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Nature of Dark side of our Universe

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Abstract. In this paper I have made a great discovery ,I realized there was space-time before the Big Bang, the universe was shrinking and dark energy was negative before the big bang. Afterward, universe has been expanding after the Big Bang. Another prediction of this paper is that ordinary matter, which makes up 5 percent of the universe was made before the big bang, and dark matter has been made after the big bang. The reason for these predictions is that I have put the essence of dark energy into Wilmore energy and made the dark matter equivalent to the Hawking mass. Dark matter has a mathematically imaginary nature whose physical interpretation can be that it can be in and out of space-time. Another discovery is Unveiling of fine-tuning problem [1].

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Contents

1 Introduction

It was once thought that time is absolute quantity and gravity is just the force between two masses, but it was ended by a genius named Albert Einstein. He understood that time and space are not independently absolute, and that gravitational force is the force associated with space-time. In other words, his field equation explains the relationship between the curvature of space-time and matter, which means that gravity means that the curvature of space-time causes the motion of planets, stars, and galaxies in our universe.
The exact solution of the Einstein field equation was done by Friedmann – Lemaître – Robertson – Walker, called the FLRW model. This accelerated model and expansion of the universe generates a Cosmological constant model. This model predicts the acceleration of our universe, is compatible with observations. But there are some problems with this model, the most important problem is the fine-tuning problem [1]. The fine-tuning is the large difference between the relativistic quantum mechanics and the cosmological constant model for calculating the density of the vacuum. In this article I have solved this problem by noting that dark energy has a geometric origin .This is not just a hypothesis, I have proved in this article , But how did I prove this? I used the definition of scalar curvature and the equation of continuity and some creativity in mathematical equations[2]. And then I realized that Pressure to density ratio of whole universe is equal to$-\frac{2}{3} or w_{total} = -\frac{2}{3} = w_{\text{radiation}} + w_{DE} + w_{DM} = \frac{1}{3} - 1 + 0$ so this data($-\frac{2}{3}$) is Compatible with observations [2], so we can understand Dark Energy has geometrical origin. In my model I used Wilmore energy as Dark energy ,for solving fine-tuning problem I added Wilmore energy instead of cosmological constant into Einstein–Hilbert action. Geometry of relativistic quantum mechanics is Minkowski's geometry, So in this geometry there is no Wilmore energy because the space-time radius is constant(The space-time radius is the scale

factor). My model can predict that before the Big Bang the vacuum density was very high when the universe was in a infancy , so we can conclude that relativistic quantum mechanics can only measure the density of the vacuum before the Big Bang. The reason for this result is that the radius in Minokowski's space-time is constant. My model can predict that the vacuum density is very low in the current time. In my model, I used the mean curvature flow to define the radius of the universe. In this model the universe was initially shrinking before Big Bang, the universe shrank to a point where the radius of the universe reached zero then the Big Bang happened then the universe has been expanding . In my model, the dark matter is made after the big bang, while the ordinary material is made before the big bang. In this model, the dark matter is an imaginary number while the ordinary matter is a positive number. I used the Hawking mass formula to define dark matter and ordinary matter.
The physical interpretation of imaginary quantities can be that all of these quantities can go beyond space-time to travel in and out of space-time. Like the fourth dimension, or the time dimension, or the wave function in quantum mechanics. And in my model the mass of the dark matter and the radius of the universe after the Big Bang.
My model can predict the age of the universe, which is slightly different from the cosmological constant theory .
After applying Wilmore energy into Einstein–Hilbert action, we can predict density of our universe for each cosmic times. With this approach, quantum gravity is born. This model better than other model for expanding of our universe such as F(R) gravity and F(R,T) gravity [9],[10], because this model has more universality and has simpler scalar curvature than all Modified Theories.
Definition of Willmore energy: In The Willmore Energy and Willmore Conjecture book has been written:" Willmore energy is a type of energy that is studied in differential geometry ,physics and mechanics.It was first introduced by Sim'eon-Denis Poisson and Marie-Sophie Germain,independently,at the beginning of the nineteenth century,but its complete formalism was due to Thomas Willmore.In essence,this type of energy quantitatively measures the deviation of a surface from local sphericity.It is important to note that there is some ambiguity in the literature between the terms bending energy and Willmore energy,and different sources provide different definitions". [3]
After Generalization of general relativity from small scales till large scale, we can see real world.

