Quaternionic Field Theory

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Abstract

The correct specification of the concept of physical fields requires a platform in which these physical fields can be defined. This platform represents a base model that emerges from a Hilbert lattice, a vector space, and a number system. The number system must be an associative division ring. Dynamic fields require the selection of the quaternionic number system. Quaternionic fields are constructed eigenspaces of normal operators in a quaternionic Hilbert space. The base model supports symmetry-related fields and a field that always and everywhere exists. It acts as a repository for dynamic geometric data.

1 The Base Model

The base model is part of the Hilbert Book Model [1] [2] [3] [4] [5]. The base model delivers the platform for modeling quaternionic field theory. Thus, we stop treating the Hilbert Book model after introducing its base model. A PowerPoint presentation that treats highlights of this paper is available for download at

http://www.e-physics.eu/Base%20model.ppsx

First, we derive a standard base model from a set of separable Hilbert spaces that emerge from a Hilbert lattice and share the underlying vector space. Only a subtle difference exists between a vector space and a separable Hilbert space. We exploit this fact to let a single vector space support a huge set of separable Hilbert spaces. In the Hilbert Book Model each elementary particle resides on a private separable Hilbert space. One of the separable Hilbert spaces acts as a background platform. It has an infinite dimension and owns a unique non-separable Hilbert space that embeds its separable companion. The standard base model applies the quaternionic number system. Quaternionic eigenvalues suit as storage bins of a scalar timestamp and a three-dimensional location vector. Together with the standard base model this non-separable Hilbert space constitutes the base model that supports quaternionic fields in the eigenspaces of normal operators. Sequencing the scalar parts of the eigenvalues turns the base model into a dynamic model in which a scanning subspace divides the model into a historic part, a static status quo and a future part. The base model acts as a repository for dynamic geometric data of point-like objects and for dynamic quaternionic fields.

The quaternionic fields are the mathematical equivalents of physical fields. Quaternionic fields are described by quaternionic functions. The dynamic behavior of the quaternionic field is described by quaternionic differential and integral calculus. This paper applies the quaternionic nabla to define the first-order change of a quaternionic field. The quaternionic nabla separates the change of a quaternionic field into five terms.

1.1 The Hilbert lattice

A Hilbert lattice [7] is the lattice of closed linear subspaces of a Hilbert space (with preorder given by the inclusion) over real, complex or quaternion numbers. This is an orthocomplemented lattice and in fact an orthomodular lattice.

The logic it embodies is Birkhoff-von Neumann quantum logic.

1.2 Vector space

A vector space over a mathematical field F is a set V together with two operations that satisfy the eight axioms listed below. In the following, $V \times V$ denotes the Cartesian product of V with itself, and $\rightarrow$ denotes a mapping from one set to another.

- The first operation, called **vector addition** or simply **addition** $+ : V \times V \rightarrow V$, takes any two vectors $\vec{v}$ and $\vec{w}$ and assigns to them a third vector which is commonly written as $\vec{v}+\vec{w}$, and called the sum of these two vectors. (The resultant vector is also an element of the set V .)

- The second operation, called **scalar multiplication** $\cdot : F \times V \to V$, takes any scalar a and any vector $\vec{v}$ and gives another vector $a \cdot \vec{v}$. (Similarly, the vector $a \cdot \vec{v}$ is an element of the set V . Scalar multiplication is not to be confused with the scalar product, also called *inner product* or *dot product*, which is an additional structure present on some specific, but not all vector spaces. Scalar multiplication is a multiplication of a vector *by* a scalar; the other is a multiplication of two vectors *producing* a scalar.)

Elements of V are commonly called *vectors*. Elements of F are commonly called *scalars*.

1.2.1 Axioms

- *Associativity of addition* $\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$
- *Commutativity of addition* $\vec{u}+\vec{v}=\vec{v}+\vec{u}$
- *Identity element of addition* There exists an element $\vec{0} \in V$, called the *zero vector*, such that $\vec{0}+\vec{v}=\vec{v}$ for all $\vec{v} \in V$.
- *Inverse elements of addition* For every $\vec{v} \in V$, there exists an element $-\vec{v} \in V$, called the *additive inverse* of $\vec{v}$, such that $-\vec{v}+\vec{v}=\vec{0}$
- *Compatibility of scalar multiplication with field multiplication* $a \cdot (b \cdot \vec{v}) = (ab) \cdot \vec{v}$
- *Identity element of scalar multiplication* $1 \cdot \vec{v}=\vec{v}$, where 1 denotes the multiplicative identity in *F*
- *Distributivity of scalar multiplication with respect to vector addition* $a \cdot (\vec{u}+\vec{v}) = a \cdot \vec{u}+a \cdot \vec{v}$
- *Distributivity of scalar multiplication with respect to field multiplication* $(a+b) \cdot \vec{v}=a \cdot \vec{v}+b \cdot \vec{v}$

1.3 Quaternions

Quaternions were discovered by Rowan Hamilton in 1843. Later, in the twentieth century, quaternions fell in oblivion.

Hilbert spaces can only cope with number systems whose members form a division ring. Quaternionic number systems represent the most versatile associative division ring. Quaternionic number systems exist in many versions that differ in the way that coordinate systems can sequence them. Quaternions can store a combination of a scalar timestamp and a three-dimensional spatial location. Thus, they are ideally suited as storage bins for dynamic geometric data.

In this paper, we represent quaternion q by a real one-dimensional part q_r and a three-dimensional imaginary part $\vec{q}$.

The summation is commutative and associative

The following quaternionic multiplication rule describes most of the arithmetic properties of the quaternions.

$$\begin{aligned} c = c_r + \vec{c} = ab = \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) \\ = a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \end{aligned} \tag{1.2.1}$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system.

