Some Problems about CP Violation in the Neutral Kaon Decay

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Abstract – The CP violation concluded from the neutral Kaon decays in 1964 causes our curiosity about whether it is true or not. The experimentally observed particles are thought to be the short-lived K_1 and long-lived K_2 mesons as mentioned in 1964, not K_1^0 and K_2^0, the two eigenstates of CP. According to the conservation of CP, K_1^0 is responsible for the 2π decay and K_2^0 for the 3π. In the 1964 explanation, the short-lived K_1 was thought to decay totally and only the long-lived K_2 was survival after traveling 57 feet. Then the conclusion considered K_2 to be the superposition of both K_1^0 and K_2^0 states so it claimed to reveal the CP violation on K_2. However, the so-called CP violation doesn't take place on K_2^0 because the 2π decay events are indeed originated from K_1^0 and K_2^0 is responsible for the 3π-decay events. The observations of the 2π-decay events in K_2 indicate that it contains K_1^0 component. In our explanation, the experimentally observed particles shall be K_1^0 and K_2^0, not K_1 and K_2. As long as the K_1^0's energy is large enough, it can move a very long distance before decay. This situation is like muon passing through a much long distance to reach the Earth and then take place decay. We also demonstrate a case that the survival probability of the K_1^0 meson traveling 57 feet is about $2.41\text{x}10^{-3}$, close to the branching ratio about $2\text{x}10^{-3}$ of the two-body decay of the neutral K_2 meson exhibited in 1964 (Ref. 2). If so, the CP violation really doesn't take place in the neutral Kaon decay. Besides, the estimations of the K_1^0's and K_2^0's average lifetimes have to include the data in 1964 which may lead to significant corrections.

Keywords: Kaon, meson, CP violation, muon, pion

The K meson was discovered in 1947 and a total of four K mesons were found, namely K^0, $\bar{K}^0$, K^-, and K^+ [1]. In 1964, it was further found that the neutral kaon decay experienced a little deviation which is so-called CP violation [1-3]. $\bar{K}^0$ is the K^0's anti-particle and both of them can turn into each other through the second-order weak interaction [1,3], in which the process is

$$K^0 \leftrightarrow \bar{K}^0. \tag{1}$$

The original thought is that the Kaon decay seriously obeys CP symmetry. The eigenstates of CP are K_1^0 and K_2^0 states, which are the combinations of K^0 and $\bar{K}^0$ states [1], expressed as

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle), \tag{2}$$

and

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle). \tag{3}$$

K_1^0 and K_2^0 states have different decay processes, and the former decays to 2π and the latter to 3π because of CP-conservation [1,3]:

$$K_1^0 \to 2\pi. \tag{4}$$

and

$$K_2^0 \to 3\pi. \tag{5}$$

The exchange of K_1^0 and K_2^0 decay processes in Eqs. (4) and (5) is forbidden [3]. The experimentally observed particles are long-lived K_2 and short-lived K_1 mesons [1-3]. According to the previous data [3], scientists thought K_2's average lifetime much longer than K_1 so it shouldn't observer the 2π decay after a long distance as shown in Fig. 1 [2]. However, this assumption is based on the average lifetimes for both K_1 and K_2 which respectively are [3]

$$\tau_1 = (8.954 \pm 0.004) \times 10^{-11}\, sec. \tag{6}$$

and

$$\tau_2 = (5.116 \pm 0.021) \times 10^{-8}\, sec. \tag{7}$$

The above lifetimes are also suitable for K_1^0 and K_2^0. In 1964, the CP violation in the Kaon decay was summarized and the long-lived non-perfect eigenstate of CP was proposed as [1,3]

$$|K_2\rangle = A(\epsilon|K_1^0\rangle + |K_2^0\rangle) = \frac{1}{\sqrt{1+|\epsilon|^2}}(\epsilon|K_1^0\rangle + |K_2^0\rangle), \tag{8}$$

where $A\epsilon$ is the K_1^0's probability amplitude and $|\epsilon|^2$ is proportional to the ratio of K_1^0 received by the detector, and A is the normalized factor. However, such expression means the occupation of K_1^0 to be [3]