2 Step1:Finding Pressure to density ratio of whole universe with Mathematical Method

For this target I use scalar curvature definition and continuity equation and I assume Pressure to density ratio of whole universe is constant ,because this assumption is most successful than other Models ,I have been meaning cosmological constant model. [4],[5]

$$w = \frac{P}{\rho}, \tag{2.1}$$

$$\frac{\dot{\rho}}{\rho} = -3\frac{\dot{\alpha}}{\alpha}(w+1). \tag{2.2}$$

Stress–energy tensor in FLRW model is defined by [5]:

$$\mathcal{T} = 3P - \rho = \rho(3w-1) \rightarrow \rho = \frac{\mathcal{T}}{(3w-1)}. \tag{2.3}$$

density is defined by [4]:

$$\rho = \rho_0 \alpha^{-3(w+1)}. \tag{2.4}$$

I apply (2.3) into (2.4) , so scale factor is defined by:

$$\alpha = (\frac{\mathcal{T}}{\rho_0(3w-1)})^{\frac{1}{-3(w+1)}}. \tag{2.5}$$

So derivative of scale factor is equal to :

$$\frac{d\alpha}{dt} = \dot{\alpha} = \frac{(3w-1)\dot{\rho}}{-3(w+1)}(\frac{\mathcal{T}}{\rho_0(3w-1)})^{\frac{3w+4}{-3(w+1)}}. \tag{2.6}$$

Then I apply (2.6), (2.5) into (2.2).

$$\frac{1}{\rho} = (3w-1)(\frac{\mathcal{T}}{\rho_0(3w-1)})^{\frac{-3(w+1)}{3(w+1)}}, \tag{2.7}$$

$$\rho = (\frac{\rho(3w-1)}{\rho_0(3w-1)^2}) \to \rho_0 = \frac{1}{3w-1}. \tag{2.8}$$

so density and pressure and scalar Stress–energy tensor s of whole universe can achieve like that:

$$\rho = \rho_0 \alpha^{-3(w+1)} = \frac{\alpha^{-3(w+1)}}{3w-1}, \tag{2.9}$$

$$P = w\rho_0 \alpha^{-3(w+1)} = \frac{w\alpha^{-3(w+1)}}{3w-1}, \tag{2.10}$$

$$\mathcal{T} = \frac{3w\alpha^{-3(w+1)}}{3w-1} - \frac{\alpha^{-3(w+1)}}{3w-1} = \alpha^{-3(w+1)}, \tag{2.11}$$

$$(\rho(3w-1))^{\frac{1}{-3(w+1)}} = \alpha. \tag{2.12}$$

Now we can achieve Pressure to density ratio of whole universe by scalar curvature definition in Roberson walker metric [4] and equation (2.12).

$$\mathcal{R} = -6(\frac{\ddot{\alpha}}{\alpha} + \frac{\dot{\alpha}^2}{\alpha^2} + \frac{k}{\alpha^2}). \tag{2.13}$$

So derivatives of scale factor based on equation (2.12) are equal to :

$$\dot{\alpha}^2 = \frac{(\rho(3w-1))^{\frac{(6w+8)}{-3(w+1)}}}{9(w+1)^2}\dot{\rho}^2, \tag{2.14}$$

$$\ddot{\alpha} = \frac{(3w+4)}{9(w+1)^2}(\rho(3w-1))^{\frac{6w+7}{-3(w+1)}}\dot{\rho}^2 + \frac{1}{-3(w+1)}(\rho(3w-1))^{\frac{(3w+4)}{-3(w+1)}}\ddot{\rho}. \tag{2.15}$$

