < 0$, then $l = n^*q^N - 1 < 0$, and we don't know how to handle it. So to calculate eq-2, we have to use the general formed $\{N,n//q\}$ to move r_1 inward if $N < 0$ or outward if $N > 0$. For example, the $\{-3,4//6\}$ orbit shell's probability function will be calculated as $r^2 * |R(4,l)|^2 * |Y(l,m)|^2$, with r_1 at $\{-3,1//6\}$, and with $l = 0, 1, 2, 3, m = -l, \dots +l$, even it can be written as $r^2 * |R(4/6^3,l)|^2 * |Y(l,m)|^2$, or its $|n,l,m\rangle$ can be written as $|4/6^3,l,m\rangle$. Similarly, the $\{1,3//6\}$ orbit shell's probability function will be calculated as $r^2 * |R(3,l)|^2 * |Y(l,m)|^2$, with r_1 at $\{1,1//6\}$, and with $l = 0, 1, 2, m = -l, \dots +l$, even it can be written as $r^2 * |R(3^*6^1,l)|^2 * |Y(l,m)|^2$, or its $|n,l,m\rangle$ can be written as $|3^*6,l,m\rangle$.

Then, according to SunQM-3s8, Sun's $\{0,1//6\}$ orbit space shell can be accurately described by eq-1. Note: the small contribution from $r^2 * |R(2,1)|^2$ in the outer edge of $\{0,1\}$ orbit shell is ignored here (see SunQM-3s8). Sun's $\{-1,n=1..5//6\}$ orbit shells can be accurately described by

$$r^2 * [a_1 * |R(1/6,0)|^2 * |Y(0,0)|^2 + a_2 * |R(2/6,l=0..1)|^2 * |Y(l=0..1,m)|^2 + a_3 * |R(3/6,l=0..2)|^2 * |Y(l=0..2,m)|^2 + a_4 * |R(4/6,l=0..3)|^2 * |Y(l=0..3,m)|^2 + a_5 * |R(5/6,l=0..4)|^2 * |Y(l=0..4,m)|^2] \quad \text{eq-3}$$

where, $l = 0, \dots, n-1$, and $m = -l, \dots, +l$, and $a_1 \dots a_5$ are the linear combination coefficients. Note: the small contribution from $r^2 * |R(6/6,5)|^2 * |Y(5,5)|^2$ in the outer edge of $\{-1,n=1..5//6\}$ orbit shell is ignored here too (see SunQM-3s8). Similarly, Sun's $\{-2,n=1..5//6\}$ orbit shells can be accurately described by

$$r^2 * [a_1 * |R(1/6^2,0)|^2 * |Y(0,0)|^2 + a_2 * |R(2/6^2,l=0..1)|^2 * |Y(l=0..1,m)|^2 + a_3 * |R(3/6^2,l=0..2)|^2 * |Y(l=0..2,m)|^2 + a_4 * |R(4/6^2,l=0..3)|^2 * |Y(l=0..3,m)|^2 + a_5 * |R(5/6^2,l=0..4)|^2 * |Y(l=0..4,m)|^2]$$

eq-4

where $l = 0, \dots, n-1$, and $m = -l, \dots, +l$, and $a_1 \dots a_5$ are the linear combination coefficients which have values different than those in eq-3. And, so on so forth for probability functions of $N = -2, -3$, etc.

Now let's determine how many negative valued N super-shells are needed for Sun's probability function at the minimum acceptable accuracy, is it $\{0, 1/6\}$ orbit shell space only, or $\{0, 1/6\}$ plus $\{-1, n=1..5/6\}$ orbit shell spaces, or $\{0, 1/6\}$ plus $\{-1, n=1..5/6\}$ plus $\{-2, n=1..5/6\}$ orbit shell spaces, or even more? If we only count the $\{0, 1/6\}$ orbit shell space and not include Sun core $\{0, 1/6\}$, then the volume ratio of total Sun (with radius R) and Sun core (with radius r) is $R^3 / r^3 = 4^3 / 1^3 = 64$. However, from wiki "Solar core", "The core inside 0.20 of the solar radius, contains 34% of the Sun's mass, but only 0.8% of the Sun's volume". So it is obvious that only count Sun's $\{0, 1/6\}$ orbit shell space and ignore the Sun core is not accurate enough for the Sun's probability function. Now if we count the total Sun as $\{0, 1/6\}$ plus $\{-1, n=1..5/6\}$ orbit shell spaces, and ignore Sun's center ball part within $\{-1, 1/6\}$ (and define its radius as r'). Then the volume ratio of total Sun (with radius $R = 6^2 * 2^2 * r' = 144 r'$) vs. the ignored center ball (with radius r') is $R^3 / r'^3 = 2985984$. Then the mass in $\{0, 1/6\}$ plus $\{-1, n=1..5/6\}$ orbit shell spaces can easily pass 99% of the Sun (even though the center part of Sun has much higher mass density). Therefore, we believe that counting $\{0, 1/6\}$ plus $\{-1, n=1..5/6\}$ orbit shell spaces is enough to give the minimum acceptable accuracy for the Sun's probability function.

Accordingly, we can obtain a Sun's probability density function by adding eq-3 to eq-1, as shown in below:

$$r^2 * |\Psi(r, \theta, \phi)_{Sun}|^2 \propto r^2 * [a_1 * |R(1/6,0)|^2 * |Y(0,0)|^2 + a_2 * |R(2/6,l=0..1)|^2 * |Y(l,m)|^2 + a_3 * |R(3/6,l=0..2)|^2 * |Y(l,m)|^2 + a_4 * |R(4/6,l=0..3)|^2 * |Y(l,m)|^2 + a_5 * |R(5/6,l=0..4)|^2 * |Y(l,m)|^2 + a_6 * |R(1,0)|^2 * |Y(0,0)|^2]$$

eq-5

where, $l = 0, \dots, n-1$, and $m = -l, \dots, +l$, and $a_1 \dots a_6$ are the linear combination coefficients. By integrating eq-5, we can get the integration form of a Sun's probability function:

$$\text{Mass}(r, \theta, \phi) = \iiint r^2 * [a_1 * |R(1/6,0)|^2 * |Y(0,0)|^2 + a_2 * |R(2/6,l)|^2 * |Y(l,m)|^2 + a_3 * |R(3/6,l)|^2 * |Y(l,m)|^2 + a_4 * |R(4/6,l)|^2 * |Y(l,m)|^2 + a_5 * |R(5/6,l)|^2 * |Y(l,m)|^2 + a_6 * |R(1,0)|^2 * |Y(0,0)|^2] * \sin(\theta) \, dr \, d\theta \, d\phi, [r=0, 6.96E+8 \text{ m}; \theta=0, \pi; \phi=0, 2\pi]$$

eq-6

where, $l = 0, \dots, n-1$, $m = -l, \dots, +l$, and $a_1 \dots a_6$ are the normalized linear combination coefficients that makes the integrated eq-6's value equals to Sun's total mass. Notice that in eq-6, the normalization coefficient of each radial wave function $R(n,l)$ is no longer the same as the original $R(n,l)$ (which is normalized for H-atom's $r_1 = a_0 = 5.29E-11$ m, also see section I-b for detailed explanation). The normalization coefficient of each $R(n,l)$ in eq-6 (and eq-7) includes Sun's radial mass density distribution information. In eq-6, the integration of each $\theta\phi$ probability $\iint |Y(l,m)|^2 * \sin(\theta) \, d\theta \, d\phi$, $[\theta=0, \pi; \phi=0, 2\pi]$ with $l = 0, \dots, n-1$, and $m = -l, \dots, +l$ is always independent of r -dimension's integration, and due to the ~ 100% mass occupancy inside Sun, it always give a constant value. So we can put this constant value into the coefficient a_j (where $j = 1, 2, \dots$). Therefore, eq-6 can also be simplified as

$$1.99E+30 \text{ kg} = \int r^2 * [a_1 * |R(1/6,0)|^2 + a_2 * |R(2/6,l=0..1)|^2 + a_3 * |R(3/6,l=0..2)|^2 + a_4 * |R(4/6,l=0..3)|^2 + a_5 * |R(5/6,l=0..4)|^2 + a_6 * |R(1,0)|^2] \, dr, [r=0, 6.96E+8 \text{ m}]$$

eq-7

where $R(n,l)$ is normalized for Sun's $r_1 = 1.74E+8$ m, and also contains Sun's radial mass density distribution information, and $a_j = 1 \dots 6$ are the r -independent values, and they are the normalized linear combination constants that makes the integrated eq-7's value equals to Sun's total mass.

Now if we use H-atom's $R(n,l)$, and it should not contain Sun's radial mass density distribution information, then we have an integration formula that is similar to that in SunQM-3s8 section-I:

$$1.99E+30 \text{ kg} = \int r^2 * [|R(1/6,0)|^2 + |R(2/6,l=0..1)|^2 + |R(3/6,l=0..2)|^2 + |R(4/6,l=0..3)|^2 + |R(5/6,l=0..4)|^2 + |R(1,0)|^2] * W * D \text{ dr}, [r=0, 6.96E+8 \text{ m}] \tag{eq-8}$$

where $R(n,l)$ is normalized for H-atom's $r_1 = a_0 = 5.29E-11 \text{ m}$, and does not contain Sun's radial mass density distribution information, and $D(r)$ is the Sun's radial mass density r -distribution function (in SunQM-3 section I-f, $D(r)$ was determined to be $D \approx 1.26E+23 / r^{2.33} \text{ kg/m}^3$), and W is a normalization constant to make the integrated eq-8's value equals to Sun's total mass.

I-b. A major correction for inside Sun's (or inside planet's, or inside pre-Sun ball's) radial wave function $R(r)$

Solving Schrodinger equation for a single (nonrelativistic) particle doing orbital movement under a central attractive force (valid for both H-atom and pre-Sun ball models) gives the eigenstates of this equation

$$\Psi(r,\theta,\varphi,t) = R(r) * \Theta(\theta) * \Phi(\varphi) * T(t) \tag{eq-9}$$

The two factors of Ψ are usually grouped together as spherical harmonics (because of the RF!)

$$\Theta(\theta) * \Phi(\varphi) = Y(l,m). \tag{eq-10}$$

The solution of $Y(l,m)$ is valid for both outside the current Sun, as well as inside the current Sun and pre-Sun ball. The radial wave function $R(r)$ for H-atom (ignoring the normalization coefficient) is valid for outside the current Sun, but for inside the current Sun or a pre-Sun ball, there is significant deviation (due to that the center mass of the gravity potential $V(r)$ inside Sun is also dependent on r). For example, if inside the Sun, the center Mass can be simplified to have a $M \propto r^b$ (where $0 > b < 1$) relationship, then the G-potential $V(r) = \int F \text{ dr} \propto \int G * M(r^b) * m / r^2 \text{ dr} = G * M * m \int 1 / r^{2-b} \text{ dr} = -G * M * m / r^{1-b} / (1-b) + \text{constant}$. If $b = 0$, then it goes back to $V(r) = -GMm / r$. For inside the Sun, if we guess $b \approx 0.5$, then $V(r) \propto -2 * G * M * m / r^{0.5} + \text{constant}$. So we need to solve the Schrodinger equation for inside the current Sun with this equation (rather than the H-atom equivalent with gravity's $V(r) = -GMm / r$), and to get an accurate $R(r)$. As a citizen scientist level QM physicist, I don't have the ability to solve it. However, I do believe that the curve shape of the inside Sun's true $R(r)$ is very similar as that of outside Sun's $R(r)$, or the H-atom's $R(r)$. For this reason, in the SunQM series papers, I use H-atom's $R(r)$ to mimic inside Sun's (or inside planet's, or inside pre-Sun ball's) radial wave function with two minor modifications:

- 1) In H-atom's $R(r)$ formula, Bohr radius a_0 is replaced by Sun or planet's r_1 ;
- 2) When plotting the $R(r)$ curve vs. r for Sun (or for planet), at the outside of Sun (or planet) surface, the r is log compressed with the formula $r (r > b) = b + \log(r - b)$, where $b = r_{\text{surface}}$.

