# Some Problems About The Neutrino Oscillation And New Explanation For The Neutrino Observations

Ting-Hang Pei

Thpei142857@gmail.com

**Abstract –** We review the neutrino oscillation and find some problems about it. The original theory predicts the mass differences existing on three kinds of neutrinos. However, one neutrino transfers to another and then transfers back to itself again that experiences the mass non-conserved procedure if no external energy or mass participates in the transferring process. It also violates one of the conservations of energy and momentum. Furthermore, the speeds of neutrinos before and after transfer must be different that results in the special physical phenomena of self-acceleration and self-deceleration. Even the Lorentz violation is proposed in the standard model extension to discuss the neutrino oscillation without the mass difference, the all other originally elementary particles predicting by the standard model will lose their criteria. After reviewing the results of Super-Kamiokande Collaboration and Sudbury Neutrino Observatory, both results strongly imply the ratio of number between three kinds of neutrinos is $v_e$:$v_\mu$:$v_\tau$=1:1:1. According to this, we propose a new explanation for the observation data.

## I.     Introduction

The neutrino was first proposed by W. Pauli in 1930 for explanation of missing energy and momentum in $\beta$-decays [1,2]. Since neutrinos interact very weakly with other known particles, they are much difficult to detect. The experimental group led by Cowan and Reines detected the electron neutrinos $v_e$ firstly produced by nuclear reactors in 1956 [1-4]. Next, the conclusion that the chiral characteristics of neutrinos produced by weak-interaction decay are all left-handed was obtained in 1958 [1,2,5]. Nowadays in the standard model, all the neutrinos only appear in the left-handed form. The $v_\mu$ neutrinos associated with the $\mu$ charged lepton were confirmed by experiments in 1962 [6]. Until 2000, the neutrino of the third-generation lepton, $v_\tau$, was detected in Fermilab [7].

Neutrino has a special characteristic, the Neutrino Oscillation, referring to the phenomenon that different types of neutrinos can transfer from each other which was first proposed in 1957 [8]. It is believed that neutrinos are similar to the mixing and oscillation phenomena of neutral K meson and its antiparticle [1,2]. The mixing and

oscillation of the different generation neutrinos was first discussed in 1962 [10]. The neutrino oscillation further indicates neutrinos having non-zero mass and it must exist the mixing between the different flavors of neutrinos. However, the difficulty to detect neutrinos is that they really interact with matter very weakly. Averagely speaking, one neutrino has to pass through water more than several hundred light-years to take place one interaction. Therefore, the detection of neutrinos is a challenge that requires a lot of detectors and the cutting-edge technology.

Recently, neutrino detections in Super-Kamiokande (SK) Collaboration [10] and Sudbury Neutrino Observatory (SNO) [11,12] revealed the observation data which could prove the neutrino oscillation. But an immediate problem is that the existence of the mass difference between neutrinos causes neutrinos before and after transfer exhibit different speeds. If there were no other mass or energy involving this transfer, these neutrinos with different mass will result in the violation of some physical conversations. Therefore, we propose some serious problems about the neutrino oscillation and offer new explanation for the neutrino observations in SK and SNO.

## II.  The Problems About The Neutrino Oscillation

In 1932, electron neutrino $v_e$ was first investigated by Sir James Chadwick in the neutron beta-decay [2]

$$n^0 \rightarrow p^+ + e^- + \bar{v}_e. \tag{1}$$

The Feynman diagram is shown in Fig. 1(a) which involves the weak interaction. In 1956, Cowan and Reins measured this reaction near the nuclear reactor [2]

$$\bar{v}_e + p^+ \rightarrow n^0 + e^+. \tag{2}$$

The electron antineutrino was first found in the experiments. In 1962, the muon antineutrino was also investigated from the expected reaction at Brookhaven [2]

$$\bar{v}_\mu + p^+ \rightarrow n^0 + \mu^+. \tag{3}$$

In 1956, the neutrino flux was recorded as high as $5 \times 10^{13} \, particles/cm^2 \cdot sec$ [2]. Total $10^{14}$ antineutrinos from $\pi^-$ decays were used in the experiments but only 29 instances were identified in 1962 [2]. Seriously speaking, the neutrino physics is like a field to study the highly invisible particles.