So scalar curvature based on equation (2.13) is equal to :

$$\mathcal{R} = -6(\frac{(3w+4)}{9(w+1)^2}(\rho(3w-1))^{-2}\dot{\rho}^2 + \frac{1}{-3(w+1)}(\rho(3w-1))^{-1}\ddot{\rho} + \frac{(\rho(3w-1))^{-2}\dot{\rho}^2}{9(w+1)^2} + k(\rho(3w-1))^{\frac{2}{(w+1)}}. \tag{2.16}$$

Now we should replace for $\dot{\rho}$ and $\ddot{\rho}$ in equation (2.16).

$$\rho = \frac{\alpha^{-3(w+1)}}{3w-1}, \tag{2.17}$$

$$\dot{\rho} = \frac{-1}{3w-1}3(w+1)\alpha^{-3(w+1)-1}\dot{\alpha} \rightarrow \dot{\rho}^2 = \frac{9(w+1)^2\alpha^{-6(w+1)-2}\dot{\alpha}^2}{(3w-1)^2}, \tag{2.18}$$

$$\ddot{\rho} = \frac{1}{3w-1}3(w+1)((3w+4)\alpha^{-3(w+1)-2}\dot{\alpha}^2 - \frac{1}{3w-1}3(w+1)\alpha^{-3(w+1)-1}\ddot{\alpha}. \tag{2.19}$$

Now I apply (2.17), (2.18), (2.19) into (2.16).

$$\mathcal{R} = -6(\frac{(3w+4)}{(3w-1)^2}\frac{\dot{\alpha}^2}{\alpha^2} - \frac{1}{3w-1}((3w+4)\frac{\dot{\alpha}^2}{\alpha^2} + \frac{1}{3w-1}\frac{\ddot{\alpha}}{\alpha} + \frac{1}{(3w-1)^2}\frac{\dot{\alpha}^2}{\alpha^2} + k\alpha^{-2}). \tag{2.20}$$

we can consider equation (2.20) and equation (2.13) as same equation, so we can make equality :

$$-6((\frac{3w+4}{(3w-1)^2} - \frac{3w+4}{3w-1} + \frac{1}{(3w-1)^2})\frac{\dot{\alpha}^2}{\alpha^2} + \frac{1}{3w-1}\frac{\ddot{\alpha}}{\alpha} + k\alpha^{-2}) = -6(\frac{\ddot{\alpha}}{\alpha} + \frac{\dot{\alpha}^2}{\alpha^2} + k\alpha^{-2}). \tag{2.21}$$

Coefficient of $\frac{\dot{\alpha}^2}{\alpha^2}$ is equal to 1 so:

$$\frac{(3w+4)}{(3w-1)^2} + \frac{1}{(3w-1)^2} - \frac{(3w+4)}{3w-1} = \frac{-9w^2-6w+9}{(3w-1)^2} = 1, \tag{2.22}$$

$$-9w^2 - 6w + 9 = 9w^2 - 6w + 1, \tag{2.23}$$

$$-18w^2 = -8, \tag{2.24}$$

$$w^2 = \frac{8}{18} = \frac{4}{9} \rightarrow w = \pm\frac{2}{3}. \tag{2.25}$$

So Pressure to density ratio of whole universe was achieved , but we can accept $w = -\frac{2}{3}$ because it is Compatible with observations[2]. I mean :

$$w_{total} = -\frac{2}{3} = w_{\text{radiation}} + w_{DE} + w_{DM} = \frac{1}{3} - 1 + 0. \tag{2.26}$$

So existence of dark energy has not relate to any physical interpretation because, I achieved Pressure to density ratio of whole universe without matter and energy interpretation , so all we have is math and geometry therefor, dark Energy has geometrical origin.

