Unification for Gravity and Electromagnetic Field in Kerr-Newman Solution

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ABSTRACT

Solutions of unified theory equations of gravity and electromagnetism has to satisfy Einstein-Maxwell equation. Specially, solution of the unified theory is generally Kerr-Newman solution in vacuum. We finally found the revised Einstein gravity tensor equation with new term (2-order contravariant metric tensor two times product and the constant matrix) is right in Kerr-Newman solution.

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Key words:General relativity theory,

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1.Introduction

This theory's aim is that we discover the revised Einstein gravity equation had Kerr-Newman solution in vacuum..

First, we know the revised Einstein gravity equation[1].

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R + \Lambda g_{\mu\upsilon} I (g^{\theta\theta})^2 = -\frac{8\pi G}{c^4} T_{\mu\upsilon}$$

In this time,

$$\Lambda = k\frac{GQ^2}{c^4}, \quad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{1}$$

If Eq(1) take covariant differential operator,

$$(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R)_{;\mu} + \Lambda g_{\mu\upsilon} I 2 g^{\theta\theta} g^{\theta\theta}{}_{;\mu} = -\frac{8\pi G}{c^4} T_{\mu\upsilon;\mu} = 0 \tag{2-i}$$

$$(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R)_{;\upsilon} + \Lambda g_{\mu\upsilon} I 2 g^{\theta\theta} g^{\theta\theta}{}_{;\upsilon} = -\frac{8\pi G}{c^4} T_{\mu\upsilon;\upsilon} = 0 \tag{2-ii}$$

In this time, in Kerr-Newman solution

$$g_{\theta\theta} = 1 / g^{\theta\theta} = \rho^2 = r^2 + a^2 \cos^2\theta$$

$$g^{\theta\theta}{}_{;\rho} = \frac{\partial g^{\theta\theta}}{\partial x^\rho} + 2\Gamma^\theta{}_{\sigma\rho} g^{\sigma\theta} = \frac{\partial g^{\theta\theta}}{\partial r} + 2\Gamma^\theta{}_{\theta r} g^{\theta\theta}$$

$$= \frac{\partial}{\partial r}(\frac{1}{\rho^2}) + 2 \cdot \frac{r}{\rho^2} \cdot \frac{1}{\rho^2} = -2\frac{1}{\rho^3} \cdot \frac{r}{\rho} + \frac{2r}{\rho^4} = 0 \tag{3}$$

$$g^{\theta\theta}{}_{;\rho} = \frac{\partial g^{\theta\theta}}{\partial x^\rho} + 2\Gamma^\theta{}_{\sigma\rho} g^{\sigma\theta} = \frac{\partial g^{\theta\theta}}{\partial \theta} + 2\Gamma^\theta{}_{\theta\theta} g^{\theta\theta}$$

$$= \frac{\partial}{\partial\theta}(\frac{1}{\rho^2}) - 2 \cdot \frac{1}{\rho^2} \cdot \frac{1}{2}\rho\frac{2a^2}{\rho}\cos\theta\sin\theta \cdot \frac{1}{\rho^2}$$

$$= -2 \cdot \frac{1}{\rho^3} \cdot -\frac{2a^2\cos\theta\sin\theta}{\rho} - \frac{4a^2}{\rho^4}\cos\theta\sin\theta = 0 \tag{4}$$

If $g^{\theta\theta}{}_{;\rho} = V_\rho$, the vector transformation is

$$0 = V_\rho = \frac{\partial x'^\alpha}{\partial x^\rho} V'_\alpha \quad , \quad V'_\alpha = 0 \tag{5}$$

Therefore, if the coordinate is not the Kerr-Newman's coordinate, the covariant differential of

$g^{\theta\theta} = \frac{1}{\rho^2}$ is still zero in the changed coordinates.

2. The revised Einstein gravity equation and Kerr-Newman solution

In this theory, Eq(1) can change the following equation.

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) + \Lambda g_{\mu\upsilon}/(g^{\theta\theta})^2 \tag{6}$$

In this time, in vacuum, specially, in Kerr-Newman solution,

$$T_{\mu\upsilon} = 0, \quad T^{\lambda}{}_{\lambda} = g^{\mu\upsilon}T_{\mu\upsilon} \neq 0 \tag{7}$$

Therefore, Eq(1) is

$$T_{\mu\upsilon} = 0 \ , -\Lambda g_{\mu\upsilon}/(g^{\theta\theta})^2 = R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}R = \frac{2G}{c^5}(F_{\mu\rho}F_{\upsilon}{}^{\rho} - \frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma})$$

$$= \frac{2G}{c^5}(g_{\mu\upsilon}F_{\upsilon\rho}F^{\upsilon\rho} - \frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma}) \tag{8}$$

In this time, According to [2],

$$E = F_{01} = -F_{10} = \frac{Q(r^2 - a^2\cos^2\theta)}{(r^2 + a^2\cos^2\theta)^2} = \frac{Q(r^2 - a^2\cos^2\theta)}{\rho^4}$$

$$B = F_{23} = -F_{32} = \frac{2Qar\cos\theta}{(r^2 + a^2\cos^2\theta)^2} = \frac{2Qar\cos\theta}{\rho^4} \tag{9}$$

Hence,

$$\frac{2G}{c^5}(g_{\mu\upsilon}F_{\upsilon\rho}F^{\upsilon\rho} - \frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma})$$

$$= -\frac{G}{c^5}g_{\mu\upsilon}I(B^2 + E^2), \ I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$= -\frac{G}{c^5}\begin{pmatrix} g_{00} & 0 & 0 & -g_{03} \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & -g_{22} & 0 \\ g_{30} & 0 & 0 & -g_{33} \end{pmatrix}(B^2 + E^2), B^2 + E^2 = \frac{Q^2}{\rho^4}$$

$$= -\frac{G}{c^5}\begin{pmatrix} g_{00} & 0 & 0 & -g_{03} \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & -g_{22} & 0 \\ g_{30} & 0 & 0 & -g_{33} \end{pmatrix}\frac{Q^2}{\rho^4} = -\Lambda g_{\mu\upsilon}/(g^{\theta\theta})^2$$

$$\Lambda = k\frac{GQ^2}{c^4} \tag{10}$$

3. Conclusion

We finally found the revised Einstein equation of unified theory (the gravity and electromagnetic field) is right in Kerr-Newman solution.

References

[1]S.Yi, ”Unified Theory of Gravity and Electromagnetic Field in Reissner-Nodstrom Solution, Kerr-Newman Solution”,**6**,11(2019),pp1-3

[2]K.Rosquist, ”Interacting Kerr-Newman Fields”,International Center for Relativistic Astrophysics, MG12,Paris (2009)

[3]S.Yi, “Spherical Solution of Classical Quantum Gravity”, International Journal of Advanced Research in Physical Science,**6**,8,(2019),pp3-6

[4]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[5]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[6]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[7]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[8]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)