In quantum field theory (QFT), neutrinos are produced by the weak interactions. Each neutrino and its corresponding charged lepton are generated by the $W^-$ or $W^+$ decays, where $W^-$ and $W^+$ are the charged, spin-1, and massive gauge bosons [1,2,13,14]. In the pion decay for the first generation in the lepton section, the reaction is [1,2, 13,14]

$$\pi^- \rightarrow e^- + \bar{v}_e, \tag{4}$$

and it takes place through the weak charged current [1,2,13,14] as shown in Fig. 1(b). The interaction is the $\Phi^3$ structure and its Lagrangian is [1,2,13,14]

$$L_{W\pi} = -g_{W\pi}\left(J_\mu^{W^-} W^{+\mu} + J_\mu^{W^+} W^{-\mu}\right), \tag{5}$$

where $J_\mu^{W^-}$ and $J_\mu^{W^+}$ are weak charged currents, $W^-$ and $W^+$ are charged spin-1 gauge bosons, and $g_{W\pi}$ is the coupling constant. It is similar to the one that can describe the pair-production process by a photon [13,14]

$$L_{EM} = eJ_{EM}^\mu A_\mu, \tag{6}$$

where $e$ is the unit charge, $J_{EM}^\mu$ is the electromagnetic current, and $A_\mu$ is the photon field. The weak charged current has the two parts which are shown as

$$J_\mu^{W^-} = J_{e\mu}^{W^-} + J_{H\mu}^{W^-}, \tag{7}$$

where

$$J_{e\mu}^{W^-} = \overline{\Psi}_e \gamma^\mu (1 - \gamma^5) \Psi_{\nu_e}, \tag{8}$$

and

$$J_{H\mu}^{W^-} = f_\pi \partial_\mu \Phi_\pi, \tag{9}$$

$\gamma^\mu$ ($\mu = 0,1,2,3$) are the Dirac's 4matrices, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\Phi_\pi$ is the positively charged pion field, and $f_\pi$ is the pion decay constant. The general Lagrangian of the charged current for three generations of leptons is [13,14]

$$L = -\frac{g}{\sqrt{2}} \overline{\Psi}_L \gamma^\mu (1 - \gamma^5) \Psi_{L_\nu} W^- + H.C., \tag{10}$$

where all the first elements of the left-handed doublets for the three-generation leptons are $\Psi_L = \left(\Psi_e, \Psi_\mu, \Psi_\tau\right)^T$, and all the second elements of the left-handed doublets for the three-generation leptons are $\Psi_{L_\nu} = \left(\Psi_{e_\nu}, \Psi_{\mu_\nu}, \Psi_{\tau_\nu}\right)^T$. If the neutrinos have mass, their quantity would be very small. Direct measurements, such as the electron energy spectrum of the Tritium's beta decay, give the electron neutrino mass less than 1 eV [15].
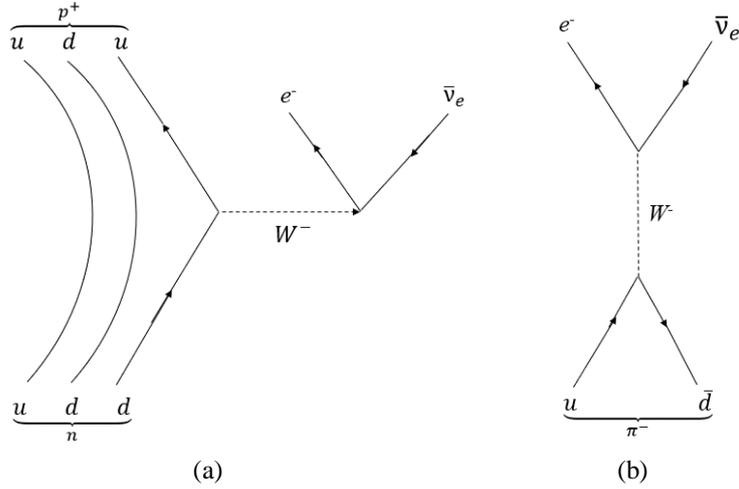
Figure 1. (a) The neutron decay through the weak interaction [1,2]. (b) The negative pion decay in the weak interaction [1,2].