3 Step2: defining radius of universe as function of time by mean curvature flow and finding exact amount of age of universe with Mathematical Method

What is the mean curvature ? Mean curvature flow is an example of a geometric flow of hypersurfaces in a Riemannian manifold. Spheres and cylinders are the easiest and actually some of the few nontrivial explicitly computable examples of mean curvature flows[6]. Let me consider a sphere of radius $\alpha(t)$ which is scale factor in Roberson walker metric , In mean curvature, radius is changing by time as same as our universe that is expanding by scale factor , I would like to say our universe works like a mean curvature flow , So I can conclude manifold of our universe is mean curvature , it is just not Similarity , we can calculate age of universe that is Compatible with observations . radius of mean curvature is defined [6]:

$$\alpha(t) = \sqrt{\alpha_0^2 - 2nt}, \tag{3.1}$$

α_0^2 is initial radius , we can calculate by wavelength [1] , and n is dimensions(n=4) ,

$$\alpha(t) = \frac{1}{1+z}, \tag{3.2}$$

$$\alpha_0 = 1. \tag{3.3}$$

Hubble law [1], it can be write :

$$H = \frac{\dot{\alpha}}{\alpha} = \frac{\frac{-n}{\sqrt{1-2nt}}}{\sqrt{1-2nt}} = \frac{-n}{1-2nt}. \tag{3.4}$$

we should put Hubble constant into equation (3.4). that is based on observations .Then we can find age of universe . Hubble constant in the current time : $H_0 = 1.18 \times 10^{-61}$.[11]
so age universe is equal to :

$$t = \frac{1}{16 \times 1.18 \times 10^{-61}} + \frac{1}{8}. \tag{3.5}$$

it is Compatible with observations ,it has some a little difference between this model and cosmological constant model [4].

4 Step3: interpretation of scale factor or radius of universe

$$\alpha(t) = \sqrt{1-2nt}. \tag{4.1}$$

First of all it shows us Space time has existence before big bang because when radius is equal to zero time has existence

$$if : \alpha(t) = 0 \rightarrow t = \frac{1}{2n}. \tag{4.2}$$

It means that at beginning of time until $t = \frac{1}{2n}$ our universe had shrunk then big bang was started , after big bang radius of universe getting imaginary quantity because our universe after big bang has no center. But before big bang our universe has center , the point is $t = \frac{1}{4n}$.

5 Step4: Introducing new candidate for dark matter by Hawking mass and dark energy by Wilmore energy

Willmore energy definition. [7] :

$$\mathcal{W}(\Sigma) = \frac{1}{4}\int \mathcal{H}^2 d\Sigma. \tag{5.1}$$

$\mathcal{H}$ is mean curvature , scalar curvature in this model equal to:

$$\mathcal{R} = -6(\frac{\ddot{\alpha}}{\alpha} + \frac{\dot{\alpha}^2}{\alpha^2} + \frac{k}{\alpha^2}) = -6(\frac{-n^2}{(1-2nt)^2} + \frac{n^2}{(1-2nt)^2} + \frac{k}{1-2nt}) = \frac{-6k}{1-2nt}, \tag{5.2}$$

$$\mathcal{R} = \frac{-6k}{1-2nt} = \frac{-6k}{\alpha^2}. \tag{5.3}$$

As I told dark energy has geometrical origin now I want to use wilmore energy as dark energy, because wilmore energy has expanding nature[3]. mean curvature definition[8]:

$$\mathcal{H} = \frac{n}{\alpha(t)}. \tag{5.4}$$

n is [8]:

$$n = -\dot{\alpha}(t)\alpha(t) = 2. \tag{5.5}$$

With applying Eq (5.4) wilmore energy is achieved :

$$\mathcal{W}(\Sigma) = \frac{1}{4}\int_{\Sigma} (\frac{n^2}{\alpha^2(t)})d\Sigma = \frac{1}{4}\int_{\Sigma} (\frac{n^2}{\alpha^2(t)}8\pi\alpha d\alpha, \tag{5.6}$$

$$\mathcal{W}(\Sigma) = \frac{1}{4}\int_{\Sigma} \frac{8\pi n^2}{\alpha}d\alpha = 2\pi n^2 \ln\alpha. \tag{5.7}$$