$$|\langle K_1^0|K_2\rangle|^2 = \frac{|\epsilon|^2}{1+|\epsilon|^2}. \tag{9}$$

It also means that after a long distance, the occupation of K_1^0 is still non-zero as the reveal in Ref. 2. The 2π-decay process is thought from K_1^0 [3] and the so-called CP violation was thought not to take place on K_2^0. In the quantum theory, the K_2-state in Eq. (8) is more like the superposition of two states, K_1^0 and K_2^0 states. Then this expression in Eq. (8) obviously links the truth that the 2π-decay events are original from K_1^0 and K_2^0 is responsible for the 3π-decay events. Why was the CP violation claimed on the K_2 meson? The reason is that they thought the totally disappeared K_1 meson after traveling 57 feet. Is it true or not? The kinetic reference tells us that the survival probability of a relativistic particle is not zero no matter how long distance it travels [5]. It might be very close to zero but not really zero, statistically speaking! Therefore, according to their discussions, the experimentally observed particles are actually K_1^0

and K_2^0, not the K_2. The reason is that the records in experiments are directly related to the charged pions [1-3], whose decay rules follow the conservation of CP. Eq. (8) clearly tells us that the 2π-decay events must come from K_1^0, and the 3π-decay events must come from K_2^0. One thing we have to do is to present a case that the survival probability of K_1^0 can be close to the experimental value and we will show it later.

Although it was claimed the non-perfect eigenstate of CP more than 50 years, we are still curiosity about the results of the experiments [2]. In Fig. 1, when the Kaon's moving distances is not too long or considering the initial time less than 10^{-10} *sec*. after neutral Kaon's birth, the number of K_1^0 and K_2^0 should be equal or close to each other. As time goes by, K_1^0 experiences decay quickly to 2π and many K_2^0 still exist until 10^{-8} *sec*. If we use a superposition state $K_2(z)$ to represent the mixture of K_1^0 and K_2^0, then

$$|K_2(z)\rangle = \frac{1}{\sqrt{1+|\epsilon(z)|^2}}(|K_2^0\rangle + \epsilon(z)|K_1^0\rangle), \tag{10}$$

where z is the distance in the neutral Kaon's moving direction. ϵ can also be the function of time t. The ratio of K_1^0 in this case is

$$|\langle K_1^0|K_2(z)\rangle|^2 = \frac{|\epsilon(z)|^2}{1+|\epsilon(z)|^2}. \tag{11}$$

It is obviously that this ratio is a function of the moving distance z and it is not a constant because of the rapid K_1^0 decay in z. K_1^0 decays gradually and rapidly in the real experiments so $|\epsilon|^2$ decreases in z or t. This expression tells us that K_1^0 still possibly exists even the moving distance is very long and the revolution time reaches 10^{-8} *sec*. after they were born. The Eq. (8) shown in 1964 [2] also agrees the survive of K_1^0 until 10^{-8} *sec*. It also reveals that K_1^0's lifetime can overlap K_2^0's as shown in Fig. 2, and the estimation of the K_1^0's average lifetime shall include the data in 1964 [2] which can correct the K_1^0's average lifetime in Eq. (6) meaningfully. In 1964, the experimental setup was fixed at 57 feet from the internal target to the end of the collimator [2], and the 2π-decay data were recorded in 45 of 22700 events. If we increase or decrease the collimator or the length in the experimental setup, the recorded data will be changed. Therefore, the conclusion of CP violation from the experimental results in 1964 [2] seems to have some fundamental questions: is it really the verification of CP violation? What if the 45 of 22700 events in 1964 were included in the statistics of K_1^0's average lifetime? If the CP violation doesn't occur on K_2^0 and K_2 is only a superposition state of K_1^0 and K_2^0, can we still conclude the CP violation occurred in these experiments?

