On Quantum Field Theory and Gravitation

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03. Quantum mechanics, field theories, and special relativity

11. General theory of fields and particles

04.30.-w Gravitational waves

Abstract

Quantum Field Theory is derived from the relativistic symmetry group, the Lorentz group (LG). Results presented here aim at a basic understanding rather then at treating specific systems. They include:

- Categorizing field types in terms of irreducible representations of the LG (infinitely many)
- A particular field type, “bosonic quark”, yielding a meson-type candidate for dark matter
- Structure of spinor-type fields as 4-vector amplitudes
- Flavor as consequence of the permutation group S3 for the three space dimensions
- Spin-dynamics equations for general spinor types (spin propagation), yielding phase speeds smaller then one, for spin 1 upwards
- Consequently, no free field solutions for spin 1 upwards (asymptotic freedom)
- Identification of interaction as propagation of fields of equal transformation type (phase speed)
- Gravitation as a consequence of the square-root-density (sqd) nature of fields (density-mediated propagation)
- The gravitational field is the scalar field which is the only field that is affected by Lorentz transformations exclusively through its sqd property.

Symmetries and Representations

Quantum fields, $\Phi(x)$, are local quantities associated with the points, x, of a metric space. The quantum property signifies that they obey certain normalization conditions. In the absence of propagation they can be categorized into finite-dimensional vectors transforming according to irreducible representations of the local symmetries of the metric space.

Physical processes take place in 4-dimensional space-time, *R4*. The ‘isotropic’ symmetry is the Lorentz group, *L*, which preserves the indefinite (3,1) metric. The elementary fields are derived from the principal bundle of *L* over *R4*. They are the associated fiber bundles in which the fibers are finite-dimensional irreducible representations of *L*. These representations are related to the ones of the group *O(4)*, the group which preserves the Cartesian metric in *R4*. *O(4)* has two components, the normal, connected subgroup *SO(4)* of proper transformations with determinant one and it's separated coset of improper transformations.

The group *SO(4)* is locally a direct product of two groups, labeled *l* and *r*, that are both isomorphic to the group *SO(3)* of proper transformations of *R3*, which preserve Cartesian metric [1,2]:

$$SO(4)={}^{l}SO(4)\otimes{}^{r}SO(4) \quad \text{or} \quad o(4)={}^{l}o(4)\oplus{}^{r}o(4) \ , \tag{1}$$

for the corresponding Lee algebra. Clearly, *l*- and *r*-transformations commute.

The difference between the two factors can be understood when keeping the element in one factor (say with superscript *r*) fixed to its identity. The effect of a transformation in the other factor is to make a simultaneous rotation in two orthogonal, 2-dimensional sub-spaces by the same angle. If the identity is kept in the other (*l*) factor space, the rotations in the 2-dimensional sub-spaces have the opposite relations to each other. Thus, the two situations can be described by a chiral parameter, left- or right-handed (*l* or *r*), a nomenclature that becomes meaningful in the *SO(3,1)* case, where a time direction is singled out.

The representations of *SO(4)* can thus be described by product-representations *(j,k)* where the non-negative half-integers *j* and *k* label the well-known representations of *SO(3)*. The first position (*j*) refers to a left-chiral property, not affected by transformations of ${}^{r}SO(4)$, the second (*k*) to a right-chiral one. If we consider, in addition, improper transformations, their effect is to exchange chiralities, i.e. *(j,k)* goes to *(k,j)*. Consequently, eigenstates of mirror operations, can only belong to a single representation if $j = k$. For $j \neq k$ one obtains combinations of two representations of *O(4)*.

When considering the physical relevant space-time, *R4* with indefinite metric, the symmetry group changes from *O(4)* to *O(3,1)*. The indefinite metric can be considered, formally, as a Cartesian one when the time variable is considered as the zeroth coordinate multiplied by the imaginary unit *i*. This gives rise to the so-called unitary trick, which relates the representations of *SO(3,1)* to the ones of *SO(4)* [3]. Thus, we may tentatively use the irreducible transformations of *SO(4)* for a classification of elementary excitations in Minkowski space-time as follows.