A quaternionic conjugation exists

$$q^* = (q_r + \vec{q})^* = q_r - \vec{q} \tag{1.2.2}$$

$$\left(ab\right)^* = b^* a^* \tag{1.2.3}$$

The norm $|q|$ equals

$$|q| = \sqrt{q_r^2 + \langle \vec{q}, \vec{q} \rangle} \tag{1.2.4}$$

$$q^{-1} = \frac{1}{q} = \frac{q}{|q|^2} \tag{1.2.5}$$

$$q = |q| \exp\left(q_{\varphi} \frac{\vec{q}}{|\vec{q}|} \right) \tag{1.2.6}$$

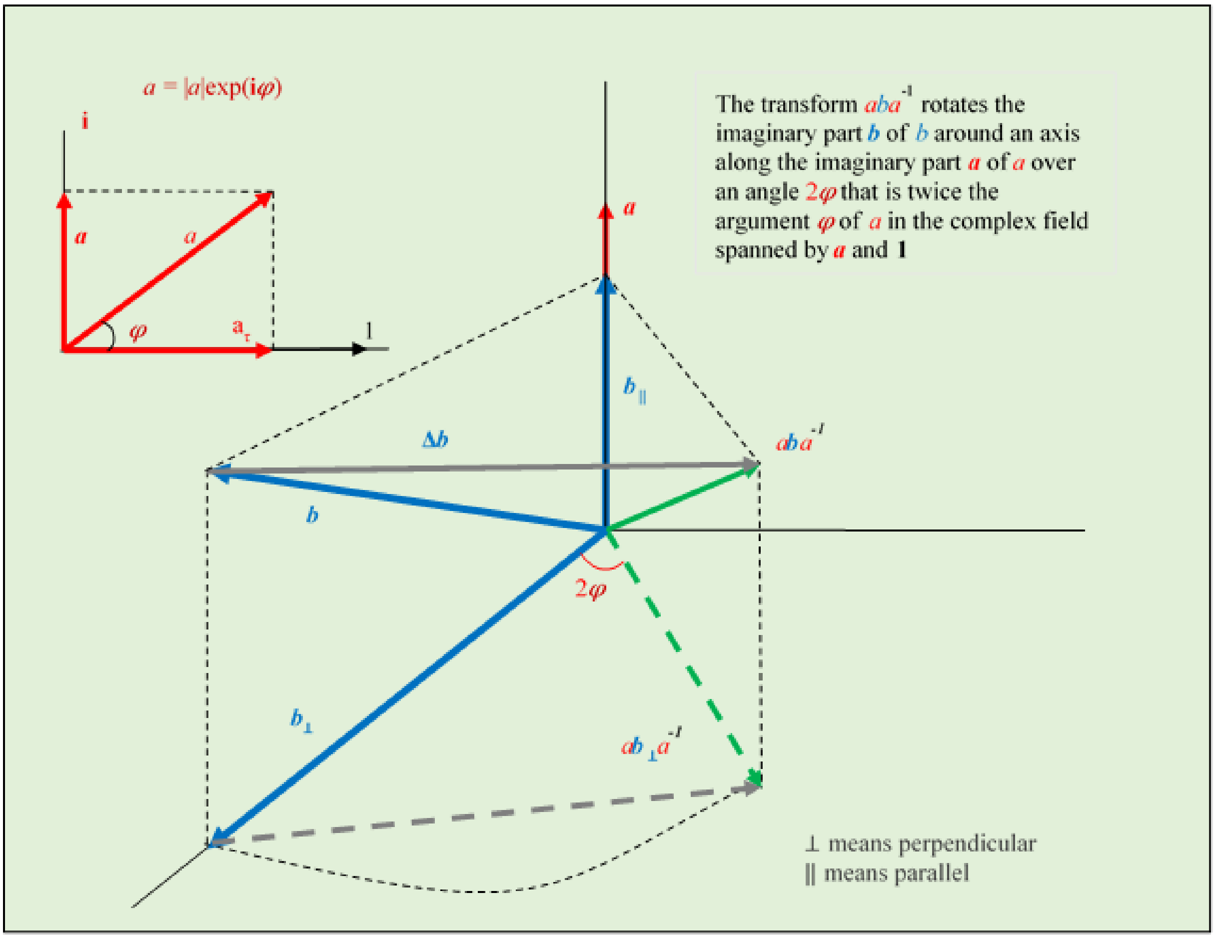
$\frac{\vec{q}}{|\vec{q}|}$ is the spatial direction of q.

A quaternion and its inverse can rotate a part of a third quaternion. The imaginary part of the rotated quaternion that is perpendicular to the imaginary part of the first quaternion is rotated over an angle that is twice the angle of the argument φ between the real part and the imaginary part of the first quaternion. This makes it possible to shift the imaginary part of the third quaternion to a different dimension. For that reason, must $\varphi = \pi / 4$.

Each quaternion *c* can be written as a product of two complex numbers *a* and *b* of which the imaginary base vectors are perpendicular

$$\begin{aligned} c &= \left(a_r + a_1\vec{i}\right)\left(b_r + b_2\vec{j}\right) \\ &= a_r b_r + \left(a_1 + b_r\right)\vec{i} + \left(a_r + b_2\right)\vec{j} + a_1 b_2 \vec{k} \\ &= c_r + c_1\vec{i} + c_2\vec{j} + c_3\vec{k} \end{aligned} \tag{1.2.7}$$

Where $\vec{k} = \vec{i} \times \vec{j}$



1.3.1 Quaternionic storage bins

Quaternions are ideally suited as storage bins of a scalar timestamp and a three-dimensional location.

1.4 Bra's and ket's

Paul Dirac introduced a handy formulation for the inner product that applies a bra and a ket.

The bra $\langle \vec{f} |$ is a covariant vector, and the ket $| \vec{g} \rangle$ is a contravariant vector. The inner product $\langle \vec{f} | \vec{g} \rangle$ acts as a metric. It has a quaternionic value. Since the product of quaternions is not commutative, care must be taken with the format of the formulas.

For bra vectors hold

$$\langle \vec{f} | + \langle \vec{g} | = \langle \vec{g} | + \langle \vec{f} | = \langle \vec{f} + \vec{g} | \qquad (1.2.8)$$

$$\left(\left\langle \vec{f}+\vec{g}\right|\right)+\left\langle \vec{h}\right|=\left\langle \vec{f}\right|+\left(\left\langle \vec{g}+\vec{h}\right|\right)=\left\langle \vec{f}+\vec{g}+\vec{h}\right| \tag{1.2.9}$$

For ket vectors hold

$$\left|\vec{f}\right\rangle+\left|\vec{g}\right\rangle=\left|\vec{g}\right\rangle+\left|\vec{f}\right\rangle=\left|\vec{f}+\vec{g}\right\rangle \tag{1.2.10}$$

$$\left(\left|\vec{f}+\vec{g}\right\rangle\right)+\left|\vec{h}\right\rangle=\left|\vec{f}\right\rangle+\left(\left|\vec{g}+\vec{h}\right\rangle\right)=\left|\vec{f}+\vec{g}+\vec{h}\right\rangle \tag{1.2.11}$$

For the inner product holds

$$\left\langle \vec{f}\mid\vec{g}\right\rangle=\left\langle \vec{g}\mid\vec{f}\right\rangle^{*} \tag{1.2.12}$$

For quaternionic numbers α and β hold

$$\langle \alpha f\mid g\rangle=\langle g\mid \alpha f\rangle^{*}=\left(\langle g\mid f\rangle\alpha\right)=\alpha^{*}\langle f\mid g\rangle \tag{1.2.13}$$

$$\left\langle \alpha\vec{f}\mid\vec{g}\right\rangle=\left\langle \vec{g}\mid\alpha\vec{f}\right\rangle^{*}=\left(\left\langle \vec{g}\mid\vec{f}\right\rangle\alpha\right)=\alpha^{*}\left\langle \vec{f}\mid\vec{g}\right\rangle \tag{1.2.14}$$

$$\left\langle \vec{f}\mid\beta\vec{g}\right\rangle=\left\langle \vec{f}\mid\vec{g}\right\rangle\beta \tag{1.2.15}$$

$$\begin{aligned}\left\langle(\alpha+\beta)\,\vec{f}\mid\vec{g}\right\rangle&=\alpha^{*}\left\langle \vec{f}\mid\vec{g}\right\rangle+\beta^{*}\left\langle \vec{f}\mid\vec{g}\right\rangle\\&=(\alpha+\beta)^{*}\left\langle \vec{f}\mid\vec{g}\right\rangle\end{aligned} \tag{1.2.16}$$

Thus

$$\alpha\left|\vec{f}\right\rangle \tag{1.2.17}$$

$$\left\langle \alpha\vec{f}\right|=\alpha^{*}\left\langle \vec{f}\right| \tag{1.2.18}$$

$$\left|\alpha\vec{g}\right\rangle=\left|\vec{g}\right\rangle\alpha \tag{1.2.19}$$

We made a choice. Another possibility would be $\left\langle \alpha\vec{f}\right|=\alpha\left\langle \vec{f}\right|$ and $\left|\alpha\vec{g}\right\rangle=\alpha^{*}\left|\vec{g}\right\rangle$

In mathematics a topological space is called separable if it contains a countable dense subset; that is, there exists a sequence $\left\{\left|\vec{f}_i\right\rangle\right\}_{i=\infty}^{i=0}$ of elements of the space such that every nonempty open subset of the space contains at least one element of the sequence.

Its values on this countable dense subset determine every continuous function on the separable space $\mathfrak{H}$.

The Hilbert space $\mathfrak{H}$ is separable. That means that a countable row of elements $\left\{\left|\vec{f}_n\right\rangle\right\}$ exists that spans the whole space.

If $\left\langle \vec{f}_m \mid \vec{f}_n \right\rangle = \delta(m,n)$ [1 if n=m; otherwise 0], then $\left\{\left|\vec{f}_n\right\rangle\right\}$ is an orthonormal base of Hilbert space $\mathfrak{H}$.

A ket base $\left\{\left|\vec{k}\right\rangle\right\}$ of $\mathfrak{H}$ is a minimal set of ket vectors $\left|\vec{k}\right\rangle$ that span the full Hilbert space $\mathfrak{H}$.