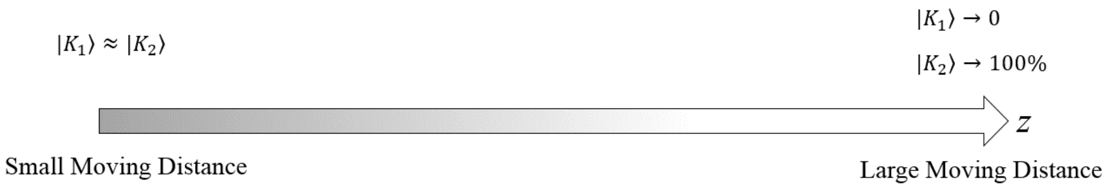

Figure 1. K_1^0 and K_2^0 decay as the moving distance increase and K_1^0 decreases much faster than K_2^0 due to the much shorter lifetime so theoretically speaking, K_1^0 will disappear and only K_2^0 will survive as long as the moving distance is long enough.

Except for the truth of no CP violation on K_1^0 and K_2^0, furthermore, the conclusion of CP violation might ignore two things. One is the identification of the K_1^0's average lifetime as mentioned previously. It is obviously that if the information about the average lifetime is incorrect, then this conclusion would be problematic. As we know, the lifetime and mass of K^- and K^+ are almost equal. It is recorded that this mass difference is only about (0.032±0.009) MeV/c^2 between them [3]. This difference is within the statistical error and is even much larger than the difference between K_1^0 and K_2^0, which is only 3.5x10^{-12} MeV/c^2 [1,3], less than 10^{-14} K_1^0's or K_2^0's mass. This very tiny mass difference is thought to be induced by the weak interaction. Both masses are much closer than the K^+ and K^- pair. Therefore, when we find the 2π-decay events in the long-lived K_2 state, it makes the K_1^0's statistical distribution in 1964 lack such data in time. When we add the data in 1964, then the average lifetime in Eq. (6) may be meaningfully different.

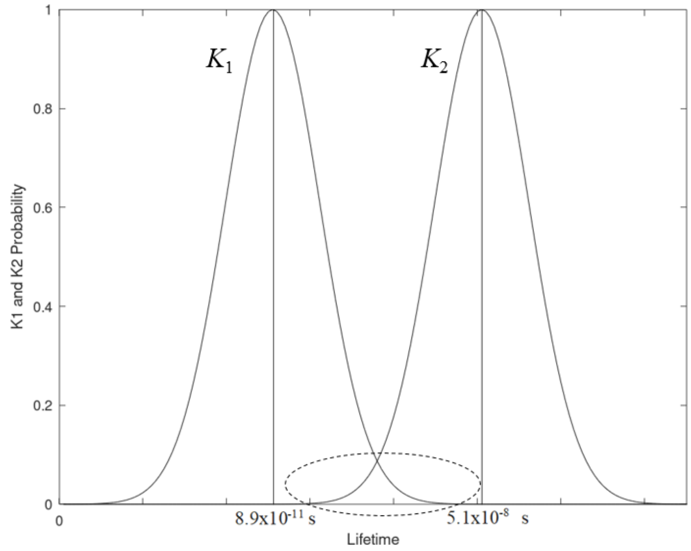

Figure 2. The demonstration of the overlap between K_1^0 and K_2^0 in the lifetime statistics. The region denoted by the dashed-line ellipse means that very few K_1^0 can survive close to the K_2^0's lifetime so K_1^0 possibly appears after 10^{-8} *sec*. It is one of the several possibilities that 45 events about the 2π decay were recorded in the total amount of 22700 in 1964 [1-3].