We need to emphasize that the more accurate $R(r)$ function (which we don't have it now) is only needed for inside Sun's (or planet's) $R(r)$, it is not needed for the outside Sun's region (where the H-atom's $R(r)$ formula works fine after the r_1 re-normalization).

II. To build a (time-independent) 3D probability density $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ for Solar system's region from $\{0,2\}$ to $\{5,1\}$ at median or low resolutions

Note: for most readers, you can skip this section (with median or low resolution description), and directly read the section III (with high resolution description).

II-a. At median resolution (good resolution for belts, poor resolution for planets)

To simplify the primary $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ for Sun and all planets/belts, those n orbits with (practically) zero mass should be omitted. We only need to show those n orbit regions with mass. According to Table 1, these n regions with mass are $n < 1$, and $n = 1, 18, 24, 30, 36, 48, 64, 96, 128, 160, 192, 384, 576, 768, \text{ and } 959$ (the Oort cloud is not included for the moment). For the current Sun's $\{0, 1//6\}$ orbit shell space, it is in $|1, 0, 0\rangle$ QM state. For the rest $n(s)$ that > 1 , due to they all have $< 1\%$ mass occupancy, the spinning Sun's nLL QM effect causes all mass stay in $|n, l=n-1, m \leq l\rangle$ QM state. The result in SunQM-3s10 confirmed that all Asteroid belt's mass is in the $|48, 47, 47\rangle$ QM state, and Kuiper belt's cold-KBO mass is in the $|192, 191, 191\rangle$ QM state. SunQM-3s10 also defined that if all mass of an object is perfectly in a single $|nLL\rangle$ state, then this $|nLL\rangle$ is the Eigen QM state of this object. So $|48, 47, 47\rangle$ is the Eigen QM state of Asteroid belt, and $|192, 191, 191\rangle$ is the Eigen QM state of the cold-KBO. We can confidently to say that the mass in QM states of $n=18, 24, 30, 36, 48, 64, 96, 128, 160, 192, 384, 576, 768, \text{ and } 959$ can be described by $|nLL\rangle$ as $|18, 17, 17\rangle, |24, 23, 23\rangle, |30, 29, 29\rangle, |36, 35, 35\rangle, |48, 47, 47\rangle, |64, 63, 63\rangle, |96, 95, 95\rangle, |128, 127, 127\rangle, |160, 159, 159\rangle, |192, 191, 191\rangle, |384, 383, 383\rangle, |576, 575, 575\rangle, |768, 767, 767\rangle, \text{ and } |959, 958, 958\rangle$. Because $|48, 47, 47\rangle$ and $|192, 191, 191\rangle$ are the Eigen QM state of Asteroid belt and the cold-KBO, $|384, 383, 383\rangle, |576, 575, 575\rangle, |768, 767, 767\rangle, \text{ and } |959, 958, 958\rangle$ are also expected to be the Eigen QM states for all four undiscovered belts at $\{3, n=2..5//6\}$ (if they have not accreted into planets by now). For eight planets, $|18, 17, 17\rangle, |24, 23, 23\rangle, |30, 29, 29\rangle, |36, 35, 35\rangle, |64, 63, 63\rangle, |96, 95, 95\rangle, |128, 127, 127\rangle, |160, 159, 159\rangle$ are not the Eigen QM states, these are the low-resolution QM description, not perfect but still OK. This is because as shown in Figure 3 of SunQM-3s10, an n state in r -dimension can be described from high resolution to low resolution by using high-frequency n' or sub-base frequency n . Let's use Earth as the example, section-III of this paper will show that although Earth is better to be described by a high-frequency $n'=5*6^{11}=1.81E+9$, (because it describes both Earth's orbit- r and surface- r), the relative low frequency $n=30$ also describes Earth's orbit- r , but not the surface- r . Both two descriptions give the same Earth's orbit- r at $r = r_1 * n^2 = 1.74E+8 * 30^2 = 4.76E-8 * 1.81E+9^2 \approx 1.57E+11$ m. So before the mass in $\{1, 5//6\}$ orbit shell (or $\{0, 30//6\}$ orbit shell) starts to accrete, $|30, 29, 29\rangle$ would be a Eigen description for a belt (made of pre-Earth's mass).

For a set of QM description that gives good resolution for belts but poor resolution for planets, we call it a median resolution. Therefore, for eight planets, $|18, 17, 17\rangle, |24, 23, 23\rangle, |30, 29, 29\rangle, |36, 35, 35\rangle, |64, 63, 63\rangle, |96, 95, 95\rangle, |128, 127, 127\rangle, |160, 159, 159\rangle$ are the medium resolution description of their QM state, Now we can have a median resolution $|n, l, m\rangle$ QM state description for the whole solar system as

$$|n, l, m\rangle_{\text{SolarSystem}} = a * |n, l, m\rangle_{\text{SunCore}} + b * |1, 0, 0\rangle + c_1 * |18, 17, 17\rangle + c_2 * |24, 23, 23\rangle + c_3 * |30, 29, 29\rangle + c_4 * |36, 35, 35\rangle + c_5 * |48, 47, 47\rangle + c_6 * |64, 63, 63\rangle + c_7 * |96, 95, 95\rangle + c_8 * |128, 127, 127\rangle + c_9 * |160, 159, 159\rangle + c_{10} * |192, 191, 191\rangle + c_{11} * |384, 383, 383\rangle + c_{12} * |576, 575, 575\rangle + c_{13} * |768, 767, 767\rangle + c_{14} * |959, 958, 958\rangle$$

eq-11

where a, b, c_1, \dots, c_{14} are the coefficients of the linear combination. also where $|n, l, m\rangle_{\text{SunCore}} = [a_{11} * |1/6, l, m\rangle + a_{12} * |2/6, l, m\rangle + a_{13} * |3/6, l, m\rangle + a_{14} * |4/6, l, m\rangle + a_{15} * |5/6, l, m\rangle + \dots]$ and a_{11}, a_{12}, \dots are the coefficients of the linear combination. Eq-11 described a Sun's RF ball up to $\{0, 2\}$, plus 14 belts (two for Asteroid and Kuiper, 8 for planets, and 4 for undiscovered $\{3, n=2..5//6\}$ planer/belt) with a good accuracy. Then the corresponding primary 3D probability density $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ for Solar system up to $\{3, 5//6\}$ is the linear combination of:

$$r^2 * |\Psi(r, \theta, \varphi)_{\text{SolarSystemMedianResolution}}|^2 \propto r^2 * \{ [eq-3] + |b * R(1,0)|^2 * |Y(0,0)|^2 + |c_1 * R(n=18,17)|^2 * |Y(17,17)|^2 + |c_2 * R(n=24,23)|^2 * |Y(23,23)|^2 + |c_3 * R(n=30,29)|^2 * |Y(29,29)|^2 + |c_4 * R(n=36,35)|^2 * |Y(35,35)|^2 + |c_5 * R(48,47)|^2 * |Y(47,47)|^2 + |c_6 * R(n=64,63)|^2 * |Y(63,63)|^2 + |c_7 * R(n=96,95)|^2 * |Y(95,95)|^2 + |c_8 * R(n=128,127)|^2 * |Y(127,127)|^2 + |c_9 * R(n=160,159)|^2 * |Y(159,159)|^2 + |c_{10} * R(n=192,191)|^2 * |Y(191,191)|^2 + |c_{11} * R(384,383)|^2 * |Y(383,383)|^2 + |c_{12} * R(576,575)|^2 * |Y(575,575)|^2 + |c_{13} * R(768,767)|^2 * |Y(767,767)|^2 + |c_{14} * R(959,958)|^2 * |Y(958,958)|^2 \}$$

eq-12

where a, b, c_1, \dots, c_{14} are the coefficients of the linear combination, and [eq-3] is the formula of eq-3 (without r^2).

We also can describe the primary $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ for solar system up to $\{3,5//6\}$ using a integration form:

$$\text{Mass}(r,\theta,\varphi) = 1.99E+30 \text{ kg} = \iiint [\text{eq-12}] * \sin(\theta) \text{ dr d}\theta \text{ d}\varphi, [r=0, 1550*1.49E+11 \text{ m}; \theta=0, \pi; \varphi=0, 2\pi] \quad \text{eq-13}$$

where the radial integration from 0 m, to 1550 AU = 1550* 1.49E+11 m (where $\{3,5//6\}$ orbit ends), and 1.99E+30 kg is the total mass of our solar system. Notice that the integration of each item in eq-13 will generate the mass for each planet, belt, and Sun as listed in column 2 in Table 1.

Although eq-12 and eq-13 give good description for our solar system, they do not have the enough resolution to describe the eight planets (for their size). Therefore in section-III, we will build more accurate description for eight planets using planet's Eigen n ' based $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ probability function so that both planets' orbit- r and surface- r can be described with the acceptable accuracy.

II-b. At low resolution (good resolution for each N super shells, poor resolution for each belts and planets)

According to SunQM-3s10's Figure 4a (or Figure 5a), using $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ probability curve to describe $\{N=1..3,n//6\}$ region can be at level of each n orbit, or at level of each N super shell, or even at level that treat the whole $\{N=1..3,n//6\}$ region as a single super-super shell. In section II-a, we have built the 3D probability density $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ to describe $\{N=1..3,n//6\}$ region at level of each n orbit (see eq-12 and eq-13). In current section, we will build the $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ to describe $\{N=1..3,n//6\}$ region at level of each N super shell, or each N super shell is described by a single $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ probability curve. According to Figure 4a of SunQM-3s10, $N=1$ super shell region can be described by $|n,l,m\rangle = |4*6,l=4-1=3,m=3\rangle$ QM state with r_1 at $\{1,1//6\}$, or $r^2 * |R(n=4*6,l=3)|^2 * |Y(3,3)|^2$ curve. Also the $N=2$ super shell region can be described by $|2*6^2,l=2-1=1,m=1\rangle$ QM state with r_1 at $\{2,1//6\}$, or $r^2 * |R(n=2*6^2,l=1)|^2 * |Y(1,1)|^2$ curve. Similarly, the $N=3$ super shell region can be described by $|2*6^3,l=2-1=1,m=1\rangle$ QM state with r_1 at $\{3,1//6\}$, or $r^2 * |R(n=2*6^3,l=1)|^2 * |Y(1,1)|^2$ curve. Therefore, we can have the eq-11 equivalent equation with lower resolution as

$$|n,l,m\rangle_{\text{SolarSystem}} = a * |n,l,m\rangle_{\text{SunCore}} + b * |1,0,0\rangle + c_1 * |4*6^1,3,3\rangle + c_2 * |2*6^2,1,1\rangle + c_3 * |2*6^3,1,1\rangle \quad \text{eq-14}$$

where a, b, c_1, \dots, c_3 are the coefficients of the linear combination. The corresponding primary $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ is:

$$r^2 * |\Psi(r,\theta,\varphi)_{\text{SolarSystemLowResolution}}|^2 \propto r^2 * \{ [\text{eq-3}] + |b * R(1,0)|^2 * |Y(0,0)|^2 + |c_1 * R(4*6^1,3)|^2 * |Y(3,3)|^2 + |c_2 * R(2*6^2,1)|^2 * |Y(1,1)|^2 + |c_3 * R(2*6^3,1)|^2 * |Y(1,1)|^2 \} \quad \text{eq-15}$$

and its integration form is:

$$\text{Mass}(r, \theta, \varphi) = 1.99E+30 \text{ kg} = \iiint [\text{eq-15}] * \sin(\theta) \text{ dr d}\theta \text{ d}\varphi, [r=0, 1550*1.49E+11 \text{ m}; \theta=0, \pi; \varphi=0, 2\pi] \quad \text{eq-16}$$