Eq (5.7) is dark energy equation .Constant of integral in Eq (5.7) is zero because when radius is equal to zero then dark energy is equal to zero. Now lets go to Hawking mass : Hawking mass is all mass in our universe that is defined by:[7]

$$m(\Sigma) = \frac{\Sigma^{\frac{1}{2}}}{(16\pi)^{\frac{3}{2}}}(16\pi - \int_{\Sigma}\mathcal{H}^2 d\Sigma = \frac{\Sigma^{\frac{1}{2}}}{(16\pi)^{\frac{3}{2}}}(16\pi - 4\mathcal{W}(\Sigma)). \tag{5.8}$$

that $(\Sigma = 4\pi\alpha^2)$ With applying Eq (4.1) and (5.7), Hawking mass is achieved :

$$m(\Sigma) = \frac{1}{2}\alpha - 4\alpha\ln\alpha = \frac{1}{2}\sqrt{(1-4t)} - \sqrt{(1-4t)}\ln\sqrt{(1-4t)} \tag{5.9}$$

As you see Hawking mass in the current time is imaginary , it means that all masses that are created in universe in the current time , they are imaginary , and if I put early stages times of universe(before big bang) then, I will achieve positive matter . It means that ordinary matter was created before big bang and dark matter has been creating after big bang.

6 Step5: Unveiling of fine-tuning problem

In fine-tuning problem Vacuum density is main topic .in this model I want to calculate Vacuum density so:

$$m_{vacuum}(\Sigma) = 0, \tag{6.1}$$

$$\frac{\Sigma^{\frac{1}{2}}}{(16\pi)^{\frac{3}{2}}}(16\pi - 4\mathcal{W}_{vacuum}(\Sigma)) = 0, \tag{6.2}$$

$$\mathcal{W}_{vacuum}(\Sigma) = 4\pi. \tag{6.3}$$

V is 4 dimension Volume of a ball of radius α

$$\rho_{vacuum} = \frac{\mathcal{W}_{vacuum}(\Sigma)}{V} = \frac{4\pi}{\frac{\pi^2}{2}\alpha^4} = \frac{8}{\pi\alpha^4} = \frac{8}{\pi(1-2nt)^2} = \frac{8}{\pi(1-4t)^2}. \tag{6.4}$$

Result of Eq (6.4) is Compatible with observations , If you put current time you will understand , but if I put before big bang time I will achieve Relativistic Quantum mechanics prediction for vacuum density . It means that radius of universe has played main roll. geometry of Relativistic Quantum mechanics is Minkowski's spacetime and radius is constant therefor, relativistic Quantum mechanics just can predict vacuum density before big bang.

7 Step6: Adding Wilmore energy to Einstein–Hilbert action to update Friedmann equations for describing quantum gravity

$$S = \frac{1}{16\pi}\int \sqrt{-g}(\mathcal{R} + 2k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma))dx^4 + \int \sqrt{-g}\mathcal{L}_m dx^4. \tag{7.1}$$

$k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma)$ is dark energy term.
τ is proper time in action above. Now I should take variation from action above.but first of all I should know How can take variation from Wilmore energy : based on Eq (5.7) and Eq (5.3) we have

$$\mathcal{R} = \frac{-6k}{\alpha^2}, \mathcal{W}(\Sigma) = 2\pi n^2 \ln \alpha. \tag{7.2}$$

Since taking variation is partial , so variation of Wilmore energy:

$$\delta\mathcal{W}(\Sigma) \approx 0. \tag{7.3}$$

Dimension of wilmore energy can explain by 2 sides :

$$\begin{aligned} &k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma) = \left[L^{-7}\right]\left[T^2\right]\left[L^5\right]\left[T^{-2}\right] = \left[L^{-2}\right] \\ &.k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma) = \left[L^{5}\right]\left[L^{-7}\right]\left[T^2\right]\left[T^{-2}\right] = \left[L^{-2}\right]. \end{aligned} \tag{7.4}$$