However, SK only tells us that oscillating from $v_e$ to $v_\mu$ or $v_\tau$ can explain the flow problem of the solar neutrinos, but it does not prove that the missing part of $v_e$ just transfers to $v_\mu$ and $v_\tau$. Fortunately, the experiments in SNO can give us more information about it. The SNO's experiments use heavy water $D_2O$ as a target to detect neutrinos. They mainly measure three reaction processes:

$$v_e + d^+ \rightarrow p^+ + p^+ + e^- \tag{11}$$

$$v_x + d^+ \rightarrow p^+ + n^0 + v_x \tag{12}$$

$$v_x + e^- \rightarrow v_x + e^- \tag{13}$$

The first reaction is the charged-current (CC) process only for $v_e$, the second is the neutral-current (NC) process for three kinds of neutrinos, and the third is the elastic process (ES) also for three kinds of neutrino. The first and third statistical data are respectively

$$\Phi^{cc} = 1.75 \pm 0.07(stat.)^{+0.12}_{-0.11}(sys.) \pm 0.05(theor.) \times 10^6 \ cm^{-2}s^{-1} \tag{14}$$

and

$$\Phi^{ES} = 2.39 \pm 0.34(stat.)^{+0.16}_{-0.14} \times 10^6 \ cm^{-2}s^{-1}. \tag{15}$$

In general, the neutrinos of the weakly acting eigenstates are not the eigenstates of mass $v_m$. But quantum mechanics tells us that due to the completeness of the eigenstates, the eigenstates of the weak interactions and mass eigenstates can represent each other $N_L = V_{PMNS}N_L^m$, that is, $v_e$ is a linear combination of states of different masses of $v_m$. For three generations of neutrinos, $V_{PMNS}$ is a $3 \times 3$ positive matrix, which describes the characteristics of three generations of neutrino mixing. $V_{PMNS}$ is often written as [13,14]

$$V_{PMNS} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix}. \tag{16}$$

$V_{PMNS}$ can be described by four independent parameters, three mixing angles $\theta_{12}$, $\theta_{23}$, $\theta_{13}$, and one phase term $\delta$. The commonly used standard form is

$$V_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{17}$$

where $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$. The oscillation probability from $\nu_\alpha$ to $\nu_\beta$ is given by the following formula

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} Re(V_{\alpha i}^* V_{\alpha j} V_{\beta i} V_{\beta j}^*)\sin^2(\Delta m_{ij}^2 x/4E)$$

$$- 2\sum_{i>j} Im(V_{\alpha i}^* V_{\alpha j} V_{\beta i} V_{\beta j}^*)\sin^2(\Delta m_{ij}^2 x/2E), \tag{18}$$

where $x$ is the propagation distance from the origin at $t=0$ and $E$ is the total energy of neutrino. $E$ is assumed to be a constant before and after the transfer.

Next, consider the two types of neutrinos that are easier to understand, i.e. $\nu_e$ and $\nu_\mu$, and the mixing angle is $\theta$. Since the neutrinos have very low activity interacting with other substances, the energy of the propagation process can be conserved, that is, the individual energy of the propagation process is $E_i = (m_i^2 c^4 + c^2 p_i^2)^{1/2}$ where $i=1,2$. Their relation with the mass eigenstates $\nu_{m1}$ and $\nu_{m2}$ can be obtained by using the mixing matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}. \tag{19}$$

However, both momenta are $p_1$ and $p_2$, respectively. It means that two parts of $\nu_e$ move inconsistent in space if $v_1 \neq v_2$. That will cause $\nu_e$ separated as shown in Fig. 2, so two parts must have the same speed at $t=0$. It means

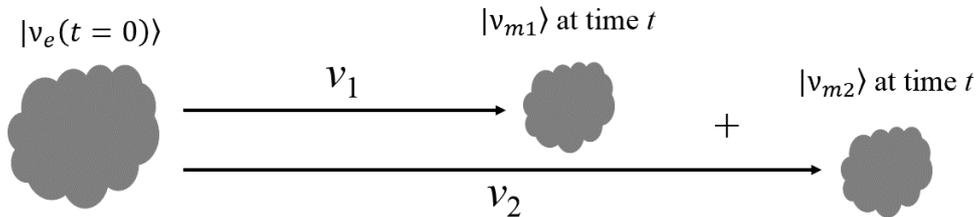$$\frac{p_1 c^2}{E_1} = v_1 = v_2 = \frac{p_2 c^2}{E_2}. \tag{20}$$



Figure 2. According to the *PMNS* matrix, if two Parts of neutrino $\nu_e$ moving in different speeds in space, they will separate from each other.

