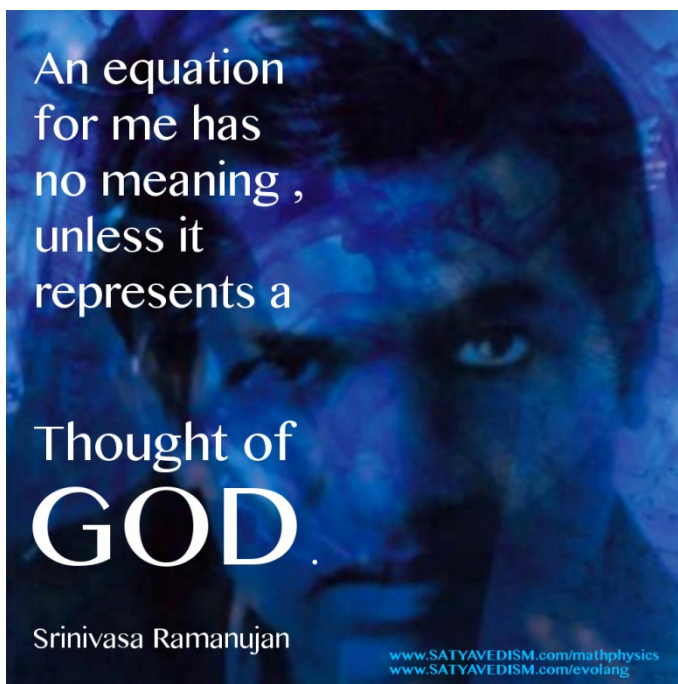


# Ramanujan approximations to $\pi$ , invariant class and other expressions: further mathematical connections with some sectors of Particle Physics, String Theory and Physics of Black Holes (entropy)

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## Abstract

In this research paper, we have obtained further mathematical connections with some sectors of Particle Physics, String Theory and Physics of Black Holes (entropy) and the Ramanujan approximation to  $\pi$ , invariant class and other expressions extracted from some pages of original manuscript



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From:

<https://www.cittanuova.it/ramanujanhardy-e-il-piacere-di-scoprire/>

*One is completely astonished in front of the notebooks of Srinivasa Ramanujan, seeing "the beauty and the singularity of his results" (still today part of the contents of the notebooks is not completely understood). Ramanujan developed a theory of reality around Zero (representing absolute Reality) and Infinity (the multiple manifestations of that reality): their mathematical product represented all the numbers, each of which corresponded to individual acts of creation. for him "the numbers and their mathematical relationships let us understand how everything was in harmony in the universe"*

From:

S. Ramanujan "Modular equations and approximations to  $\pi$ " - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have the following modular equation:

$$G_{441}^2 = \left( \frac{\sqrt{3} + \sqrt{7}}{2} \right) (2 + \sqrt{3})^{\frac{1}{3}} \left\{ \frac{2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}}}{2} \right\} \left\{ \frac{\sqrt{3 + \sqrt{7}} + (6\sqrt{7})^{\frac{1}{4}}}{\sqrt{3 + \sqrt{7}} - (6\sqrt{7})^{\frac{1}{4}}} \right\}$$

$$\left( \frac{(\sqrt{3} + \sqrt{7})}{2} \right) (2 + \sqrt{3})^{\frac{1}{3}} \left( \frac{(2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}})}{2} \right) \left[ \frac{(\sqrt{3 + \sqrt{7}} + ((6\sqrt{7})^{\frac{1}{4}}))}{(\sqrt{3 + \sqrt{7}} - ((6\sqrt{7})^{\frac{1}{4}}))} \right]$$

Input:

$$\left( \frac{1}{2} (\sqrt{3} + \sqrt{7}) \right) \sqrt[3]{2 + \sqrt{3}} \left( \left( \frac{1}{2} \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right) \right) \times \frac{\sqrt{3 + \sqrt{7}} + \sqrt[4]{6\sqrt{7}}}{\sqrt{3 + \sqrt{7}} - \sqrt[4]{6\sqrt{7}}} \right)$$

Result:

$$\frac{\sqrt[3]{2 + \sqrt{3}} (\sqrt{3} + \sqrt{7}) \left( \sqrt[4]{6\sqrt{7}} + \sqrt{3 + \sqrt{7}} \right) \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right)}{4 \left( \sqrt{3 + \sqrt{7}} - \sqrt[4]{6\sqrt{7}} \right)}$$

Decimal approximation:

172.6408721781608642202334242951081787952105395862630665234...

The result is 172,640... that multiplied by 10 is equal to 1726,40, result very near to the mass of meson  $f_0(1710)$  candidate glueball

Alternate forms:

More

$$\text{root of } x^{16} - 5145536x^{15} - 14823304x^{14} - 401380672x^{13} + 1371000092x^{12} - 1789864384x^{11} + 5730236232x^{10} - 14643739968x^9 + 20866389574x^8 - 14643739968x^7 + 5730236232x^6 - 1789864384x^5 + 1371000092x^4 - 401380672x^3 - 14823304x^2 - 5145536x + 1 \text{ near } x = 5.14554 \times 10^6 \text{ } ^{(1/3)}$$

$$\begin{aligned} & \left( (\sqrt{3} + \sqrt{7}) \left( \text{root of } x^4 - 6x^2 + 2 \text{ near } x = 2.37608 + \sqrt[4]{6} \sqrt[8]{7} \right) \right. \\ & \quad \left. \text{root of } x^6 - 4x^3 + 1 \text{ near } x = 1.55113 \right. \\ & \quad \left( \text{root of } x^4 - 14x^2 + 112 \text{ near } x = 2.96505 - 1.33847i + \right. \\ & \quad \left. \text{root of } x^4 - 14x^2 + 112 \text{ near } x = 2.96505 + 1.33847i + \right. \\ & \quad \left. 2\sqrt{2} + \sqrt{14} \right) \Big/ \\ & \left( 4\sqrt{2} \left( \text{root of } x^4 - 6x^2 + 2 \text{ near } x = 2.37608 - \sqrt[4]{6} \sqrt[8]{7} \right) \right) \\ & \frac{\sqrt[3]{2+\sqrt{3}} (2+\sqrt{7})(\sqrt{3}+\sqrt{7}) \left( \sqrt[4]{6} \sqrt[8]{7} + \sqrt{3+\sqrt{7}} \right)}{4 \left( \sqrt[4]{6} \sqrt[8]{7} - \sqrt{3+\sqrt{7}} \right)} \\ & \frac{\sqrt[3]{2+\sqrt{3}} (\sqrt{3}+\sqrt{7}) \sqrt{7+4\sqrt{7}} \left( \sqrt[4]{6} \sqrt[8]{7} + \sqrt{3+\sqrt{7}} \right)}{4 \left( \sqrt[4]{6} \sqrt[8]{7} - \sqrt{3+\sqrt{7}} \right)} \end{aligned}$$

Minimal polynomial:

$$\begin{aligned} & x^{48} - 5145536x^{45} - 14823304x^{42} - 401380672x^{39} + 1371000092x^{36} - \\ & 1789864384x^{33} + 5730236232x^{30} - 14643739968x^{27} + \\ & 20866389574x^{24} - 14643739968x^{21} + 5730236232x^{18} - 1789864384x^{15} + \\ & 1371000092x^{12} - 401380672x^9 - 14823304x^6 - 5145536x^3 + 1 \end{aligned}$$

Continued fraction:

[172; 1, 1, 1, 3, 1, 1, 1, 3, 1, 1, 1, 4, 17, 12, 2, 1, 1, 1, 2, 66, 3, 2, 1, 11, 19, 1, 1, 2, ...]

$$8 * \left[ \left( \frac{(\sqrt{3} + \sqrt{7})}{2} \right) (2 + \sqrt{3})^{1/3} \right] \left[ \frac{(2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}})}{2} \right] \left[ \frac{(\sqrt{3 + \sqrt{7}} + ((6\sqrt{7})^{1/4}))}{(\sqrt{3 + \sqrt{7}} - ((6\sqrt{7})^{1/4}))} \right]$$

Input:

$$8 \left( \frac{1}{2} (\sqrt{3} + \sqrt{7}) \right) \sqrt[3]{2 + \sqrt{3}} \left( \frac{1}{2} \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right) \right) \times \frac{\sqrt{3 + \sqrt{7}} + \sqrt[4]{6\sqrt{7}}}{\sqrt{3 + \sqrt{7}} - \sqrt[4]{6\sqrt{7}}}$$

Result:

$$\frac{2\sqrt[3]{2+\sqrt{3}}(\sqrt{3}+\sqrt{7})\left(\sqrt[4]{6}\sqrt[8]{7}+\sqrt{3+\sqrt{7}}\right)\left(2+\sqrt{7}+\sqrt{7+4\sqrt{7}}\right)}{\sqrt{3+\sqrt{7}}-\sqrt[4]{6}\sqrt[8]{7}}$$

Decimal approximation:

1381.126977425286913761867394360865430361684316690104532187...

Alternate forms:

$$\left(\sqrt{2}\left(\sqrt{3}+\sqrt{7}\right)\left(\text{root of } x^4-6x^2+2 \text{ near } x=2.37608\right)+\sqrt[4]{6}\sqrt[8]{7}\right) \frac{\left(\text{root of } x^6-4x^3+1 \text{ near } x=1.55113\right)\left(\text{root of } x^4-14x^2+112 \text{ near } x=2.96505-1.33847i\right)+\left(\text{root of } x^4-14x^2+112 \text{ near } x=2.96505+1.33847i\right)+2\sqrt{2}+\sqrt{14}}{\left(\text{root of } x^4-6x^2+2 \text{ near } x=2.37608\right)-\sqrt[4]{6}\sqrt[8]{7}}$$

root of

$$x^{16}-2634514432x^{15}-3885840203776x^{14}-53872401858953216x^{13}+94214408927247859712x^{12}-62975254475204080959488x^{11}+103226759036720460268044288x^{10}-135064661735823849511289094144x^9+98538738742758214488145216405504x^8-35406390686075807206287368295284736x^7+7093688866156589439125081773433683968x^6-1134461330352365038560292680360569864192x^5+444914966921409978994965768260400869015552x^4-66690803273670337566199583651708651788304384x^3-1261027242677153702302623667960439863987142656x^2-224119701652082946244541276675331952089914933248x+22300745198530623141535718272648361505980416$$

near  $x=2.63452 \times 10^9$

^(1/3)

$$\frac{2\sqrt[3]{2+\sqrt{3}}(2+\sqrt{7})(\sqrt{3}+\sqrt{7})\left(\sqrt[4]{6}\sqrt[8]{7}+\sqrt{3+\sqrt{7}}\right)}{\sqrt[4]{6}\sqrt[8]{7}-\sqrt{3+\sqrt{7}}}$$


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$$\frac{2\sqrt[3]{2+\sqrt{3}}(\sqrt{3}+\sqrt{7})\sqrt{7+4\sqrt{7}}\left(\sqrt[4]{6}\sqrt[8]{7}+\sqrt{3+\sqrt{7}}\right)}{\sqrt[4]{6}\sqrt[8]{7}-\sqrt{3+\sqrt{7}}}$$

Minimal polynomial:

$$\begin{aligned}
& x^{48} - 2\,634\,514\,432\,x^{45} - 3\,885\,840\,203\,776\,x^{42} - \\
& 53\,872\,401\,858\,953\,216\,x^{39} + 94\,214\,408\,927\,247\,859\,712\,x^{36} - \\
& 62\,975\,254\,475\,204\,080\,959\,488\,x^{33} + 103\,226\,759\,036\,720\,460\,268\,044\,288\,x^{30} - \\
& 135\,064\,661\,735\,823\,849\,511\,289\,094\,144\,x^{27} + \\
& 98\,538\,738\,742\,758\,214\,488\,145\,216\,405\,504\,x^{24} - \\
& 35\,406\,390\,686\,075\,807\,206\,287\,368\,295\,284\,736\,x^{21} + \\
& 7\,093\,688\,866\,156\,589\,439\,125\,081\,773\,433\,683\,968\,x^{18} - \\
& 1\,134\,461\,330\,352\,365\,038\,560\,292\,680\,360\,569\,864\,192\,x^{15} + \\
& 444\,914\,966\,921\,409\,978\,994\,965\,768\,260\,400\,869\,015\,552\,x^{12} - \\
& 66\,690\,803\,273\,670\,337\,566\,199\,583\,651\,708\,651\,788\,304\,384\,x^9 - \\
& 1\,261\,027\,242\,677\,153\,702\,302\,623\,667\,960\,439\,863\,987\,142\,656\,x^6 - \\
& 224\,119\,701\,652\,082\,946\,244\,541\,276\,675\,331\,952\,089\,914\,933\,248\,x^3 + \\
& 22\,300\,745\,198\,530\,623\,141\,535\,718\,272\,648\,361\,505\,980\,416
\end{aligned}$$

Continued fraction:

[1381; 7, 1, 7, 37, 2, 7, 2, 2, 18, 2, 1, 7, 1, 1, 1, 25, 2, 1, 38, 1, 2, 4, 2, 3, 3, 1, 1, 1, ...]

The result 1381,12697 is very near to the rest mass of Sigma baryon, that is 1382.8±0.4

$$\left[ \frac{14}{\pi} * \left( \frac{(\sqrt{3} + \sqrt{7})}{2} \right) (2 + \sqrt{3})^{1/3} \left( \frac{(2 + \sqrt{7}) + \sqrt{7 + 4\sqrt{7}}}{2} \right) \right] \left[ \frac{(\sqrt{3} + \sqrt{7}) + ((6\sqrt{7})^{1/4})}{(\sqrt{3} + \sqrt{7}) - ((6\sqrt{7})^{1/4})} \right]$$

Input:

$$\frac{14}{\pi} \left( \frac{1}{2} (\sqrt{3} + \sqrt{7}) \right) \sqrt[3]{2 + \sqrt{3}} \left( \frac{1}{2} \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right) \right) \times \frac{\sqrt{3 + \sqrt{7}} + \sqrt[4]{6\sqrt{7}}}{\sqrt{3 + \sqrt{7}} - \sqrt[4]{6\sqrt{7}}}$$

Result:

$$\frac{7 \sqrt[3]{2 + \sqrt{3}} (\sqrt{3} + \sqrt{7}) \left( \sqrt[4]{6\sqrt{7}} + \sqrt{3 + \sqrt{7}} \right) \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right)}{2 \left( \sqrt{3 + \sqrt{7}} - \sqrt[4]{6\sqrt{7}} \right) \pi}$$

Decimal approximation:

769.3461492318103349693167253133484167280379887511514827185...

Property:

$$\frac{7 \sqrt[3]{2 + \sqrt{3}} (\sqrt{3} + \sqrt{7}) \left( \sqrt[4]{6\sqrt{7}} + \sqrt{3 + \sqrt{7}} \right) \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right)}{2 \left( -\sqrt[4]{6\sqrt{7}} + \sqrt{3 + \sqrt{7}} \right) \pi}$$

is a transcendental number

The result 769,346 is very near to the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for  $m_q = 4.7 \text{ MeV}/c^2 = 0.0047 \text{ GeV}/c^2$ , that is the mass of quark down is  $4.8 \pm 0.5 \pm 0.3 = 4.7 \text{ MeV}/c^2$ .

$$\left[ \frac{13}{\pi} * \left( \frac{(\sqrt{3} + \sqrt{7})}{2} \right) (2 + \sqrt{3})^{1/3} \left( \frac{(2 + \sqrt{7}) + \sqrt{7 + 4\sqrt{7}}}{2} \right) \right] \left[ \frac{(\sqrt{3 + \sqrt{7}}) + ((6\sqrt{7})^{1/4})}{(\sqrt{3 + \sqrt{7}}) - ((6\sqrt{7})^{1/4})} \right]$$

Input:

$$\frac{13}{\pi} \left( \frac{1}{2} (\sqrt{3} + \sqrt{7}) \right)^3 \sqrt[3]{2 + \sqrt{3}} \left( \frac{1}{2} \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right) \right) \times \frac{\sqrt{3 + \sqrt{7}} + \sqrt[4]{6\sqrt{7}}}{\sqrt{3 + \sqrt{7}} - \sqrt[4]{6\sqrt{7}}}$$

Result:

$$\frac{13 \sqrt[3]{2 + \sqrt{3}} (\sqrt{3} + \sqrt{7}) \left( \sqrt[4]{6\sqrt[8]{7}} + \sqrt{3 + \sqrt{7}} \right) \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right)}{4 \left( \sqrt{3 + \sqrt{7}} - \sqrt[4]{6\sqrt[8]{7}} \right) \pi}$$

Decimal approximation:

714.3928528581095967572226735052521012474638466974978053815...

