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A new appraoch to proof The Riemann hypothesis using new operator

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Abstract: In this note we present a new approach to proof the Riemann hypothesis one of the most important open problem in pure mathematics using a new operator derived from unitary operator groups acts on Riemann-Siegal function and it uses partition function for Hamiltonian operator , The interest idea is to compute the compositional inverse of Riemann zeta function at $s = -\frac{1}{2}$ such as we show that:$\zeta^{-1}(-\frac{1}{2}) = \zeta(\frac{1}{2} + i\beta) = 0$ for some $\beta > 0$

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1 | INTRODUCTION:

In 1859, the German mathematician Georg Friedrich Bernhard Riemann proposed a hypothesis [Riemann, 1859, pp. 1–9] about prime numbers that would later bear his name, the Riemann hypothesis. The prime numbers do not appear to follow any obvious pattern. However, Riemann observed a close relation between the behavior of an elaborate function, the so-called Riemann zeta function, and the frequency of prime numbers. Riemann calculated a few zeros of this function, and quickly noted that the interesting ones lay on a certain vertical straight line in the complex plane. Riemann subsequently conjectured that all non-trivial zeros lie on this line. Today, over 1013 zeros are known [Gourdon, 2004, pp. 19–25], and all of them agree with the hypothesis. The Riemann hypothesis has important implications for the distribution of prime numbers and is strongly connected to the prime number theorem, which gives a good approximation of the density of prime numbers. In particular, the Riemann hypothesis gives a precise answer to how good the approximation given by the prime number theorem is. In a sense, the Riemann hypothesis conveys the idea that the prime numbers are distributed as regularly as possible. This regularity would tell a great deal about the average behavior of prime numbers in the long run.

Riemann hypothesis was attacked by many attempts of many mathematicians but no attempt of proof accepted yet , Mayeb the only area in science (physics) is able to describe the Riemann hypothesis in a way which encourage people to work every day to proof RH is Quantum mechanic ,The latter attack Riemann Hypothesis by Unitary operator Groups precisly Hamiltonian operator ,There are many interesting papers discussed Riemann Hypothesis using Hamiltonian operator ,The distribution of energy level is totally identitic to prime numbers distributions then Prime numbers present energy level of atom .The main Goal of this paper is to compute the compositional inverse of Riemann zeta function for negative half integer getting a new operator acts on Riemann Siegal formula .

Main result:

$$\zeta^{-1}(-\frac{1}{2}) = \zeta(\frac{1}{2} + i\beta) = 0$$

, for some real $\beta > 0$

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Before giving a proof to the given main result we should show that

Lemma 1

$$\zeta(0) = -\frac{1}{2}$$

Proof By the functional equation of the zeta function:

$$\zeta(s) = 2^s \pi^{s-1} \sin \frac{\pi s}{2} \Gamma(1-s)\zeta(1-s)$$

We now use the fact that the zeta function has a simple pole at $s = 1$ with residue 1 (this is, imo, one of the most beautiful elementary things that can be proved about this wonderful function), and this means that

$$\lim_{s \to 1} (s-1)\zeta(s) = 1$$

[1]For the definition of Riemann zeta function for positive s is not requred here because we are interested only for negative strip ,may readers check out in the web to get many definitions of Riemann zeta function

Now, using the functional equation for the Gamma Function $s\Gamma(s) = \Gamma(s+1)$, we multiply the functional equation for zeta by $(1-s)$ and then pass to the limit when $s \to 1$:

$$(1-s)\zeta(s) = 2^s\pi^{s-1}\sin\frac{\pi s}{2}\left[(1-s)\Gamma(1-s)\right]\zeta(1-s) \implies$$

$$\lim_{s\to 1}(1-s)\zeta(s) = -1 = \lim_{s\to 1}\ \Gamma(2-s)2^s\pi^{s-1}\zeta(1-s) = 1{\cdot}2\zeta(0) \implies$$