Next, using Dirac's notation，the neutrino $v_e$ (0) generated at $t = 0$ is

$$|v_e(0)\rangle = \cos\theta|v_{m1}\rangle + \sin\theta|v_{m2}\rangle. \tag{21}$$

After the generation of neutrinos, they will propagate at the mass eigenstates in the form of plane waves in vacuum. At time $t$ and the propagation distance $x$, the neutrino state will become

$$|v_e(t)\rangle = \cos\theta\, e^{i(p_1 x - E_1 t)/\hbar}|v_{m1}\rangle + \sin\theta\, e^{i(p_2 x - E_2 t)/\hbar}|v_{m2}\rangle. \tag{22}$$

Hence, the probability amplitudes of $v_e$ and $v_\mu$ measured at $t$ are

$$\langle v_e(0)|v_e(t)\rangle = \cos^2\theta e^{i(p_1 x - E_1 t)/\hbar} + \sin^2\theta e^{i(p_2 x - E_2 t)/\hbar} \tag{23}$$

and

$$\langle v_\mu(0)|v_e(t)\rangle = -\sin\theta\cos\theta\, e^{i(p_1 x - E_1 t)/\hbar} + \sin\theta\cos\theta\, e^{i(p_2 x - E_2 t)/\hbar}. \tag{24}$$

Therefore, the probability of the transfer from $v_e$ to $v_\mu$ is

$$P(v_e \to v_\mu) = \left|\langle v_\mu(0)|v_\mu(t)\rangle\right|^2 = \sin^2(2\theta) \cdot \sin^2\left(\frac{\Delta p x - \Delta E t}{2\hbar}\right), \tag{25}$$

where $x/t = v = v_1 = v_2$, $\Delta p = (p_2 - p_1)$, and $\Delta E = (E_2 - E_1)$. The above equation shows oscillation in time dependent on the initial condition $(v\Delta p - \Delta E)$ so the period $T$ that $v_e$ totally becomes $v_\mu$ is

$$T = \frac{\pi\hbar}{v\Delta p - \Delta E}\,. \tag{26}$$

Considering the average of momentum in the $x$ direction

$$\overline{p(t)} = \sum_{i=1,2}\left|\left\langle v_e(t)\left|i\hbar\frac{\partial}{\partial x}\right|v_e(t)\right\rangle\right|^2 = p_1\cos^2\theta + p_2\sin^2\theta. \tag{27}$$

This result tells us the conservation of momentum for the free $v_e$. Furthermore, the average total mass $M$ in all time is

$$\overline{M} = \sum_{i=1,2}|\langle v_{mi}|\hat{m}|v_e(t)\rangle|^2 = m_1\cos^2\theta + m_2\sin^2\theta, \tag{28}$$

where $\hat{m}$ is the mass operator and $m_1$ and $m_2$ are the eigenvalues of the two mass eigenstates. It obviously that the average total mass $M$ is a constant and independent of time. If the neutrino oscillation happens, mass will have no difference after one neutrino transfers to another because the average total mass is conserved. According to the transition period $T$, $v_e$ totally becomes to $v_\mu$ and then becomes to itself again after

another period time $T$ as shown in Fig.3. Its mass will change from $m_e$ to $m_\mu$ and back to $m_e$ again. If there were no additional mass or energy and $m_\mu \neq m_e$, then the conservation of mass is directly broken. It means that the neutrino oscillation will not happen at the mass difference equal to nonzero. Besides, the existence of mass difference after this transfer also causes one serious problem. At the conservation of energy, the speed before and after transfer must be different because two neutrinos have different mass if no other particles participate in the transfer. Then neutrino will perform self-acceleration or self-deceleration without external force. This directly violate the conservation of momentum especially in the elementary particle physics.