Property:

$$\frac{13 \sqrt[3]{2 + \sqrt{3}} (\sqrt{3} + \sqrt{7}) \left( \sqrt[4]{6\sqrt[8]{7}} + \sqrt{3 + \sqrt{7}} \right) \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right)}{4 \left( -\sqrt[4]{6\sqrt[8]{7}} + \sqrt{3 + \sqrt{7}} \right) \pi}$$

is a transcendental number

$$2 * 5 * \left( \frac{(\sqrt{3} + \sqrt{7})}{2} \right) (2 + \sqrt{3})^{1/3} \left( \frac{(2 + \sqrt{7}) + \sqrt{7 + 4\sqrt{7}}}{2} \right) \left[ \frac{(\sqrt{3 + \sqrt{7}}) + ((6\sqrt{7})^{1/4})}{(\sqrt{3 + \sqrt{7}}) - ((6\sqrt{7})^{1/4})} \right]$$

Input:

$$\left( 2 * 5 \left( \frac{1}{2} (\sqrt{3} + \sqrt{7}) \right) \right)^3 \sqrt[3]{2 + \sqrt{3}} \left( \frac{1}{2} \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right) \right) \times \frac{\sqrt{3 + \sqrt{7}} + \sqrt[4]{6\sqrt{7}}}{\sqrt{3 + \sqrt{7}} - \sqrt[4]{6\sqrt{7}}}$$

Result:

$$\frac{5 \sqrt[3]{2 + \sqrt{3}} (\sqrt{3} + \sqrt{7}) \left( \sqrt[4]{6\sqrt[8]{7}} + \sqrt{3 + \sqrt{7}} \right) \left( 2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}} \right)}{2 \left( \sqrt{3 + \sqrt{7}} - \sqrt[4]{6\sqrt[8]{7}} \right)}$$

Decimal approximation:

1726.408721781608642202334242951081787952105395862630665234...

The result 1726,408 is very near to the value of the mass of the candidate “glueball”  $f_0(1710)$  that is 1723 (+ 6 – 5).

Now, we have the following modular equation:

$$\frac{32}{\pi} = (5\sqrt{5} - 1) + \frac{47\sqrt{5} + 29}{64} \left(\frac{1}{2}\right)^3 \left(\frac{\sqrt{5} - 1}{2}\right)^8 + \frac{89\sqrt{5} + 59}{64^2} \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^3 \left(\frac{\sqrt{5} - 1}{2}\right)^{16} + \dots$$

$$[(((5\sqrt{5})-1))+((47\sqrt{5}+29)/64)) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16))]]$$

Input:

$$(5\sqrt{5} - 1) + \left(\frac{1}{64} (47\sqrt{5} + 29)\right) \times \frac{1}{8} \left(\frac{1}{2} (\sqrt{5} - 1)\right)^8 + \frac{89\sqrt{5} + 59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2} (\sqrt{5} - 1)\right)^{16}$$

Result:

$$-1 + 5\sqrt{5} + \frac{(\sqrt{5} - 1)^8 (29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5} - 1)^{16} (59 + 89\sqrt{5})}{137438953472}$$

Decimal approximation:

10.18591635745234529933672773439453907823343935074991009694...

The result 10,1859 multiplied by  $10^2$  is equal to 1018,59, that is very near to the rest mass of Phi meson  $1019.445 \pm 0.020$

Alternate forms:

$$\frac{15(1041875\sqrt{5} - 905609)}{2097152}$$

$$\frac{15628125\sqrt{5}}{2097152} - \frac{13584135}{2097152}$$

$$\frac{15628125\sqrt{5} - 13584135}{2097152}$$

From this expression, multiplied by the result of the precedent modular equation, we obtain:

$$(172.640872) * [(((5\sqrt{5})-1))+((47\sqrt{5}+29))/64) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16))]] - 32$$

Input interpretation:

$$172.640872 \left[ \left( 5\sqrt{5} - 1 \right) + \left( \frac{1}{64} (47\sqrt{5} + 29) \right) \times \frac{1}{8} \left( \frac{1}{2} (\sqrt{5} - 1) \right)^8 + \frac{89\sqrt{5} + 59}{64^2} \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16} \right] - 32$$

Result:

1726.50548...

The result 1726,50548 is very near to the value of the mass of the candidate “glueball”  $f_0(1710)$  that is 1723 (+ 6 – 5).

$$1/[(((5\sqrt{5})-1))+((47\sqrt{5}+29))/64) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16))]]^0.2$$

Input:

$$\frac{1}{\left( 5\sqrt{5} - 1 \right) + \left( \frac{1}{64} (47\sqrt{5} + 29) \right) \times \frac{1}{8} \left( \frac{1}{2} (\sqrt{5} - 1) \right)^8 + \frac{89\sqrt{5} + 59}{64^2} \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16}}{0.2}$$

Result:

0.628637...

From Wikipedia

In mathematics, the **continuum hypothesis** is a hypothesis put forward by Georg Cantor which concerns the possible dimensions for infinite sets. Cantor introduced the concept of cardinality and cardinal number (which we can imagine as a "dimension" of the set) to compare transfinite sets with each other, and demonstrated the existence of infinite sets of different cardinality, such as natural numbers and real numbers. The hypothesis of the continuous states that: *There is no set whose cardinality is strictly included between that of integers and real numbers.* Mathematically speaking, given that the cardinality of the integers  $|Z|$  is  $\aleph_0$  (aleph-zero) and the cardinality of real numbers  $|R|$  is  $2^{\aleph_0}$ , the continuum hypothesis states:



$$\nexists A : \aleph_0 < |A| < 2^{\aleph_0}$$

where  $|A|$  indicates the cardinality of  $A$ .

The name of this hypothesis derives from the straight line of the real numbers, called "the continuum". There is also a generalization of the continuum hypothesis, called "generalized hypothesis of the continuum", which states that for every transfinite cardinal  $T$ :

$$\nexists A : |T| < |A| < 2^{|T|}$$

In mathematics the cardinality of the continuum is the cardinal number of the set of real numbers  $\mathbb{R}$  (which, sometimes, is called the continuum). This cardinal number is often indicated with the character  $c$ :  $c = |\mathbb{R}|$ . Georg Cantor introduced the concept of cardinality of a set to compare the dimensions of infinite sets. He proved that the set of real numbers is uncountable, that is, that  $c$  is greater than the cardinality of natural numbers, indicated with  $\aleph_0$  (aleph-null):  $\aleph_0 < c$ .

In other words, the real numbers are many more (immeasurably more) than the integers numbers: so much so that the latter can be counted, while the real numbers are not (for example it is perfectly legitimate to speak of the "first 100 integers numbers"; the same expression applied to real numbers is meaningless). Cantor demonstrated this statement using a technique known as the diagonal argument. The famous continuum hypothesis states that there is also the second aleph number, i.e.

$\aleph_1$  (aleph-one). In other words, the continuum hypothesis states that there is no set  $A$  having cardinality strictly included between  $\aleph_0$  and  $c$ :

$$\nexists A : \aleph_0 < |A| < c.$$

Many sets studied in mathematics have cardinality equal to  $c$ . For example: the set of real numbers  $\mathbb{R}$ , any interval (not degenerate) open or closed in  $\mathbb{R}$ , such as the unit interval  $[0,1]$ , the set of irrational numbers, the set of transcendental numbers. **The Cantor set**, introduced by the German mathematician Georg Cantor, is a subset of the interval  $[0,1]$  of real numbers. The Cantor set contains as many points as the interval  $[0,1]$  contains: both have the cardinality of the continuum. To prove this, it is sufficient to construct a surjective function  $f$  from the first to the second set. The existence of a surjective function implies that the starting set (the Cantor set) cannot have cardinality smaller than the arrival one (the interval). Since the Cantor set is a subset of the interval, it cannot have even higher cardinality, and therefore the two sets have the same cardinality. The Cantor set is a fractal (of a deterministic type). Taking two sets of Cantor in the intervals  $[0,1]$  and  $[2,3]$ , and contracting the interval  $[0,3]$  by a factor  $1/3$ , we get the Cantor set again. It has a "non-integer dimension",

intermediate between the dimensions 0 and 1 of the point and of the straight line respectively. In fact its Hausdorff dimension is equal to  $\ln(2) / \ln(3)$ :

$$\frac{\log(2)}{\log(3)}$$

Decimal approximation:

0.630929753571457437099527114342760854299585640131880427870...

Alternative representations:

$$\frac{\log(2)}{\log(3)} = \frac{\log_e(2)}{\log_e(3)}$$

$$\frac{\log(2)}{\log(3)} = \frac{\log(a) \log_a(2)}{\log(a) \log_a(3)}$$

$$\frac{\log(2)}{\log(3)} = \frac{2 \coth^{-1}(3)}{2 \coth^{-1}(2)}$$

Series representations:

$$\frac{\log(2)}{\log(3)} = \frac{2\pi \left[ \frac{\arg(2-x)}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}}{2\pi \left[ \frac{\arg(3-x)}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{\log(2)}{\log(3)} = \frac{2\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{2\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k}}$$

$$\frac{\log(2)}{\log(3)} = \frac{\left[ \frac{\arg(2-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(2-z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{\left[ \frac{\arg(3-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(3-z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{\log(2)}{\log(3)} = \frac{\int_1^2 \frac{1}{t} dt}{\int_1^3 \frac{1}{t} dt}$$

$$\frac{\log(2)}{\log(3)} = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

The result is 0,630929... that added to 1 give 1,630929 and is near to the precedent result 0,628637.

We have also that:

$$\frac{1}{32.291^2} \left( \left( \left( \left( \left( \left( 172.640872 \right) * \left[ \left( \left( 5\sqrt{5}-1 \right) + \left( \left( 47\sqrt{5}+29 \right) / 64 \right) \right] * \frac{1}{8} * \left( \left( \sqrt{5}-1 \right) / 2 \right) \right) \right) \right)^8 + \left( \left( \left( 89\sqrt{5}+59 \right) / 64^2 \right) \left( \frac{3}{8} \right)^3 \left( \left( \sqrt{5}-1 \right) / 2 \right) \right)^{16} \right) \right) \right) - 32 \right)$$

Input interpretation:

$$\frac{1}{32.291^2} \left( 172.640872 \left( \left( 5\sqrt{5}-1 \right) + \left( \frac{1}{64} \left( 47\sqrt{5}+29 \right) \right) \right) \times \frac{1}{8} \left( \frac{1}{2} \left( \sqrt{5}-1 \right) \right)^8 + \frac{89\sqrt{5}+59}{64^2} \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} \left( \sqrt{5}-1 \right) \right)^{16} \right) - 32$$

Result:

1.65579...

The result 1.65579 is practically equal to the 14-th root of Ramanujan's class invariant 1164.2696, to the numerical result for  $\theta_{(2)}$  as a function of  $\theta_{(0)}$  1,6557 for the D7-brane in  $\text{AdS}_2 \times \text{S}^2$ -sliced thermal  $\text{AdS}_5$  and a good approximation to the mass of the proton.

$$\frac{10}{\left[ \left( \left( \left( 5\sqrt{5}-1 \right) + \left( \left( 47\sqrt{5}+29 \right) / 64 \right) \right) * \frac{1}{8} * \left( \left( \sqrt{5}-1 \right) / 2 \right) \right) \right]^8 + \left( \left( \left( 89\sqrt{5}+59 \right) / 64^2 \right) \left( \frac{3}{8} \right)^3 \left( \left( \sqrt{5}-1 \right) / 2 \right) \right)^{16} \right]^{0.2}}$$

Input:

$$\frac{10}{\left( \left( 5\sqrt{5}-1 \right) + \left( \frac{1}{64} \left( 47\sqrt{5}+29 \right) \right) \right) \times \frac{1}{8} \left( \frac{1}{2} \left( \sqrt{5}-1 \right) \right)^8 + \frac{89\sqrt{5}+59}{64^2} \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} \left( \sqrt{5}-1 \right) \right)^{16} \right)^{0.2}}$$

Result:

6.28637...

The result 6,28637 is practically equal to the length of a circle of radius 1, i.e.  $2\pi$

$$76 * ((5\sqrt{5})-1)+((47\sqrt{5}+29))/64)) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16))))$$

Input:

$$76(5\sqrt{5}-1) + \left(\frac{1}{64}(47\sqrt{5}+29)\right) \times \frac{1}{8} \left(\frac{1}{2}(\sqrt{5}-1)\right)^8 + \frac{89\sqrt{5}+59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}$$

Result:

$$76(5\sqrt{5}-1) + \frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137438953472}$$

Decimal approximation:

773.7114079198734814527768535086231273684653242050720566985...

The result 773,711 is very near to the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for  $m_q = 4.7 \text{ MeV}/c^2 = 0.0047 \text{ GeV}/c^2$ , that is the mass of quark down is  $4.8 \pm 0.5 \pm 0.3 = 4.7 \text{ MeV}/c^2$ .

$$1/72 * 54\pi ((5\sqrt{5})-1)+((47\sqrt{5}+29))/64)) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16))))$$

Input:

$$\frac{1}{72} \times 54\pi(5\sqrt{5}-1) + \left(\frac{1}{64}(47\sqrt{5}+29)\right) \times \frac{1}{8} \left(\frac{1}{2}(\sqrt{5}-1)\right)^8 + \frac{89\sqrt{5}+59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}$$

Result:

$$\frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137438953472} + \frac{3}{4}(5\sqrt{5}-1)\pi$$

Decimal approximation:

23.99243722116377586278028176529688623588187995466296118481...

The result 23,99243... $\approx$  24, represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates

Property:

$$\frac{(-1 + \sqrt{5})^8 (29 + 47\sqrt{5})}{131072} + \frac{27(-1 + \sqrt{5})^{16} (59 + 89\sqrt{5})}{137438953472} + \frac{3}{4} (-1 + 5\sqrt{5})\pi$$

is a transcendental number

$$1/142 * 54\pi ((5\sqrt{5})-1)+((47\sqrt{5}+29))/64)) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16))))$$

Input:

$$\frac{1}{142} \times 54\pi (5\sqrt{5} - 1) + \left(\frac{1}{64} (47\sqrt{5} + 29)\right) \times \frac{1}{8} \left(\frac{1}{2} (\sqrt{5} - 1)\right)^8 + \frac{89\sqrt{5} + 59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2} (\sqrt{5} - 1)\right)^{16}$$

Result:

$$\frac{(\sqrt{5} - 1)^8 (29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5} - 1)^{16} (59 + 89\sqrt{5})}{137438953472} + \frac{27}{71} (5\sqrt{5} - 1)\pi$$

Decimal approximation:

12.16792840014457492486296087642990747926492736214220358167...

Property:

$$\frac{(-1 + \sqrt{5})^8 (29 + 47\sqrt{5})}{131072} + \frac{27(-1 + \sqrt{5})^{16} (59 + 89\sqrt{5})}{137438953472} + \frac{27}{71} (-1 + 5\sqrt{5})\pi$$

is a transcendental number

The result 12,1679 is very near to 12,19 that is the value of the black hole entropy.

$$54\pi ((5\sqrt{5})-1)+((47\sqrt{5}+29))/64)) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16))))$$

Input:

$$54\pi (5\sqrt{5} - 1) + \left(\frac{1}{64} (47\sqrt{5} + 29)\right) \times \frac{1}{8} \left(\frac{1}{2} (\sqrt{5} - 1)\right)^8 + \frac{89\sqrt{5} + 59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2} (\sqrt{5} - 1)\right)^{16}$$

Result:

$$\frac{(\sqrt{5} - 1)^8 (29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5} - 1)^{16} (59 + 89\sqrt{5})}{137438953472} + 54(5\sqrt{5} - 1)\pi$$

Decimal approximation:

1727.059550557100688092529270358966598010340680494581220540...