Any ket vector $\left|\vec{f}\right\rangle$ in $\mathfrak{H}$ can be written as a linear combination of elements of $\left\{\left|\vec{k}\right\rangle\right\}$.

$$\left|\vec{f}\right\rangle = \sum_k \left|\vec{k}\right\rangle \left\langle \vec{k} \mid \vec{f}\right\rangle \tag{1.2.20}$$

A bra base $\left\{\left\langle\vec{b}\right|\right\}$ of $\mathfrak{H}^\dagger$ is a minimal set of bra vectors $\left\langle\vec{b}\right|$ that span the full Hilbert space $\mathfrak{H}^\dagger$.

Any bra vector $\left\langle\vec{f}\right|$ in $\mathfrak{H}^\dagger$ can be written as a linear combination of elements of $\left\{\left\langle\vec{b}\right|\right\}$.

$$\left\langle\vec{f}\right| = \sum_b \left\langle \vec{f} \mid \vec{b}\right\rangle \left\langle\vec{b}\right| \tag{1.2.21}$$

Usually, a base selects vectors such that their norm equals 1. Such a base is called an orthonormal base

1.5 Hilbert space

A **Hilbert space** H is a real, complex or quaternionic vector space that provides an inner product and that is also a complete metric space with respect to the distance function induced by the inner product.

To say that H is a quaternionic **inner product space** means that H is a quaternionic vector space on which there is an inner product $\langle \vec{u} \mid \vec{v} \rangle$ associating a quaternion to each pair of elements $\vec{u}, \vec{v}$ of H that satisfies the following properties:

1. The inner product is conjugate symmetric; that is, the inner product of a pair of elements is equal to the quaternionic conjugate of the inner product of the swapped elements: $\langle \vec{u} \mid \vec{v} \rangle = \langle \vec{v} \mid \vec{u} \rangle^*$
2. The inner product is linear in its first argument. For all quaternions $\langle a\vec{u} + b\vec{v} \mid \vec{w} \rangle = \langle a\vec{u} \mid \vec{w} \rangle + \langle b\vec{v} \mid \vec{w} \rangle = \langle \vec{u} \mid \vec{w} \rangle a^* + \langle \vec{v} \mid \vec{w} \rangle b^*$
3. The inner product of an element with itself is positive definite: $\begin{cases} \langle \vec{u} \mid \vec{u} \rangle > 0 & \vec{u} \neq \vec{0} \\ \langle \vec{u} \mid \vec{u} \rangle = 0 & \vec{u} = \vec{0} \end{cases}$

It follows from properties 1 and 2 that a quaternionic inner product is conjugate linear in its second argument, meaning that

$$\langle \vec{u} \mid a\vec{v} + b\vec{w} \rangle = \langle \vec{u} \mid a\vec{v} \rangle + \langle \vec{v} \mid b\vec{w} \rangle = \langle \vec{u} \mid \vec{v} \rangle a + \langle \vec{v} \mid \vec{w} \rangle b \qquad (1.3.1)$$

A real inner product space is defined in the same way, except that H is a real vector space and the inner product takes real values. Such an inner product will be bilinear: that is, linear in each argument.

The ***norm*** is the real-valued function

$$\|\vec{u}\| = \sqrt{\langle \vec{u} \mid \vec{u} \rangle} \qquad (1.3.2)$$

and the ***distance*** d between two points $\vec{u}, \vec{v}$ in H is defined in terms of the norm by

$$d(\vec{u}, \vec{v}) = \|\vec{u} - \vec{v}\| \qquad (1.3.3)$$

$$d\left(\vec{u},\vec{w}\right) \leq d\left(\vec{u},\vec{v}\right) + d\left(\vec{v},\vec{w}\right) \tag{1.3.4}$$

With a distance function defined in this way, any inner product space is a metric space. Sometimes is known as a **pre-Hilbert space**. Any pre-Hilbert space that is additionally also a complete space is a Hilbert space.

In mathematical analysis, a metric space *M* is called **complete** (or a **Cauchy space**) if every Cauchy sequence of points in *M* has a limit that is also in *M* or, alternatively, if every Cauchy sequence in *M* converges in *M*.

An orthonormal base is defined as

$$\left\{\vec{u}_k\right\} : \left\langle \vec{u}_i \mid \vec{u}_j \right\rangle = \delta_{ij} \tag{1.3.5}$$

The Hilbert space H is separable if every orthonormal base is countable.

An orthonormal basis for an inner product space V with finite dimension is a basis for V whose vectors are orthonormal, that is, they are all unit vectors and orthogonal to each other. A separable Hilbert space has a countable orthonormal basis that spans H.

1.5.1 Operators

Operators act on a subset of the elements of the Hilbert space.

An operator L is linear when for all vectors $\left|\vec{f}\right\rangle$ and $\left|\vec{g}\right\rangle$ for which L is defined and for all quaternionic numbers α and β

$$\begin{aligned} L\left|\alpha \vec{f}\right\rangle + L\left|\beta \vec{g}\right\rangle &= L\left|\vec{f}\right\rangle \alpha + L\left|\vec{g}\right\rangle \beta = L\left(\left|\vec{f}\right\rangle \alpha + \left|\vec{g}\right\rangle \beta\right) \\ &= L\left(\left|\alpha \vec{f}\right\rangle + \left|\beta \vec{g}\right\rangle\right) \end{aligned} \tag{1.3.6}$$

The operator B is **colinear** when for all vectors $\left|\vec{f}\right\rangle$ for which Bs defined and for all quaternionic numbers α there exists a quaternionic number γ such that

$$\alpha B\left|\vec{f}\right\rangle = B\left|\vec{f}\right\rangle \gamma\alpha\gamma^{-1} \equiv B\left|\gamma\alpha\gamma^{-1}\vec{f}\right\rangle \tag{1.3.7}$$

If $|\vec{a}\rangle$ is an eigenvector of the operator A with quaternionic eigenvalue α,

$$A|\vec{a}\rangle = |\vec{a}\rangle\alpha \tag{1.3.8}$$

then $|\beta a\rangle$ is an eigenvector of A with quaternionic eigenvalue $\beta^{-1}\alpha\beta$.

$$A|\beta\vec{a}\rangle = A|\vec{a}\rangle\beta = |\vec{a}\rangle\alpha\beta = |\beta\vec{a}\rangle\beta^{-1}\alpha\beta \tag{1.3.9}$$

$A^\dagger$ is the **adjoint** of the **normal** operator A

$$\langle \vec{f} \mid A\vec{g} \rangle = \langle \vec{f}A^\dagger \mid \vec{g} \rangle = \langle \vec{g} \mid A^\dagger \vec{f} \rangle^* \tag{1.3.10}$$

$$A^{\dagger\dagger} = A \tag{1.3.11}$$

$$(A+B)^\dagger = A^\dagger + B^\dagger \tag{1.3.12}$$

$$(AB)^\dagger = B^\dagger A^\dagger \tag{1.3.13}$$

If $A = A^\dagger$ then A is a **self-adjoint** operator.

A linear operator L is normal if $LL^\dagger$ exists, and $LL^\dagger = L^\dagger L$

For the normal operator N holds

$$\langle N\vec{f} \mid N\vec{g} \rangle = \langle NN^\dagger \vec{f} \mid \vec{g} \rangle = \langle \vec{f} \mid NN^\dagger \vec{g} \rangle \tag{1.3.14}$$

Thus

$$N = N_r + \vec{N} \tag{1.3.15}$$

$$N^\dagger = N_r - \vec{N} \tag{1.3.16}$$

$$N_r = \frac{N + N^\dagger}{2} \tag{1.3.17}$$

$$\vec{N} = \frac{N - N^\dagger}{2} \tag{1.3.18}$$

$$NN^\dagger = N^\dagger N = N_r N_r + \langle \vec{N}, \vec{N} \rangle = |N|^2 \tag{1.3.19}$$

N_r is the Hermitian part of N.

$\vec{N}$ is the anti-Hermitian part of N.