Representation (j, k)	Field Type	Pure Solution
(0, 0)	gravitational	graviton, static field
(½, 0), (0, ½)	lepton	no
(½, ½)	electromagnetic	photon, static field
(1, 0), (0, 1)	boson-quark	no
(½, 1), (1, ½)	quark	no
(1, 1)	gluon	no
...		no

As mentioned above improper symmetry operations mediate between particles (left chiral) and antiparticles (right chiral). Thus the electron *(½,0)* corresponds to the positron *(0, ½)*, etc. Eigenstates of the mirror operators are thus the symmetric and antisymmetric combination of a particle with its anti-particle. For the cases $j = k$ we have two irreducible representations *(j,j)+* and *(j,j)-*, symmetric and anti-symmetric representations. These can be transformed into a pair of chiral states by taking suitable combinations of these + and – representations. However, it must be emphasized here, that this whole consideration is taken over from the *O(4)* case, where a distinction into various flavors for leptons and quarks does not make sense yet.

Remark

The simple cases are the pure *l*- or *r*-type representations, k=0 or j=0, respectively, which correspond to the representations of *O(3)*. For the mixed *(j,k)* representations we can get some feeling by keeping one coordinate axis (say 0) fixed. Then the set of transformations is reduced to rotations in the (1,2,3)-space, and we can perform a Clebsch-Gordan decomposition of the *(j,k)*-representation to a sum of D_n representations of *O(3)* with $n = |j-k|, \ldots, (j+k)$. This decomposition differs for different choices of the fixed axis, or differently stated, under the full *O(4)* group, mixing among these various representations D_n occurs.

The O(3,1) Case

For the *O(3,1)* case additional and essential complications occur. To get a idea of these complications we may start, in the *O(4)*-case, from a infinitesimal *l*- (*r*-) transformation, δT, involving a rotation, $\delta\varphi$, in a plane specified by the 0-axis and a orthogonal (unit-)vector q in the (1,2,3)-subspace:

$$\partial^l T_q = \partial\phi \frac{i}{2}(S_q + S_{0,q}) \quad , (\quad \partial^r T_q = \partial\phi \frac{i}{2}(S_q - S_{0,q}) \quad), \tag{2}$$

where $iS_{0,q}$ is a infinitesimal rotation in the $(0,q)$ plane and iS_q a infinitesimal rotation about the q-axis in the restricted (1,2,3) space. If we were to exponentiate this transformation to an angle $\varphi = \pi$ we would end up at the (proper) symmetry element *PT*. Hence, in a representation *D*, we can consider exponentiation of expression (2) as a rotation in a plane defined by a vector and its image obtained by a transformation $D_q(\pi/2)$. The infinitesimal transformation is a infinitesimal step towards the *PT* related field component. Clearly, this step is different for different directions q, and the various possible steps are characterized by the manifold of unit vectors q.

In going to *O(3,1)* the 0-component is multiplied by the imaginary unit and correspondingly the infinitesimal operators $S_{0,q}$ as well, such that the infinitesimal transformation now exponentiates to a boost, a aperiodic transformation. Furthermore, the *PT* related component of the field must now lie in a separated component of the field, because the *PT*-element lies now in a separated component of *O(3,1)*. Because of the aperiodicity of boosts this field-component must have the topological structure of a 3-dimensional vector as imposed by the boost direction q. For a given (local) boost it is convenient to choose the boost direction as quantization axis with various m-values, magnetic quantum numbers. The m-dependent boost-distortion of a field can then be described by a time component (*~cosh(mb)*) and a q-directed space component (*~sinh(mb)*) such that the m-components of the field are actually 4-vectors, the norms of which have invariant values ($\cosh(x)^2 - \sinh(x)^2 = 1$). Thus, by splitting off a position-dependent quantum-mechanical probability amplitude we have a set of (*2m+1)* 4-vectors with a constant norm in a generalized Hilbert-type space. We can choose the boost direction as one coordinate axis such that two of the space-components vanish. In the rest system, the space components vanish altogether. Here the parameter

$$b = \frac{1}{2}\text{artanh}\left(v\right) \qquad (3)$$

describes the (local) boost amount in terms of the (local) speed parameter v.

Flavor

Instead of writing down a general transformation here, we treat simple cases, firstly by keeping $k = 0$. We can again assemble the *2m+1* time-components to a ‘time-component’ of the field and the *2m+1* (3-vector) space-components to the ‘space component’, of the field.

One must realize that, with respect to the time component, the three space components of a field acquire direction-dependent phases mediated by the representation-specific unit-size operators (see expression (2)).

$${}^{j}D_{\vec{q}} = e^{i\pi^{j}S_{\vec{q}}} \quad . \qquad (4)$$

Consequently, the three space components have different phases which must be accounted for when changing the coordinate system. For integer j the ${}^{j}D_q$’s square to the unit matrix, such that $-{}^{j}D_q$ is also a possible solution, because there is no connection between time- and space-like components of the field.