We may ask whether the large deviation of lifetime in Fig. 2 is possible? We have to remind it that the conclusions of the most particle-physics experiments are made from a lot of data so the statistical problems such as average, errors, and standard deviation naturally exist. The mass range of the neutral K_2 statistically from the experimental data is an explicit case as shown in Fig. 3 [2]. After the neutral K_2 decays in He gas, the experimental distribution from 5,211 events is approximately redrawn in Fig. 3, where the maximum number locates at 450 MeV/c^2, roughly 10% mass deviation from the identified neutral Kaon mass of 499.0 MeV/c^2 at that time [2]. The range of the mass distribution is about from 330.0 MeV/c^2 to 550.0 MeV/c^2, and the maximum mass deviation of the neutral K_2 is about 210 MeV/c^2. The deviation from the lowest mass to the identified neutral Kaon mass is about 33.0%. From the statistics of the experiments, it cannot precisely tell us the mass of the neutral kaon. Especially the Monte-Carlo simulations match the experimental results much well so this broad distribution in mass reflects the intrinsic uncertainty of the neutral K_2 experiments. Therefore, the experimentalists chose three mass ranges between 484.0 MeV and 514.0 MeV with cosθ>0.99999 to derive the average mass of the neutral K_2 meson at 499.1 MeV [2]. However, this determination uses the mass range deviates the main distribution around 450.0 MeV and disregards all other information about the mass distribution. It doesn't have strong reason but just meet the angular distribution of the K_1 meson. Furthermore, the two spectrometers for detecting charged pions only allows the entrance of small scattering-angle pions, not omni-directional detection in the experiments. Such maximum deviation about 33.0% in the mass distribution makes us think about more possibilities. We wonder about how these data convince us the existence of the CP violation in the neutral Kaon decays? Hence, the extension of the K_1^0's lifetime even the overlap between the K_1^0's and K_2^0's lifetimes in Fig. 2 becomes possible. The statement in the previous paragraphs is physically reasonable.

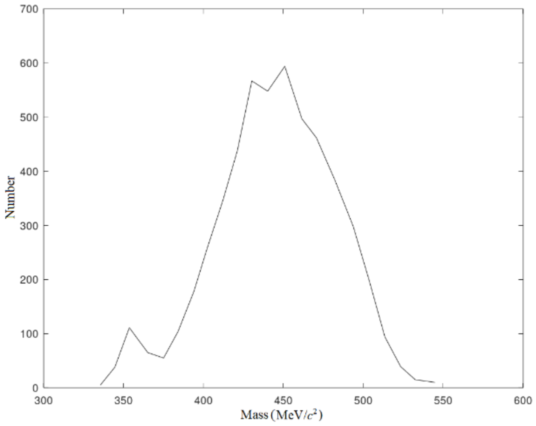

Figure 3. The redrawing data of the invariant mass m^* range for the neutral K_2 from the Fig. 2(a) in the Ref. 2 [2]. It is the experimental distribution about the neutral K_2 decays in He gas where each charged particle is assumed to have the mass of the charged pion [2].

The second thing is that we have to calculate whether K_1^0 can move longer than the predicted distance and survive with most K_2^0 as long as the K_1^0 has enough energy? The average lifetime is defined in the rest coordinate frame, and most of Kaons are close to the speed of light c so they can travel much longer than the distance only equal to c times lifetime τ. For example, the muon detection is a good demonstration. It is well known that most of muons in nature originally come from the high-energy cosmic ray. Due to collisions with molecules in the air, protons decay to muons and other elementary particles. Considering the muon decay originally produced by cosmic rays, the lifetime of muon is very short that it shall detect much few muons on the ground by prediction. However, the lifetime is the value in the rest coordinate frame, and therefore, in reality, more high-speed muons can reach ground after they are generated at very high places above the sea level. The muon's average lifetime is 2.197x10^{-6} *sec*. in the rest coordinate frame [1-4] and the relativistic effect makes them be able to move more than 15 km, not only 660 *m*, and more detections on the Earth. In this muon decay case, the Lorentz factor,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \tag{12}$$

is as large as 23 because of its velocity *v* very close to *c*. If some K_1^0 and K_2^0 particles have energy high enough, they can also travel through a much long distance like muon. In 1964's experiments, the neutral Kaon decays after the collimator took place and the distance from the internal target to the end of the collimator is 57 feet. We are curiosity about whether the data of the K_1^0 meson moving a very long distance is still survival and also included in its average lifetime as muon moves about 15 km long to reach the Earth?