For two normal operators A and B holds

$$AB = A_r B_r - \langle \vec{A}, \vec{B} \rangle + A_r \vec{B} + \vec{A} B_r \pm \vec{A} \times \vec{B} \qquad (1.3.20)$$

For a unitary transformation U holds

$$\langle U\vec{f} \mid U\vec{g} \rangle = \langle \vec{f} \mid \vec{g} \rangle \qquad (1.3.21)$$

The closure of separable Hilbert space $\mathfrak{H}$ means that converging rows of vectors of $\mathfrak{H}$ converge to a vector in $\mathfrak{H}$.

1.5.1.1 Operator construction

$|\vec{f}\rangle\langle\vec{g}|$ is a constructed operator.

$$|\vec{g}\rangle\langle\vec{f}| = \left(|\vec{f}\rangle\langle\vec{g}|\right)^\dagger \qquad (1.3.22)$$

For the orthonormal base $\{|\vec{q}_i\rangle\}$ consisting of eigenvectors of the reference operator, holds

$$\langle \vec{q}_n \mid \vec{q}_m \rangle = \delta_{nm} \qquad (1.3.23)$$

The reverse bra-ket method enables the definition of new operators that are defined by quaternionic functions.

$$\langle \vec{g} \mid F \mid \vec{h} \rangle = \sum_{i=1}^{N} \left\{ \langle \vec{g} | \vec{q}_i \rangle F(q_i) \langle \vec{q}_i | \vec{h} \rangle \right\} \qquad (1.3.24)$$

The symbol F is used both for the operator F and the quaternionic function $F(q)$. This enables the shorthand

$$F \equiv |\vec{q}_i\rangle F(q_i) \langle \vec{q}_i| \qquad (1.3.25)$$

It is evident that

$$F^\dagger \equiv |\vec{q}_i\rangle F^*(q_i) \langle \vec{q}_i| \qquad (1.3.26)$$

For ***reference operator*** $\mathfrak{R}$ holds

$$\mathfrak{R} = |\vec{q}_i\rangle q_i \langle \vec{q}_i| \qquad (1.3.27)$$

If $\{q_i\}$ consists of all rational values of the version of the quaternionic number system that $\mathfrak{H}$ applies then the eigenspace of $\mathfrak{R}$ represents the private parameter space of the separable Hilbert space $\mathfrak{H}$. It is also the parameter space of the function $F(q)$ that defines the operator F in the formula **Fout! Verwijzingsbron niet gevonden.**.

1.6 Non-separable Hilbert space

Every infinite-dimensional separable Hilbert space $\mathfrak{H}$ owns a unique non-separable companion Hilbert space $\mathcal{H}$. This is achieved by the closure of the eigenspaces of the reference operator and the defined operators. In this procedure, on many occasions, the notion of the dimension of subspaces loses its sense.

Gelfand triple and ***Rigged Hilbert space*** are other names for the general non-separable Hilbert spaces.

In the non-separable Hilbert space, for operators with continuum eigenspaces, the reverse bra-ket method turns from a summation into an integration.

$$\left\langle \vec{g} \,|\, F \,|\, \vec{h} \right\rangle \equiv \int\iiint \left\{ \left\langle \vec{g} | \vec{q} \right\rangle F(q) \left\langle \vec{q} | \vec{h} \right\rangle \right\} dV d\tau \qquad (1.4.1)$$

Here we omitted the enumerating subscripts that were used in the countable base of the separable Hilbert space.

The shorthand for the operator F is now

$$F \equiv |\vec{q}\rangle F(q) \langle \vec{q}| \qquad (1.4.2)$$

For eigenvectors $|q\rangle$, the function $F(q)$ defines as

$$F(q) = \langle \vec{q} \,|\, F\vec{q} \rangle = \int\iiint \left\{ \langle \vec{q} \,|\, \vec{q}\,' \rangle F(q') \langle \vec{q}\,' \,|\, \vec{q} \rangle \right\} dV' d\tau' \qquad (1.4.3)$$

The reference operator $\mathcal{R}$ that provides the continuum background parameter space as its eigenspace follows from

$$\left\langle \vec{g} \mid \mathcal{R}\vec{h} \right\rangle \equiv \int\iiint \left\{ \left\langle \vec{g} \middle| \vec{q} \right\rangle q \left\langle \vec{q} \middle| \vec{h} \right\rangle \right\} dV d\tau \tag{1.4.4}$$

The corresponding shorthand is

$$\mathcal{R} \equiv |\vec{q}\rangle q \langle \vec{q}| \tag{1.4.5}$$

The reference operator is a special kind of defined operator. Via the quaternionic functions that specify defined operators, it becomes clear that every infinite-dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that can be considered to embed its separable companion.

The reverse bracket method combines Hilbert space operator technology with quaternionic function theory and indirectly with quaternionic differential and integral technology.

1.7 Private parameter space

Each Hilbert space manages a private parameter space in the eigenspace of the reference operator. This parameter space applies the members of the version of the number system that the Hilbert space selects for specifying its inner product.

The parameter space is applied for the quaternionic functions that define the eigenspaces of the category of normal operators that share the eigenvectors of the reference operator and use the target values of the quaternionic functions as their eigenvalues. The parameter space determines the ***symmetry*** of the Hilbert space via the Cartesian and polar coordinate systems that arrange the elements of the selected number system.

1.7.1 Floating platforms

Apart from the background Hilbert space, all separable Hilbert spaces float with the geometric center of their private parameter space over the background parameter space. The axes of the floating Cartesian coordinate systems are parallel to the background Cartesian coordinate axes.

In physical reality, the floating platforms move with the image of their geometric center of their private parameter space over the background parameter space. No straightforward reason can be found for this movement of the floating platforms relative to the background platform. From observations, we know that in free space conglomerates of objects move at a uniform speed. Theoretically, it is not clear why the floating platforms move uniformly relative to the background platform when they do not interact with other objects.

1.8 Quaternionic Fields

Quaternionic fields are represented by eigenspaces of constructed operators. In separable Hilbert spaces, the constructed fields are sampled versions of continuum fields. The private parameter space is a flat field. It is also possible that some enclosed regions a continuum field are sampled fields, which are eigenspaces of constructed operators in an embedded separable Hilbert space that is embedded in a non-separable Hilbert space.

1.9 Symmetry versions

Between versions of the quaternionic number system, the sequencing of the numbers along the Cartesian axes can change direction. The polar coordinates can start with the azimuth or with the polar angle. The azimuth has a range of π radians. The polar angle has a range of 2π radians. Both can run up or down. Only the symmetry difference between the floating platforms and the background platform is important. That difference results in a symmetry-related charge that locates at the geometric center of the private parameter space of the floating platform. This symmetry-related charge results in a symmetry-related field. It is represented by a source or a sink at the location of the charge.

1.9.1 Restrictions to coordinate axes

No clear reason is found for the fact that the selection of Cartesian coordinate axes is restricted to axes that are parallel to the Cartesian axes of the background platform. A reason might be that in this way the sources and sinks of symmetry-related fields can be related to the symmetry differences between platforms.

2 Quaternionic differential calculus

The quaternionic analysis is not so well accepted as complex function analysis

2.1 Field equations

Maxwell equations apply the three-dimensional nabla operator in combination with a time derivative that applies coordinate time. The Maxwell equations derive from the results of experiments. For that reason, those equations contain physical units.

In this treatment, the quaternionic partial differential equations apply the quaternionic nabla. The equations do not derive from the results of experiments. Instead, the formulas apply the fact that the quaternionic nabla behaves as a quaternionic multiplying operator. The corresponding formulas do not contain physical units. This approach generates essential differences between Maxwell field equations and quaternionic partial differential equations.

The quaternionic partial differential equations form a complete and self-consistent set. They use the properties of the three-dimensional spatial nabla.

The corresponding formulas are taken from Bo Thidé's EMTF book., section Appendix F4.

Another online resource is Vector calculus identities.

The quaternionic differential equations play in a Euclidean setting that is formed by a continuum quaternionic parameter space and a quaternionic target space. The parameter space is the eigenspace of the reference operator of a quaternionic non-separable Hilbert space. The target space is eigenspace of a defined operator that resides in that same Hilbert space. The defined operator is specified by a quaternionic function that completely defines the field. Each basic field owns a private defining quaternionic function. All basic fields that are treated in this chapter are defined in this way.