Even some research points out the neutrino oscillation possibly existing at Lorentz and *CPT* violation [16], the direct violation of Lorentz invariance still makes some problem. The Lorentz violation of neutrino means the forever existence of this violation since the neutrino's birth. Such unique spacetime for the neutrino makes it inconsistent with other elementary particles describing by the standard model based on the Lorentz invariance so as to result in neutrino oscillation questionable and doubtful. Then without the Lorentz invariance, how to describe and calculate the following reaction [2]?
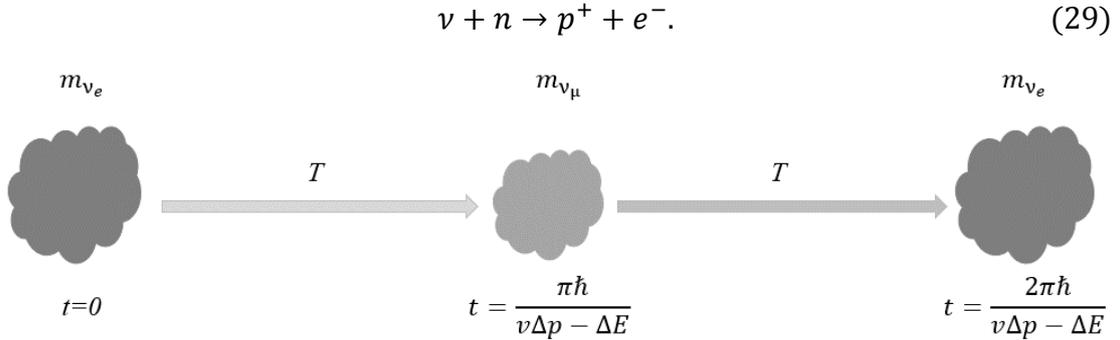
$$\nu + n \rightarrow p^+ + e^-. \tag{29}$$



Figure 3. $\nu_e$ totally transfers to $\nu_\mu$ after time $T$ and becomes itself after $2T$. If there were no additional mass or energy and the mass of both neutrinos $m_{\nu_\mu} \neq m_{\nu_e}$, then the conservation of mass is directly broken in this case which is inconsistent with Eq. (22).

### III.    The Estimation Of The Upper Mass Limit For Neutrino From The Supernova SN 1987A Event

If neutrino has mass, then it must be affected by gravity so the Lorentz violation would be questionable and unbelievable based on the instantaneously inertial coordinate frame of General Relativity. In 1987A supernova event [17-21], neutrinos were detected three hours earlier than photons that causes a problem: if neutrinos have mass, why they came to earth faster than photons? The role of this supernova, SN 1987A, is 168,000 light years far away from the Earth [22]. It means that the early arrival neutrinos move averagely faster than photons even their speeds should be slightly slower than the speed of light. The time difference of three hours means the average speed difference between neutrinos and photons from SN 1987A is $2\text{x}10^{-9}$ or

0.6 $m$/sec. If the rest mass of one neutrino $m_v$ were non-zero, then the relativistic effect must be considered. Supposing the speed of neutrino very close to the speed of light, the difference is as small as $10^{-8}$ in speed. According to the Lorentz factor,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \tag{30}$$

this difference in speed gives $\gamma \approx 7,000$. If the supernova exhausted one solar mass to transfer energy to all neutrinos and produce $10^{60}$ neutrinos, then the upper mass limit of each neutrino will be

$$m_v c^2 \approx (1.99 \times 10^{30}) \times (3 \times 10^8)^2/10^{60}/\gamma \approx 150 \; eV. \tag{31}$$

However, the energy of one neutrino occupies the nuclear reaction very small, the upper mass limit should be at least $10^3$ times smaller than the above value. Roughly speaking, the upper mass limit of one neutrino is

$$m_v \leq 0.15 \; eV. \tag{32}$$

This rough estimation is from SN 1987A event. Theoretically speaking, the neutrinos must be affected by the gravitation if they have non-zero mass. However, the early arrival neutrinos revealed that their average speeds were larger than the photons due to the three-hour difference. As we know, the escaping velocity to get rid of the solar gravitation, the third cosmic velocity, is $1.67 \times 10^4 \; m/\sec \approx 5.57 \times 10^{-5} \; c$, so each neutrino escaping the gravitation of the supernova would also lose its speed at the same order. If the photons interact with the interstellar materials very few, their average speeds should be still very close to $c$. Only when photons pass through the interstellar materials and interact with them, photons will slow down. On the Other hand, in the empty space far away from gravitation, they will propagate in the velocity of light. But it is not the same thing for the neutrinos with non-zero mass. Once the neutrinos leave the initial gravitation, their velocities are affected or decrease until they arrive the detectors on the Earth, about 168,000 years long in time. Their average speeds must be slower than the speed of light during the whole traveling time. Therefore, an easy way to explain the phenomenon of the arrival neutrinos three hours earlier than photons is the zero-mass neutrino. Because the average speed of neutrinos is $2 \times 10^{-9}$ faster than the average speeds of photons, it is a reasonable factor that neutrinos are hard to interact with matters and they are not affected by gravitation due to zero mass!