II-c. At very low resolution (good resolution for the whole $\{N=1..3,n//6\}$ region as a unit, poor resolution for belts and planets, even for each N super shell)

In current section, we will build the $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ probability curve to describe the whole $\{N=1..3,n//6\}$ region with a single $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ probability curve. According to Figure 4a of SunQM-3s10, it

should be $|4/6^2, l=4-1=3, m=3\rangle$ QM state with r_1 at $\{3, 1/6\}$, or $r^2 * |R(n=4/6^2, l=3)\rangle^2 * |Y(3,3)\rangle^2$ curve. Therefore, we can have the eq-11 equivalent equation with a very lower resolution as

$$|n, l, m\rangle_{\text{SolarSystem}} = a * |n, l, m\rangle_{\text{SunCore}} + b * |1, 0, 0\rangle + c * |4/6^2, 3, 3\rangle \quad \text{eq-17}$$

where a, b, c are the coefficients of the linear combination. The corresponding primary $r^2 * |R(n, l)\rangle^2 * |Y(l, m)\rangle^2$ is:

$$r^2 * |\Psi(r, \theta, \varphi)_{\text{SolarSystemVeryLowResolution}}|^2 \propto r^2 * \{ [eq-3] + |b * R(1, 0)\rangle^2 * |Y(0, 0)\rangle^2 + |c * R(4/6^2, 3)\rangle^2 * |Y(3, 3)\rangle^2 \} \quad \text{eq-18}$$

and its integration form is:

$$\text{Mass}(r, \theta, \varphi) = 1.99E+30 \text{ kg} = \iiint [eq-18] * \sin(\theta) \text{ dr d}\theta \text{ d}\varphi, [r=0, 1550*1.49E+11 \text{ m}; \theta=0, \pi; \varphi=0, 2\pi] \quad \text{eq-19}$$

II-d. To build a Oort cloud at $\{N=4, n=1..5\}$ orbit space using $|n, l, m\rangle$ QM state with $l = 0.., n-1, m = -l, \dots +l$

From wiki ‘‘Oort cloud’’, ‘‘Oort cloud is a theoretical cloud of predominantly icy planetesimals believed to surround the Sun. The region can be subdivided into a spherical outer Oort cloud of 20,000–50,000 AU, and a torus-shaped inner Oort cloud of 2,000–20,000 AU’’. Therefore in Figure 2 of SunQM-1, the inner Oort cloud is assigned as in $\{4, n=1..3/6\}$ orbit shells with $\Delta\theta' \approx \pm 30^\circ$, and the outer Oort cloud is assigned as in $\{4, n=4..5/6\}$ orbit shells with $\Delta\theta' \approx \pm 90^\circ$. So Oort cloud is not in a $|nLL\rangle$ QM state. It is in a general $|n, l, m\rangle$ form with $l = 0 \dots n-1, m = -l \dots +l$, although for the inner Oort, both the low valued $l(s)$ and the low valued $|\pm m| (s)$ are missing. Then we can write Oort cloud’s primary form $|n, l, m\rangle$ as a combination of

$$|n, l, m\rangle_{\text{Oort}} = d_1 * |1*5.33*6^3, l, m\rangle + d_2 * |2*5.33*6^3, l, m\rangle + d_3 * |3*5.33*6^3, l, m\rangle + d_4 * |4*5.33*6^3, l, m\rangle + d_5 * |5*5.33*6^3, l, m\rangle. \quad \text{eq-20}$$

where $d_1 \dots d_5$ are the linear combination coefficients, and $l = 0 \dots n-1, m = -l \dots +l$, and both the low valued $l(s)$ and $|\pm m| (s)$ are missing, and the missing weight is heavy at the small n, l, m side, and light at large n, l, m side. Note: for accuracy, the original $n * 6^4$ is now become $n * 5.33 * 6^3$ due to the compression in $N=2$ super shell. Note: here we need to use the primary form $|n^*q^N, l, m\rangle$. Then Oort cloud’s primary 3D probability density $r^2 * |R(n, l)\rangle^2 * |Y(l, m)\rangle^2$ is:

$$r^2 * |\Psi(r, \theta, \varphi)_{\text{OortCloud}}|^2 \propto r^2 * [|d_1 * R(1*5, 33*6^3, l)\rangle^2 * |Y(l, m)\rangle^2 + |d_2 * R(2*5, 33*6^3, l)\rangle^2 * |Y(l, m)\rangle^2 + |d_3 * R(3*5, 33*6^3, l)\rangle^2 * |Y(l, m)\rangle^2 + |d_4 * R(4*5, 33*6^3, l)\rangle^2 * |Y(l, m)\rangle^2 + |d_5 * R(5*5, 33*6^3, l)\rangle^2 * |Y(l, m)\rangle^2] \quad \text{eq-21}$$

and its integration form is:

$$\text{Mass}(r, \theta, \varphi) = 1.99E+30 \text{ kg} = \iiint [eq-21] * \sin(\theta) \text{ dr d}\theta \text{ d}\varphi, [r=0, 8000*1.49E+11 \text{ m}; \theta=0, \pi; \varphi=0, 2\pi] \quad \text{eq-22}$$

So, once this result is added to the result in section I-a (or I-b, or I-c), we have built a (time-independent) 3D probability density $r^2 * |R(n, l)\rangle^2 * |Y(l, m)\rangle^2$ for the whole Solar system (in median, low, or very low resolution) including Oort cloud.

III. To build a (time-dependent, semi-QM deduced) 3D probability density $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ for a planet in region from $\{1,3//6\}$ to $\{2,5//6\}$ at high resolution

From $\{1,3//6\}$ to $\{2,5//6\}$, the mass occupancy is $< 1\%$, so this region is dominated by the spinning Sun's QM nLL disk-lyzation effect. Now let's use planet Earth at orbit $\{1,5//6\} = \{0,30//6\}$ as the example. After the pre-Sun ball quantum collapsed into current Sun (see SunQM-1s1 for details), the mass (with $< 1\%$ mass occupancy) in orbit space $\{1,5//6\}$ disk-lyzed into a belt (or a ring) governed by the spinning Sun QM's nLL effect (see SunQM-3s1, and SunQM-3s2 for details). Then the QM effect caused the mass in this belt to accrete into a single planet (see SunQM-4s1 and SunQM-4s3 for details). The original Earth was $\sim 25x$ of mass of current Earth (see SunQM-1s1 for details), Then the expanded ice-evap-line ripped off the whole (original) outer atmosphere (after or even before the original Earth was formed), and left a core to be our current Earth (see SunQM-3s6 for details).

In eq-11 and eq-12, Earth is described as $|30,29,29\rangle$ QM state or a 3D probability density of $r^2 * |R(30,29)|^2 * |Y(29,29)|^2$. It gives the good description for Earth's orbit-r, the poor description for Earth's surface-r, because the probability curve is too broad. So we need an $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ function that can describe both Earth's orbit-r and Earth's surface-r with good accuracy. From the knowledge we learned from Figure 3 of SunQM-3s10, we know that we can achieve this goal by using the high frequency multiplier n' . Since the mass ratio of Moon to Earth is $7.3E+22 \text{ kg} / 5.97E+24 \text{ kg} \approx 1.2\%$, so we can set our goal as: to looking for n' to make $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ curve's width (with 99% of Earth mass included) equals to 2 times of Earth's surface-r in all $r, \theta, \phi, 3D$ -dimensions. In SunQM-3s10, an Eigen quantum n' is defined as the maximum n' that can describe one orbit space's $> 90\%$ mass in a single $|nLL\rangle = |n', n'-1, n'-1\rangle$ QM state. Therefore, the n' we are looking for is the Eigen n' of this planet.

III-a. (full-QM) determination of Solar $\{N, n\}$ QM's r-dimensional probability density for Earth using multiplier $n'_r = n * q^w r$

Here we try to figure out n'_r for the $r^2 * |R(n', l=n'-1)|^2$ curve, so that the width at 1% of its peak covers Earth's diameter $d = 2r = 1.28E+7 \text{ m}$. According to SunQM-3s10's eq-1, $|nLL\rangle$ QM state's $R(n, l=n-1)$ formula is

$$R(n, l=n-1) \propto r_1^{-(3/2)} * (r / r_1)^{(n-1)} * \exp(-r / r_1 / n) \quad \text{eq-23}$$

After normalize to the maximum value, it becomes (see SunQM-3s10, eq-5)

$$r^2 * |R(n, n-1)|^2 \propto [r / r_n * \exp(1 - r / r_n)]^{(2 * n)} \quad \text{eq-24}$$

If we plot eq-24, we can see that this is an exponential rising curve times an exponential declining curve, with maximum always at r_n , and the higher the n value, the narrower the peak. Eq-23 and eq-24 are valid for both base-frequency n and high-frequency n' , with the limitation that $R(n, l=n-1)$.

Now the task becomes to adjust n' value to make eq-24 curve peak's half-width (at 1% of its peak value) Δr roughly equals a planet's r_{surface} . Since this n' is specifically in r-dimension, we name it as n'_r . For a planet, we know its orbital r_n and its r_{surface} (here we use $b = r_{\text{surface}}$ in the formula), so we need to find an n'_r so that at $r_n \pm b$, its probability of eq-24 equals to 0.01. Using eq-24, we have

$$\{[(r_n \pm b) / r_n * \exp[1 - (r_n \pm b) / r_n]]^{(2 * n'_r)} = 0.01 \quad \text{eq-25}$$

or,

$$n'_r = \text{LN}(0.1) / [\text{LN}(1 \pm b / r_n) - (\pm b / r_n)] \quad \text{eq-26}$$

For Earth, $r_n = 1.57E+11$ m (or more accurately, $1.565E+11$ m), $b = r_{\text{surface}} = 6.38E+6$ m, so we calculate n'_r at $r_n \pm b$ (either $r = 1.565E+11 + 6.38E+6$ m, or $r = 1.565E+11 - 6.38E+6$ m) as $n'_r = \text{LN}(0.1) / [\text{LN}(1 + 6.38E+6 / 1.565E+11) - (6.38E+6 / 1.565E+11)] = 2.77E+9$ (see Table 1 column 13), and $r_1' = 1.565E+11 / n'^2 = 2.01E-8$ m. Then, for $\{N,n/q\}$ QM, an n' need to satisfy $n' = n * q^w$ (where w is integer). Therefore, we have $2.77E+9 = 5 * 6^w$, $w = \log(2.77E+9 / 5) / \log(6) = 11.24$, or round w down to 11. So, at $w = 11$, $n' = n * q^w$, we have the final $n'_r = 5 * 6^{11} \approx 1.81E+9$ (see Table 1 column 16). This is also the Eigen n' of Earth in r-dimension.