$$\zeta(0) = -\frac{1}{2}$$

Now we pass to the compositional inverse of Riemann zeta function in the region $|s| < \frac{1}{2}$ precisly at $s = -\frac{1}{2}$ using countor integral (Cauchy integral), The inverse of Riemann zeta function at positive values is approximated by polynomial as well in this paper [01] , We may show that the compositional inverse of Riemann zeta function exist in the neigheberhood of $s = 0$ such that it is locally injective , just to check this interesting paper [05] entitled " Self-intersections of the Riemann zeta function on the critical line" (Local injectivity, proposition 3) it show that the curve $\zeta(\frac{1}{2}+it)$ is locally injective implies that it has a compositional inverse at $s=-\frac{1}{2}$.

Lemma 2 *Let U be the image by ζ of $|s|<1/2$ and for $z\in U$*

$$F(z)=\frac{1}{2i\pi}\int_{|s|=1/2}\frac{\zeta'(s)s}{\zeta(s)-z}ds$$

Proof for $z\in U,\exists|a|<1/2, z=\zeta(a)$ then for $|s|\leq 1/2$, $s\mapsto\frac{\zeta'(s)s}{z-\zeta(s)}$ has a unique simple pole at $s=a$ of residue a so that $F(z)=Res(\frac{\zeta'(s)s}{\zeta(s)-z},a)=a.$

Since $\zeta(0)=-1/2$ then $F(-1/2)=0$.

When extending this local branch F of ζ^{-1} by analytic continuation it will have a branch point at each $z=\zeta(a),\zeta'(a)=0$.

ζ isn't injective and for any $a\neq 0$ there are infinitely many s such that $\zeta(s)=a$, infinitely many of them are on the strip $\Re(s)\in(\sigma-\epsilon,\sigma+\epsilon)$ for any $\sigma>1$ such that $1/\zeta(\sigma)<|a|<\zeta(\sigma)$.

We are ready now to give a simple proof for our main result which it states that the compositional inverse of Riemann zeta function at $s=-\frac{1}{2}$ is really its image at $s=\frac{1}{2}+i\beta$. The question we may ask here is : Does there exists in Quantum mechanic an operator satisfies: $A^{-1}(t) = A(-t + i\beta)$, β is a real number non-null? we say yes which it follow this

Proof Assume we have unitary operator[03] : $W(t) = \exp(itH)$, so

$$W(t)W(-t) = \mathbb{1},$$

we could define

$$U(t) \equiv W(t)\ e^{-\beta H/2},$$

so that

$$U(t)U(-t + i\beta) = W(t)\ e^{-\beta H/2} W(-t + i\beta)\ e^{-\beta H/2} = \mathbb{1},$$

Such that H is the standard Hamiltonian operator

Note: The compositional inverse is the same as multiplication inverse in Quantum mechanic

Then we are showed that there exist an operator $U(t) \equiv W(t)\ e^{-\beta H/2}$, for which $\zeta^{-1}(-\frac{1}{2}) = \zeta(\frac{1}{2} + i\beta) = 0$ for $\beta > 0$, Then since The Riemann zeta function is injective at $s = -\frac{1}{2}$ then Riemann hypothesis Holds , Now the discussion of that operator is montioned in this paper [02] such that Authors showed that :$e^{-\beta H/2}$ is the partition function with $H = p^2 + V(x)$, in the phase space (x, p) ,moment and position ,That partition function was considered to be The Riemann Siegal formula as $Z(\beta)$ function by the computation of it traces , (See page 10 , 4.Conclusions and final remarks)

Finally : We conclude from

$$U(t) \equiv W(t)\ \ e^{-\beta H/2}$$

that U is real because $e^{-\beta H/2}$ is real as a reason H is Hermitian operaot and $W(t) = \exp(itH)$ is also real then product of two real is real implies the compositional inverse of U which act on Riemann Siegal formual is real implies Riemann hypothesis is hold

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