Property:

$$\frac{(-1 + \sqrt{5})^8 (29 + 47\sqrt{5})}{131072} + \frac{27(-1 + \sqrt{5})^{16} (59 + 89\sqrt{5})}{137438953472} + 54(-1 + 5\sqrt{5})\pi$$

is a transcendental number

The result 1727,0595 is very near to the value of the mass of the candidate “glueball”  $f_0(1710)$  that is 1723 (+ 6 – 5).

$$32 / ((5\sqrt{5})-1)+((47\sqrt{5}+29))/64)) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16$$

Input:

$$\frac{32}{5\sqrt{5}-1} + \left(\frac{1}{64}(47\sqrt{5}+29)\right) \times \frac{1}{8} \left(\frac{1}{2}(\sqrt{5}-1)\right)^8 + \frac{89\sqrt{5}+59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}$$

Result:

$$\frac{32}{5\sqrt{5}-1} + \frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137438953472}$$

Decimal approximation:

3.148889989307964167496244769746256269340822855416830797226...

$$\pi * ((5\sqrt{5})-1)+((47\sqrt{5}+29))/64)) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16 - 5$$

Input:

$$\pi \left(5\sqrt{5}-1\right) + \left(\frac{1}{64}(47\sqrt{5}+29)\right) \times \frac{1}{8} \left(\frac{1}{2}(\sqrt{5}-1)\right)^8 + \frac{89\sqrt{5}+59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5}-1)\right)^{16} - 5$$

Result:

$$-5 + \frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137438953472} + (5\sqrt{5}-1)\pi$$

Decimal approximation:

26.98805747156723554461008922348312901416572408865318775456...

Property:

$$-5 + \frac{(-1 + \sqrt{5})^8 (29 + 47\sqrt{5})}{131072} + \frac{27(-1 + \sqrt{5})^{16} (59 + 89\sqrt{5})}{137438953472} + (-1 + 5\sqrt{5})\pi$$

is a transcendental number

The result  $26,98805... \approx 27$ , the number that multiplied by 64 is equal to 1728, and is a number very important in string theory (see Appendix)

$$\pi * ((5\sqrt{5}-1)+((47\sqrt{5}+29)/64)) * 1/8 * (((\sqrt{5}-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5}-1)/2))^16 - 6$$

Input:

$$\pi \left( 5\sqrt{5} - 1 \right) + \left( \frac{1}{64} (47\sqrt{5} + 29) \right) \times \frac{1}{8} \left( \frac{1}{2} (\sqrt{5} - 1) \right)^8 + \frac{89\sqrt{5} + 59}{64^2} \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16} - 6$$

Result:

$$-6 + \frac{(\sqrt{5} - 1)^8 (29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5} - 1)^{16} (59 + 89\sqrt{5})}{137438953472} + (5\sqrt{5} - 1)\pi$$

Decimal approximation:

25.98805747156723554461008922348312901416572408865318775456...

Property:

$$-6 + \frac{(-1 + \sqrt{5})^8 (29 + 47\sqrt{5})}{131072} + \frac{27(-1 + \sqrt{5})^{16} (59 + 89\sqrt{5})}{137438953472} + (-1 + 5\sqrt{5})\pi$$

is a transcendental number

The result  $25,988 \approx 26$  is the 26-dimensional bosonic string. (ghost and antighost fields describing an open string in 26 dimensions with  $0 \leq \sigma \leq \pi$ )

Witten presented this formal structure and argued that all the needed axioms are satisfied when  $\mathcal{A}$  is taken to be the space of string fields

$$\mathcal{A} = \{ \Psi[x(\sigma); c(\sigma), b(\sigma)] \} \quad (56)$$

which can be described as functionals of the matter, ghost and antighost fields describing an open string in 26 dimensions with  $0 \leq \sigma \leq \pi$ . Such a string field can be written as a formal sum over open string Fock space states with coefficients given by an infinite family of space-time fields

$$\Psi = \int d^{26}p \left[ \phi(p) |0_1; p\rangle + A_\mu(p) \alpha_{-1}^\mu |0_1; p\rangle + \dots \right] \quad (57)$$

$$((5\sqrt{5}-1)+((47\sqrt{5}+29)/64)) * 1/8 * (((\sqrt{5}-1)/2))^8 +$$

$$(((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5}-1)/2))^16 -(\pi-4)$$

Input:

$$\left(5\sqrt{5}-1\right) + \left(\frac{1}{64}(47\sqrt{5}+29)\right) \times \frac{1}{8} \left(\frac{1}{2}(\sqrt{5}-1)\right)^8 +$$

$$\frac{89\sqrt{5}+59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5}-1)\right)^{16} - (\pi-4)$$

Result:

$$3+5\sqrt{5} + \frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137438953472} - \pi$$

Decimal approximation:

11.04432370386255206087408435111503619403626995137480427596...

Property:

$$3+5\sqrt{5} + \frac{(-1+\sqrt{5})^8(29+47\sqrt{5})}{131072} + \frac{27(-1+\sqrt{5})^{16}(59+89\sqrt{5})}{137438953472} - \pi$$

is a transcendental number

The result 11,04432... $\approx$  11 concerning the  $D = 11$  supergravity.

The  $\mathcal{N} = 1$ ,  $D = 11$  supergravity theory has the following field and

particle contents,

$$D = 11 \begin{cases} G_{\mu\nu} & SO(9) & 44_B & \text{metric-graviton} \\ A_{\mu\nu\rho} & & 84_B & \text{antisymmetric rank 3} \\ \psi_{\mu\alpha} & & 128_F & \text{Majorana gravitino} \end{cases} \quad (69)$$

Here and below, the numbers following the little group (for the massless representations)  $SO(9)$  represent the number of physical degrees of freedom in the multiplet. For example, the graviton in  $D = 11$  is given by the rank 2 symmetric traceless representation of  $SO(9)$ , of dimension  $9 \times 10/2 - 1 = 44$ . The Majorana spinor  $\psi_{\mu\alpha}$  as a vector has 9 physical components, but it also satisfies the  $\Gamma$ -tracelessness condition  $(\Gamma^\mu)^{\beta\alpha}\psi_{\mu\alpha} = 0$ , which cuts the number down to 8. The 32 component spinor satisfies a Dirac equation, which cuts its number of physical components down to 16, yielding a total of  $8 \times 16 = 128$ . The subscripts  $B$  and  $F$  refer to the bosonic or fermionic nature of the state.



**The principal numbers of D = 11 supergravity theory are 8, 16, 32, 44, 84 and 128** (84 + 16 + 8) = 108; (64 + 64) = 128; (36 + 8) = 44. Note that 36, 64, 16, 8 and 32 are divisible for 1728

$$(\ln 1.601045) ((5\sqrt{5})-1)+((47\sqrt{5}+29))/64)) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16$$

Input interpretation:

$$\log(1.601045) \left( 5\sqrt{5} - 1 \right) + \left( \frac{1}{64} (47\sqrt{5} + 29) \right) \times \frac{1}{8} \left( \frac{1}{2} (\sqrt{5} - 1) \right)^8 + \frac{89\sqrt{5} + 59}{64^2} \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16}$$

- $\log(x)$  is the natural logarithm

Result:

4.797020...

Alternative representations:

$$\log(1.60105) \left( 5\sqrt{5} - 1 \right) + \frac{\left( \frac{1}{2} (\sqrt{5} - 1) \right)^8 (47\sqrt{5} + 29)}{8 \times 64} + \frac{\left( \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16} \right) (89\sqrt{5} + 59)}{64^2} = \log_e(1.60105) \left( -1 + 5\sqrt{5} \right) + \frac{\left( \frac{1}{2} (-1 + \sqrt{5}) \right)^8 (29 + 47\sqrt{5})}{8 \times 64} + \frac{\left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (-1 + \sqrt{5}) \right)^{16} (59 + 89\sqrt{5})}{64^2}$$

$$\log(1.60105) \left( 5\sqrt{5} - 1 \right) + \frac{\left( \frac{1}{2} (\sqrt{5} - 1) \right)^8 (47\sqrt{5} + 29)}{8 \times 64} + \frac{\left( \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16} \right) (89\sqrt{5} + 59)}{64^2} = \log(\alpha) \log_\alpha(1.60105) \left( -1 + 5\sqrt{5} \right) + \frac{\left( \frac{1}{2} (-1 + \sqrt{5}) \right)^8 (29 + 47\sqrt{5})}{8 \times 64} + \frac{\left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (-1 + \sqrt{5}) \right)^{16} (59 + 89\sqrt{5})}{64^2}$$

$$\begin{aligned} & \log(1.60105) \left(5\sqrt{5} - 1\right) + \frac{\left(\frac{1}{2}(\sqrt{5} - 1)\right)^8 (47\sqrt{5} + 29)}{8 \times 64} + \\ & \frac{\left(\left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{16}\right) (89\sqrt{5} + 59)}{64^2} = -\text{Li}_1(-0.601045) \left(-1 + 5\sqrt{5}\right) + \\ & \frac{\left(\frac{1}{2}(-1 + \sqrt{5})\right)^8 (29 + 47\sqrt{5})}{8 \times 64} + \frac{\left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(-1 + \sqrt{5})\right)^{16} (59 + 89\sqrt{5})}{64^2} \end{aligned}$$

Series representations:

$$\begin{aligned} & \log(1.60105) \left(5\sqrt{5} - 1\right) + \\ & \frac{\left(\frac{1}{2}(\sqrt{5} - 1)\right)^8 (47\sqrt{5} + 29)}{8 \times 64} + \frac{\left(\left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{16}\right) (89\sqrt{5} + 59)}{64^2} = \\ & \left(-\sum_{k=1}^{\infty} \frac{(-0.601045)^k}{k}\right) \left(-1 + 5\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right) + \\ & \frac{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)^8 \left(29 + 47\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)}{131072} + \\ & \frac{27 \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)^{16} \left(59 + 89\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)}{137438953472} \end{aligned}$$

$$\begin{aligned} & \log(1.60105) \left(5\sqrt{5} - 1\right) + \\ & \frac{\left(\frac{1}{2}(\sqrt{5} - 1)\right)^8 (47\sqrt{5} + 29)}{8 \times 64} + \frac{\left(\left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{16}\right) (89\sqrt{5} + 59)}{64^2} = \\ & \left(-\sum_{k=1}^{\infty} \frac{(-0.601045)^k}{k}\right) \left(-1 + 5\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right) + \\ & \frac{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)^8 \left(29 + 47\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)}{131072} + \\ & \frac{27 \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)^{16} \left(59 + 89\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)}{137438953472} \end{aligned}$$

$$\begin{aligned}
& \log(1.60105) \left( 5\sqrt{5} - 1 \right) + \\
& \frac{\left( \frac{1}{2} (\sqrt{5} - 1) \right)^8 (47\sqrt{5} + 29)}{8 \times 64} + \frac{\left( \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16} \right) (89\sqrt{5} + 59)}{64^2} = \\
& \log(1.60105) \left( -1 + 5 \exp\left( i\pi \left[ \frac{\arg(5-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) + \\
& \frac{1}{131072} \left( -1 + \exp\left( i\pi \left[ \frac{\arg(5-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right)^8 \\
& \left( 29 + 47 \exp\left( i\pi \left[ \frac{\arg(5-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) + \\
& \frac{1}{137438953472} 27 \\
& \left( -1 + \exp\left( i\pi \left[ \frac{\arg(5-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right)^{16} \\
& \left( 59 + 89 \exp\left( i\pi \left[ \frac{\arg(5-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right)
\end{aligned}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

Integral representations:

$$\begin{aligned}
& \log(1.60105) \left( 5\sqrt{5} - 1 \right) + \\
& \frac{\left( \frac{1}{2} (\sqrt{5} - 1) \right)^8 (47\sqrt{5} + 29)}{8 \times 64} + \frac{\left( \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16} \right) (89\sqrt{5} + 59)}{64^2} = \\
& -1 + 5\sqrt{5} \int_1^{1.60105} \frac{1}{t} dt + \frac{(-1 + \sqrt{5})^8 (29 + 47\sqrt{5})}{131072} + \frac{27(-1 + \sqrt{5})^{16} (59 + 89\sqrt{5})}{137438953472}
\end{aligned}$$

$$\begin{aligned}
& \log(1.60105) \left( 5\sqrt{5} - 1 \right) + \\
& \frac{\left( \frac{1}{2} (\sqrt{5} - 1) \right)^8 (47\sqrt{5} + 29)}{8 \times 64} + \frac{\left( \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16} \right) (89\sqrt{5} + 59)}{64^2} = \\
& \frac{-1 + 5\sqrt{5}}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.509085s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \frac{(-1 + \sqrt{5})^8 (29 + 47\sqrt{5})}{131072} + \\
& \frac{27(-1 + \sqrt{5})^{16} (59 + 89\sqrt{5})}{137438953472} \text{ for } -1 < \gamma < 0
\end{aligned}$$

Where 1,60105 is the 15-th root of Ramanujan class invariant 1164,2696. Indeed, we have:

$$\sqrt[15]{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3} = 1,601045117 \dots$$

From:

**The toroidal Hausdorff dimension of 2d Euclidean quantum gravity**

J.Ambjørn, T.Budd - ©2013 Elsevier B.V. All rights reserved.

The lengths of shortest non-contractible loops are studied numerically in 2d Euclidean quantum gravity on a torus coupled to conformal field theories with central charge less than one. We find that the distribution of these geodesic lengths displays a scaling in agreement with a Hausdorff dimension given by the formula of Y.Watabiki.

From the above Ramanujan modular equation:

$$(\ln 1.601045) ((5\sqrt{5})-1)+((47\sqrt{5}+29)/64) * 1/8 * (((\sqrt{5})-1)/2))^8 + ((89\sqrt{5}+59)/64^2) (3/8)^3 (((\sqrt{5})-1)/2))^16$$

Input interpretation:

$$\log(1.601045) (5\sqrt{5} - 1) + \left(\frac{1}{64} (47\sqrt{5} + 29)\right) \times \frac{1}{8} \left(\frac{1}{2} (\sqrt{5} - 1)\right)^8 + \frac{89\sqrt{5} + 59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2} (\sqrt{5} - 1)\right)^{16}$$

- $\log(x)$  is the natural logarithm

Result:

4.797020...

We note that the result 4,797 is a very good approximation to the value of fractal connectivity dimension 4,828427

However, contrary to the particle case, we can define a non-trivial *intrinsic* Hausdorff dimension. This dimension is natural and of interest when we view non-critical string theory as two-dimensional (Euclidean) quantum gravity coupled to some conformal theory with central charge  $c < 1$ . For such a conformal field theory one does not in general have a natural definition of  $D_H$  which refers explicitly to the Gaussian fields  $X_\mu$ , but we can, by analogy with (4), define the *intrinsic* Hausdorff dimension as

$$\langle r^2 \rangle_V \sim V^{2/d_h}. \quad (7)$$

The average is defined with respect to the partition function  $Z_V$ :

$$Z_V = \int \mathcal{D}[g]_V Z_V(g, c), \quad (8)$$

where  $Z(g, c)$  denotes the partition function for the conformal field theory with central charge  $c$  we consider, in the “background geometry” defined by the 2d metric  $g_{ab}(\xi)$ . The functional integration in (8) is over two-dimensional geometries with volume  $V$ , as defined in Eq. (2), and the average in (7) is now

$$\begin{aligned} \langle r^2 \rangle_V &= \frac{1}{2V^2 Z_V} \int \mathcal{D}[g] Z_V(g, c) \int d^2\xi \\ &\quad \times \int d^2\xi' \sqrt{g(\xi)} \sqrt{g(\xi')} r_g^2(\xi, \xi') \end{aligned} \quad (9)$$

where  $r_g(\xi, \xi')$  denotes the geodesic distance between  $\xi$  and  $\xi'$  in the geometry defined by  $g_{ab}(\xi)$ .