## IV.    New Explanation For The Neutrino Observations

According to the SNO's observations in 2001, the occupation of $v_e$ from the sun is about 0.32, close to 1/3. If no neutrino oscillation takes place, $v_\mu$ and $v_\tau$ will occupy about 2/3 neutrino flux from the sun. One thing is possible that the solar model needs

to be corrected, and the other thing is to boldly predict that only one kind of neutrino exists which can be a linear combination of three different neutrino states. Because neutrino is hard to detect and the present recorded data cannot completely avoid such possibility. For example, the decays of $Z^0$ bosons can produce three kinds of neutrino. The atmospheric observations from SK in 1998 also revealed the close 1:1 ratio between $v_\mu$ and $v_e$, and the missing part of $v_\mu$ was very possibly to be $v_\tau$ roughly equal to $v_\mu$ [10]. Both results of SK and SNO imply three equal neutrinos in number and lead to a unified neutrino state as a linear combination of three neutrino states

$$\left|v_{unified}\right\rangle = \frac{1}{\sqrt{3}}\left|v_e\right\rangle + \frac{1}{\sqrt{3}}\left|v_\mu\right\rangle + \frac{1}{\sqrt{3}}\left|v_\tau\right\rangle. \tag{33}$$

This situation is similar to the neutral $K$ mesons which are linear combinations of $K^0$ and $\overline{K}_0$ We have to also consider the possibility of the cross terms that all neutrinos may interact with all leptons so the correction of Lagrangian is multiplied by a matrix $U$

$$U = \begin{pmatrix} 1 - \delta_{e2} - \delta_{e3} & \delta_{e2} & \delta_{e3} \\ \delta_{\mu1} & 1 - \delta_{\mu2} - \delta_{\mu2} & \delta_{\mu3} \\ \delta_{\tau1} & \delta_{\tau2} & 1 - \delta_{\tau3} - \delta_{\tau3} \end{pmatrix}, \tag{34}$$

where $\delta_{e2}, \delta_{e3}, \delta_{\mu1}, \delta_{\mu3}, \delta_{\tau1}$, and $\delta_{\tau2}$ are possibly non-zero values, and each element should be not negative. Therefore, the matrix will appear in the Lagrangian which becomes

$$L = -\frac{g}{\sqrt{2}}\bar{\psi}_L U \gamma^\mu (1 - \gamma^5)\left(\sqrt{3}\Psi_{L_v}\right)W^- + H.C. \tag{35}$$

In fact, the inverse muon decay [23] reveals the possibility of the form in Eq. (34). The reaction is

$$v_\mu + e^- \rightarrow \mu^- + v_e. \tag{36}$$

The role of the muon neutrino is changed to the electron neutrino after this reaction so it makes the unified neutrino model become reasonable.


## V.    Conclusions

In conclusion, the neutrino oscillation violates several conservations no matter the mass difference exists or not. The original theory predicts the mass differences existing on three kinds of neutrino. However, one neutrino transfers to another and then transfers back to itself again that causes the mass non-conservation if no external energy or mass participates in the transferring process. It also violates one of the conservations of energy and momentum. Furthermore, the speeds of neutrinos before and after transfer

must be different that results in self-acceleration and deceleration. Even the Lorentz violation is proposed in the standard model extension, the all other originally elementary particles predicting by the standard model will lose their criteria. Once neutrino is produced, its spacetime will violate the Lorentz symmetry everywhere. Neutrino shall not have so special spacetime independent of other elementary particles. Thus, the Lorentz violation is still not reasonable to explain the neutrino oscillation even the mass differences are not existent by assumption. The non-zero mass of neutrinos might cause a problem because they must be affected by gravity. If so, the fact that the neutrinos arrived Earth three hours than photons in the supernova SN 1987A event would be not easy to reasonably explain. An easy way to explain the observations from the supernova SN 1987A is the zero mass of neutrinos.

After reviewing the results of SK and SNO, both results strongly imply the ratio of number between three kinds of neutrinos is $v_e$:$v_\mu$:$v_\tau$=1:1:1. According to this, we propose a new explanation for the observation data. Only one unified neutrino exists in nature which is a linear combination of three neutrino states. This situation is similar to the neutral $K$ mesons which are linear combinations of $K^0$ and $\overline{K}_0$. Each lepton not only interacts with the corresponding neutrino state, but also interacts with other lepton neutrino states.

**Acknowledgement**

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