Physical field theories tend to use a non-Euclidean setting, which is known as spacetime setting. This is because observers can only perceive in spacetime format. Thus, Maxwell equations use coordinate time, where the quaternionic differential equations use proper time. In both settings, the observed event is presented in Euclidean format. The hyperbolic Lorentz transform converts the Euclidean format to the perceived spacetime format. Chapter 3 treats the Lorentz transform. The Lorentz transform introduces time dilation and length contraction. Quaternionic differential calculus describes the interaction between discrete objects and the continuum at the location where events occur. Converting the results of this calculus by the Lorentz transform will describe the information that the observers perceive. Observers perceive in spacetime format. This format features a Minkowski signature. The Lorentz transform converts from the Euclidean storage format at the situation of the observed event to the perceived spacetime format. Apart from this coordinate transformation, the perceived scene is influenced by the fact that the retrieved information travels through a field that can be deformed and acts as the living space for both the observed event and the observer. Consequently, the information path deforms with its carrier field, and this affects the transferred information. In this chapter, we only treat what happens at the observed event. So, we ignore the Lorentz coordinate transform, and we are not affected by the deformations of the information path.

The Hilbert Book Model archives all dynamic geometric data of all discrete creatures that exist in the model in eigenspaces of separable Hilbert spaces whose private parameter spaces float over the background parameter space, which is the private parameter space of the non-separable Hilbert space. For example, elementary particles reside on a private floating platform that is implemented by a private separable Hilbert space.

Quantum physicists use Hilbert spaces for the modeling of their theory. However, most quantum physicists apply complex-number based Hilbert spaces. Quaternionic quantum mechanics appears to represent a natural choice. Quaternionic Hilbert spaces store the dynamic geometric data in the Euclidean format in quaternionic eigenvalues that consist of a real scalar-valued timestamp and a spatial, three-dimensional location.

In the Hilbert Book Model, the instant of storage of the event data is irrelevant if it coincides with or precedes the stored timestamp. Thus, the model can store all data at an instant, which precedes all stored timestamp values. This impersonates the Hilbert Book Model as a creator of the universe in which the observable events and the observers exist. On the other hand, it is possible to place the instant of archival of the event at the instant of the event itself. It will then coincide with the archived timestamp. In both interpretations, after sequencing the timestamps, the repository tells the life story of the discrete objects that are archived in the model. This story describes the ongoing embedding of the separable Hilbert spaces into the non-separable Hilbert space. For each floating separable Hilbert space this embedding occurs step by step and is controlled by a private stochastic process, which owns a characteristic function. The result is a stochastic hopping path that walks through the private parameter space of the platform. A coherent recurrently regenerated hop landing location swarm characterizes the corresponding elementary object.

Elementary particles are elementary modules. Together they constitute all other modules that occur in the model. Some modules constitute modular systems. A dedicated stochastic process controls the binding of the components of the module. This process owns a characteristic function that equals a dynamic superposition of the characteristic functions of the stochastic processes that control the components. Thus, superposition occurs in Fourier space. The superposition coefficients act as gauge factors that implement displacement generators, which control the internal locations of the components. In other words, the superposition coefficients may install internal oscillations of the components. These oscillations are described by differential equations.

2.2 Fields

In the Hilbert Book Model fields are eigenspaces of operators that reside in the non-separable Hilbert space. Continuous or mostly continuous functions define these operators, and apart from some discrepant regions, their eigenspaces are continuums. These regions might reduce to single discrepant point-like artifacts. The parameter space of these functions is constituted by a version of the quaternionic number system. Consequently, the real number valued coefficients of these parameters are mutually independent, and the differential change can be expressed in terms of a linear combination of partial differentials. Now the total differential change df of field f equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x} \vec{i} dx + \frac{\partial f}{\partial y} \vec{j} dy + \frac{\partial f}{\partial z} \vec{k} dz \qquad (2.2.1)$$

In this equation, the partial differentials $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ are quaternions.

The quaternionic nabla ∇ assumes the ***special condition*** that partial differentials direct along the axes of the Cartesian coordinate system. Thus

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \qquad (2.2.2)$$

The Hilbert Book Model assumes that the quaternionic fields are moderately changing, such that only first and second-order partial differential equations describe the model. These equations can describe fields of which the continuity gets disrupted by point-like artifacts. Spherical pulse responses, one-dimensional pulse responses, and Green's functions describe the reaction of the field on such disruptions.

2.3 Field equations

Generalized field equations hold for all basic fields. Generalized field equations fit best in a quaternionic setting.

Quaternions consist of a real number valued scalar part and a three-dimensional spatial vector that represents the imaginary part.

The multiplication rule of quaternions indicates that several independent parts constitute the product.

$$\begin{aligned} c = c_r + \vec{c} = ab &= \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) \\ &= a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \end{aligned} \qquad (2.3.1)$$

The $\pm$ indicates that quaternions exist in right-handed and left-handed versions.

The formula can be used to check the completeness of a set of equations that follow from the application of the product rule.

We define the quaternionic nabla as

$$\nabla \equiv \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \qquad (2.3.2)$$

$$\vec{\nabla} \equiv \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} \tag{2.3.3}$$

$$\nabla_r \equiv \frac{\partial}{\partial \tau} \tag{2.3.4}$$

$$\begin{aligned} \phi = \phi_r + \vec{\phi} = \nabla \psi &= \left(\frac{\partial}{\partial \tau} + \vec{\nabla} \right) \left(\psi_r + \vec{\psi} \right) \\ &= \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \end{aligned} \tag{2.3.5}$$

$$\phi_r = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \tag{2.3.6}$$

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{2.3.7}$$

Further,

$\vec{\nabla} \psi_r$ is the gradient of ψ_r

$\left\langle \vec{\nabla}, \vec{\psi} \right\rangle$ is the divergence of $\vec{\psi}$

$\vec{\nabla} \times \vec{\psi}$ is the curl of $\vec{\psi}$

The change $\nabla \psi$ divides into five terms that each has a separate meaning. That is why these terms in Maxwell equations get different names and symbols. Every basic field offers these terms!

$$\vec{E} = -\nabla_r \vec{\psi} - \vec{\nabla} \psi_r \tag{2.3.8}$$

$$\vec{B} = \vec{\nabla} \times \psi \tag{2.3.9}$$

It is also possible to construct higher-order equations. For example

$$\vec{J} = \vec{\nabla} \times \vec{B} - \nabla_r \vec{E} \tag{2.3.10}$$

The equation (2.3.6) has no equivalent in Maxwell's equations. Instead, its right part is used as a gauge.

Two special second-order partial differential equations use the terms $\frac{\partial^2 \psi}{\partial \tau^2}$ and $\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi$

$$\phi = \left\{ \frac{\partial^2}{\partial \tau^2} - \langle \vec{\nabla}, \vec{\nabla} \rangle \right\} \psi \tag{2.3.11}$$

$$\rho = \left\{ \frac{\partial^2}{\partial \tau^2} + \langle \vec{\nabla}, \vec{\nabla} \rangle \right\} \psi \tag{2.3.12}$$

The equation (2.3.11) is the quaternionic equivalent of the wave equation.