A remarkable formula for  $d_h$  was derived by Y. Watabiki [3], using Liouville theory and the heat kernel expansion:

$$d_h^W(c) = 2 \frac{\sqrt{49-c} + \sqrt{25-c}}{\sqrt{25-c} + \sqrt{1-c}}. \quad (10)$$

This formula is not the only one proposed. Already in the original articles where quantum Liouville theory was defined an alternative formula was suggested [4]

$$d_h' = -\frac{2}{\gamma}, \quad \gamma = \frac{c-1 - \sqrt{(1-c)(25-c)}}{12}. \quad (11)$$

The two formulas agree for  $c = 0$  where  $d_h^W = d_h' = 4$ . However, they have a quite different behavior for  $c \rightarrow -\infty$  where  $d_h' \rightarrow 0$  while  $d_h^W \rightarrow 2$ . Formally one expects  $d_h \rightarrow 2$  since this is the limit where the quantum Liouville theory should behave semiclassically and matter and geometry should be only weakly coupled, and this is indeed the behavior of  $d_h^W$ . On the other hand it is quite difficult to provide any geometric interpretation of  $d_h' \rightarrow 0$ . The difference between the two predictions becomes even more pronounced when we consider the region  $0 < c < 1$ . For  $c \rightarrow 1$ ,  $d_h' \rightarrow \infty$  while  $d_h^W \rightarrow 2 + 2\sqrt{2} = 4.83$ .

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**Analytic Study of Fractal Structure of Quantized Surface in Two-Dimensional Quantum Gravity** - Yoshiyuki WATABIKI

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We have that:

$$D_{\text{con}} = -2 \frac{a_1}{a-1} = 2 \times \frac{\sqrt{25-c} + \sqrt{19-c}}{\sqrt{25-c} + \sqrt{1-c}}. \quad (8.1)$$

For example, we have,

$$D_{\text{con}} = \begin{cases} 2(1+\sqrt{2}) & \text{for } c=1, \\ (7+\sqrt{97})/4 & \text{for } c=1/2, \\ 4 & \text{for } c=0, \\ (3+\sqrt{17})/2 & \text{for } c=-2, \\ 3 & \text{for } c=-44/5, \\ 2 & \text{for } c=-\infty. \end{cases} \quad (8.2)$$

Now:

$$-\ln(1/e^{(1.65578/4)}) ((5\sqrt{5}-1))+((47\sqrt{5}+29)/64) * 1/8 * (((\sqrt{5}-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5}-1)/2))^16$$

Input interpretation:

$$-\log\left(\frac{1}{e^{1.65578/4}}\right)(5\sqrt{5}-1) + \left(\frac{1}{64}(47\sqrt{5}+29)\right) \times \frac{1}{8} \left(\frac{1}{2}(\sqrt{5}-1)\right)^8 + \frac{89\sqrt{5}+59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}$$

- $\log(x)$  is the natural logarithm

Result:

4.21968...

Alternative representations:

$$\begin{aligned}
& -\log\left(\frac{1}{e^{1.65578/4}}\right)\left(5\sqrt{5}-1\right)+\frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8\times 64}+ \\
& \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2}=-\log_e\left(\frac{1}{e^{1.65578/4}}\right)\left(-1+5\sqrt{5}\right)+ \\
& \frac{\left(\frac{1}{2}(-1+\sqrt{5})\right)^8(29+47\sqrt{5})}{8\times 64}+\frac{\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(-1+\sqrt{5})\right)^{16}(59+89\sqrt{5})}{64^2}
\end{aligned}$$

$$\begin{aligned}
& -\log\left(\frac{1}{e^{1.65578/4}}\right)\left(5\sqrt{5}-1\right)+\frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8\times 64}+ \\
& \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2}=-\log(a)\log_a\left(\frac{1}{e^{1.65578/4}}\right)\left(-1+5\sqrt{5}\right)+ \\
& \frac{\left(\frac{1}{2}(-1+\sqrt{5})\right)^8(29+47\sqrt{5})}{8\times 64}+\frac{\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(-1+\sqrt{5})\right)^{16}(59+89\sqrt{5})}{64^2}
\end{aligned}$$

$$\begin{aligned}
& -\log\left(\frac{1}{e^{1.65578/4}}\right)\left(5\sqrt{5}-1\right)+\frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8\times 64}+ \\
& \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2}=\text{Li}_1\left(1-\frac{1}{e^{1.65578/4}}\right)\left(-1+5\sqrt{5}\right)+ \\
& \frac{\left(\frac{1}{2}(-1+\sqrt{5})\right)^8(29+47\sqrt{5})}{8\times 64}+\frac{\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(-1+\sqrt{5})\right)^{16}(59+89\sqrt{5})}{64^2}
\end{aligned}$$

Series representations:

$$\begin{aligned}
& -\log\left(\frac{1}{e^{1.65578/4}}\right)\left(5\sqrt{5}-1\right)+ \\
& \frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8\times 64}+\frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2}= \\
& \left(\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{1}{e^{0.413945}}\right)^k}{k}\right)\left(-1+5\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)+ \\
& \frac{\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^8\left(29+47\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}{131072}+ \\
& \frac{27\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^{16}\left(59+89\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}{137438953472}
\end{aligned}$$

$$\begin{aligned}
& -\log\left(\frac{1}{e^{1.65578/4}}\right)(5\sqrt{5}-1) + \\
& \frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8 \times 64} + \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2} = \\
& \left(\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{1}{e^{0.413945}}\right)^k}{k}\right) \left(-1 + 5\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + \\
& \frac{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)^8 \left(29 + 47\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)}{131072} + \\
& \frac{27 \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)^{16} \left(59 + 89\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)}{137438953472}
\end{aligned}$$

$$\begin{aligned}
& -\log\left(\frac{1}{e^{1.65578/4}}\right)(5\sqrt{5}-1) + \\
& \frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8 \times 64} + \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2} = \\
& -\log\left(\frac{1}{e^{0.413945}}\right) \left(-1 + 5 \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) + \\
& \frac{1}{131072} \left(-1 + \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^8 \\
& \left(29 + 47 \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) + \\
& \frac{1}{137438953472} 27 \\
& \left(-1 + \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^{16} \\
& \left(59 + 89 \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)
\end{aligned}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

Integral representation:

$$\begin{aligned}
& -\log\left(\frac{1}{e^{1.65578/4}}\right)(5\sqrt{5}-1) + \frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8 \times 64} + \\
& \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2} = -(-1 + 5\sqrt{5}) \int_1^{\frac{1}{e^{0.413945}}} \frac{1}{t} dt + \\
& \frac{(-1 + \sqrt{5})^8(29 + 47\sqrt{5})}{131072} + \frac{27(-1 + \sqrt{5})^{16}(59 + 89\sqrt{5})}{137438953472}
\end{aligned}$$



The result 4,21968 is very near to the value  $(7+\sqrt{97})/4$  for  $c=1/2$ , regarding to the value of fractal connectivity dimension 4,21221445 and very near to the mass of Dark Matter particle:

Using (22) this translates to an upper bound on the mass of the DM particle:

$$m \lesssim 4.2 \left( \frac{\sigma/m}{\text{cm}^2/\text{g}} \right)^{1/4} \text{ eV}. \quad (24)$$

Smaller and less massive galaxies result in a somewhat weaker bound.

The bound (24) on the DM particle mass is the main result of this Section. It shows that for values of  $\sigma/m$  satisfying the merging-cluster bound  $\sim 1 \text{ cm}^2/\text{g}$  [85–88],  $m$  must be somewhat below 4 eV. The dependence on the cross section is rather weak, however, scaling as the  $1/4$  power. It should be mentioned that the upper bound (24) would be somewhat tighter had we assumed a  $\rho \propto r^{-2}$  transition density profile outside the superfluid core, instead of  $\rho \propto r^{-3}$ .

(from: Phenomenological consequences of superfluid dark matter with baryon-phonon coupling- arXiv:1711.05748v1)

$$-\ln(1/e^{((\sqrt{\pi}))/5}) ((5\sqrt{5})-1)+((47\sqrt{5}+29))/64) * 1/8 * (((\sqrt{5})-1)/2))^8 + (((89\sqrt{5}+59)/64^2)) (3/8)^3 (((\sqrt{5})-1)/2))^16$$

Input:

$$-\log\left(\frac{1}{e^{\sqrt{\pi}/5}}\right)(5\sqrt{5}-1) + \left(\frac{1}{64}(47\sqrt{5}+29)\right) \times \frac{1}{8} \left(\frac{1}{2}(\sqrt{5}-1)\right)^8 + \frac{89\sqrt{5}+59}{64^2} \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}$$

- $\log(x)$  is the natural logarithm

Exact result:

$$\frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137438953472} + \frac{1}{5}(5\sqrt{5}-1)\sqrt{\pi}$$

Decimal approximation:

$$3.614412997378304625176254659190111895993197417863998842651\dots$$

Property:

$$\frac{(-1 + \sqrt{5})^8 (29 + 47\sqrt{5})}{131072} + \frac{27(-1 + \sqrt{5})^{16} (59 + 89\sqrt{5})}{137438953472} + \frac{1}{5} (-1 + 5\sqrt{5}) \sqrt{\pi}$$

is a transcendental number

Alternate forms:

$$-\frac{11486983}{2097152} + \frac{5142365\sqrt{5}}{2097152} - \frac{\sqrt{\pi}}{5} + \sqrt{5\pi}$$

$$\frac{-57434915 + 25711825\sqrt{5} - 2097152\sqrt{\pi} + 10485760\sqrt{5\pi}}{10485760}$$

$$\frac{5142365\sqrt{5} - 11486983}{2097152} + \frac{1}{5} (5\sqrt{5} - 1) \sqrt{\pi}$$

Alternative representations:

$$-\log\left(\frac{1}{e^{\sqrt{\pi}/5}}\right) (5\sqrt{5} - 1) + \frac{\left(\frac{1}{2}(\sqrt{5} - 1)\right)^8 (47\sqrt{5} + 29)}{8 \times 64} +$$

$$\frac{\left(\left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{16}\right) (89\sqrt{5} + 59)}{64^2} = -\log_e\left(\frac{1}{e^{\sqrt{\pi}/5}}\right) (-1 + 5\sqrt{5}) +$$

$$\frac{\left(\frac{1}{2}(-1 + \sqrt{5})\right)^8 (29 + 47\sqrt{5})}{8 \times 64} + \frac{\left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(-1 + \sqrt{5})\right)^{16} (59 + 89\sqrt{5})}{64^2}$$

$$-\log\left(\frac{1}{e^{\sqrt{\pi}/5}}\right) (5\sqrt{5} - 1) + \frac{\left(\frac{1}{2}(\sqrt{5} - 1)\right)^8 (47\sqrt{5} + 29)}{8 \times 64} +$$

$$\frac{\left(\left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{16}\right) (89\sqrt{5} + 59)}{64^2} = -\log(a) \log_a\left(\frac{1}{e^{\sqrt{\pi}/5}}\right) (-1 + 5\sqrt{5}) +$$

$$\frac{\left(\frac{1}{2}(-1 + \sqrt{5})\right)^8 (29 + 47\sqrt{5})}{8 \times 64} + \frac{\left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(-1 + \sqrt{5})\right)^{16} (59 + 89\sqrt{5})}{64^2}$$

$$\begin{aligned}
& -\log\left(\frac{1}{e^{\sqrt{\pi}/5}}\right)(5\sqrt{5}-1) + \frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8 \times 64} + \\
& \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2} = \text{Li}_1\left(1 - \frac{1}{e^{\sqrt{\pi}/5}}\right)(-1+5\sqrt{5}) + \\
& \frac{\left(\frac{1}{2}(-1+\sqrt{5})\right)^8(29+47\sqrt{5})}{8 \times 64} + \frac{\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(-1+\sqrt{5})\right)^{16}(59+89\sqrt{5})}{64^2}
\end{aligned}$$

Series representations:

$$\begin{aligned}
& -\log\left(\frac{1}{e^{\sqrt{\pi}/5}}\right)(5\sqrt{5}-1) + \\
& \frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8 \times 64} + \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2} = \\
& \left(\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + e^{-\sqrt{\pi}/5}\right)^k}{k}\right) \left(-1 + 5\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right) + \\
& \frac{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)^8 \left(29 + 47\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)}{131072} + \\
& \frac{27 \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)^{16} \left(59 + 89\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)}{137438953472}
\end{aligned}$$

$$\begin{aligned}
& -\log\left(\frac{1}{e^{\sqrt{\pi}/5}}\right)(5\sqrt{5}-1) + \\
& \frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8 \times 64} + \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2} = \\
& \left(\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + e^{-\sqrt{\pi}/5}\right)^k}{k}\right) \left(-1 + 5\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right) + \\
& \frac{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)^8 \left(29 + 47\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)}{131072} + \\
& \frac{27 \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)^{16} \left(59 + 89\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)}{137438953472}
\end{aligned}$$

$$\begin{aligned}
& -\log\left(\frac{1}{e^{\sqrt{\pi}/5}}\right)\left(5\sqrt{5}-1\right)+\frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8\times 64}+ \\
& \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2}=\left(\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+e^{-\sqrt{\pi}/5}\right)^k}{k}\right) \\
& \left(-1+5\exp\left(i\pi\left[\frac{\arg(5-x)}{2\pi}\right]\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^kx^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)+ \\
& \frac{1}{131072}\left(-1+\exp\left(i\pi\left[\frac{\arg(5-x)}{2\pi}\right]\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^kx^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8 \\
& \left(29+47\exp\left(i\pi\left[\frac{\arg(5-x)}{2\pi}\right]\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^kx^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)+ \\
& \frac{1}{137438953472}27 \\
& \left(-1+\exp\left(i\pi\left[\frac{\arg(5-x)}{2\pi}\right]\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^kx^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{16} \\
& \left(59+89\exp\left(i\pi\left[\frac{\arg(5-x)}{2\pi}\right]\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^kx^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)
\end{aligned}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

Integral representation:

$$\begin{aligned}
& -\log\left(\frac{1}{e^{\sqrt{\pi}/5}}\right)\left(5\sqrt{5}-1\right)+\frac{\left(\frac{1}{2}(\sqrt{5}-1)\right)^8(47\sqrt{5}+29)}{8\times 64}+ \\
& \frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}\right)(89\sqrt{5}+59)}{64^2}=-\left(-1+5\sqrt{5}\right)\int_1^{e^{-\sqrt{\pi}/5}}\frac{1}{t}dt+ \\
& \frac{(-1+\sqrt{5})^8(29+47\sqrt{5})}{131072}+\frac{27(-1+\sqrt{5})^{16}(59+89\sqrt{5})}{137438953472}
\end{aligned}$$

The result 3,61441 is very near to the value  $(3+\sqrt{17})/2$  for  $c=-2$ , regarding to the value of fractal connectivity dimension 3,5615528

Now, we have the following Ramanujan modular equation:

$$e^{\frac{1}{4}\pi\sqrt{102}}=800\sqrt{3}+196\sqrt{51},$$

We have that:

$[800\sqrt{3} + 196\sqrt{51}]$

Input:

$$800\sqrt{3} + 196\sqrt{51}$$

Decimal approximation:

2785.360618049500434429839436232681888192413750389240333753...

Alternate forms:

$$4(200\sqrt{3} + 49\sqrt{51})$$

$$4\sqrt{3}(200 + 49\sqrt{17})$$

$$4\sqrt{3(80817 + 19600\sqrt{17})}$$

and:

$\exp(1/4\pi\sqrt{102})$

Input:

$$\exp\left(\frac{1}{4}\pi\sqrt{102}\right)$$

Exact result:

$$e^{1/2\sqrt{51/2}\pi}$$

Decimal approximation:

2785.360618048297258720619547762375886413771158374418011952...