For half-integer j the squares are minus the unit matrix. In this case we have two distinguished fields related by ${}^{j}D_q$’s of different signs. A example are charged leptons an their corresponding neutrinos. As an illustration we write down these matrices for the lepton case ($j = ½$):

$${}^{\frac{1}{2}}D_x = \pm\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \ , \ {}^{\frac{1}{2}}D_y = \pm\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \ , \ {}^{\frac{1}{2}}D_z = \pm\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \ , \ {}^{\frac{1}{2}}D_d = \frac{\pm 1}{\sqrt{3}}\begin{pmatrix} i & i+1 \\ i-1 & -i \end{pmatrix} \ , \qquad (5)$$

which all square to minus the unity (matrix d corresponds to the space diagonal). This feature originates in the two-valued nature of these representations under rotations, which is maintained separately in time- and space-components of the field (two sheets). Consequently, there are two distinct ways of connecting the double-sheets of the separated time- and space-components. This is the origin of one kind of flavor caused by the aperiodicity of boosts.

To arrive at a full description of flavor one can generalize this consideration by the observation, that, again due to the separated nature of time and space components of the field, the assignment of the D-matrices (5) (belonging to the coordinate axes x, y, z) to the three space components of the field is arbitrary, which leads to a additional symmetry, *S3*, consisting of the permutation of three elements (assignments), which then yields the three one-dimensional representations of the (Abelian) alternating group, *A3*. For integer $j+k$ they extend to even and odd representations for the full *S3*, while for half-integer $j+k$ the corresponding diversification consists of alternative sheet-connection between time- and space-components of the fields, as mentioned above.

Propagation

To handle propagation we must find the way to describe fields at neighboring points in space-time. Because we have infinitesimal steps and assume differentiable fields, they must be related via a infinitesimal transformation, a representation of a member of the Lie-algebra, *o(4)* or *o(3,1)*.

One can build a 4-vector operator out of the three *l*- (or *r*-) basis vectors of the Lie-algebra

$$ {}^{l}L_i=\frac{i}{2}(S_i+S_{0,i}) \quad , \quad {}^{r}L_i=\frac{i}{2}(S_i-S_{0,i}) \quad , \text{(i = 1, 2, 3)}, \tag{6} $$

by taking as time- (0-) component the square-root of the corresponding Casimir operators

$$ {}^{l,r}C \equiv ({}^{l,r}L_0)^2=\sum_{i=1}^{3}({}^{l,r}L_i)^2 \quad . \tag{7} $$

Because the Casimir operator is a constant within a representation, namely *-j(j+1)* or *-k(k+1)* for *l*- or *r*-type representations, respectively, we may write down the representation-specific 4-vector operators

$$ ({}^{l=j}L_\mu)=(-i\sqrt{(j(j+1))},{}^{l=j}L_{1,}{}^{l=j}L_{2,}{}^{l=j}L_3) \quad , \tag{8} $$

and correspondingly for (*r=k*). The somewhat loose notation *l=j* should signify that they act specifically on a *l*-type field of irreducible representation *j*. Clearly, the vectors (7) are zero-vectors within that irreducible representation.

Now we can write down a expression for the propagation of a field. Multiplying the gradient with the infinitesimal transformation vector (8) we obtain the expression for spin-propagation

$$ ({}^{l=j}L_\mu)\partial^\mu\psi=a({}^{l=j}L_\mu)\partial^\mu\,{}^{j}u+({}^{l=j}L_\mu)\,{}^{j}u\,\partial^\mu a=M\psi \quad , \tag{9} $$

where the second form applies when the field is written as a (real) amplitude $a(x_\mu)$ times a unit-field ${}^{j}u(x_\mu)$ in the sense of the above-introduced (4-vector) norm.

Because propagation of a field must commute with the symmetry group, the operator must be a (solution-specific) constant, *M* (wave number), which must follow from quantization conditions. Furthermore, under a parity operation the spacial components of the gradient change sign, the left-handed operators change to right-handed ones and the *(j,0)* representation changes to the *(0,j)* type (anti-field). Hence, in order to have a invariant propagation, we see, that we must choose different signs for the time-component of (9) for left- and right-handed fields. In addition, the sign of the (mass-) constant, *M*, must change such that the propagation equation remains equivalent.

For a spin one-half field (*j*=½) expression (9) resembles the Dirac equation. However, the 4-vector nature of the field here is totally different from how the Dirac-field is interpreted conventionally (namely as a representation of the Clifford algebra *Cliff(3,1)*). This (mis-)interpretation is the origin of a 50-year lasting confusion on the nature of the “weak interaction” [4] and, for the same reason, flavor is still considered a mystery [5,6].