The equation (2.3.12) can be divided into two first-order partial differential equations.

$$\begin{aligned} \chi = \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi &= \left(\nabla_r + \vec{\nabla} \right) \left(\nabla_r - \vec{\nabla} \right) (\psi_r + \vec{\psi}) \\ &= \left(\nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle \right) \psi \end{aligned} \tag{2.3.13}$$

This composes from $\chi = \nabla^* \varphi$ and $\varphi = \nabla \psi$

The prove of (2.3.13) applies the equality

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{a} \right) = \vec{\nabla} \langle \vec{\nabla}, \vec{a} \rangle - \langle \vec{\nabla}, \vec{\nabla} \rangle \vec{a} \tag{2.3.14}$$

Such that

$$\begin{aligned} \vec{\nabla} \left(\vec{\nabla} a \right) &= \vec{\nabla} \left(\vec{\nabla} \times \vec{a} \right) - \langle \vec{\nabla}, \vec{a} \rangle + \vec{\nabla} a_r \\ &= \vec{\nabla} \times \vec{\nabla} \times \vec{a} - \vec{\nabla} \langle \vec{\nabla}, \vec{a} \rangle - \langle \vec{\nabla}, \vec{\nabla} \rangle a_r \\ &= \vec{\nabla} \langle \vec{\nabla}, \vec{a} \rangle - \langle \vec{\nabla}, \vec{\nabla} \rangle \vec{a} - \vec{\nabla} \langle \vec{\nabla}, \vec{a} \rangle - \langle \vec{\nabla}, \vec{\nabla} \rangle a_r \\ &= - \langle \vec{\nabla}, \vec{\nabla} \rangle a \end{aligned} \tag{2.3.15}$$

$\frac{\partial^2}{\partial \tau^2} - \langle \vec{\nabla}, \vec{\nabla} \rangle$ is the quaternionic equivalent of d'Alembert's operator $\square$.

The operator $\frac{\partial^2}{\partial \tau^2} + \langle \vec{\nabla}, \vec{\nabla} \rangle$ does not yet have an accepted name.

The Poisson equation equals

$$\rho = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi \tag{2.3.16}$$

A very special solution of this equation is the Green's function $\frac{1}{4\pi\left(\vec{q}-\vec{q'}\right)}$ of the affected field

$$\nabla \frac{1}{\left|\vec{q}-\vec{q'}\right|} = -\frac{\left(\vec{q}-\vec{q'}\right)}{\left|\vec{q}-\vec{q'}\right|^3} \tag{2.3.17}$$

$$\langle \vec{\nabla}, \vec{\nabla} \rangle \frac{1}{\left|\vec{q}-\vec{q'}\right|} \equiv \left\langle \vec{\nabla}, \vec{\nabla} \frac{1}{\left|\vec{q}-\vec{q'}\right|} \right\rangle = -\left\langle \vec{\nabla}, \vec{\nabla} \frac{\left(\vec{q}-\vec{q'}\right)}{\left|\vec{q}-\vec{q'}\right|^3} \right\rangle \tag{2.3.18}$$
$$= 4\pi\delta\left(\vec{q}-\vec{q'}\right)$$

The spatial integral over Green's function is a volume.

(2.3.11) offers a dynamic equivalent of the Green's function, which is a spherical shock front. It can be written as

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q'}\right| - c(\tau-\tau')\right)}{\left|\vec{q}-\vec{q'}\right|} \tag{2.3.19}$$

A one-dimensional type of shock front solution is

$$\psi = \vec{f}\left(\left|\vec{q}-\vec{q'}\right| - c(\tau-\tau')\right) \tag{2.3.20}$$

The equation (2.3.11) is famous for its wave type solutions

$$\nabla_r \nabla_r \psi = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi = -\omega^2 \psi \tag{2.3.21}$$

Periodic harmonic actuators cause the appearance of waves,

Planar and spherical waves are the simpler wave solutions of this equation.

$$\psi(\vec{q},\tau) = \exp\left\{\vec{n}\left(\langle \vec{k}, \vec{q}-\vec{q}_0 \rangle - \omega\tau + \varphi\right)\right\} \tag{2.3.22}$$

$$\psi(\vec{q},\tau)=\frac{\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\}}{\left|\vec{q}-\vec{q}_0\right|} \tag{2.3.23}$$

The Helmholtz equation considers the quaternionic function that defines the field separable.

$$\psi(q_r,\vec{q})=A(\vec{q})T(q_r) \tag{2.3.24}$$

$$\frac{\left\langle\vec{\nabla},\vec{\nabla}\right\rangle A}{A}=\frac{\nabla_r\nabla_r T}{T}=-k^2 \tag{2.3.25}$$

$$\left\langle\vec{\nabla},\vec{\nabla}\right\rangle A=-k^2 A \tag{2.3.26}$$

$$\nabla_r\nabla_r T=-k^2 T \tag{2.3.27}$$

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A(r,\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left\{\left(a_{lm}j_l(kr)\right)+b_{lm}Y_l^m(\theta,\varphi)\right\} \tag{2.3.28}$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics. These solutions play a role in the spectra of atomic modules.

A more general solution is a superposition of these basic types.

(2.3.11) offers a dynamic equivalent of the Green's function, which is a spherical shock front. It can be written as

$$\psi=\frac{f\left(\vec{q}-\overrightarrow{q'}+c(\tau-\tau')\right)}{\left|\vec{q}-\overrightarrow{q'}\right|} \tag{2.3.29}$$

A one-dimensional type of shock front solution is

$$\psi=\vec{f}\left(\vec{q}-\overrightarrow{q'}+c(\tau-\tau')\right) \tag{2.3.30}$$

Equation (2.3.12) offers no waves as part of its solutions.

During travel, the amplitude and the lateral direction $\frac{\vec{f}}{\left|\vec{f}\right|}$ of the one-dimensional shock fronts are fixed. The longitudinal direction is along $\frac{\vec{q}-\vec{q'}}{\left|\vec{q}-\vec{q'}\right|}$.

The shock fronts that are triggered by point-like actuators are the tiniest field excitations that exist. The actuator must fulfill significant restricting requirements. For example, a perfectly isotropic actuator must trigger the spherical shock front. The actuator can be a quaternion that belongs to another version of the quaternionic number system than the version, which the background platform applies. The symmetry break must be isotropic. Electrons fulfill this requirement. Neutrinos do not break the symmetry but have other reasons why they cause a valid trigger. Quarks break symmetry, but not in an isotropic way.

2.4 Energy operators

In contemporary quantum physics, the del operator stands for the momentum operator $\vec{p}=h\vec{\nabla}$. In the quaternionic function theory, this is automatically an imaginary operator.

In contemporary quantum physics T is the kinetic energy operator.

$$T=\frac{p^2}{2m}=-\frac{h^2}{2m}\left\langle\vec{\nabla},\vec{\nabla}\right\rangle \tag{2.4.1}$$

The gravitation potential of an elementary particle is a superposition ψ_E of solutions ψ_L of the equation

$$\left(\nabla_r\nabla_r+\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\psi_L=4\pi\delta\left(\vec{q}-\vec{q'}\right)\theta\left(\tau\pm\tau'\right) \tag{2.4.2}$$

that are integrated over the regeneration cycle of the location density distribution of the footprint of the particle. Here the Dirac delta function and the step function represent the location and the timestamps of the hop landings.

The fact that the stochastic process, which controls the elementary particle owns a characteristic function makes that the superposition ψ_E of the solutions ψ_L is a wave package. The superposition is defined in Fourier space. ψ_E is a solution of

$$\left(\nabla_r\nabla_r - \left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\psi_E = 0 \qquad (2.4.3)$$

Thus, the gravitation potential ψ_E of an elementary particle is also a superposition of wave-like solutions ψ_ω of the equation

$$\nabla_r\nabla_r\psi_\omega = \left\langle\vec{\nabla},\vec{\nabla}\right\rangle\psi_\omega = \omega^2\psi_\omega \qquad (2.4.4)$$

The composition of composite objects that are constituted by elementary particles is also controlled by a stochastic process that owns a characteristic function. That characteristic function is a dynamic superposition of the characteristic functions of the components of the composite. So, its gravitational potential ψ is a dynamic wave package and it is a dynamic superposition of functions of type ψ_E. Again, the superposition is defined in Fourier space.

According to the Klein Gordon equation, which presents the vision of current physics, ψ is a solution of

$$\left(\nabla_r\nabla_r - \left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\psi = -\left(\frac{m}{\hbar}\right)^2\psi \qquad (2.4.5)$$

Here we have taken $c^2 = 1$. This shows a significant difference between conventional physics and the Hilbert Book Model in how these theories handle the concept of mass. The HBM applies Newton’s gravitation potential to relate mass to the gravitation potential of massive objects.

$$\phi(r) = \frac{MG}{r} \tag{2.4.6}$$

3 Lorentz transform

3.1 The transform

The shock fronts move with speed c. In the quaternionic setting, this speed is unity.

$$x^2 + y^2 + z^2 = c^2\tau^2 \tag{3.1.1}$$

Swarms of spherical pulse response triggers move with lower speed v.