We know that at r equals to $r_n = 1.57E+11$ m, the probability is maximum, or = 100%. Then we can ask at what probability % eq-24 gives (a peak's half-width) Δr exactly equals to Earth's r_{surface} ? The answer is, it equals to $[(1 \pm 6.38E+6 / 1.57E+11) * \exp((1 - (1 \pm 6.38E+6 / 1.57E+11)))^{(2 * 1.81E+9)} \approx 5\%$ of maximum. If we choose $w=12$ (rather than $w=11$), then $n' = n * q^w$, $n' = 5 * 6^{12} \approx 1.09E+10$, or at $r = 1.57E+11 + 6.38E+6$ m, probability = $[(1 + 6.38E+6 / 1.57E+11) * \exp((1 - (1 + 6.38E+6 / 1.57E+11)))^{(2 * 1.09E+10)} = 1.52E-8$ of maximum probability. It obvious that $1.52E-8$ is too strict. So here we choose $w=11$ for Earth's n' . For Earth's $w=11$, $r_1' = r_n' / (n * q^w)^2 = 1.565E+11 / (5 * 6^{11})^2 = 4.76E-8$ m (see Table 1 column 18). Check SunQM-1s2 Table 1, we find that $4.76E-8$ m equals to the Hot-G r track at $\{-10,1\}$, and then it is recorded in Table 1 column 19. In Table 1 columns 13 through 19, all other planets' $n'_r(s)$ were determined by using eq-26. In column 14's calculation, for $N=1$ super-shell $\{1,n/6\}$ planets,

$$n' = n * 6^w, w = \log(n' / n) / \log(6) \tag{eq-27}$$

and for $N=2, N=3$ super-shell (or n at $\{2,2//6\}$ and above),

$$n' * (5.33 / 6) = n * 6^w, w = \log(n' * (5.33 / 6) / n) / \log(6) \tag{eq-28}$$

III-b. (full-QM) determination of Solar $\{N,n\}$ QM's θ -dimensional probability density for a planet using multiplier $n'_\theta = n * q^w$

QM text books (e.g., from Davis J Griffiths' book "Introduction to Quantum mechanics", 2nd ed. 2005. pp135, combining eq-2.7, eq-4.15 and eq-4.19) tell us that the wave function for the Schrodinger equation can be written as eq-9: $\Psi(r,\theta,\varphi,t) = R(r) * \Theta(\theta) * \Phi(\varphi) * T(t)$ where $\Theta(\theta) * \Phi(\varphi) = Y(l,m)$ as shown in eq-10. For $|nLL\rangle = |n,l=n-1,m=n-1\rangle$, we know that (see John S. Townsed, A Modern Approach to Quantum Mechanics, 2nd ed., 2012, pp334, eq-9.146, or from wiki "Table of spherical harmonics"),

$$Y(l=n-1,m=n-1) \propto \exp(im\varphi) * [\sin(\theta)]^l = \exp[i * (n-1) * \varphi] * [\sin(\theta)]^{(n-1)} \tag{eq-29}$$

Or

$$|\Theta(\theta)|^2 \propto [\sin(\theta)]^{[2(n-1)]} \tag{eq-30}$$

where n can be either base frequency n or high-frequency n'_θ . Note: both eq-29 and eq-30 are only valid for nLL QM state which means $l = n-1, m = n-1$. Similar as what we have done in section III-a, let's define that 1% of the probability density peak is the acceptable (probability density peak's) size for a planet, and let's define $\theta = \pi/2 - \theta'$, and define n'_θ is the n' in θ -dimension, then eq-30 under $n'_\theta \gg 1$ can be rewritten as

$$\sin(\theta)^{[2 * (n'_\theta - 1)]} = [\sin(\pi/2 - \theta')]^{[2 * (n'_\theta - 1)]} = [\cos(\theta')]^{[2 * (n'_\theta - 1)]} \approx [\cos(\theta')]^{(2 * n'_\theta)} = 0.01 \tag{eq-31}$$

or (see Table 1 column 23)

$$\theta' = \arccos\{0.01^{[1 / (2 * n'_\theta)]}\} \tag{eq-32}$$

Then, we can calculate planet's surface-r as $\sin(\theta') * r_n$, where r_n is planet's orbit r (see Table 1 column 11). In Table 2, using $n' = n * q^w = 5 * 6^w$ for Earth, we search w value to fit Earth's surface-r = 6.38E+6 m. The result shows that at w=11, it gives the best fitting. So, $w(\theta) = w(r) = 11$, and $n'_\theta = n'_r = 1.81E+9$. This is also the Eigen n' of Earth in θ -dimension.

In the same way, in columns 20 through 24 of Table 1, values of $w(\theta)$ and n'_θ were determined for all other planets. As expected, all planets have $w(\theta) = w(r)$, and $n'_\theta = n'_r$.

Table 2. Searching the right $w(r)$ for Earth

n=	5	5	5
q=	6	6	6
w=	10	11	12
$n'=n*q^w$	3.02E+08	1.81E+09	1.09E+10
$0.01^{1/(2n'_\theta)}$	0.999999992	0.999999999	1.000000000
arccos	0.000123419	0.000050386	0.000020570
r=	1.57E+11	1.57E+11	1.57E+11
b=	1.94E+07	7.91E+06	3.23E+06

III-c. (semi-QM) determination of Solar $\{N, n\}$ QM's φ -dimensional probability density for Earth with time-dependent orbital movement

QM text books tell us that for the $|n, l, m\rangle = |n, l=n-1, m=n-1\rangle$ state in eq-29,

$$\Phi(\varphi) \propto \exp(im\varphi) \tag{eq-33}$$

and

$$T(t) \propto \exp(-i\omega t), \text{ where } \omega = E/\hbar \tag{eq-34}$$

In the traditional QM probability calculation, $|\exp(im\varphi)|^2 = \exp(im\varphi) * \exp(-im\varphi) = 1$, and $|\exp(-i\omega t)|^2 = \exp(-i\omega t) * \exp(+i\omega t) = 1$ (see David J. Griffiths, "Introduction to Quantum Mechanics", 2nd ed., 2015, pp345). Therefore, both the probability peak and its time-dependent movement in φ -dimension cancelled out (for any situation, even for an orbital moving planet). We know that Earth on orbit $\{1,5\}$ should correspond to a probability density peak, and this peak should have a time-dependent orbital movement in φ -dimension. Therefore, the traditional QM probability calculation method is no longer correct for this situation. A new method to calculate the Solar $\{N, n\}$ QM's φ -dimensional probability density (and its time-dependency) has to be established. However, due to that the newly established method (see SunQM-4s1) against some current QM rules, many readers may not accept it. So in the current section, let us first use the classical wave physics (with some traditional QM) to describe solar QM's φ -dimension probability density peak and the time-dependency.

In Solar $\{N, n\}$ QM, a planet doing orbital movement is described as this planet is in the nLL QM state (see SunQM-3s1). According to (de Broglie's matter wave theory explained) Bohr model (see Douglas C. Giancoli, Physics for Scientists & Engineers with Modern Physics, 4th ed. 2009, pp 1010, Figure 37-29), an electron doing orbital movement around a proton can be described as a 1D (standing) wave traveling in the φ -dimension-only space. Using the exactly the same concept, here we also describe a planet doing orbital movement around Sun as the planet's matter wave (a 1D standing wave) traveling in the φ -dimension-only space.

In classical physics (see Giancoli, pp404, eq.15-10a and eq.15-10c), the mathematical description of a 1D traveling wave is

$$D(x,t) = A \sin (2\pi / \lambda *(x - v * t)) = A \sin (2\pi / \lambda * x - 2\pi v / \lambda * t) \tag{eq-35}$$

where $D(x)$ is the displacement of wave at position x , and A is the amplitude, $v = \lambda * f$ is the (phase) velocity of the moving wave. Or

$$D(x,t) = A \sin(k * x - \omega * t), \text{ where } k = 2\pi / \lambda, \omega = 2\pi * f. \quad \text{eq-36}$$

Note: although in Giancoli's text book, ω is named as the angular frequency, in SunQM series papers, we name ω as angular frequency/velocity (because its meaning is more like angular velocity), and name f in $\omega = 2\pi * f$ as the orbital frequency. Comparing to the traditional QM probability form of

$$|\Phi(\varphi)|^2 * |T(t)|^2 \propto |\exp(im\varphi)|^2 * |\exp(-i\omega t)|^2 = |\exp[i * (m\varphi - \omega t)]|^2 = |\cos(m\varphi - \omega t) + i * \sin(m\varphi - \omega t)|^2 \quad \text{eq-37}$$

Let's rewrite eq-36 in that, 1) choose $\cos(x)$ rather than $\sin(x)$ for the wave, 2) choose $\lambda = 2\pi$, or $k = 2\pi / \lambda = 1$ (Note: $\lambda = 2\pi$ should mean in φ -dimension 1D wave function $n=1$, or ground state), 3) use $\varphi = x$, so that

$$D(\varphi,t) = A * \cos(\varphi - \omega t) \quad \text{eq-38}$$

For the orbital moving Earth, its ω can be calculated either as $\omega = 2\pi * f = 2\pi / [365 \text{ days}] = 1.99E-7$ (arc/second), or as $\omega * r = v$, or $\omega = v / r = [29800 \text{ (m/s)}] / [1.49E+11 \text{ (m)}] = 2.0E-7$ (arc/second). Notice that this ω is the orbit v related angular frequency/velocity (which correlates to QM matter wave's group velocity, not the phase velocity).

For a $|nLL\rangle = |n, n-1, n-1\rangle$ QM state's $Y(l,m)$, it is eq-29, or

$$Y(l=n-1, m=n-1) \propto [\cos(\varphi) + i * \sin(\varphi)]^{(n-1)} * [\sin(\theta)]^{(n-1)} \quad \text{eq-39}$$

Or

$$|Y(l=n-1, m=n-1)|^2 \propto |\cos(\varphi) + i * \sin(\varphi)|^{[2 * (n-1)]} * |\sin(\theta)|^{[2 * (n-1)]} \quad \text{eq-40}$$

Notice that here we use $|\cos(\varphi) + i * \sin(\varphi)|^2$ but not $[\cos(\varphi) + i * \sin(\varphi)]^2$ because the former one involves the conjugated production. Section III-b showed that when $n \gg 1$, the probability density of $|\sin(\theta)|^{(n-1)}$ became very thin plane at around x-y plane, so we know that the probability function of eq-40 must describe an object that is doing φ rotation in x-y plan (because $|\cos(\varphi) + i * \sin(\varphi)|^2$ describing a circular movement).

After reviewing the physics meaning of the Euler's formula $\exp(\varphi) = \cos(\varphi) + i * \sin(\varphi)$, I realized that just like that the θ -dimension's matter wave function $\Theta(\theta)$ is a 1D periodic (real number) function (see eq-30), and the r -dimension's matter wave function $R(r)$ is a 1D (real number) function (see eq-23 or eq-24), the original φ -dimension's matter wave function $\Phi(\varphi)$ (in nLL QM state) should also be a 1D (φ -dimension) periodic (real number) function. The only reason to use a complex number in φ -dimension's matter wave function $\Phi(\varphi) \propto \exp(im\varphi)$ is that the complex numbered Euler's formula is exceptionally simple for describing a circular movement (which in most cases is in RF with θ -dimension). However, now the complex numbered wave function $[\cos(\varphi) + i * \sin(\varphi)]^{(n-1)}$ become an obstacle (at least for me) to calculate the mass density. So now we need to get rid off the complex number, and use the (original) 1D φ -dimensional periodic (real number) function as the wave function. From eq-39, we see $\Theta(\theta) \propto [\sin(\theta)]^{(n-1)}$, so that $|\Theta(\theta)|^2 \propto [\sin(\theta)]^{[2 * (n-1)]}$ (Note: for nLL QM state only). Since a planet's projection in the Solar system's $\theta\varphi$ -2D-dimension is a circle, the probability density of $|\Theta(\theta)|^2$ and $|\Phi(\varphi)|^2$ (which directly correlate to the mass density) also should have the same formula. Therefore, $|\Phi(\varphi)|^2$ can be either $[\cos(\varphi)]^{[2 * (n-1)]}$ or $[\sin(\varphi)]^{[2 * (n-1)]}$. Here we choose

$$|\Phi(\varphi)|^2 \propto |\cos(\varphi) + i * \sin(\varphi)|^{[2 * (n-1)]} \propto [\cos(\varphi)]^{[2 * (n-1)]} \quad \text{eq-41}$$

which equivalent to

$$\Phi(\varphi)_{\text{equivalent}} \propto [\cos(\varphi)]^{(n-1)} \quad \text{eq-42}$$

where n can be either base frequency n or high-frequency n'_φ . Notice that this is valid for nLL QM state only!