Furthermore, considering plane wave solutions, we see that the propagation expression (9) yields *j*-dependent phase velocities:

$$ v_p=\frac{1}{\sqrt{(j(j+1))}} \quad , \tag{10} $$

which are smaller than one for $j \geq 1$. Since the group velocity is limited by the phase velocity, quanta of free fields can only exist for *j*- (or *k*-) values smaller than one. Differently stated, the dispersion relations for spinor fields are all off-shell, above-shell for *j* (or *k*) = ½, below shell for $j \geq 1$ (asymptotic freedom).

Gravitation

In order to handle gravitation, we have to remember that the norm squared of a field is a density and fields themselves are square roots of densities. Lorentz transformations also affect the density property and consequently we have to augment the propagation operator (9) by a appropriate term. Of course, this term must be identical for all field types and, in particular, it is the only term that affects the scalar field. This is precisely how gravitation is supposed to act, and thus, we end up by identifying the scalar field with the gravitational field.

A density changes with a boost via Lorentz-contraction and is proportional to the Lorentz factor

$$\gamma=\frac{1}{\sqrt{(1-v^2)}} \quad , \tag{11}$$

with v (of the local boost) given by (3). Thus, for a square-root density we obtain a infinitesimal boost operator

$$\partial(\gamma^{\frac{1}{2}})=\frac{-v}{2(1-v^2)^{\frac{5}{4}}}\partial v=-v(1-v^2)^{-\frac{1}{4}}\partial b \quad . \tag{12}$$

Because time dilation is reciprocal to Lorentz contraction its infinitesimal value is just opposite such that we can write down the density(-mediated) propagation contribution to expression (9)

$$i\,v(1-v^2)^{-\frac{1}{4}}(\vec{b}\vec{u})(\partial t-u_l\partial^l)\psi \quad , \tag{13}$$

where $\vec{b}$ is the vector of local boost of the field and $\vec{u}$ the unit direction vector of the infinitesimal boost. Analogously to spin-propagation (9), we have here a scalar product of a time-like infinitesimal boost operator with the gradient. In this case the dispersion relation is on-shell for changes parallel to the actual boost.

A peculiarity of density-propagation (13), as compared to spin-propagation (9), is that it diminishes quadratically for small boost values.

One may now get a understanding of the reasons why General Relativity theory (GR) yields “correct” results. A boosted scalar (gravitational) field has a local metric corresponding to its (local) Lorentz-boost value. Obviously, GR corresponds to assigning this local, field-specific metric to the geometry of space-time.

Interaction is Propagation!

Interaction among fields is conventionally treated by additional terms that connect fields of different types (representations), or even of the same type (e.g. Higgs field), whatever that means. However, one must acknowledge that there is just a single field in the cosmos which can be partitioned into a direct product of fields of the various irreducible representations. This is evident from the fact that particles of the same type are indistinguishable. Along this line of thinking follows, that propagation of the type (9) actually affects all fields having a factor of the same sub-representation (co-oscillating). Thus, propagation takes place between the electron field (j=1/2) and the electromagnetic field which has a j=1/2 factor, or also directly between the electron and the “charged” quark fields, which is conventionally called “weak” interaction.

The concept of “weak” interaction is very useful for keeping track of topological symmetries that connect charged and uncharged leptons, or charged and uncharged quarks, etc, but the interaction is really (j=1/2)-propagation among fields at the same location.

At this point it must be mentioned that the concept of broken-charge quarks is rather unfortunate, because it hides the fact that there are two topologically different species (permutation-flavors, see above), only one of which propagates to the electromagnetic field. However, equivalently to the direct interaction among charged quarks and electrons, neutral quarks and and neutrinos also interact directly via the same spin-propagation term.

Gravitational interaction corresponds to propagation via the density-expression (13) which takes place among irreducible fields of any type.