For the geometric centers of these swarms still holds:

$$x^2 + y^2 + z^2 - c^2\tau^2 = x'^2 + y'^2 + z'^2 - c^2\tau'^2 \tag{3.1.2}$$

If the locations $\{x, y, z\}$ and $\{x', y', z'\}$ move with uniform relative speed v, then

$$ct' = ct\cosh(\omega) - x\sinh(\omega) \tag{3.1.3}$$

$$x' = x\cosh(\omega) - ct\sinh(\omega) \tag{3.1.4}$$

$$\cosh(\omega) = \frac{\exp(\omega) + \exp(-\omega)}{2} = \frac{c}{\sqrt{c^2 - v^2}} \tag{3.1.5}$$

$$\sinh(\omega) = \frac{\exp(\omega) - \exp(-\omega)}{2} = \frac{v}{\sqrt{c^2 - v^2}} \tag{3.1.6}$$

$$\cosh(\omega)^2 - \sinh(\omega)^2 = 1 \tag{3.1.7}$$

This is a hyperbolic transformation that relates two coordinate systems.

This transformation can concern two platforms P and P' on which swarms reside and that move with uniform relative speed .

However, it can also concern the storage location P that contains a timestamp t and spatial location $\{x,y,z\}$ and platform P' that has coordinate time t and location $\{x',y',z'\}$.

In this way, the hyperbolic transform relates two individual platforms on which the private swarms of individual elementary particles reside.

It also relates the stored data of an elementary particle and the observed format of these data for the elementary particle that moves with speed relative to the background parameter space.

The Lorentz transform converts a Euclidean coordinate system consisting of a location $\{x,y,z\}$ and proper timestamps τ into the perceived coordinate system that consists of the spacetime coordinates $\{x',y',z',ct'\}$ in which t' plays the role of proper time. The uniform velocity v causes time dilation $\Delta t' = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}}$ and length contraction $\Delta L' = \Delta L\sqrt{1-\frac{v^2}{c^2}}$

3.2 Minkowski metric

Spacetime is ruled by the Minkowski metric.

In flat field conditions, proper time τ is defined by

$$\tau = \pm\frac{\sqrt{c^2t^2 - x^2 - y^2 - z^2}}{c} \tag{3.2.1}$$

And in deformed fields, still

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{3.2.2}$$

Here ds is the spacetime interval and $d\tau$ is the proper time interval. dt is the coordinate time interval

3.3 Schwarzschild metric

Polar coordinates convert the Minkowski metric to the Schwarzschild metric. The proper time interval $d\tau$ obeys [89] [90]

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2 d\varphi^2\right)$$

(3.3.1)

Under pure isotropic conditions, the last term on the right side vanishes.

According to mainstream physics, in the environment of a black hole, the symbol r_s stands for the Schwarzschild radius.

$$r_s = \frac{2GM}{c^2} \qquad (3.3.2)$$

The variable r equals the distance to the center of mass of the massive object with mass M .

The Hilbert Book model finds a different value for the boundary of a spherical black hole. That radius is a factor of two smaller.

4 Line, surface and volume integrals

4.1 Line integrals

The curl can be presented as a line integral

$$\left\langle \vec{\nabla} \times \vec{\psi}, \vec{n} \right\rangle \equiv \lim_{A \to 0} \left(\frac{1}{A} \oint_C \langle \vec{\psi}, d\vec{r} \rangle \right) \tag{4.1.1}$$

4.2 Surface integrals

With respect to a local part of a closed boundary that is oriented perpendicular to vector $\vec{n}$ the partial differentials relate as

$$\begin{aligned} \vec{\nabla}\psi &= -\left\langle \vec{\nabla}, \vec{\psi} \right\rangle + \vec{\nabla}\psi_r \pm \vec{\nabla} \times \vec{\psi} \Leftrightarrow \vec{n}\psi \\ &= -\langle \vec{n}, \vec{\psi} \rangle + \vec{n}\psi_r \pm \vec{n} \times \vec{\psi} \end{aligned} \tag{4.2.1}$$

This is exploited in the surface-volume integral equations that are known as Stokes and Gauss theorems.

$$\iiint \vec{\nabla}\psi \, dV = \oiint \vec{n}\psi \, dS \tag{4.2.2}$$

$$\iiint \left\langle \vec{\nabla}, \vec{\psi} \right\rangle dV = \oiint \langle \vec{n}, \vec{\psi} \rangle \, dS \tag{4.2.3}$$

$$\iiint \vec{\nabla} \times \vec{\psi} \, dV = \oiint \vec{n} \times \vec{\psi} \, dS \tag{4.2.4}$$

$$\iiint \vec{\nabla}\psi_r \, dV = \oiint \vec{n}\psi_r \, dS \tag{4.2.5}$$

This result turns terms in the differential continuity equation into a set of corresponding integral balance equations.

The method also applies to other partial differential equations. For example

$$\begin{aligned} \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) &= \vec{\nabla} \left\langle \vec{\nabla}, \vec{\psi} \right\rangle - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \vec{\psi} \Leftrightarrow \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) \\ &= \vec{n} \langle \vec{n}, \vec{\psi} \rangle - \langle \vec{n}, \vec{n} \rangle \vec{\psi} \end{aligned} \tag{4.2.6}$$

$$\iiint_V \left\{ \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) \right\} dV = \oiint_S \left\{ \vec{\nabla} \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \right\} dS - \oiint_S \left\{ \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \vec{\psi} \right\} dS$$

(4.2.7)

One dimension less, a similar relation exists.

$$\iint_S \left(\left\langle \vec{\nabla} \times \vec{a}, \vec{n} \right\rangle \right) dS = \oint_C \left\langle \vec{a}, d\vec{l} \right\rangle \tag{4.2.8}$$

4.3 Using volume integrals to determine the symmetry-related charges

In its simplest form in which no discontinuities occur in the integration domain Ω, the generalized Stokes theorem runs as

$$\int_\Omega d\omega = \int_{\partial\Omega} \omega = \oint_\Omega \omega \tag{4.3.1}$$

We separate all point-like discontinuities from the domain Ω by encapsulating them in an extra boundary. Symmetry centers represent spherically shaped or cube-shaped closed parameter space regions H_n^x that float on a background parameter space $\mathfrak{R}$. The boundaries ∂H_n^x separate the regions from the domain H_n^x. The regions H_n^x are platforms for local discontinuities in basic fields. These fields are continuous in the domain $\Omega - H$.

$$H = \bigcup_{n=1}^{N} H_n^x \tag{4.3.2}$$

The symmetry centers $\mathfrak{S}_n^x$ are encapsulated in regions H_n^x, and the encapsulating boundary ∂H_n^x is not part of the disconnected boundary, which encapsulates all continuous parts of the quaternionic manifold ω that exists in the quaternionic model.

$$\int_{\Omega - H} d\omega = \int_{\partial\Omega \cup \partial H} \omega = \int_{\partial\Omega} \omega - \sum_{k=1}^{N} \int_{\partial H_n^x} \omega \tag{4.3.3}$$

In fact, it is sufficient that ∂H_n^x surrounds the current location of the elementary module. We will select a boundary, which has the shape of a small cube of which the sides run through a region of the parameter spaces where the manifolds are continuous.

If we take everywhere on the boundary the unit normal to point outward, then this reverses the direction of the normal on ∂H_n^x which negates the integral. Thus, in this formula, the contributions of boundaries $\{\partial H_n^x\}$ are subtracted from the contributions of the boundary $\partial\Omega$. This means that $\partial\Omega$ also surrounds the regions $\{\partial H_n^x\}$

This fact renders the integration sensitive to the ordering of the participating domains.