Property:

$e^{1/2\sqrt{51/2}\pi}$  is a transcendental number

Series representations:

$$e^{(\pi\sqrt{102})/4} = e^{1/4\pi\sqrt{101} \sum_{k=0}^{\infty} 101^{-k} \binom{1/2}{k}}$$

$$e^{(\pi\sqrt{102})/4} = \exp\left(\frac{1}{4}\pi\sqrt{101} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{101}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{(\pi \sqrt{102})/4} = \exp \left( \frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 101^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}{8 \sqrt{\pi}} \right)$$

Integral representation:

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(\alpha) \text{ and } |\arg(z)| < \pi)$$

The result 2785,36 is very near to the value of rest mass of charmed Omega baryon  $2765.9 \pm 2.0$ . Note that the number divided by  $10^3$  is equal to 2,78536, value very near to the proton magnetic  $\mu_P$   $1.410607 \times 10^{-26} \text{ JT}^{-1}$  moment  $2.792847 \mu_N$

$$1/2 * [800\sqrt{3} + 196\sqrt{51}]$$

Input:

$$\frac{1}{2} (800 \sqrt{3} + 196 \sqrt{51})$$

Decimal approximation:

1392.680309024750217214919718116340944096206875194620166876...

Alternate forms:

$$2(200\sqrt{3} + 49\sqrt{51})$$

$$400\sqrt{3} + 98\sqrt{51}$$

$$2\sqrt{3}(200 + 49\sqrt{17})$$

The result 1392,68 is very near to the rest mass of Sigma baryon  $1387.2 \pm 0.5$

$$((\ln [800\sqrt{3} + 196\sqrt{51}]))^{1/4}$$

Input:

$$\sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})}$$

Decimal approximation:

1.678214587630776290450907843147794577584621231779518438476...

- $\log(x)$  is the natural logarithm

Property:

$\sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})}$  is a transcendental number

Alternate forms:

$$\sqrt[4]{\log(4\sqrt{3}(200 + 49\sqrt{17}))}$$

$$\sqrt[4]{\log(4\sqrt{3}(80817 + 19600\sqrt{17}))}$$

$$\sqrt[4]{\frac{1}{2}(4\log(2) + \log(3) + 2\log(200 + 49\sqrt{17}))}$$

Alternative representations:

$$\sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = \sqrt[4]{\log_e(800\sqrt{3} + 196\sqrt{51})}$$

$$\sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = \sqrt[4]{\log(a)\log_a(800\sqrt{3} + 196\sqrt{51})}$$

$$\sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = \sqrt[4]{-\text{Li}_1(1 - 800\sqrt{3} - 196\sqrt{51})}$$

Series representations:

$$\sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = \sqrt[4]{\log(-1 + 800\sqrt{3} + 196\sqrt{51}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-800\sqrt{3}-196\sqrt{51}}\right)^k}{k}}$$

$$\sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = \left(2i\pi \left\lfloor \frac{\arg(800\sqrt{3} + 196\sqrt{51} - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (800\sqrt{3} + 196\sqrt{51} - x)^k x^{-k}}{k} \right)^{(1/4)} \text{ for } x < 0$$

$$\sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = \left( \log(z_0) + \left[ \frac{\arg(800\sqrt{3} + 196\sqrt{51} - z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (800\sqrt{3} + 196\sqrt{51} - z_0)^k z_0^{-k}}{k} \right)^{1/4}$$

Integral representations:

$$\sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = \sqrt[4]{\int_1^{800\sqrt{3} + 196\sqrt{51}} \frac{1}{t} dt}$$

$$\sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = \frac{\sqrt[4]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+800\sqrt{3} + 196\sqrt{51})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}}{\sqrt[4]{2\pi}}$$

for  $-1 < \gamma < 0$

The result 1,67821458... is a good approximation to the fourteenth root of Ramanujan's class invariant 1164.2696, (that is 1,65578), to the numerical result for  $\theta_{(2)}$  as a function of  $\theta_{(0)}$  1,6557 for the D7-brane in  $AdS_2 \times S^2$ -sliced thermal  $AdS_5$  and very near to the mass of the neutron.

While:

$$10^3 ((\ln [800\sqrt{3} + 196\sqrt{51}]))^{1/4}$$

Input:

$$10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})}$$

- $\log(x)$  is the natural logarithm

Exact result:

$$1000 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})}$$

Decimal approximation:

1678.214587630776290450907843147794577584621231779518438476...

Property:

$$1000 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} \text{ is a transcendental number}$$

Alternate forms:



$$1000 \sqrt[4]{\log\left(4\sqrt{3}\left(200+49\sqrt{17}\right)\right)}$$

$$1000 \sqrt[4]{\log\left(4\sqrt{3}\left(80817+19600\sqrt{17}\right)\right)}$$

$$500 \times 2^{3/4} \sqrt[4]{4\log(2)+\log(3)+2\log\left(200+49\sqrt{17}\right)}$$

Alternative representations:

$$10^3 \sqrt[4]{\log\left(800\sqrt{3}+196\sqrt{51}\right)} = 10^3 \sqrt[4]{\log_e\left(800\sqrt{3}+196\sqrt{51}\right)}$$

$$10^3 \sqrt[4]{\log\left(800\sqrt{3}+196\sqrt{51}\right)} = 10^3 \sqrt[4]{\log(a)\log_a\left(800\sqrt{3}+196\sqrt{51}\right)}$$

$$10^3 \sqrt[4]{\log\left(800\sqrt{3}+196\sqrt{51}\right)} = 10^3 \sqrt[4]{-\text{Li}_1\left(1-800\sqrt{3}-196\sqrt{51}\right)}$$

Series representation:

$$10^3 \sqrt[4]{\log\left(800\sqrt{3}+196\sqrt{51}\right)} =$$

$$1000 \sqrt[4]{\log\left(-1+800\sqrt{3}+196\sqrt{51}\right) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-800\sqrt{3}-196\sqrt{51}}\right)^k}{k}}$$

$$10^3 \sqrt[4]{\log\left(800\sqrt{3}+196\sqrt{51}\right)} = 1000 \left( 2i\pi \left[ \frac{\arg(800\sqrt{3}+196\sqrt{51}-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (800\sqrt{3}+196\sqrt{51}-x)^k x^{-k}}{k} \right)^{\wedge (1/4)} \text{ for } x < 0$$

$$10^3 \sqrt[4]{\log\left(800\sqrt{3}+196\sqrt{51}\right)} =$$

$$1000 \left( \log(z_0) + \left[ \frac{\arg(800\sqrt{3}+196\sqrt{51}-z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (800\sqrt{3}+196\sqrt{51}-z_0)^k z_0^{-k}}{k} \right)^{\wedge (1/4)}$$

Integral representations:

$$10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = 1000 \sqrt[4]{\int_1^{800\sqrt{3} + 196\sqrt{51}} \frac{1}{t} dt}$$

$$10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} =$$

$$\frac{500 \times 2^{3/4} \sqrt[4]{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+800\sqrt{3} + 196\sqrt{51})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}}{\sqrt[4]{\pi}} \quad \text{for } -1 < \gamma < 0$$

The result 1678,21458 is very near to the rest mass of Omega baryon  $1672.45 \pm 0.29$

$$(48) + 10^3 ((\ln [800\sqrt{3} + 196\sqrt{51}]))^{1/4}$$

Input:

$$48 + 10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})}$$

- $\log(x)$  is the natural logarithm

Exact result:

$$48 + 1000 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})}$$

Decimal approximation:

1726.214587630776290450907843147794577584621231779518438476...

Property:

$48 + 1000 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})}$  is a transcendental number

Alternate forms:

$$48 + 1000 \sqrt[4]{\log\left(4\sqrt{3}\left(80817 + 19600\sqrt{17}\right)\right)}$$

$$8\left(6 + 125 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})}\right)$$

$$8\left(6 + 125 \sqrt[4]{\log\left(4\sqrt{3}\left(200 + 49\sqrt{17}\right)\right)}\right)$$

Alternative representations:

$$48 + 10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = 48 + 10^3 \sqrt[4]{\log_e(800\sqrt{3} + 196\sqrt{51})}$$

$$48 + 10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = 48 + 10^3 \sqrt[4]{\log(a) \log_a(800\sqrt{3} + 196\sqrt{51})}$$

$$48 + 10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = 48 + 10^3 \sqrt[4]{-\text{Li}_1(1 - 800\sqrt{3} - 196\sqrt{51})}$$

Series representations:

$$48 + 10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} =$$

$$48 + 1000 \sqrt[4]{\log(-1 + 800\sqrt{3} + 196\sqrt{51}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-800\sqrt{3}-196\sqrt{51}}\right)^k}{k}}$$

$$48 + 10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} =$$

$$48 + 1000 \left( 2i\pi \left[ \frac{\arg(800\sqrt{3} + 196\sqrt{51} - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (800\sqrt{3} + 196\sqrt{51} - x)^k x^{-k}}{k} \right)^{\wedge (1/4)} \text{ for } x < 0$$

$$48 + 10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} =$$

$$48 + 1000 \left( \log(z_0) + \left[ \frac{\arg(800\sqrt{3} + 196\sqrt{51} - z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (800\sqrt{3} + 196\sqrt{51} - z_0)^k z_0^{-k}}{k} \right)^{\wedge (1/4)}$$

Integral representations:

$$48 + 10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} = 48 + 1000 \sqrt[4]{\int_1^{800\sqrt{3}+196\sqrt{51}} \frac{1}{t} dt}$$

$$48 + 10^3 \sqrt[4]{\log(800\sqrt{3} + 196\sqrt{51})} =$$

$$48 + \frac{500 \times 2^{3/4} \sqrt[4]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+800\sqrt{3}+196\sqrt{51})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}}{\sqrt[4]{\pi}} \text{ for } -1 < \gamma < 0$$

The result 1726.21458 is very near to the value of the mass of the candidate “glueball”  $f_0(1710)$  that is 1723 (+ 6 – 5).

$$12 + ((1/(\text{Pi} \cdot 1.08643^2)) [800\sqrt{3} + 196\sqrt{51}])$$

Input interpretation:

$$12 + \frac{1}{\pi \times 1.08643^2} (800 \sqrt{3} + 196 \sqrt{51})$$

Result:

763.152...

Series representations:

$$12 + \frac{800 \sqrt{3} + 196 \sqrt{51}}{\pi \cdot 1.08643^2} = 12 + \sum_{k=0}^{\infty} \frac{677.776 \times 50^{-k} \binom{\frac{1}{2}}{k} (25^k \sqrt{2} + 0.245 \sqrt{50})}{\pi}$$

$$12 + \frac{800 \sqrt{3} + 196 \sqrt{51}}{\pi \cdot 1.08643^2} = 12 + \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)_k \left(677.776 \left(-\frac{1}{2}\right)^k \sqrt{2} + 166.055 \left(-\frac{1}{50}\right)^k \sqrt{50}\right)}{\pi k!}$$

$$12 + \frac{800 \sqrt{3} + 196 \sqrt{51}}{\pi \cdot 1.08643^2} = 12 + \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \sqrt{z_0} \left(677.776 (3 - z_0)^k + 166.055 (51 - z_0)^k\right) z_0^{-k}}{\pi k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The result 763.152 is very near to the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for  $m_q = 4.7 \text{ MeV}/c^2 = 0.0047 \text{ GeV}/c^2$ , that is the mass of quark down is  $4.8 \pm 0.5 \pm 0.3 = 4.7 \text{ MeV}/c^2$ .

And also:

$$1/(\sqrt{13}) * [800\sqrt{3} + 196\sqrt{51}]$$

Input:

$$\frac{1}{\sqrt{13}} (800 \sqrt{3} + 196 \sqrt{51})$$

Result:

$$\frac{800 \sqrt{3} + 196 \sqrt{51}}{\sqrt{13}}$$

Decimal approximation:

772.5200406950416775940358804749682174548274875382908900248...

Alternate forms:

$$\frac{4}{13} (200 \sqrt{39} + 49 \sqrt{663})$$

$$\sqrt{\frac{3879216}{13} + \frac{940800 \sqrt{17}}{13}}$$

$$4 \sqrt{\frac{3}{13}} (200 + 49 \sqrt{17})$$

The result 772,52 is very near to the rest mass of the charged Rho meson  $775.4 \pm 0.4$  and is very near to the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for  $m_q = 4.7 \text{ MeV}/c^2 = 0.0047 \text{ GeV}/c^2$ , that is the mass of quark down is  $4.8 \pm 0.5 \pm 0.3 = 4.7 \text{ MeV}/c^2$ .

$$1/1739 [800\text{sqrt}(3) + 196\text{sqrt}(51)]$$

Input:

$$\frac{1}{1739} (800 \sqrt{3} + 196 \sqrt{51})$$

Result:

$$\frac{800 \sqrt{3} + 196 \sqrt{51}}{1739}$$

Decimal approximation:

1.601702483064692601742288347459851574578731311322162354084...

Alternate forms:

$$\frac{4(200 \sqrt{3} + 49 \sqrt{51})}{1739}$$

$$\sqrt{\frac{3879216}{3024121} + \frac{940800 \sqrt{17}}{3024121}}$$

$$\frac{4 \sqrt{3} (200 + 49 \sqrt{17})}{1739}$$

The result 1,60170 is very near to the value of electric charge of the positron and 1,60105 is the 15-th root of Ramanujan class invariant 1164,2696. Indeed, we have:

$$\sqrt[15]{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3} = 1,601045117 \dots$$

Pi/729 [800sqrt(3) + 196sqrt(51)]

Input:

$$\frac{\pi}{729} (800 \sqrt{3} + 196 \sqrt{51})$$

Result:

$$\frac{1}{729} (800 \sqrt{3} + 196 \sqrt{51}) \pi$$

Decimal approximation:

12.00338608403653856561154538042653658719596537727264509177...

Property:

$$\frac{1}{729} (800 \sqrt{3} + 196 \sqrt{51}) \pi \text{ is a transcendental number}$$

The result 12,003386 is very near to the value of the black hole entropy 12,19

2Pi/729 [800sqrt(3) + 196sqrt(51)]

Input:

$$2 \times \frac{\pi}{729} (800 \sqrt{3} + 196 \sqrt{51})$$

Result:

$$\frac{2}{729} (800 \sqrt{3} + 196 \sqrt{51}) \pi$$

Decimal approximation:

24.00677216807307713122309076085307317439193075454529018354...

Property:

$$\frac{2}{729} (800 \sqrt{3} + 196 \sqrt{51}) \pi \text{ is a transcendental number}$$

The result  $24,006 \approx 24$  represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates

$$\left(\frac{2}{\exp(1.61404238^4)}\right) [800\sqrt{3} + 196\sqrt{51}]$$

Input interpretation:

$$\frac{2}{\exp(1.61404238^4)} (800\sqrt{3} + 196\sqrt{51})$$

Result:

6.2874959...

Series representations:

$$\frac{(800\sqrt{3} + 196\sqrt{51}) 2}{\exp(1.61404^4)} = \sum_{k=0}^{\infty} \frac{2^{3-k} \times 25^{-k} \binom{\frac{1}{2}}{k} (8 \times 25^{1+k} \sqrt{2} + 49 \sqrt{50})}{\exp(6.78672)}$$

$$\frac{(800\sqrt{3} + 196\sqrt{51}) 2}{\exp(1.61404^4)} = \sum_{k=0}^{\infty} \frac{2^{3-k} \left(-\frac{1}{2}\right)_k (200 (-1)^k \sqrt{2} + 49 \left(-\frac{1}{25}\right)^k \sqrt{50})}{\exp(6.78672) k!}$$

$$\frac{(800\sqrt{3} + 196\sqrt{51}) 2}{\exp(1.61404^4)} = \sum_{k=0}^{\infty} \frac{8 (-1)^k \left(-\frac{1}{2}\right)_k \sqrt{z_0} (200 (3 - z_0)^k + 49 (51 - z_0)^k) z_0^{-k}}{\exp(6.78672) k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The result 6,2874959... is very near to the length of a circle  $C = 2\pi r$  with  $r = 1$

$$\left(\frac{2}{\exp(1.61091^4)}\right) [800\sqrt{3} + 196\sqrt{51}]$$

Input interpretation:

$$\frac{2}{\exp(1.61091^4)} (800\sqrt{3} + 196\sqrt{51})$$

Result:

6.62661...