Another reason to support eq-41 is, for $|nLL\rangle$ QM state, if we pair $Y(l=n-1, +m=n-1)$ and $Y(l=n-1, -m=n-1)$, then $Y(l, +m) + Y(l, -m)$ is proportional to either $\cos(m\varphi)$ or $i * \sin(m\varphi)$, and $\exp(\pm im\varphi)$ will be removed from $\Phi(\varphi)$ the wave function. For example,

$$Y(1,1) + Y(1,-1) = (-1/2) * \sqrt{3/2/\pi} * \exp(i\varphi) * \sin(\theta) + (1/2) * \sqrt{3/2/\pi} * \exp(-i\varphi) * \sin(\theta) = -\sqrt{3/2/\pi} * i * \sin(\varphi) * \sin(\theta), \text{ because } -\exp(i\varphi) + \exp(-i\varphi) = -2 * i * \sin(\varphi);$$

$$Y(2,2) + Y(2,-2) = (1/4) * \sqrt{15/2/\pi} * \exp(i2\varphi) * [\sin(\theta)]^2 + (1/4) * \sqrt{15/2/\pi} * \exp(-i2\varphi) * [\sin(\theta)]^2 = (1/2) * \sqrt{15/2/\pi} * \cos(2\varphi) * [\sin(\theta)]^2, \text{ because } \exp(i2\varphi) + \exp(-i2\varphi) = 2 * \cos(2\varphi);$$

$$Y(3,3) + Y(3,-3) = (-1/8) * \sqrt{35/\pi} * \exp(i3\varphi) * [\sin(\theta)]^3 + (1/8) * \sqrt{35/\pi} * \exp(-i3\varphi) * [\sin(\theta)]^3 = (-1/4) * \sqrt{35/\pi} * i * \sin(3\varphi) * [\sin(\theta)]^3, \text{ because } -\exp(i3\varphi) + \exp(-i3\varphi) = -2 * i * \sin(3\varphi), \text{ etc.}$$

Then the probability calculation of $|Y(l, \pm m)|^2 \propto |\Phi(\varphi)|^2 \propto [\cos(m\varphi)]^2$ when m is an even number, or $|\Phi(\varphi)|^2 \propto |-i * \sin(m\varphi)|^2 = |i * \sin(m\varphi)| * |-i * \sin(m\varphi)| = [\sin(m\varphi)]^2$ when m is an odd number. And then treat the probability of $[\cos(m\varphi)]^2$ equivalents as the probability of $[\sin(m\varphi)]^2$, and also use SunQM-4s1 Figure 2 (Note: figure number may change after publication)'s result, then we will also get eq-41.

Using eq-41, then eq-40 can be simplified as

$$|Y(l=n-1, m=n-1)|^2 \propto \{[\cos(\varphi)]^2\}^{(n-1)} * \{[\sin(\theta)]^2\}^{(n-1)} \tag{eq-43}$$

In eq-43, we force $n'_\varphi = n'_\theta = n'$ (and write n' as n). So Earth's Eigen n' in φ -dimension is forced to be the same as that in θ -dimension. We know that the power index $(n-1)$ in $\{[\sin(\theta)]^2\}^{(n-1)}$ is to make $[\sin(\theta)]^2$ curve peak become more narrower. Similarly, the power index $(n-1)$ in $\{[\cos(\varphi)]^2\}^{(n-1)}$ is to make $[\cos(\varphi)]^2$ curve peak become more narrower. Because $|\Theta(\theta)|^2$ and $|\Phi(\varphi)|^2$ have the same formula (notice that $[\sin(\varphi)]^2$ and $[\cos(\varphi)]^2$ are the same in turns of probability calculation) and same power index, eq-43 generates a perfect circle for a planet's projection in the Solar system's $\theta\varphi$ -2D-dimension, and this is exactly what we expected.

Combining all results from sections II-a, b and c, now we have a time-independent 3D probability density peak which has both orbit- r and the body- r closely matches to that of a planet:

$$r^2 * |\Psi(r, \theta, \varphi)_{\text{Planet}}|^2 \propto r^2 * |R(n, l)|^2 * |[\sin(\theta)]^l * [\cos(\varphi)]^m|^2 = [r/r_n * \exp(1 - r/r_n)]^{(2 * n')} * [\sin(\theta)]^{[(2 * (n' - 1)) * [\cos(\varphi)]^{(2 * (n' - 1))}] \tag{eq-44}$$

where $n'_r = n'_\varphi = n'_\theta = n'$ (see Table 1).

To add the time dependent orbital movement in φ -dimension for this probability density peak in $|nLL\rangle$ QM state, we can simply replace φ by $\varphi - \omega t$, where ω is the angular frequency/velocity of this planet's orbital movement, so that eq-44 becomes

$$r^2 * |\Psi(r, \theta, \varphi - \omega t)_{\text{Planet}}|^2 \propto [r/r_n * \exp(1 - r/r_n)]^{(2 * n')} * [\sin(\theta)]^{[(2 * (n' - 1)) * [\cos(\varphi - \omega t)]^{(2 * (n' - 1))}] \tag{eq-45}$$

Eq-45 is the final semi-QM deduced 3D probability density $r^2 * |R(n, l)|^2 * |Y(l, m)|^2$ for a planet in Solar $\{N, n\}$ QM structure. Notice that eq-45 is originated from

$$r^2 * |\Psi(r, \theta, \varphi - \omega t)_{\text{Planet}}|^2 \propto [r/r_n * \exp(1 - r/r_n)]^{(2 * n'_r)} * [\sin(\theta)]^{[(2 * (n'_\theta - 1)) * [\cos(\varphi - \omega t)]^{(2 * (n'_\varphi - 1))}] \tag{eq-46}$$

and for planets, we have $n'_r = n'_\theta = n'_\phi = n'$ (see Table 1 columns 16 & 21), so eq-46 becomes eq-45. Notice that in eq-45, in the power index of $(2 * n')$ or $2 * (n'-1)$, the multiplier n' is switchable to base frequency n . Notice that when $n' \gg 1$ (e.g., Earth's $n' = 5 * 6^{11} = 1813985280 \gg 1$), then $n'-1 \approx n'$. So we can simplify eq-46 to be:

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{Planet}}|^2 \propto [r / r_n * \exp(1 - r / r_n) * \sin(\theta) * \cos(\phi - \omega t)]^{(2 * n')} \tag{eq-47}$$

After input Earth's r_n , ω and n' values, we have

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{Earth}}|^2 \propto [r / 1.57E+11 * \exp(1 - r / 1.57E+11) * \sin(\theta) * \cos(\phi - \omega t)]^{(2 * 1.81E+9)} \tag{eq-48}$$

Notice that $r_n = 1.57E+11$ m, $\omega = 1.86E-7$ arc/s, and $n' = 1.81E+9$ are modeled $\{1, 5//6\}$ orbit's values (obtained from Table 1 column 11 and 26), not exactly the Earth's values (because the whole calculation in this paper is based on Solar $\{N, n/q\}$ QM model structure that mimics the Solar system). Remember at this time, the ϕ -dimension probability peak and the time-dependent movement is deduced by using the semi-QM and semi-classical physics (because eq-35, eq-36, eq-38 are come from the classical physics), and remember eq-45 and eq-47 only valid for nLL QM state, and eq-47 only valid at $n' \gg 1$.

When looking into the formula of eq-47, we are amazed on how simple the formula is and how straightforward meaning it is: $r^2 * |R(r)|^2 \propto [r / r_n * \exp(1 - r / r_n)]^{(2 * n')}$ produces an exponential rising curve times an exponential declining curve, with the peak always at $r = r_n$, and the higher the n' , the narrower the peak. $|\Theta(\theta)|^2 \propto [\sin(\theta)]^{(2 * n')}$ produces a peak at $\theta = \pi/2$, and the higher the n' , the narrower the peak. $|\Phi(\phi - \omega t)|^2 \propto [\cos(\phi - \omega t)]^{(2 * n')}$ produces a peak at $\phi - \omega t = 0$, or $\phi = \omega t$, and the higher the n' , the narrower the peak.

Figure 1 shows that eq-47 with $2n' = 64$ gives a narrow peak in $\theta\phi$ -2D-dimension and moving in $+\phi$ direction with $\phi = \omega t = 0, 0.5$, and 1 . However, we can see one problem in the figure: $[\cos(\phi - \omega t)]^{(2 * n')}$ generates two probability peaks in ϕ -dimension: one at $\phi - \omega t = 0$ (or $\phi = \omega t$), and the second one at $\phi - \omega t = \pm\pi$ (or $\phi = \omega t \pm \pi$). We know that in Earth's $\{1, 5\}$ orbit, there should be only one mass density peak. So only the first probability peak represents the true mass density peak (which corresponds to planet matter wave's positive maximum amplitude). The second probability peak represents the mass density minimum (which correspond to planet matter wave's negative maximum amplitude) and need to be ignored. In the full-QM deduction in SunQM-4s1, the mass density minimum will no longer produce any probability density peak.

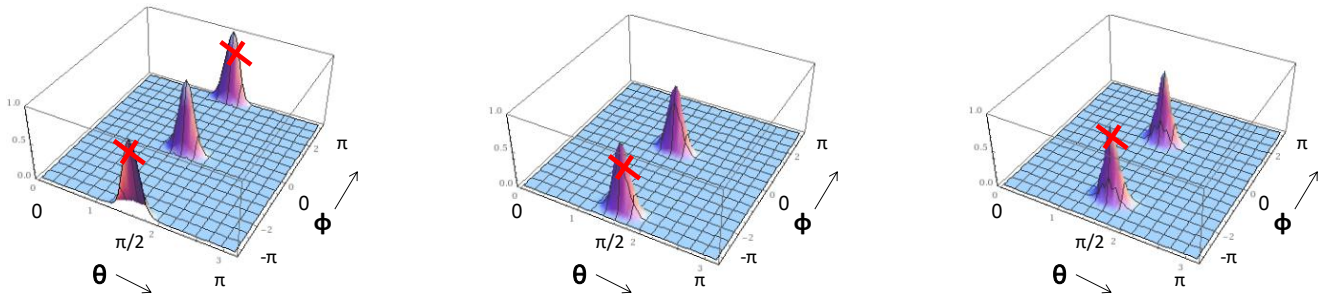


Figure 1. Using WolframAlpha plot to show eq-47's $[\sin(\theta) * \cos(\phi - \omega t)]^{(2 * n')}$ at $2 * n' = 64$ gives a narrow peak in $\theta\phi$ -2D dimension and moving in $+\phi$ direction. Figure 1a (left), $\omega t = 0$; Figure 1b (middle), $\omega t = 0.5$; Figure 1c (right), $\omega t = 1$.