Since the neutral Kaon and its anti-particle belong to a strong eigenstate with no definite lifetime [1,3], the two eigenstates of *CP* should have a relatively large lifetime deviation in statistics. The report in 1964 showed that the K_1's mean momentum *p* was 1100 MeV/*c* [2]. Using the relativistic principle,

$$E = \gamma m_0 c^2 = [(m_0 c^2)^2 + c^2 p^2]^{1/2}, \tag{13}$$

its velocity equals to 0.91099 *c* where m_0 is the K_1^0's or K_2^0's rest mass, 498 MeV/c^2 [1,3,4]. It exhibits another possibility that the K_1^0 can move a very long distance to decay to 2π so it happens in the long-lived K_2 state as shown in Eqs. (8) and (9). If the incident proton transfers almost all its energy to the neutral Kaon, γ can be as large as 60 so its average movement is about 1.60 *m* longer than the length of the collimator in 1964. Its occupancy is a function of *z* or *t* as shown in Eq. (10) and (11) and gradually and rapidly decays in *z* or *t*. Actually, the collimator is about 1.2 *m* in length, and it only needs 3/4 total energy of the incident proton, about 22.5 GeV in which γ is 45, to pass through the collimator.

On the other hand, we can calculate the survival probability of the K_1^0 meson after 57 feet, the distance from the internal target to the end of the collimator. The probability of a particle of a mass M and four momentum $(E, \boldsymbol{p})$ after traveling a distance x_0 is [5]

$$P(x_0) = e^{-Mx_0\Gamma/|\boldsymbol{p}|}, \tag{14}$$

where Γ is the inverse of the proper lifetime for this particle. When γ=45 or $M/|\boldsymbol{p}|\sim 1/\gamma c$, x_0=17.1 m, and Γ=1/(8.954x10^{-11} *sec*.) are substituted into Eq. (14), then we have

$$P(17.1\,m) \approx 7.18 \times 10^{-7}. \tag{15}$$

Furthermore, as long as we extend the lifetime of the K_1^0 meson to its 2.5 times, it can give

$$P(17.1\,m) \approx 3.49 \times 10^{-3}. \tag{16}$$

When we consider the decay channel of two charged pions, the branching ratio is 0.69 [3] and the probability becomes

$$P(17.1\,m|K_1^0 \rightarrow \pi^- + \pi^+) \approx 2.41 \times 10^{-3}, \tag{17}$$

close to the ratio the two-body to three-body decays of the neutral K_2 meson in 1964 [2]. Eq. (17) clearly represent the possibility of the neutral K_1^0 meson still survival after traveling 17.1 m and the probability is about 3.49x10^{-3} close to the branching ratio of the two-body decay about 2.0x10^{-3} announced in 1964. Here, we present another possibility that the branching ratio of the two-body decay is from the neutral K_1^0 meson, not the K_2^0 meson. Because the neutral Kaon decays in the He gas, the large energy lost is possible before decay due to the scattering with the He atoms. The charged pions generating from the neutral Kaon decays also possibly lose their energy by the same scattering mechanism. Therefore, the neutral Kaon mesons might have higher energy than the records and the mass distribution of the K_2 meson shown in Fig. 3 also reveals the possibility of the large deviation in mass. Therefore, the original neutral Kaon may have energy as high as 22.5 GeV and the lifetime of the K_1^0 meson is possibly 2.5 times as large as its average value. If so, the channel decay of the two charged pions can be from K_1^0 and the CP violation will not exist on K_2^0!

In conclusion, we re-explain the role of the long-lived K_2 state and think it more like the superposition of both K_1^0 and K_2^0 states. The experimentally observed particles shall be K_1^0 and K_2^0, not K_1 and K_2. As long as the K_1^0's energy is large enough, it can move a long distance before decay. In this framework of the new explanation, the 2π-decay events are related to K_1^0's occupation in this mixing state. It is based on the truth that the 2π decays originate from K_1^0 and no CP violation takes place on K_2^0. In particle physics, K_1^0 is responsible for the 2π-decay events and K_2^0 for the 3π-decay events. It makes us ask whether the CP violation in the neutral Kaon's decays is real? In fact, the occupation of K_1^0 depends on the moving distance z and evolution time t

so K_2 is much more like a time-dependent or displacement-dependent superposition state. Furthermore, the calculations of the average lifetimes have to include the data in 1964 which may be not counted and considered in the statistics. The superposition where the concept of the quantum theory is applied can reasonably explain the experimental results, and the K_1^0's and K_2^0's average lifetimes can have meaningful corrections by adding the data in 1964.

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