Series representations:

$$\frac{(800\sqrt{3} + 196\sqrt{51}) 2}{\exp(1.61091^4)} = \sum_{k=0}^{\infty} \frac{2^{3-k} \times 25^{-k} \binom{\frac{1}{2}}{k} (8 \times 25^{1+k} \sqrt{2} + 49 \sqrt{50})}{\exp(6.73419)}$$

$$\frac{(800\sqrt{3} + 196\sqrt{51})2}{\exp(1.61091^4)} = \sum_{k=0}^{\infty} \frac{2^{3-k} \left(-\frac{1}{2}\right)_k \left(200(-1)^k \sqrt{2} + 49\left(-\frac{1}{25}\right)^k \sqrt{50}\right)}{\exp(6.73419) k!}$$

$$\frac{(800\sqrt{3} + 196\sqrt{51})2}{\exp(1.61091^4)} = \sum_{k=0}^{\infty} \frac{8(-1)^k \left(-\frac{1}{2}\right)_k \sqrt{z_0} \left(200(3-z_0)^k + 49(51-z_0)^k\right) z_0^{-k}}{\exp(6.73419) k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The result 6,62661... is practically equal to the value of Planck's constant that is  $h = 6.626\ 070\ 15 \times 10^{-34}$  J·s

Now, we have calculate various integrals from the expression of the right-hand side of the Ramanujan modular equation:

$$e^{\frac{1}{4}\pi\sqrt{102}} = 800\sqrt{3} + 196\sqrt{51},$$

We have that:

integrate  $[\exp(1/4\text{Pi}*\text{sqrt}(102))]x$

$$\int \exp\left(\frac{\pi\sqrt{102}}{4}\right) x dx \approx \text{constant} + 1392.68 x^2$$

integrate  $[((\text{Pi}^2)/5) \exp(1/4\text{Pi}*\text{sqrt}(102))]x$

$$\int \frac{1}{5} \left(\pi^2 \exp\left(\frac{\pi\sqrt{102}}{4}\right)\right) x dx \approx \text{constant} + 2749.04 x^2$$

integrate  $[((\text{Pi}^2)/8) \exp(1/4\text{Pi}*\text{sqrt}(102))]x$

$$\int \frac{1}{8} \left(\pi^2 \exp\left(\frac{\pi\sqrt{102}}{4}\right)\right) x dx \approx \text{constant} + 1718.15 x^2$$

integrate  $[((\text{Pi}^2)/18) \exp(1/4\text{Pi}*\text{sqrt}(102))]x$

$$\int \frac{1}{18} \left(\pi^2 \exp\left(\frac{\pi\sqrt{102}}{4}\right)\right) x dx \approx \text{constant} + 763.622 x^2$$

integrate  $[((\text{Pi}^2)/108) \exp(1/4\text{Pi}*\text{sqrt}(102))]x$



$$\int \frac{1}{108} \left( \pi^2 \exp\left(\frac{\pi \sqrt{102}}{4}\right) \right) x dx \approx \text{constant} + 127.27 x^2$$

integrate  $[((\text{Pi}^2)/10) \exp(1/4\text{Pi}*\text{sqrt}(102))]x$

$$\int \frac{1}{10} \left( \pi^2 \exp\left(\frac{\pi \sqrt{102}}{4}\right) \right) x dx \approx \text{constant} + 1374.52 x^2$$

integrate  $[((\text{Pi}^2)/9) \exp(1/4\text{Pi}*\text{sqrt}(102))]x$

$$\int \frac{1}{9} \left( \pi^2 \exp\left(\frac{\pi \sqrt{102}}{4}\right) \right) x dx \approx \text{constant} + 1527.24 x^2$$

integrate  $[((\text{Pi}^3)/8) \exp(1/4\text{Pi}*\text{sqrt}(102))]x$

$$\int \frac{1}{8} \left( \pi^3 \exp\left(\frac{\pi \sqrt{102}}{4}\right) \right) x dx \approx \text{constant} + 5397.73 x^2$$

All the results of the integrals are good approximations to the following rest mass of baryons and/or mesons:

1382.8±0.4 1387.2±0.5 (Sigma baryon) 1531.80±0.32 (Xi baryon) 2765.9±2.0 (charmed Omega baryon) 5412.8±1.3 (Strange B meson) 775.4±0.4 (Charged Rho meson). The mass of Higgs boson is 125.18 ± 0.16

Furthermore, the result 763,622 is very near to the rest mass of the charged Rho meson 775.4±0.4 and is very near to the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for  $m_q = 4.7 \text{ MeV}/c^2 = 0.0047 \text{ GeV}/c^2$ , that is the mass of quark down is  $4.8 \pm 0.5 \pm 0.33 = 4.7 \text{ MeV}/c^2$ . The result 1718,15 is very near to the value of the mass of the candidate “glueball”  $f_0(1710)$  that is 1723 (+ 6 – 5).

Now, we have analyzed the following Ramanujan modular equation:

$$e^{\frac{1}{4}\pi\sqrt{78}} = 300\sqrt{3} + 208\sqrt{6},$$

$\exp(\text{Pi}/4(\text{sqrt}(78)))$

$$\exp\left(\frac{\pi}{4}\sqrt{78}\right)$$

Exact result:

$$e^{1/2\sqrt{39/2}\pi}$$

Decimal approximation:

1029.109108745708701845208873263603669484774707500189796766...

The result 1029,109 is very near to the rest mass of Phi meson  $1019.445\pm 0.020$

Property:

$e^{1/2\sqrt{39/2}\pi}$  is a transcendental number

Series representations:

$$e^{(\sqrt{78}\pi)/4} = e^{1/4\pi\sqrt{77}\sum_{k=0}^{\infty} 77^{-k}\binom{1/2}{k}}$$

$$e^{(\sqrt{78}\pi)/4} = \exp\left(\frac{1}{4}\pi\sqrt{77}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{77}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{(\sqrt{78}\pi)/4} = \exp\left(\frac{\pi\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}77^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{8\sqrt{\pi}}\right)$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\Gamma(-a-s)}{z^s}ds}{(2\pi i)\Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$$[300\sqrt{3} + 208\sqrt{6}]$$

Input:

$$300\sqrt{3} + 208\sqrt{6}$$

Decimal approximation:

1029.109108769564232483268989990587119611758652119701575130...

Alternate forms:

$$4(75\sqrt{3} + 52\sqrt{6})$$

$$4\sqrt{3} (75 + 52\sqrt{2})$$

$$4\sqrt{3(11033 + 7800\sqrt{2})}$$

The result 1029,109 is very near to the rest mass of Phi meson  $1019.445 \pm 0.020$

$$((((\pi/2 [300\sqrt{3} + 208\sqrt{6}] + 108))))$$

Input:

$$\frac{\pi}{2} (300\sqrt{3} + 208\sqrt{6}) + 108$$

Result:

$$108 + \frac{1}{2} (300\sqrt{3} + 208\sqrt{6})\pi$$

Decimal approximation:

1724.520807926401228386813904373678080283400468201726868621...

Property:

$$108 + \frac{1}{2} (300\sqrt{3} + 208\sqrt{6})\pi \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{2} (\pi (300\sqrt{3} + 208\sqrt{6}) + 216)$$

$$108 + 150\sqrt{3}\pi + 104\sqrt{6}\pi$$

$$108 + 2\sqrt{3} (75 + 52\sqrt{2})\pi$$

Continued fraction:

$$\begin{array}{r}
 1724 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{11 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{16 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{13 + \cfrac{1}{101 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{29 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}} \\
 \end{array}$$

Series representations:

$$\frac{1}{2} \left( 300 \sqrt{3} + 208 \sqrt{6} \right) \pi + 108 = 108 + \sum_{k=0}^{\infty} 2 \pi \binom{\frac{1}{2}}{k} \left( 75 \times 2^{-k} \sqrt{2} + 52 \times 5^{-k} \sqrt{5} \right)$$

$$\begin{aligned}
 \frac{1}{2} \left( 300 \sqrt{3} + 208 \sqrt{6} \right) \pi + 108 = \\
 108 + \sum_{k=0}^{\infty} \frac{2^{1-k} \pi \left(-\frac{1}{2}\right)_k \left( 75 (-1)^k \sqrt{2} + 13 \left(-\frac{1}{5}\right)^k 2^{2+k} \sqrt{5} \right)}{k!}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{2} \left( 300 \sqrt{3} + 208 \sqrt{6} \right) \pi + 108 = \\
 108 + \sum_{k=0}^{\infty} \frac{2 (-1)^k \pi \left(-\frac{1}{2}\right)_k \sqrt{z_0} \left( 75 (3 - z_0)^k + 52 (6 - z_0)^k \right) z_0^{-k}}{k!}
 \end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The result 1724,5208 is very near to the value of the mass of the candidate “glueball”  $f_0(1710)$  that is 1723 (+ 6 – 5).

$$1/72 \left( \left( \left( \left( \left( \left( \pi/2 \left[ 300\text{sqrt}(3) + 208\text{sqrt}(6) \right] + 108 \right) \right) \right) \right) \right) \right)$$

Input:

$$\frac{1}{72} \left( \frac{\pi}{2} \left( 300 \sqrt{3} + 208 \sqrt{6} \right) + 108 \right)$$

Result:

$$\frac{1}{72} \left( 108 + \frac{1}{2} (300 \sqrt{3} + 208 \sqrt{6}) \pi \right)$$

Decimal approximation:

23.95167788786668372759463756074552889282500650280176206419...

Property:

$$\frac{1}{72} \left( 108 + \frac{1}{2} (300 \sqrt{3} + 208 \sqrt{6}) \pi \right) \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{144} \left( \pi (300 \sqrt{3} + 208 \sqrt{6}) + 216 \right)$$

$$\frac{1}{36} (54 + 75 \sqrt{3} \pi + 52 \sqrt{6} \pi)$$

$$\frac{3}{2} + \frac{(75 + 52 \sqrt{2}) \pi}{12 \sqrt{3}}$$

Continued fraction:

Linear form

$$\begin{aligned} & 23 + \cfrac{1}{1 + \cfrac{1}{19 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{50 + \cfrac{1}{11 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{12 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{10 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}} \end{aligned}$$

Series representations:

$$\frac{1}{72} \left( \frac{1}{2} \pi (300 \sqrt{3} + 208 \sqrt{6}) + 108 \right) = \frac{3}{2} + \sum_{k=0}^{\infty} \frac{1}{36} \pi \binom{\frac{1}{2}}{k} (75 \times 2^{-k} \sqrt{2} + 52 \times 5^{-k} \sqrt{5})$$

$$\frac{1}{72} \left( \frac{1}{2} \pi (300 \sqrt{3} + 208 \sqrt{6}) + 108 \right) = \frac{3}{2} + \sum_{k=0}^{\infty} \frac{2^{-2-k} \pi \left(-\frac{1}{2}\right)_k (75 (-1)^k \sqrt{2} + 13 \left(-\frac{1}{5}\right)^k 2^{2+k} \sqrt{5})}{9 k!}$$

$$\frac{1}{72} \left( \frac{1}{2} \pi (300 \sqrt{3} + 208 \sqrt{6}) + 108 \right) = \frac{3}{2} + \sum_{k=0}^{\infty} \frac{(-1)^k \pi \left(-\frac{1}{2}\right)_k \sqrt{z_0} (75 (3 - z_0)^k + 52 (6 - z_0)^k) z_0^{-k}}{36 k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The result 23,95167... $\approx$  24, represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates

$$1/142 \left( \left( \left( \left( \left( \pi/2 [300\sqrt{3} + 208\sqrt{6}] + 108 \right) \right) \right) \right) \right)$$

Input:

$$\frac{1}{142} \left( \frac{\pi}{2} (300 \sqrt{3} + 208 \sqrt{6}) + 108 \right)$$

Result:

$$\frac{1}{142} \left( 108 + \frac{1}{2} (300 \sqrt{3} + 208 \sqrt{6}) \pi \right)$$

Decimal approximation:

12.14451273187606498863953453784280338227746808592765400437...

Property:

$$\frac{1}{142} \left( 108 + \frac{1}{2} (300 \sqrt{3} + 208 \sqrt{6}) \pi \right) \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{284} \left( \pi (300 \sqrt{3} + 208 \sqrt{6}) + 216 \right)$$

$$\frac{1}{71} (54 + 75 \sqrt{3} \pi + 52 \sqrt{6} \pi)$$

$$\frac{54}{71} + \frac{75 \sqrt{3} \pi}{71} + \frac{52 \sqrt{6} \pi}{71}$$

Continued fraction:

Linear form

$$\begin{array}{l}
 12 + \frac{1}{\dots} \\
 6 + \frac{1}{\dots} \\
 1 + \frac{1}{\dots} \\
 11 + \frac{1}{\dots} \\
 2 + \frac{1}{\dots} \\
 7 + \frac{1}{\dots} \\
 1 + \frac{1}{\dots} \\
 2 + \frac{1}{\dots} \\
 1 + \frac{1}{\dots} \\
 21 + \frac{1}{\dots} \\
 8 + \frac{1}{\dots} \\
 1 + \frac{1}{\dots} \\
 4 + \frac{1}{\dots} \\
 1 + \frac{1}{\dots} \\
 5 + \frac{1}{\dots} \\
 4 + \frac{1}{\dots} \\
 1 + \frac{1}{\dots} \\
 8 + \frac{1}{\dots} \\
 6 + \frac{1}{\dots} \\
 2 + \frac{1}{\dots} \\
 122 + \frac{1}{\dots}
 \end{array}$$

Series representations:

$$\begin{aligned}
 & \frac{1}{142} \left( \frac{1}{2} \pi (300 \sqrt{3} + 208 \sqrt{6}) + 108 \right) = \\
 & \frac{54}{71} + \sum_{k=0}^{\infty} \frac{1}{71} \times 10^{-k} \pi \binom{\frac{1}{2}}{k} \left( 3 \times 5^{2+k} \sqrt{2} + 13 \times 2^{2+k} \sqrt{5} \right)
 \end{aligned}$$

$$\frac{1}{142} \left( \frac{1}{2} \pi (300 \sqrt{3} + 208 \sqrt{6}) + 108 \right) = \frac{54}{71} + \sum_{k=0}^{\infty} \frac{\pi \left(-\frac{1}{2}\right)_k \left(75 \left(-\frac{1}{2}\right)^k \sqrt{2} + 52 \left(-\frac{1}{5}\right)^k \sqrt{5}\right)}{71 k!}$$

$$\begin{aligned}
 & \frac{1}{142} \left( \frac{1}{2} \pi (300 \sqrt{3} + 208 \sqrt{6}) + 108 \right) = \\
 & \frac{54}{71} + \sum_{k=0}^{\infty} \frac{(-1)^k \pi \left(-\frac{1}{2}\right)_k \sqrt{z_0} \left(75 (3 - z_0)^k + 52 (6 - z_0)^k\right) z_0^{-k}}{71 k!}
 \end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The result 12,144512 is very near to the value of the black hole entropy 12,19

$$32 \frac{1}{72} \left( \left( \left( \left( \frac{\pi}{2} \left[ 300\sqrt{3} + 208\sqrt{6} \right] + 108 \right) \right) \right) \right)$$

Input:

$$32 \times \frac{1}{72} \left( \frac{\pi}{2} \left( 300 \sqrt{3} + 208 \sqrt{6} \right) + 108 \right)$$

Result:

$$\frac{4}{9} \left( 108 + \frac{1}{2} \left( 300 \sqrt{3} + 208 \sqrt{6} \right) \pi \right)$$

Decimal approximation:

766.4536924117338792830284019438569245704002080896563860540...