IV. To build a complete (semi-QM) 3D probability density map to describe the whole Solar system with time-dependent orbital movement

In Table 1 column 16 and column 21, eight known planets' and four potential planets' n'_r and n'_θ values were determined (by using the same method as that for Earth's). All of them have $n'_r = n'_\theta = n'_\phi$, and $w(r) = w(\theta) = w(\phi)$. Using an online information (<https://www.theplanetstoday.com/astrology.html>), the initial ϕ values (at time of 8/14/2019) for all eight known planets were listed in Table 1 column 29 (Note: Earth's initial ϕ is set to be 0, and rest planets' initial ϕ values are relative to Earth's $\phi = 0$). Eq-47 gives the general form of a planet's time-dependent 3D probability density formula (with $n' \gg 1$) where values of r_n , ϕ , ω , n' for each planet can be obtained from columns of 11, 29, 26, 16 in Table 1. Therefore, we are able to construct the time-dependent 3D probability density peaks for each planet as (with $t=0$ on Aug. 14, 2019):

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{Mercury}}|^2 \propto [r / 5.64E+10 * \exp(1 - r / 5.64E+10) * \sin(\theta) * \cos(1.08 - 8.61E-7 * t)]^{(2 * 1.09E+9)} \quad \text{eq-49}$$

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{Venus}}|^2 \propto [r / 1.00E+11 * \exp(1 - r / 1.00E+11) * \sin(\theta) * \cos(3.14 - 3.63E-7 * t)]^{(2 * 1.45E+9)} \quad \text{eq-50}$$

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{Earth}}|^2 \propto [r / 1.57E+11 * \exp(1 - r / 1.57E+11) * \sin(\theta) * \cos(0 - 1.86E-7 * t)]^{(2 * 1.81E+9)} \quad \text{eq-51}$$

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{Mars}}|^2 \propto [r / 2.25E+11 * \exp(1 - r / 2.25E+11) * \sin(\theta) * \cos(3.33 - 1.08E-7 * t)]^{(2 * 1.31E+10)} \quad \text{eq-52}$$

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{Jupiter}}|^2 \propto [r / 7.12E+11 * \exp(1 - r / 7.12E+11) * \sin(\theta) * \cos(5.31 - 1.92E-8 * t)]^{(2 * 7.26E+8)} \quad \text{eq-53}$$

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{Saturn}}|^2 \propto [r / 1.60E+12 * \exp(1 - r / 1.60E+12) * \sin(\theta) * \cos(5.74 - 5.69E-9 * t)]^{(2 * 6.53E+9)} \quad \text{eq-54}$$

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{Uranus}}|^2 \propto [r / 2.85E+12 * \exp(1 - r / 2.85E+12) * \sin(\theta) * \cos(1.27 - 2.40E-9 * t)]^{(2 * 5.22E+10)} \quad \text{eq-55}$$

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{Neptune}}|^2 \propto [r / 4.45E+12 * \exp(1 - r / 4.45E+12) * \sin(\theta) * \cos(0.52 - 1.23E-9 * t)]^{(2 * 6.53E+10)} \quad \text{eq-56}$$

For Asteroid belt, according to SunQM-3s10 eq-6, its Eigen quantum number $n' = 48$, so we have

$$r^2 * |\Psi(r, \theta, \phi - \omega t)_{\text{AsteroidBelt}}|^2 \propto r^2 * |R(48, 47)|^2 * |Y(47, 47)|^2 \propto [r / 4.01E+11 * \exp(1 - r / 4.01E+11)]^{(2 * 48)} * [\sin(\theta)]^{[2 * (48-1)]} * |\Phi(\phi - \omega t)|^2 \quad \text{eq-57}$$

where $\omega = \omega_n = 4.54E-8$ arc/s is the (averaged) angular frequency/velocity of the rotational Asteroid belt at orbit $\{0, 48/6\}$ in ϕ -dimension. For a belt, the probability density (or mass density) is de-localized everywhere in ϕ -dimension, so we can use the traditional QM's $|\Phi(\phi)|^2 \propto \exp(-i\phi) * \exp(i\phi) = 1$. Then we have a standing belt

$$r^2 * |\Psi(r, \theta, \phi)_{\text{AsteroidBelt}}|^2 \propto [r / 4.01E+11 * \exp(1 - r / 4.01E+11)]^{(2 * 48)} * [\sin(\theta)]^{(2 * 47)} \quad \text{eq-58}$$

A more accurate description of an orbital rotating Asteroid belt (and Kuiper belt) will be given in SunQM-4s1 under the full-QM deduction.

For the cold-KBO in Kuiper belt, according to SunQM-3s10 eq-13, its Eigen quantum number $n' = 192$, so we have

$$r^2 * |\Psi(r,\theta,\varphi-\omega t)_{\text{KuiperBeltColdKBO}}|^2 \propto r^2 * |R(192,191)|^2 * |Y(191,191)|^2 = [r / 6.40E+12 * \exp(1 - r / 6.40E+12)]^{(2 * 192)} * [\sin(\theta)]^{(2 * (192-1))} * |\Phi(\varphi-\omega t)|^2 \quad \text{eq-59}$$

where $\omega = \omega_n = 7.11E-10$ arc/s is the (averaged) angular frequency of the rotational Kuiper belt at orbit $\{0,192//6\}$ in φ -dimension. Or, the standing version

$$r^2 * |\Psi(r,\theta,\varphi)_{\text{KuiperBeltColdKBO}}|^2 \propto [r / 6.40E+12 * \exp(1 - r / 6.40E+12)]^{(2 * 192)} * [\sin(\theta)]^{(2 * 191)} \quad \text{eq-60}$$

For the undiscovered $\{3,n=2..5//6\}$ planets/belts, if they have formed planets, then they have the forms of eq-61a through eq-64a, except the initial φ positions are unknown. If they are still in belt form, then they have the forms of eq-61b through eq-64b. If one day we can model out these initial φ positions, then we can really help the experimental astronomers to find the real $\{3,n=2..5//6\}$ planets (see section VIII for more discussion).

$$r^2 * |\Psi(r,\theta,\varphi-\omega t)_{\{3,2\}\text{Planet}}|^2 \propto [r / 2.56E+13 * \exp(1 - r / 2.56E+13) * \sin(\theta) * \cos(\varphi_{\{3,2\}} - 8.89E-11 * t)]^{(2 * 5.64E+12)} \quad \text{eq-61a}$$

$$r^2 * |\Psi(r,\theta,\varphi)_{\{3,2\}\text{Belt}}|^2 \propto [r / 2.56E+13 * \exp(1 - r / 2.56E+13)]^{(2 * 384)} * [\sin(\theta)]^{(2 * 383)} \quad \text{eq-61b}$$

$$r^2 * |\Psi(r,\theta,\varphi-\omega t)_{\{3,3\}\text{Planet}}|^2 \propto [r / 5.76E+13 * \exp(1 - r / 5.76E+13) * \sin(\theta) * \cos(\varphi_{\{3,3\}} - 2.63E-11 * t)]^{(2 * 5.08E+13)} \quad \text{eq-62a}$$

$$r^2 * |\Psi(r,\theta,\varphi)_{\{3,3\}\text{Belt}}|^2 \propto [r / 5.76E+13 * \exp(1 - r / 5.76E+13)]^{(2 * 576)} * [\sin(\theta)]^{(2 * 575)} \quad \text{eq-62b}$$

$$r^2 * |\Psi(r,\theta,\varphi-\omega t)_{\{3,4\}\text{Planet}}|^2 \propto [r / 1.02E+14 * \exp(1 - r / 1.02E+14) * \sin(\theta) * \cos(\varphi_{\{3,4\}} - 1.11E-11 * t)]^{(2 * 4.06E+14)} \quad \text{eq-63a}$$

$$r^2 * |\Psi(r,\theta,\varphi)_{\{3,4\}\text{Belt}}|^2 \propto [r / 1.02E+14 * \exp(1 - r / 1.02E+14)]^{(2 * 768)} * [\sin(\theta)]^{(2 * 767)} \quad \text{eq-63b}$$

$$r^2 * |\Psi(r,\theta,\varphi-\omega t)_{\{3,5\}\text{Planet}}|^2 \propto [r / 1.60E+14 * \exp(1 - r / 1.60E+14) * \sin(\theta) * \cos(\varphi_{\{3,5\}} - 5.69E-12 * t)]^{(2 * 5.08E+14)} \quad \text{eq-64a}$$

$$r^2 * |\Psi(r,\theta,\varphi)_{\{3,5\}\text{Belt}}|^2 \propto [r / 1.60E+14 * \exp(1 - r / 1.60E+14)]^{(2 * 959)} * [\sin(\theta)]^{(2 * 958)} \quad \text{eq-64b}$$

Then we can use a matrix production to constitute a complete probability density function for Solar $\{N,n\}$ QM structure (as shown in eq-65). Eq-65 will produce a 3D map of probability density peaks for a complete Solar system, including a Sun, eight known planets (which doing orbital movement), two known belts (in standing), and four undiscovered planets/belts. Notice that in eq-65, the coefficient matrix is a diagonal only matrix, all non-diagonal cells have values of zero. The (most right) vector space column is composed by the probability density functions (in bold) with the equation numbers of eq-5, eq-49, eq-50, eq-51, eq-52, eq-53, eq-54, eq-55, eq-56, eq-58, eq-60, eq-61, eq-62, eq-63, eq-64, and eq-21.

$$r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{SolarSystem}}|^2 = \begin{matrix} a^2 \\ c_1^2 \\ c_2^2 \\ c_3^2 \\ c_4^2 \\ c_5^2 \\ c_6^2 \\ c_7^2 \\ c_8^2 \\ c_9^2 \\ c_{10}^2 \\ c_{11}^2 \\ c_{12}^2 \\ c_{13}^2 \\ c_{14}^2 \\ c_{15}^2 \end{matrix} \left[\begin{array}{l} \text{Sun, eq-5} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Mercury}}|^2, \text{eq-49} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Venus}}|^2, \text{eq-50} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Earth}}|^2, \text{eq-51} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Mars}}|^2, \text{eq-52} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Jupiter}}|^2, \text{eq-53} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Saturn}}|^2, \text{eq-54} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Uranus}}|^2, \text{eq-55} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Neptune}}|^2, \text{eq-56} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{AsteroidBelt}}|^2, \text{eq-58} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{KuiperBelt}}|^2, \text{eq-60} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\{3,2\}\text{Planet}}|^2, \text{eq-61} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\{3,3\}\text{Planet}}|^2, \text{eq-62} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\{3,4\}\text{Planet}}|^2, \text{eq-63} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\{3,5\}\text{Planet}}|^2, \text{eq-64} \\ \text{Oort cloud, eq-21} \end{array} \right] \quad \text{eq-65}$$

Here we name eq-65 as the ‘‘Solar $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ master matrix formula’’. Eq-65 is the Eigen description of our Solar system using Schrodinger equation’s solution. It can also be written as the integration form:

$$\text{Mass}(r,\theta,\phi-\omega t) = 1.99E+30 \text{ kg} = \iiint [\text{eq-65}] * \sin(\theta) \text{ dr d}\theta \text{ d}\phi, [r=0, 1550*1.49E+11 \text{ m}; \theta=0, \pi; \phi=0, 2\pi] \quad \text{eq-66}$$

Therefore each coefficient in eq-65’s diagonal matrix can be obtained because each item (Sun, planet, belt, cloud)’s integration should equal to this item’s mass.

V. More discussion on the Solar $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ master matrix formula eq-65

1) The r-dimensional probability density distribution of eq-65 is illustrated in Figure 2 (and Table 3). Notice that the previous low resolution diagram of probability density r-distribution (in SunQM-3s1 Figure 4 where all eight planets’ probability density peak widths were very broad) is now updated to a high resolution diagram (where all eight planets’ probability density peak widths are close to planets’ true diameters).