First side k is depending on whether the shape of the universe, second side k is the spatial curvature .
As we know Stress–energy tensor is defined by [11] :

$$\mathcal{T}_{\alpha\beta} = \frac{-2}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g^{\alpha\beta}}. \tag{7.5}$$

with Eq (7.1) we can take variation from whole action :

$$0 = \frac{1}{16\pi}(-\frac{1}{2}g_{\alpha\beta}\mathcal{R} + \mathcal{R}_{\alpha\beta} - g_{\alpha\beta}k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma) + \frac{7}{2}\mathcal{R}^{\frac{5}{2}}\mathcal{R}_{\alpha\beta}2k^{-\frac{5}{2}}\tau^2\mathcal{W}(\Sigma)) - \frac{1}{2}\mathcal{T}_{\alpha\beta}. \tag{7.6}$$

There for action becomes to New Einstein field equation :

$$(1 + 7\mathcal{R}^{\frac{5}{2}}k^{-\frac{5}{2}}\tau^2\mathcal{W}(\Sigma))\mathcal{R}_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}\mathcal{R} - g_{\alpha\beta}k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma) = 8\pi\mathcal{T}_{\alpha\beta}. \tag{7.7}$$

The interpretation of this equation is that gravity is a reaction to the expansion of the universe, so that Wilmore energy causes mass and energy bend space-time.

$$\begin{aligned} &\gamma(\mathcal{R}) = 1 + 7\mathcal{R}^{\frac{5}{2}}k^{-\frac{5}{2}}\tau^2\mathcal{W}(\Sigma) \\ &.\gamma(\mathcal{R})\mathcal{R}_{\alpha\beta} - \tfrac{1}{2}g_{\alpha\beta}\mathcal{R} - g_{\alpha\beta}k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma) = 8\pi\mathcal{T}_{\alpha\beta}. \end{aligned} \tag{7.8}$$

New Einstein field equation has quantum gravity Properties . One of clues is Coefficient of $\mathcal{R}_{\alpha\beta}$ and an other is Wilmore energy . This equation is true even for when our universe was

very small.
Now we can achieve New Friedmann equations:

$$(1 + 7\mathcal{R}^{\frac{5}{2}}k^{-\frac{5}{2}}\tau^2\mathcal{W}(\Sigma))\mathcal{R}_{00} - \frac{1}{2}g_{00}\mathcal{R} - g_{00}k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma) = 8\pi\mathcal{T}_{00}, \tag{7.9}$$

$$(1 + 7(\frac{-6k}{\alpha^2})^{\frac{5}{2}}k^{-\frac{5}{2}}\tau^2\mathcal{W}(\Sigma))\frac{\ddot{\alpha}}{\alpha} = -\frac{8\pi\rho}{3} + \frac{k}{\alpha^2} - \frac{1}{3}k^{-\frac{5}{2}}(\frac{-6k}{\alpha^2})^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma). \tag{7.10}$$

Property of radius of universe in mean curvature :

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{\dot{\alpha}^2}{\alpha^2}. \tag{7.11}$$

So second equation of Friedmann :

$$(1 + 7(\frac{-6k}{\alpha^2})^{\frac{5}{2}}k^{-\frac{5}{2}}\tau^2\mathcal{W}(\Sigma))\frac{\dot{\alpha}^2}{\alpha^2} = \frac{8\pi\rho}{3} - \frac{k}{\alpha^2} + \frac{1}{3}k^{-\frac{5}{2}}(\frac{-6k}{\alpha^2})^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma). \tag{7.12}$$

Eq (7.10) and Eq (7.12) are new Friedmann equations.

8 Conclusion

So space time has been existing before big bang, in this period(before big bang) ordinary matter was created and space time was shrinking ,I mean dark energy was negative before big bang, after big bang every thing has been changed ,positive dark energy has been appearing ,and dark matter is that imaginary matter has been creating , in this paper I used Hawking mass instead of dark matter and Wilmore energy instead of dark energy, Wilmore energy and Hawking mass have connected to each others by equation . It means that dark energy and dark matter have same origin.But my article remain some questions , for example :1- why dark matter and dark energy are connected ? .2-why space time before big bang has existence ?3-why imaginary quantities like dark matter and radius of universe after big bang can exist inside and outside of universe ?

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