Property:

$\frac{4}{9} \left( 108 + \frac{1}{2} \left( 300 \sqrt{3} + 208 \sqrt{6} \right) \pi \right)$  is a transcendental number

Alternate forms:

$$\frac{2}{9} \left( \pi \left( 300 \sqrt{3} + 208 \sqrt{6} \right) + 216 \right)$$

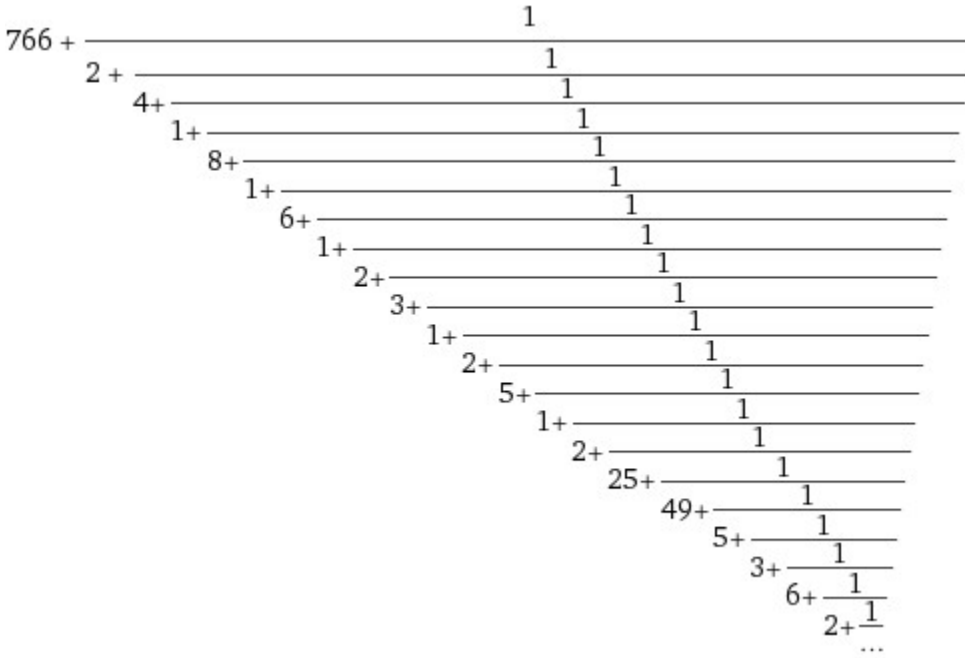
$$48 + \frac{416}{3} \sqrt{\frac{2}{3}} \pi + \frac{200 \pi}{\sqrt{3}}$$

$$48 + \frac{8(75 + 52\sqrt{2})\pi}{3\sqrt{3}}$$

Continued fraction:

Linear form





Series representations:

$$\frac{32}{72} \left( \frac{1}{2} \pi (300 \sqrt{3} + 208 \sqrt{6}) + 108 \right) = 48 + \sum_{k=0}^{\infty} \frac{8}{9} \pi \left( \frac{1}{k} \right) (75 \times 2^{-k} \sqrt{2} + 52 \times 5^{-k} \sqrt{5})$$

$$\frac{32}{72} \left( \frac{1}{2} \pi (300 \sqrt{3} + 208 \sqrt{6}) + 108 \right) = 48 + \sum_{k=0}^{\infty} \frac{2^{3-k} \pi \left(-\frac{1}{2}\right)_k (75 (-1)^k \sqrt{2} + 13 \left(-\frac{1}{5}\right)^k 2^{2+k} \sqrt{5})}{9 k!}$$

$$\frac{32}{72} \left( \frac{1}{2} \pi (300 \sqrt{3} + 208 \sqrt{6}) + 108 \right) = 48 + \sum_{k=0}^{\infty} \frac{8 (-1)^k \pi \left(-\frac{1}{2}\right)_k \sqrt{z_0} (75 (3 - z_0)^k + 52 (6 - z_0)^k) z_0^{-k}}{9 k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The value 766,4536 is very near to the rest mass of the charged Rho meson  $775.4 \pm 0.4$  and to the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for  $m_q = 4.7 \text{ MeV}/c^2 = 0.0047 \text{ GeV}/c^2$ , that is the mass of quark down is  $4.8 \pm 0.5 \pm 0.3 = 4.7 \text{ MeV}/c^2$

$$(19 \cdot \pi^2) / (1712) \left( \frac{300 \sqrt{3} + 208 \sqrt{6} + 108}{9} \right)$$

Input:

$$\frac{19\pi^2}{1712} \left( (300\sqrt{3} + 208\sqrt{6}) + 108 \right)$$

Result:

$$\frac{19(108 + 300\sqrt{3} + 208\sqrt{6})\pi^2}{1712}$$

Decimal approximation:

124.5522921870250569630044946016967215536015950539717516484...

Property:

$$\frac{19(108 + 300\sqrt{3} + 208\sqrt{6})\pi^2}{1712} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{513\pi^2}{428} + \frac{1425\pi^2\sqrt{3}}{428} + \frac{247\pi^2\sqrt{6}}{107}$$

$$\frac{19}{428} (27 + 75\sqrt{3} + 52\sqrt{6})\pi^2$$

$$\frac{19}{428} \left( 27 + \sqrt{3(11033 + 7800\sqrt{2})} \right) \pi^2$$

Continued fraction:

Linear form

$$\begin{aligned}
& 124 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6 + \frac{1}{11 + \frac{1}{22 + \frac{1}{2 + \frac{1}{7 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{58 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}} \\
\end{aligned}$$

Series representations:

$$\begin{aligned}
& \frac{\left( (300\sqrt{3} + 208\sqrt{6}) + 108 \right) (19\pi^2)}{1712} = \\
& \frac{513\pi^2}{428} + \sum_{k=0}^{\infty} \frac{19}{107} \times 2^{-2-k} \times 5^{-k} \pi^2 \binom{\frac{1}{2}}{k} \left( 3 \times 5^{2+k} \sqrt{2} + 13 \times 2^{2+k} \sqrt{5} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\left( (300\sqrt{3} + 208\sqrt{6}) + 108 \right) (19\pi^2)}{1712} = \\
& \frac{513\pi^2}{428} + \sum_{k=0}^{\infty} \frac{19 \times 2^{-2-k} \pi^2 \left(-\frac{1}{2}\right)_k \left( 75 (-1)^k \sqrt{2} + 13 \left(-\frac{1}{5}\right)^k 2^{2+k} \sqrt{5} \right)}{107 k!}
\end{aligned}$$

$$\begin{aligned}
& \frac{\left( (300\sqrt{3} + 208\sqrt{6}) + 108 \right) (19\pi^2)}{1712} = \\
& \frac{513\pi^2}{428} + \sum_{k=0}^{\infty} \frac{19 (-1)^k \pi^2 \left(-\frac{1}{2}\right)_k \sqrt{z_0} \left( 75 (3 - z_0)^k + 52 (6 - z_0)^k \right) z_0^{-k}}{428 k!}
\end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The value 124,552 is very near to the mass of the Higgs boson  $125.18 \pm 0.16$

Now, we have the following Ramanujan modular equation:

$$e^{\frac{1}{2}\pi\sqrt{46}} = 144(147 + 104\sqrt{2})$$

We have that:

$$\exp(((\text{Pi}/2)*\text{sqrt}(46)))$$

Input:

$$\exp\left(\frac{\pi}{2} \sqrt{46}\right)$$

Exact result:

$$e^{\sqrt{23/2} \pi}$$

Decimal approximation:

42347.26108215699922855859544926430987516136752699531538765...

Property:

$e^{\sqrt{23/2} \pi}$  is a transcendental number

Continued fraction:

Linear form

$$\begin{array}{r}
 42\ 347 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{15 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{41 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}} \\
 \dots
 \end{array}$$

Series representations:

$$e^{\left(\sqrt{46} \pi\right) / 2} = e^{\frac{1}{2} \pi \sqrt{46}} = \sum_{k=0}^{\infty} \frac{46^{k/2}}{k!}$$

$$e^{(\sqrt{46} \pi)/2} = \exp\left(\frac{1}{2} \pi \sqrt{45} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{45}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{(\sqrt{46} \pi)/2} = \exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 45^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{4 \sqrt{\pi}}\right)$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(a) \text{ and } |\arg(z)| < \pi)$$

(144(147+104sqrt(2)))

Input:

$$144(147 + 104\sqrt{2})$$

Decimal approximation:

42347.26231009947145085409033376443842465940600564517434389...

Alternate form:

$$21168 + 14976\sqrt{2}$$

Minimal polynomial:

$$x^2 - 42336x - 476928$$

Continued fraction:

Linear form

$$42\ 347 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4 + \frac{1}{3 + \frac{1}{17 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{10 + \frac{1}{2 + \frac{1}{4 + \frac{1}{1 + \frac{1}{3 + \frac{1}{6 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}$$

Now:

$$6 * \text{sqrt}(144(147+104\text{sqrt}(2)))$$

Input:

$$6 \sqrt{144 (147 + 104 \sqrt{2})}$$

Result:

$$72 \sqrt{147 + 104 \sqrt{2}}$$

Decimal approximation:

1234.707027259333683683446507396727521024823367827478312985...

Alternate form:

$$36 \sqrt{2} \left( \sqrt{147 - i \sqrt{23}} + \sqrt{i(\sqrt{23} + -147 i)} \right)$$

- Continued fraction:
  - Linear form

$$\begin{array}{c}
 1234 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{4 + \frac{1}{1 + \frac{1}{3 + \frac{1}{14 + \frac{1}{2 + \frac{1}{5 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}} \\
 \dots
 \end{array}$$

The result 1234.70 is very near to the value of the rest mass of the Delta baryon that is 1232±2

$(\pi^3/1.6557)^* (1/5) \left( \left( e^{144(147+104\sqrt{2})} \right)^{0.0029} \right)$  where 0,0029 is  $(0,614515959679805)^{12} = 0,0029$ . Note that  $1/(0,6141595968) = 1,62824126...$

Input interpretation:

$$\frac{\pi^3}{1.6557} \times \frac{1}{5} \left( e^{144(147+104\sqrt{2})} \right)^{0.0029}$$

Result:

$$8.08959... \times 10^{53}$$

Comparisons:

$$\approx \text{the size of the Monster group } (\approx 8.1 \times 10^{53})$$

Series representations:

$$\frac{\left( e^{144(147+104\sqrt{2})} \right)^{0.0029} \pi^3}{5 \times 1.6557} =$$

$$0.120795 \left( \exp \left[ 144 \left( 147 + 104 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (2 - z_0)^k z_0^{-k}}{k!} \right) \right] \right)^{0.0029} \pi^3$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{\left( e^{144(147+104\sqrt{2})} \right)^{0.0029} \pi^3}{5 \times 1.6557} = 0.120795$$

$$\left( \exp \left[ 144 \left( 147 + 104 \exp \left( i \pi \left[ \frac{\arg(2-x)}{2\pi} \right] \right) \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right] \right)^{0.0029}$$

$\pi^3$  for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\left( e^{144(147+104\sqrt{2})} \right)^{0.0029} \pi^3}{5 \times 1.6557} =$$

$$0.120795 \left( \exp \left[ 144 \left( 147 + 104 \left( \frac{1}{z_0} \right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (2-z_0)^k z_0^{-k}}{k!} \right] \right)^{0.0029} \pi^3$$

Or:

$$\left( \frac{\pi^3}{5} \right)^{1/14} \cdot \frac{1}{\left( \left( \sqrt{\frac{113+5\sqrt{505}}{8}} + \sqrt{\frac{105+5\sqrt{505}}{8}} \right)^3 \right)^{1/14}}$$

$$\left( \left( \left( e^{144(147+104\sqrt{2})} \right)^{5/1724.15} \right)^{1/14} \right)$$

Input interpretation:

$$\frac{\pi^3}{5} \times \frac{1}{\sqrt[14]{\left( \sqrt{\frac{1}{8}(113+5\sqrt{505})} + \sqrt{\frac{1}{8}(105+5\sqrt{505})} \right)^3}} \left( e^{144(147+104\sqrt{2})} \right)^{5/1724.15}$$

Or

$$\pi^3 \times \frac{\left( e^{144(147+104\sqrt{2})} \right)^{5/1724.15}}{\sqrt[14]{\left( \sqrt{\frac{1}{8}(113+5\sqrt{505})} + \sqrt{\frac{1}{8}(105+5\sqrt{505})} \right)^3}}$$

Result:

$$8.08223... \times 10^{53}$$

Comparisons:

$$\approx \text{the size of the Monster group } (\approx 8.1 \times 10^{53})$$

$$1728/14386 \text{ integrate integrate } \left[ \left( \frac{\pi^3}{5} \right)^{1/(1.6557)} \left( \left( \left( e^{144(147+104\sqrt{2})} \right)^{5/1724.15} \right)^{1/14} \right) \right] x$$



Input interpretation:

$$\frac{1728}{14386} \int \left( \int \frac{\pi^3}{5} \times \frac{1}{1.6557} \left( \left( e^{144(147+104\sqrt{2})} \right)^{5/1724.15} x \right) dx \right) dx$$

Result:

$$1.6181 \times 10^{52} x^3$$

The result  $1,6181 * 10^{52}$  can be considered a multiple of golden ratio 1.61803398

$$1/(((8\pi-(\sqrt{5}-1)/2))) (144(147+104\sqrt{2}))$$

Input:

$$\frac{1}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} (144(147 + 104\sqrt{2}))$$

Result:

$$\frac{144(147 + 104\sqrt{2})}{\frac{1}{2}(1 - \sqrt{5}) + 8\pi}$$

Decimal approximation:

1727.422722024477654236736075843389327371420633527326905729...

Property:

$$\frac{144(147 + 104\sqrt{2})}{\frac{1}{2}(1 - \sqrt{5}) + 8\pi} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{288(147 + 104\sqrt{2})}{1 - \sqrt{5} + 16\pi}$$

$$-\frac{288(147 + 104\sqrt{2})}{-1 + \sqrt{5} - 16\pi}$$

$$\frac{21168 + 14976\sqrt{2}}{\frac{1}{2}(1 - \sqrt{5}) + 8\pi}$$

Continued fraction:

Linear form

$$1727 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{59 + \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{11 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Series representations:

$$\frac{144(147 + 104\sqrt{2})}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} = \frac{288 \left( 147 + 104\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)}{1 + 16\pi - \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{144(147 + 104\sqrt{2})}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} = \frac{288 \left( 147 + 104 \exp\left(i\pi \left[ \frac{\arg(2-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)}{1 + 16\pi - \exp\left(i\pi \left[ \frac{\arg(5-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{144(147 + 104\sqrt{2})}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} = \frac{288 \left( 147 + 104 \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)]} z_0^{1/2+1/2 [\arg(2-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)}{1 + 16\pi - \left(\frac{1}{z_0}\right)^{1/2 [\arg(5-z_0)/(2\pi)]} z_0^{1/2+1/2 [\arg(5-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}$$

The result 1727.4227 is very near to the value of the mass of the candidate “glueball”  $f_0(1710)$  that is 1723 (+ 6 – 5).

$$1/72 * 1/(((8\pi-(\sqrt{5}-1)/2))) (144(147+104\sqrt{2}))$$

Input:

$$\frac{1}{72} \times \frac{1}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} \left( 144 \left( 147 + 104\sqrt{2} \right) \right)$$

Result:

$$\frac{2(147 + 104\sqrt{2})}{\frac{1}{2}(1 - \sqrt{5}) + 8\pi}$$

Decimal approximation:

23.99198225033996741995466772004707399126973102121287369068...

Property:

$$\frac{2(147 + 104\sqrt{2})}{\frac{1}{2}(1 - \sqrt{5}) + 8\pi} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{4(147 + 104\sqrt{2})}{1 - \sqrt{5} + 16\pi}$$

$$\frac{4(147 + 104\sqrt{2})}{-1 + \sqrt{5} - 16\pi}$$

$$\frac{588 + 416\sqrt{2}}{1 - \sqrt{5} + 16\pi}$$

Continued fraction:  
Linear form





$$1/142 * 1/(((8\pi-(\sqrt{5}-1)/2))) (144(147+104\sqrt{2}))$$

Input:

$$\frac{1}{142} \times \frac{1}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} \left( 144(147 + 104\sqrt{2}) \right)$$

Result:

$$\frac{72(147 + 104\sqrt{2})}{71\left(\frac{1}{2}(1 - \sqrt{5}) + 8\pi\right)}$$

Decimal approximation:

12.16494874665125108617419771720696709416493403892483736428...