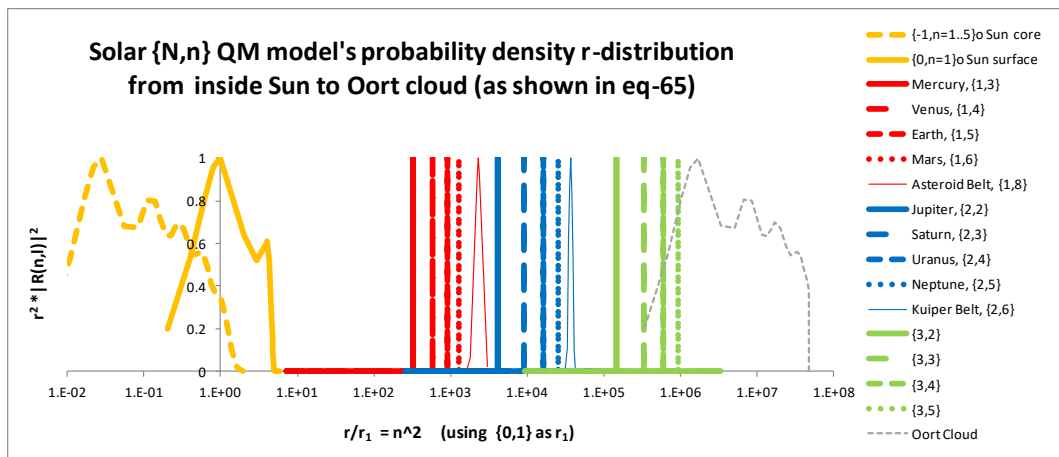


Illustration of eq-65 generated eight planets in $\theta = \pi/2$ plane

Start date: Aug. 14, 2019

End date: 60 days after

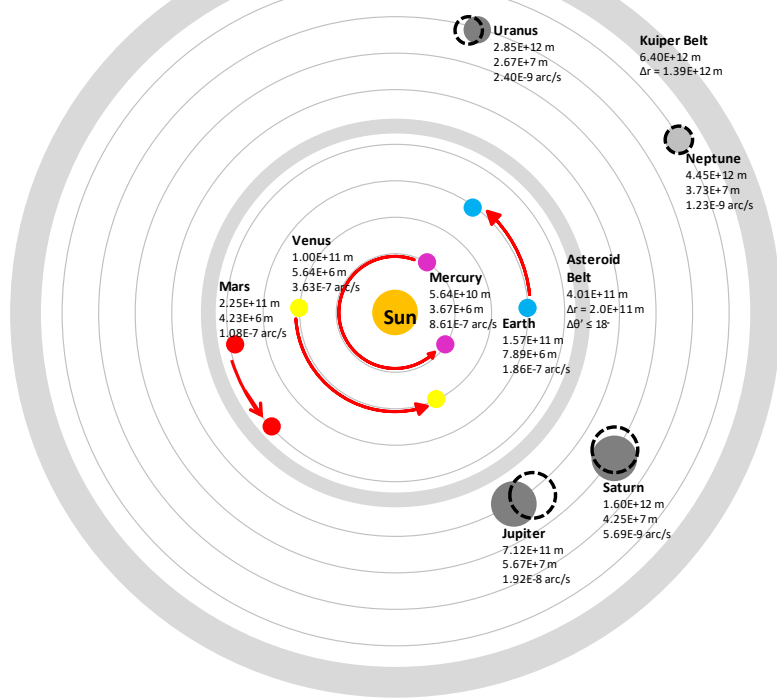


Figure 3. Illustration of eq-65 generated eight planets in $\theta = \pi/2$ plane with initial ϕ positions on Aug. 14, 2019, and the ϕ - ωt positions after 60 days of orbital movement.

3) In eq-65, only planets have the time-dependent description (as orbital movement), the rest objects (Sun, belt, cloud) are described in static. To describe Asteroid belt and Kuiper belt with the orbital rotation movement, since they are in (or mostly in) nLL QM state, it only need to multiply eq-58 (or eq-60) to $\exp(-i\omega t)$ where ω is the belt rotation's angular frequency/velocity (see SunQM-4s1 for details). For Sun and Oort cloud, since they are (mostly) not in nLL QM state, its description is much more complicated.

4) In eq-65, we can set all bases of probability density functions to be time-independent, and then to describe the time-dependent circular movement of Solar system (including planet's orbital movement and Sun's self-spin) by using a spatial rotation matrix. Notice that it is not a regular rotation matrix (that provide only one rotation velocity at one time for all r-distance), this spatial rotation matrix must have $\omega_{n-spin} = \omega_{1-spin} / n^{\wedge}x$ function (see SunQM-3s1 Table 1), and also must able to cover both planet's orbital movement and Sun's self-spin. However, this is beyond my citizen scientist's math level.

5) Here is a summary of what we have achieved in eq-65: We used a single gigantic $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ to describe the whole Solar system. To do this, we picked 6 of base-n, $\{-1,n=1..5//6\}$ and $\{0,1//6\}$, to describe the Sun, we picked 8 base-n ($\{1,n=3..6//6\}$ and $\{2,n=2..5//6\}$) to describe 8 known planets, we picked 2 base-n ($\{1,8//6\}$ and $\{2,6//6\}$) to describe Asteroid belt and Kuiper belt, we picked 4 base-n ($\{3,n=2..5//6\}$) to describe the four undiscovered planet/belt, and we picked 5 base-n ($\{4,n=1..5//6\}$) to describe the Oort cloud. Together, with many Eigen $n'(s)$ or high-frequency $n'(s)$, we can use a relatively simple QM probability density formula to describe the whole Solar system with time-dependent orbital movement at high accuracy. This is a big achievement for the $\{N,n\}$ QM.

VI. To build a complete (full-QM probability density) 3D map to describe the whole Solar system with time-dependent orbital movement

A full-QM deduced time-dependent $|\Phi(\varphi)|^2 * |T(t)|^2$ probability density function has been made. It has been moved to SunQM-4s1 because that in SunQM-3 series, we study Solar $\{N,n\}$ QM within the frame of the traditional QM, i.e., the traditional Schrodinger equation, the Born rule, etc. However, in SunQM-4 series, the traditional Schrodinger equation, the Born rule, etc. is no longer the boundary for us to study the Solar $\{N,n\}$ QM.

VII. Matter waves of galaxies, universe, protons, quarks are all (superposition) running simultaneously in Solar system's matter wave resonance chamber

In SunQM-2 section IV-c, we introduced the concept of MWRC (matter wave resonance chamber) and MWP (matter wave packet): a mass entity (like Solar system, or a proton, or our universe, etc.) can be treated as a matter wave packet (MWP) runs inside a matter wave resonance chamber (MWRC). For example, our Solar system (as a mass entity) itself is a MWRC, and our Solar system produces MWP running in its own MWRC (with high level of RF). Meanwhile, other MWPs produced by other mass entities (e.g., our universe, or galaxies, or protons, or quarks, etc.) also run inside our Solar system's MWRC (with different levels of RF) simultaneously. At the same time, our Solar system produced MWP also runs inside Milky way galaxy's MWRC (with much lower RF), or in our universe's MWRC (with very low RF), or in many proton's MWRCs (with extremely high RF), or even in many quark's MWRCs (with even higher RF).

Now let us use SunQM-3s10's Figure 3 to illustrate this idea in a more intuitive description. In that figure, the cold KBO is a small part of Solar system's MWP running inside Solar system's MWRC. Its orbit at $\{3,1//6\}$ can be Eigen described as $\{0,n=6*6*5.33//6\} = \{0,192//6\}$, or $n=192$, or in $|192,191,192\rangle$ QM state. For simplicity, let's ignore the pFactor of 5.33 and still use 6, so $\{3,1//6\}$ can be Eigen described as $\{0,n=6^3//6\} = \{0,216//6\}$, or $n=216$, or in $|216,215,215\rangle$ QM state. Then its radial wave function is $r^2 * |R(n=216,l=215)|^2$. Remember that this Eigen $n = 6^3 = 216$ is based on r_1 at Sun's $\{0,1//6\}$. The same cold-KBO can also be described by $n = 6^j$ where j is a (either positive or negative) integer number, or $n = \dots 6^{(-5)}, \dots 6^{(-1)}, 6^0, 6^1, 6^2, 6^3, 6^4, \dots 6^{18}$, etc. Recall that the Milky way galaxy at $\{8,1//6\}$, or a proton at $\{-15,1//6\}$ have $\Delta N = +8$ or $\Delta N = -15$ relative to $\{0,1//6\}$ (see SunQM-1s2 Table 1). So if we choose r_1 at $\{8,1//6\}$, then comparing to cold-KBO's $n = 6^3$, the n shifted from $n = 6^3$ to $n = 6^{(3-8)} = 6^{(-5)}$, or the cold-KBO can also be described by $r^2 * |R(n=6^{(-5)},l)|^2$. Then the cold-KBO's $r^2 * |R(n=6^{(-5)},l)|^2$ description is expected to have a strong contribution from Milky way galaxy's MWP (because its r_1 is equivalent to the size of Milky way galaxy). Similarly, if we choose r_1 at $\{-15,1//6\}$, then comparing to cold-KBO's $n = 6^3$, the n shifted from $n = 6^3$ to $n = 6^{(3-(-15))} = 6^{18}$, or the cold-KBO's can also be described by $r^2 * |R(n=6^{18},l)|^2$. Then the cold-KBO's $r^2 * |R(n=6^{18},l)|^2$ description is expected to have a strong contribution from proton's MWP (because its r_1 is equivalent to the size of a proton). Intuitively, we can easily understand that at r_1 at $\{-15,1//6\}$, the cold-KBO's $r^2 * |R(n=6^{18},l)|^2$ description will have a strong contribution from protons' MWPs because the cold-KBO is made of proton, neutron, electron, etc. However, without $\{N,n\}$ QM's help, we can hardly imagine that the cold-KBO's $r^2 * |R(n,l)|^2$ description at $n = 6^{(-5)}$ is expected to have a strong contribution from Milky way galaxy's MWPs, simply because this specific $n = 6^{(-5)}$ correlates to a r_1 at $\{8,1//6\}$.

Using the terminology of the traditional QM, this phenomenon is called superposition of QM states.

In $\{N,n\}$ QM, maybe we can think that Schrodinger equation acts as a matrix, Sun's $\{0,1//6\}$ acts as the Eigen vector, and $n=192$ acts as the Eigen value for the cold-KBO. If using $N \neq 0$'s $\{N,1//6\}$ as "Eigen vector", then there will be no "Eigen value" of $n=192$.

VIII. Can we use 3D probability density $r^2 * |R(n,l)|^2 * |Y(l,m)|^2$ map to calculate out the ϕ -positions of the four undiscovered planets at $\{3,n=2..5//6\}$ orbits?

Suppose that all mass in the $\{3,n=2..5//6\}$ orbit spaces have accreted into planets. Based on the discussion in section VII, we believe that all twelve planets' orbital movements are entangled not only through r-dimension's wave

function $r^2 * |R(n,l)|^2$ (as shown in SunQM-3s10's Figure 3), but also through the whole 3D wave function $r^2 * |R(n,l)| * |Y(l,m)|$, which means that their ϕ -positions are entangled also. When without the entanglement, the Solar system would have formed exactly as the Solar $\{N,n/6\}$ QM model with the accurate $r_n = r_1 * n^2$ orbit relationship between each circular orbits in x-y plane, and each planet's ϕ -position is unrelated to other planets' ϕ -positions. With the entanglement, these orbits have their own eccentricities, inclinations, and their averaged $r(s)$ are deviated from the $r_n = r_1 * n^2$, and the mass is part of the entanglement parameters, and ϕ -positions are also become part of the entangled parameters. A "global fitting" for all parameters of all twelve planets at same time is needed. Although some studies using the classical physics have shown some interesting results (see wiki "Planet Nine", and see [16] ~ [17]), we do believe that the correct solution can only be obtained through the $\{N,n\}$ QM kind of "global fitting" modeling. A true quantum computer may be needed to solve this true QM problem.