Unit-Speed States

A pure single-field solution cannot be at rest, because (somewhat heuristically) this would be essentially a (wave-vector, *k*, equals zero)-solution. Such a solution would be invariant under space inversion which implies a zero mass-parameter *M* and consequently, a all-together vanishing solution. However, we must consider the limit cases in which the boost of the field goes to infinity while the solution parameter *M* grows concurrently such that, by appropriately re-scaling ψ, the limiting propagation equation stays finite with a solution parameter μ. From expression (3) we obtain for this limit process

$$(1-v^2)^{-\frac{1}{4}} b \rightarrow B \equiv \frac{b}{\sqrt{2}} e^b \quad , \tag{14}$$

such that we can write $M = B\mu$ and scale down ψ with B. Then the corresponding speed-one type solutions obey the following direction-specific (unit vector q in boost direction) propagation law

$$i(\vec{q}\,\vec{u})(\partial t - u_l \partial^l)\psi = \mu\psi \quad . \tag{15}$$

Such solutions cannot be brought to rest, but for them still exist anti-solutions, associated with the opposite sign of μ. Being bosonic (see below), these solutions are normalizable to their number of quanta.

We may ask which field types allow for such a procedure. States with non-zero magnetic quantum number oscillate at ever increasing rate with increasing boost and do not approach a well-defined limit. Thus, we are left with $m = 0$ states exclusively, which means that Fermionic fields, with j+k an odd multiple of ½, cannot have speed-one solutions.

Furthermore excluded are fields with j or k larger or equal to one, because their phase velocity, v_p, is below one. Thus, we are left with the the gravitational field (j=k=0, v_p = 1) and the electromagnetic one (j=k=½, v_p = $2/\sqrt{3}$). For gravitons we have just the single scalar component [7], for photons we have the electric (asymmetric, odd-parity) m=0 component and the magnetic (symmetric, even parity) m=0 component surviving.

As an aside, gravitons of the type proposed in General Relativity (supposedly spin 2, or possibly j=k=1) could not move at v=1, because they would have a phase velocity below one!

Static Fields

As we have seen above, static (pure) fields cannot exist. We must interpret, what we consider classically a static field, as oppositely-moving waves of a varying number of quanta against the identical number of anti-quanta. This is a field solution of zero mass (μ) but non-zero mass flow, comparable to the classical picture of oppositely moving electrons and positrons with zero net charge, but non-zero electrical current (density) [8]. One must keep in mind that quanta of the same type are indistinguishable such that evaluation of operator values must include cross-terms of oppositely moving quanta.

Qualitative Description of Solutions

Pure-Field Solutions

Gravitons and photons are the only pure-field solutions. The corresponding static (stationary) solutions can also exist purely in space, but they must eventually originate and end in emitting (absorbing) structures.

Two and Three Field Solutions

The simplest example of a compound structure is the neutrino. In the absence of baryonic fields it's leptonic field exclusively interacts (propagates) via the density term (13) with the scalar (gravitational) field. The system is quite analogous to a star with its gravitational field, except that the body of the star is now a single-quantum bump of the neutrino (lepton-)field. The interaction leads to to a partial reflection of the incoming anti-gravitons into outgoing gravitons, in accordance with the above described nature of the "static" gravitational field. In addition, the neutrino field self-propagates with the strong spin-term (9) but has no partner to interact via this term. We expect this to yield a very extended (low-density) bump due to small local-boost values.

The electron is a triple system consisting of a electron-field bump with associated gravitational and electromagnetic fields, the latter obeying the same same reflection mechanism, described above for the neutrino, but now involving interaction via spin propagation (9).

We also have a new dark-matter candidate consisting of a boson-quark bound by the gluon field to a anti boson-quark. This particle should have a energy in the range of a π-meson and could only decay via density-propagation (13) (gravitational interaction) into any energetically allowed particle-antiparticle pair (leptons, photons, gravitons). However, interaction via the gravitational mode must be so weak (low-b in (13)), that it has not even a place in current elementary particle physics.

Conclusions

We have presented a new way of looking at fundamental physics. Compared to the conventional view it has a couple of advantages. Firstly, it provides a unified treatment of all fields, including the scalar gravitational field. Secondly, it observes that in addition to spin-propagation (9), which does not affect the scalar field, there must be a second propagation mechanism, density-mediated (13), which affects fields of all types alike. Thirdly, it identifies propagation and interaction as the same process, thus doing away with the endless list of interaction constants, the number of which, incidentally, would increase without limit if one acknowledges the truly infinite number of irreducible field types. The Standard Model has decided to stay with a finite number of those which reduces the number of parameters to19.

Density-mediated propagation affects all field types alike which must be viewed as a manifestation of the equality of gravitational mass and inertial mass.

Being of fundamental nature the current proposal trails behind a innumerable chain of reinterpretations of established views in elementary particle physics and, in particular, in gravitational and cosmological physics. Clearly, it is out of the scope of this rather short note, to engage in this immense task.

However, we expect that many fundamental problems, which have bothered us (and many others) in the past [4, 6-14], will now find acceptable explanations.

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