Domain Ω corresponds to part of the background parameter space $\mathfrak{R}$. As mentioned before the symmetry centers $\mathfrak{S}_n^x$ represent encapsulated regions $\{\partial H_n^x\}$ that float on the background parameter space $\mathfrak{R}$. The Cartesian axes of $\mathfrak{S}_n^x$ are parallel to the Cartesian axes of background parameter space $\mathfrak{R}$. Only the orderings along these axes may differ.

Further, the geometric center of the symmetry center $\mathfrak{S}_n^x$ is represented by a floating location on parameter space $\mathfrak{R}$.

The symmetry center $\mathfrak{S}_n^x$ is characterized by a private symmetry flavor. That symmetry flavor relates to the Cartesian ordering of this parameter space. With the orientation of the coordinate axes fixed, eight independent Cartesian orderings are possible.

The consequence of the differences in the symmetry flavor on the subtraction can best be comprehended when the encapsulation ∂H_n^x is performed by a ***cubic space form*** that is aligned along the Cartesian axes that act in the background parameter space. Now the six sides of the cube contribute differently to the effects of the encapsulation when the ordering of H_n^x differs from the Cartesian ordering of the reference parameter space $\mathfrak{R}$. Each discrepant axis ordering corresponds to one-third of the surface of the cube. This effect is represented by the ***symmetry-related charge***, which includes the ***color charge*** of the symmetry center. It is easily comprehensible related to the algorithm which below is introduced for the computation of the symmetry-related charge. Also, the relation to the color charge will be clear. ***Thus, this effect couples the ordering of the local parameter spaces to the symmetry-related charge of the encapsulated elementary module.*** The differences with the ordering of the surrounding parameter space determine the value of the symmetry-related charge of the object that resides inside the encapsulation!

4.4 Symmetry flavor

The Cartesian ordering of its private parameter space determines the symmetry flavor of the platform. For that reason, this symmetry is compared with the reference symmetry, which is the symmetry of the background parameter space. Four arrows indicate the symmetry of the platform. The background is represented by:

⬆⬆⬆⬆

Now the symmetry-related charge follows in three steps.

1. Count the difference of the spatial part of the symmetry of the platform with the spatial part of the symmetry of the background parameter space.
2. Switch the sign of the result for anti-particles.

Symmetrieversie

Ordering x y z τ	Sequence	Handedness Right/Left	Color charge	Electric charge * 3	Symmetry type.
⬆⬆⬆⬆	⓪	~~**R**~~	N	+0	neutrino
⬇⬆⬆⬆	①	**L**	R	− 1	down quark
⬆⬇⬆⬆	②	**L**	G	− 1	down quark
⬇⬇⬆⬆	③	**R**	B	+2	up quark
⬆⬆⬇⬆	④	**L**	B	−1	down quark
⬇⬆⬇⬆	⑤	**R**	G	+2	up quark
⬆⬇⬇⬆	⑥	**R**	R	+2	up quark
⬇⬇⬇⬆	⑦	**L**	N	− 3	electron
⬆⬆⬆⬇	⑧	**R**	N	+3	positron
⬇⬆⬆⬇	⑨	**L**	R	− 2	anti-up quark
⬆⬇⬆⬇	⑩	**L**	G	− 2	anti-up quark
⬇⬇⬆⬇	⑪	**R**	B	+1	anti-down quark
⬆⬆⬇⬇	⑫	**L**	B	− 2	anti-up quark
⬇⬆⬇⬇	⑬	**R**	G	+1	anti-down quark
⬆⬇⬇⬇	⑭	**R**	R	+1	anti-down quark
⬇⬇⬇⬇	⑮	~~**L**~~	N	− 0	anti-neutrino

Probably, the neutrino and the antineutrino own an abnormal handedness.

The suggested particle names that indicate the symmetry type are borrowed from the Standard Model. In the table, compared to the standard model, some differences exist with the selection of the anti-predicate. All considered particles are elementary fermions. The freedom of choice in the polar coordinate system might determine the spin. The azimuth range is 2π radians, and the polar angle range is π radians. Symmetry breaking means a difference between the platform symmetry and the symmetry of the background. Neutrinos do not break the symmetry. Instead, they probably may cause conflicts with the handedness of the multiplication rule.

4.5 Derivation of physical laws

The quaternionic equivalents of Ampère's law are

$$\vec{J} \equiv \vec{\nabla} \times \vec{B} = \nabla_r \vec{E} \Leftrightarrow \vec{J} \equiv \vec{n} \times \vec{B} = \nabla_r \vec{E} \quad (4.5.1)$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{B}, \vec{n} \right\rangle dS = \oint_C \left\langle \vec{B}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{J} + \nabla_r \vec{E}, \vec{n} \right\rangle dS \quad (4.5.2)$$

The quaternionic equivalents of Faraday's law are:

$$\nabla_r \vec{B} = \vec{\nabla} \times (\nabla_r \vec{\psi}) = -\vec{\nabla} \times \vec{E} \Leftrightarrow \nabla_r \vec{B} = \vec{n} \times (\nabla_r \vec{\psi}) = -\vec{\nabla} \times \vec{E} \quad (4.5.3)$$

$$\oint_c \left\langle \vec{E}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{\nabla} \times \vec{E}, \vec{n} \right\rangle dS = -\iint_S \left\langle \nabla_r \vec{B}, \vec{n} \right\rangle dS \quad (4.5.4)$$

$$\vec{J} = \vec{\nabla} \times \left(\vec{B} - \vec{E}\right) = \vec{\nabla} \times \vec{\varphi} - \nabla_r \vec{\varphi} = \vec{v}\rho \quad (4.5.5)$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{\varphi}, \vec{n} \right\rangle dS = \oint_C \left(\left\langle \vec{\varphi}, d\vec{l} \right\rangle \right) = \iint_S \left\langle \vec{v}\rho + \nabla_r \vec{\varphi}, \vec{n} \right\rangle dS \quad (4.5.6)$$

The equations (4.5.4) and (4.5.6) enable the derivation of the ***Lorentz force***.

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{B} \quad (4.5.7)$$

$$\frac{d}{d\tau} \iint_S \left\langle \vec{B}, \vec{n} \right\rangle dS = \iint_{S(\tau_0)} \left\langle \dot{\vec{B}}(\tau_0), \vec{n} \right\rangle ds + \frac{d}{d\tau} \iint_{S(\tau)} \left\langle \vec{B}(\tau_0), \vec{n} \right\rangle ds \quad (4.5.8)$$

The Leibniz integral equation states

$$\frac{d}{dt}\iint_{S(\tau)}\left\langle \vec{X}(\tau_0),\vec{n}\right\rangle dS$$
$$= \iint_{S(\tau_0)}\left\langle \dot{\vec{X}}(\tau_0)+\left\langle\vec{\nabla},\vec{X}(\tau_0)\right\rangle\vec{v}(\tau_0),\vec{n}\right\rangle dS - \oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{X}(\tau_0),d\vec{l}\right\rangle \tag{4.5.9}$$

With $\vec{X}=\vec{B}$ and $\left\langle\vec{\nabla},\vec{B}\right\rangle = 0$ follows

$$\frac{d\Phi_B}{d\tau}=$$
$$\frac{d}{d\tau}\iint_{S(\tau)}\left\langle \dot{\vec{B}}(\tau),\vec{n}\right\rangle dS = \iint_{S(\tau_0)}\left\langle \vec{B}(\tau_0),\vec{n}\right\rangle dS - \oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle$$
$$= -\oint_{C(\tau_0)}\left\langle E(\tau_0),d\vec{l}\right\rangle - \oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle$$

(4.5.10)

The electromotive force (EMF) ε equals

$$\varepsilon = \oint_{C(\tau_0)}\left\langle \frac{\vec{F}(\tau_0)}{q},d\vec{l}\right\rangle = -\left.\frac{d\Phi_B}{d\tau}\right|_{\tau=\tau_0}$$
$$= \oint_{C(\tau_0)}\left\langle \vec{E}(\tau_0),d\vec{l}\right\rangle + \oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle \tag{4.5.11}$$

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} \tag{4.5.12}$$

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