IX. A prediction that all mass entities (from the whole universe to a single quark) can be described by Schrodinger equation and solution

Since $\{N,n/6\}$ QM structure covers from quark $\{-17,1/6\}$ to the Virgo super cluster $\{10,1/6\}$ with good consistency (see SunQM-1s2 Table 1), and Schrodinger equation/solution has accurately described the Solar system from $\{-2,1/6\}$ to $\{5,1/6\}$ (see SunQM-3s11, and SunQM-4s1) as well as the atom system from $\{-15,1/6\}$ to $\{-11,1/6\}$ (see SunQM-1s2 Table 1), we believe that the whole universe can be described by Schrodinger equation and solution, and a single quark can also be described by Schrodinger equation and solution. After searching wiki, we found that there are some other scientists have also pointed that "*Solutions to Schrodinger's equation describe not only molecular, atomic, and subatomic systems, but also macroscopic systems, possibly even the whole universe*" (see wiki "Schrodinger equation", and also see [18]).

X. The wrap-up discussion on the phase-1 study of Solar $\{N,n\}$ QM modeling

This paper marks (almost) the end of the phase-1 study on the $\{N,n\}$ QM modeling to our Solar system. In the phase-1 study, we expanded Bohr's equation ($r_n = r_1 * n^2$), Einstein's QM equation ($E = h * f$), and Schrodinger equation from micro-world to Solar system and established a brand new $\{N,n/q\}$ QM. The final result of this study revealed that the whole Solar system can be described by a single solution of Schrodinger equation as shown in eq-65.

Just like that from Einstein field equations of general relativity, Karl Schwarzschild discovered the possible existence of black hole (see wiki "black hole"), and Georges Lemaitre discovered that the recession of nearby galaxies can be explained by an expanding universe, and this expanding universe can be further traced back to time zero as a single point (which leads to the big bang theory, see wiki "Big Bang"), here from Bohr equation and Schrodinger equation and $\{N,n\}$ QM, we have also made many astonishing new discoveries. Here we list some major ones:

1) Our observable universe from Virgo super cluster at $\{10,1/6\}$, down to Milky way galaxy at $\{8,1/6\}$, Solar system at $\{5,1/6\}$, Sun at $\{0,2/6\}$, black hole at $\{-3,1/6\}$, H-atom at $\{-12,1/6\}$, proton at $\{-15,1/6\}$, and quark at $\{-17,1/6\}$, are all mysteriously follow $\{N,n/6\}$ QM structure in size (see SunQM-1s2 table 1).

2) The whole current Solar system can be accurately described by Schrodinger equation and a single solution as shown in eq-65. From this QM description, we predict that there are four undiscovered planets/belts in our Solar system at orbit of $\{3,n=2..5\}$, each with $\sim 12x, 7x, 5x,$ and $3x$ of Earth's mass. If they had already formed planets, then their radius are predicted to be $2.18E+7$ m, $1.80E+7$ m, $1.59E+7$ m, and $1.42E+7$ m respectively with the interior $\{N,n\}$ QM structure similar as that of Neptune (see SunQM-3s6 Table 2). If they are still in belt form, then these four belts' $r \pm \Delta r$ and $\Delta \theta$ ' ranges are predicted to be 173 ± 13 AU, 390 ± 25 AU, 693 ± 38 AU, and 1081 ± 53 AU, and ± 6.3 degree, ± 5.1 degree, ± 4.4 degree, and ± 4.0 degree, respectively (see SunQM-3s10 section V).

- 3) The pre-Sun ball nebula collapsed quantumly, might first at size around of $\{6,1\}$ and then down to $\{5,1\}$, $\{4,1\}$, $\{3,1\}$, $\{2,1\}$, $\{1,1\}$, $\{0,2\}$ one by one, and this process can be described by Schrodinger equation and solution (see SunQM-1s1, SunQM-3s2).
- 4) A Sun-massed white dwarf, neutron star, and black hole also follows $\{N,n//6\}$ QM structure with size of $\{-1,1\}$, $\{-3,2\}$, and $\{-3,1\}$. The $\{N,n\}$ QM also predicted that a Sun-massed black hole (with the Schwarzschild radius = $2.95E+3$ meters), instead of having size of “singularity”, it may have a stable $\{N,n\}$ QM structure at size of $\{-5,1\}$ with $r \approx 2$ meters (see SunQM-1s2 table 1).
- 5) The quantum expansion of Sun's H-fusion ball, He-fusion ball, C-fusion ball, etc. and red giant, may also follow $\{N,n//6\}$ QM dynamics (see SunQM-1s1 table 7b).
- 6) Inside the current Sun, the inward expansion (in-pansion?) of the convective zoom may also follow $\{N,n//6\}$ QM structure (see SunQM-3s8 section II). The previous quantum in-pansion of convective zoom to $\{-1,11\}$ o orbit shell (~ 2400 Mya) and to $\{-1,10\}$ o orbit shell (~ 650 Mya) might have caused two “Snowball Earth” periods in the geological history of Earth. The Solar $\{N,n\}$ QM model predicts that in the next 5 billion years, there are four more quantum in-pansion of convective zoom from the current $\{-1,10\}$ o to $\{-1,n=9..6\}$ o orbit shells. The next quantum in-pansion of convective zoom to $\{-1,9\}$ o orbit shell is estimated to happen in ~ 650 million years.
- 7) It is predicted that there is an expanding rock-evap-line which has passed the $\{1,2\}$, it might have burned off all mass of an ancient planet at orbit $\{1,2\}$, and has been burning off most mass of Mercury at orbit $\{1,3\}$, and start to burn off the (light element) mass of Venus at orbit $\{1,4\}$ (see SunQM-3s6). It is predicted that there is an expanding ice-evap-line which has passed the $\{1,8\}$, it had burned off all of ($\sim 20,000$ km thick) original atmosphere on each of four rock planet, and most evaporated H/He/H₂O molecules were captured by Jupiter, and made Jupiter 10x more massive than the original one (see SunQM-1s1). It also produced Asteroid belt as the “ring stain” of this expanding ice-evap-line. It is predicted that there is an expanding methane-evap-line which has just arrived $\{2,6\}$, and it produced the “cold KBO” as the “ring stain” of this expanding methane-evap-line (see SunQM-3s10).
- 8) It has been shown that the formation of planet's and star's (radial) internal structure is governed by the planet's or star's radial (gravity-forced) QM (see SunQM-3s6, SunQM-3s7, and SunQM-3s8).
- 9) It has been shown that the surface mass (atmosphere, or rock, or liquid iron) movement of Sun, Jupiter, Saturn, and Earth, etc., is governed by Star's (or planet's) $\theta\phi$ -2D dimension QM (see SunQM-3s3). The famous Jupiter surface cloud bands pattern is explained as the $|5,4,m\rangle$ zonal bands embedded in the background $|400\rangle$ belt bands. The Earth's atmospheric circulation is caused by the same QM peaking/depleting effect of $|211\rangle$ state on top (not embedded) of $|100\rangle$ state.
- 10) The sunspot drift, the continental drift, and Sun's and Earth's magnetic dynamo has been explained by a (single) Yml cycle model (see SunQM-3s9). Under this model, the apparent random drift of post-Pangaea continents can be nicely depicted as an expected hydrodynamic result of a broken dam through a mouth located near the south end of South America continent.

As the result, the success of $\{N,n//q\}$ QM expanded the traditional QM, and make it more self-consistent, and more complete.

Conclusion

A number of high, medium, low resolution 3D probability density maps (based on Schrodinger equation's solution) have been constructed and they are able to describe the whole Solar system with time-dependent orbital movement. In the study, the Eigen n' of a planet has been calculated. This Eigen n' gives the planet's information not only on the orbital r , but

also for the surface r . The analysis revealed that for all planets, their Eigen $n(s)$ in all three dimensions are equal ($n'_r = n'_\theta = n'_\phi = n'$). Although the r - and θ -dimension's probability densities were deduced with the full-QM, the ϕ -dimension's probability density was deduced with only semi-QM. The successful of construction of this 3D probability density map for the Solar $\{N,n/6\}$ QM structure in the range from $\{-2,1/6\}$ to $\{5,1/6\}$ implies that both the whole universe (at $\{12,1/6\}$ or above) and a single quark (at $\{-17,1/6\}$ or below) can be described by Schrodinger equation and solution.

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- [10] SunQM-3s6: Predict mass density r -distribution for Earth and other rocky planets based on $\{N,n\}$ QM probability distribution. <http://vixra.org/pdf/1808.0639v1.pdf> (submitted on 2018-08-29)
- [11] SunQM-3s7: Predict mass density r -distribution for gas/ice planets, and the superposition of $\{N,n/q\}$ or $|qnlm\rangle$ QM states for planet/star. <http://vixra.org/pdf/1812.0302v2.pdf> (submitted on 2019-03-08)
- [12] SunQM-3s8: Using $\{N,n\}$ QM to study Sun's internal structure, convective zone formation, planetary differentiation and temperature r -distribution. <http://vixra.org/pdf/1808.0637v1.pdf> (submitted on 2018-08-29)
- [13] SunQM-3s9: Using $\{N,n\}$ QM to explain the sunspot drift, the continental drift, and Sun's and Earth's magnetic dynamo. <http://vixra.org/pdf/1812.0318v2.pdf> (submitted on 2019-01-10)

[14] SunQM-3s4: Using $\{N,n\}$ QM structure and multiplier n' to analyze Saturn's (and other planets') ring structure. . <http://vixra.org/pdf/1903.0211v1.pdf> (submitted on 2019-03-11)

[15] SunQM-3s10: Using $\{N,n\}$ QM's Eigen n to constitute Asteroid/Kuiper belts, and Solar $\{N=1..4,n\}$ region's mass density r -distribution and evolution. <http://vixra.org/pdf/1909.0267v1.pdf> (submitted on 2019-09-12)

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[19] A series of my papers that to be published (together with current paper):

SunQM-3s11: Using $\{N,n\}$ QM's probability density 3D map to build a complete Solar system with time-dependent orbital movement (semi-QM deduction).

SunQM-4: Schrodinger equation and Solar $\{N,n\}$ QM.

SunQM-4s1: (Full-QM) deduction of using $\{N,n\}$ QM's probability density 3D map to build a complete Solar system with time-dependent orbital movement.

SunQM-4s2: Using $\{N,n\}$ QM to analyze Earth's atmosphere pattern and its effect on the weather

SunQM-4s3: Using $\{N,n\}$ QM to explain how planets are formed through accretion.

SunQM-4s5: Addendums, Updates and Q/A for SunQM series papers

SunQM-5: A new version of QM based on interior $\{N,n\}$, multiplier n' , $|R(n,l)|^2 * |Y(l,m)|^2$ guided mass occupancy, and RF, and its application from string to universe (drafted in April 2018).

SunQM-5s1: White dwarf, neutron star, and black hole re-analyzed by using the internal $\{N,n\}$ QM (drafted in April 2018).

[20] Major QM books, data sources, software I used for this study:

Douglas C. Giancoli, Physics for Scientists & Engineers with Modern Physics, 4th ed. 2009.

David J. Griffiths, Introduction to Quantum Mechanics, 2nd ed., 2015.

John S. Townsend, A Modern Approach to Quantum Mechanics, 2nd ed., 2012.

Stephen T. Thornton & Andrew Rex, Modern Physics for scientists and engineers, 3rd ed. 2006.

James Binney & David Skinner, The Physics of Quantum Mechanics, 1st ed. 2014.

Wikipedia at: <https://en.wikipedia.org/wiki/>

Online free software: WolframAlpha (<https://www.wolframalpha.com/>)

Online free software: MathStudio (<http://mathstud.io/>)

Offline free software: R

Microsoft Excel, Power Point, Word.

Public TV's space science related programs: PBS-NOVA, BBC-documentary, National Geographic-documentary, etc.

Journal: Scientific American.