Property:

$$\frac{72(147 + 104\sqrt{2})}{71\left(\frac{1}{2}(1 - \sqrt{5}) + 8\pi\right)} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{144(147 + 104\sqrt{2})}{71(1 - \sqrt{5} + 16\pi)}$$

$$\frac{144(147 + 104\sqrt{2})}{71(-1 + \sqrt{5} - 16\pi)}$$

$$\frac{10584 + 7488\sqrt{2}}{71\left(\frac{1}{2}(1 - \sqrt{5}) + 8\pi\right)}$$

Continued fraction:

Linear form

$$\begin{array}{l}
12 + \frac{1}{6 + \frac{1}{16 + \frac{1}{362 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{5 + \frac{1}{4 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{14 + \frac{1}{1 + \frac{1}{5 + \frac{1}{3 + \frac{1}{457 + \frac{1}{\dots}} \\
\end{array}$$

Series representations:

$$\frac{144(147 + 104\sqrt{2})}{(8\pi - \frac{1}{2}(\sqrt{5} - 1))142} = \frac{144 \left( 147 + 104\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)}{71 \left( 1 + 16\pi - \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!} \right)}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{144(147 + 104\sqrt{2})}{(8\pi - \frac{1}{2}(\sqrt{5} - 1))142} = \frac{144 \left( 147 + 104 \exp(i\pi \left\lfloor \frac{\text{arg}(2-x)}{2\pi} \right\rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)}{71 \left( 1 + 16\pi - \exp(i\pi \left\lfloor \frac{\text{arg}(5-x)}{2\pi} \right\rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{144(147 + 104\sqrt{2})}{(8\pi - \frac{1}{2}(\sqrt{5} - 1))142} = \frac{144 \left( 147 + 104 \left(\frac{1}{z_0}\right)^{1/2 \lfloor \text{arg}(2-z_0)/(2\pi) \rfloor} z_0^{1/2+1/2 \lfloor \text{arg}(2-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)}{71 \left( 1 + 16\pi - \left(\frac{1}{z_0}\right)^{1/2 \lfloor \text{arg}(5-z_0)/(2\pi) \rfloor} z_0^{1/2+1/2 \lfloor \text{arg}(5-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!} \right)}$$

The result 12.164948 is very near to the value of the black hole entropy 12,19

$$(1-1.6644049)^2 * 1/(((8\text{Pi}-(\text{sqrt}(5)-1)/2))) (144(147+104\text{sqrt}(2)))$$

Input interpretation:

$$(1 - 1.6644049)^2 \times \frac{1}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} \left( 144 \left( 147 + 104\sqrt{2} \right) \right)$$

Result:

762.5429...

Continued fraction:

Linear form

$$762 + \frac{1}{1 + \frac{1}{1 + \frac{1}{5 + \frac{1}{3 + \frac{1}{18 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}$$

Series representations:

$$\frac{(1 - 1.6644)^2 (144 (147 + 104\sqrt{2}))}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} = \frac{826.364 \left( 1.41346 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)}{0.0625 + \pi - 0.0625 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{(1 - 1.6644)^2 (144 (147 + 104\sqrt{2}))}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} = \frac{826.364 \left( 1.41346 + \exp(i\pi \left\lfloor \frac{\text{arg}(2-x)}{2\pi} \right\rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)}{0.0625 + \pi - 0.0625 \exp(i\pi \left\lfloor \frac{\text{arg}(5-x)}{2\pi} \right\rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{(1 - 1.6644)^2 (144 (147 + 104 \sqrt{2}))}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} =$$

$$\left( 826.364 \left( 1.41346 + \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)]} z_0^{1/2+1/2 [\arg(2-z_0)/(2\pi)]} \right. \right.$$

$$\left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \right) / \left( 0.0625 + \pi - \right.$$

$$\left. 0.0625 \left(\frac{1}{z_0}\right)^{1/2 [\arg(5-z_0)/(2\pi)]} z_0^{1/2+1/2 [\arg(5-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!} \right)$$

The result 762.5429 is very near to the value of rest mass of the charged Rho meson  $775.4 \pm 0.4$  and to the value 765,171 of nonperturbative contribution to the mass of a 1S quarkonium for  $m_q = 4.7 \text{ MeV}/c^2 = 0.0047 \text{ GeV}/c^2$ , that is the mass of quark down is  $4.8 \pm 0.5 \pm 0.3 = 4.7 \text{ MeV}/c^2$

$$2 (0.6644049)^2 * 1/(((8\text{Pi}-(\text{sqrt}(5)-1)/2))) (144(147+104\text{sqrt}(2)))$$

Input interpretation:

$$2 \left( 0.6644049^2 \times \frac{1}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} \right) (144 (147 + 104 \sqrt{2}))$$

Result:

1525.086...

Continued fraction:

Linear form

$$1525 + \frac{1}{11 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\dots}}}}}}}$$

Series representations:



$$\frac{(2(144(147 + 104\sqrt{2})))0.664405^2}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} =$$

$$\frac{1652.73 \left( 1.41346 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)}{0.0625 + \pi - 0.0625 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{(2(144(147 + 104\sqrt{2})))0.664405^2}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} =$$

$$\frac{1652.73 \left( 1.41346 + \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)}{0.0625 + \pi - 0.0625 \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{(2(144(147 + 104\sqrt{2})))0.664405^2}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} =$$

$$\left( 1652.73 \left( 1.41346 + \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)]} z_0^{1/2+1/2 [\arg(2-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \right) / \left( 0.0625 + \pi - \right.$$

$$\left. 0.0625 \left(\frac{1}{z_0}\right)^{1/2 [\arg(5-z_0)/(2\pi)]} z_0^{1/2+1/2 [\arg(5-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!} \right)$$

The result 1525,086 is near to the value of rest mass of Xi baryon that is  $1531.80 \pm 0.32$

$$729 + 1/(((8\text{Pi}-(\text{sqrt}(5)-1)/2))) (144(147+104\text{sqrt}(2)))$$

Input:

$$729 + \frac{1}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} (144(147 + 104\sqrt{2}))$$

Result:

$$729 + \frac{144(147 + 104\sqrt{2})}{\frac{1}{2}(1 - \sqrt{5}) + 8\pi}$$

Decimal approximation:

2456.422722024477654236736075843389327371420633527326905729...

Property:

$$729 + \frac{144(147 + 104\sqrt{2})}{\frac{1}{2}(1 - \sqrt{5}) + 8\pi} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{9(4785 + 3328\sqrt{2} - 81\sqrt{5} + 1296\pi)}{1 - \sqrt{5} + 16\pi}$$

$$729 - \frac{288(147 + 104\sqrt{2})}{-1 + \sqrt{5} - 16\pi}$$

$$729 + \frac{288(147 + 104\sqrt{2})}{1 - \sqrt{5} + 16\pi}$$

Continued fraction:

Linear form

$$\bullet \quad 2456 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{59 + \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{11 + \cfrac{1}{\dots}}$$

Series representations:

$$729 + \frac{144(147 + 104\sqrt{2})}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} =$$

$$\left( 9 \left( 4785 + 1296\pi + 3328\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!} - \right. \right.$$

$$\left. \left. 81\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right) /$$

$$\left( 1 + 16\pi - \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$729 + \frac{144(147 + 104\sqrt{2})}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} =$$

$$\left( 9 \left( 4785 + 1296\pi + 3328 \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \right. \right.$$

$$\left. \left. 81 \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) /$$

$$\left( 1 + 16\pi - \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$729 + \frac{144(147 + 104\sqrt{2})}{8\pi - \frac{1}{2}(\sqrt{5} - 1)} =$$

$$\left( 9 \left( 4785 + 1296\pi + 3328 \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2+1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} \right. \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!} - 81 \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(5-z_0)/(2\pi) \rfloor} \right.$$

$$\left. \left. z_0^{1/2+1/2 \lfloor \arg(5-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right) /$$

$$\left( 1 + 16\pi - \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(5-z_0)/(2\pi) \rfloor} z_0^{1/2+1/2 \lfloor \arg(5-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)$$

The result 2456.4227 is very near to the value of rest mass of the charmed Sigma baryon that is  $2453.98 \pm 0.16$

$$\pi^{4/8} * \text{sqrt}(144(147+104\text{sqrt}(2)))$$

Input:

$$\frac{\pi^4}{8} \sqrt{144(147 + 104\sqrt{2})}$$

[Open code](#)

Result:

- Approximate form
- Step-by-step solution

$$\frac{3}{2} \sqrt{147 + 104\sqrt{2}} \pi^4$$

Decimal approximation:

More digits

2505.660192054728408188820261923818100972825097633076096821...

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Property:

$$\frac{3}{2} \sqrt{147 + 104\sqrt{2}} \pi^4 \text{ is a transcendental number}$$

Alternate form:

$$\frac{3 \left( \sqrt{147 - i\sqrt{23}} + \sqrt{i(\sqrt{23} + -147i)} \right) \pi^4}{2\sqrt{2}}$$

Continued fraction:

Linear form

$$2505 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{16 + \cfrac{1}{2 + \cfrac{1}{43 + \cfrac{1}{1 + \cfrac{1}{40 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{20 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Series representations:

More

$$\frac{1}{8} \sqrt{144(147 + 104\sqrt{2})} \pi^4 =$$

$$\frac{1}{8} \pi^4 \sqrt{21\,167 + 14\,976\sqrt{2}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (21\,167 + 14\,976\sqrt{2})^{-k}$$

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Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{8} \sqrt{144(147 + 104\sqrt{2})} \pi^4 =$$

$$\frac{1}{8} \pi^4 \sqrt{21\,167 + 14\,976\sqrt{2}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (21\,167 + 14\,976\sqrt{2})^{-k}}{k!}$$

[Open code](#)

$$\frac{1}{8} \sqrt{144(147 + 104\sqrt{2})} \pi^4 =$$

$$\frac{1}{8} \pi^4 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (21\,168 + 14\,976\sqrt{2} - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The result 2505,66019 is very near to the rest mass of the charmed Sigma baryon that is  $2517.9 \pm 0.6$

$$(2\pi^4)/11 * \sqrt{144(147+104\sqrt{2})}$$

Input:

$$\left(\frac{1}{11} (2\pi^4)\right) \sqrt{144(147 + 104\sqrt{2})}$$

[Open code](#)

Result:

- Approximate form
- Step-by-step solution

$$\frac{24}{11} \sqrt{147 + 104\sqrt{2}} \pi^4$$

Decimal approximation:

- More digits

3644.596642988695866456465835525553601415018323829928868104...

[Open code](#)

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Property:

$$\frac{24}{11} \sqrt{147 + 104\sqrt{2}} \pi^4 \text{ is a transcendental number}$$

Alternate forms:

$$\sqrt{\frac{84672}{121} + \frac{59904\sqrt{2}}{121}} \pi^4$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{12}{11} \sqrt{2} \left( \sqrt{147 - i\sqrt{23}} + \sqrt{i(\sqrt{23} + -147i)} \right) \pi^4$$

Continued fraction:

Linear form

$$3644 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{11 + \frac{1}{1 + \frac{1}{1 + \frac{1}{15 + \frac{1}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4 + \frac{1}{1 + \frac{1}{11 + \frac{1}{1 + \frac{1}{16 + \frac{1}{1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{30 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Series representations:

More

$$\frac{1}{11} \sqrt{144(147 + 104\sqrt{2})} (2\pi^4) = \frac{2}{11} \pi^4 \sqrt{21167 + 14976\sqrt{2}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (21167 + 14976\sqrt{2})^{-k}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{11} \sqrt{144(147 + 104\sqrt{2})} (2\pi^4) = \frac{2}{11} \pi^4 \sqrt{21167 + 14976\sqrt{2}} \sum_{k=0}^{\infty} \frac{(-1)^k \binom{-\frac{1}{2}}{k}}{k!} (21167 + 14976\sqrt{2})^{-k}$$

Open code

$$\frac{1}{11} \sqrt{144(147 + 104\sqrt{2})} (2\pi^4) =$$

$$\frac{2}{11} \pi^4 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (21168 + 14976\sqrt{2} - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The result 3644,596 is very near to the rest mass of double charmed Xi baryon, that is  $3621.40 \pm 0.78$

$$\frac{\pi}{8} \ln(144(147+104\sqrt{2}))$$

Input:

$$\frac{\pi}{8} \log(144(147 + 104\sqrt{2}))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Exact result:

$$\frac{1}{8} \pi \log(144(147 + 104\sqrt{2}))$$

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Decimal approximation:

More digits

4.183682127080426664004302610886147503737806220145001177143...

Alternate forms:

More

$$\frac{1}{8} \pi \log(21168 + 14976\sqrt{2})$$

[Open code](#)

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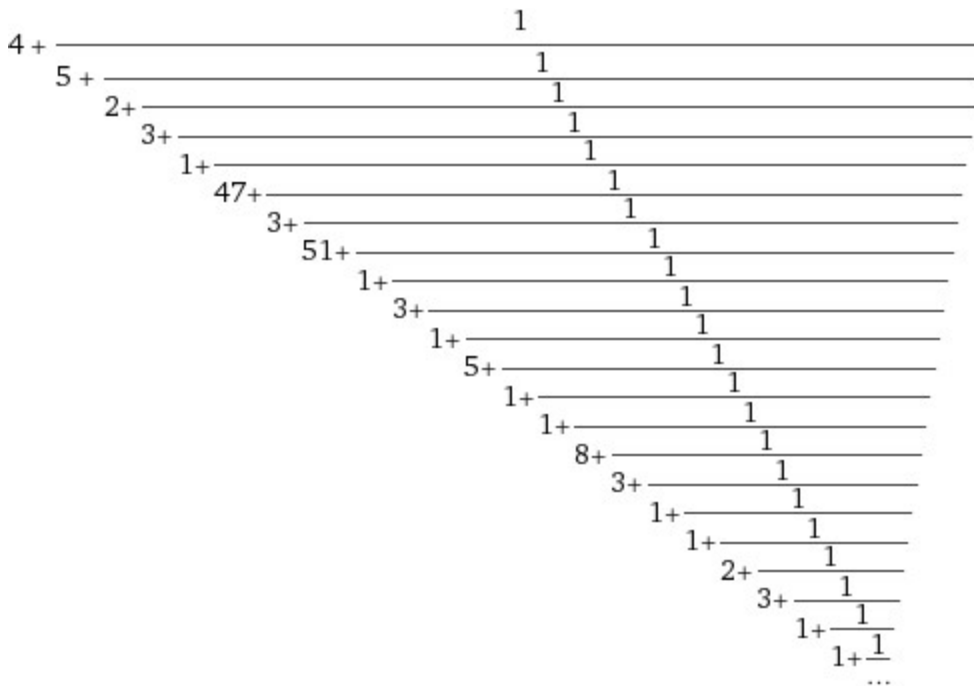
$$\frac{1}{8} \pi (\log(144) + \log(147 + 104\sqrt{2}))$$

[Open code](#)

$$\frac{1}{8} \pi (2(2\log(2) + \log(3)) + \log(147 + 104\sqrt{2}))$$

Continued fraction:

Linear form



Alternative representations:

More

$$\frac{1}{8} \log\left(144\left(147 + 104\sqrt{2}\right)\right) \pi = \frac{1}{8} \pi \log_e\left(144\left(147 + 104\sqrt{2}\right)\right)$$

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$$\frac{1}{8} \log\left(144\left(147 + 104\sqrt{2}\right)\right) \pi = \frac{1}{8} \pi \log(a) \log_a\left(144\left(147 + 104\sqrt{2}\right)\right)$$

[Open code](#)

$$\frac{1}{8} \log\left(144\left(147 + 104\sqrt{2}\right)\right) \pi = -\frac{1}{8} \pi \operatorname{Li}_1\left(1 - 144\left(147 + 104\sqrt{2}\right)\right)$$

Series representations:

More

$$\frac{1}{8} \log\left(144\left(147 + 104\sqrt{2}\right)\right) \pi = \frac{1}{8} \pi \log\left(21\,167 + 14\,976\sqrt{2}\right) - \frac{1}{8} \pi \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{21\,167 + 14\,976\sqrt{2}}\right)^k}{k}$$

[Open code](#)

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$$\frac{1}{8} \log\left(144\left(147 + 104\sqrt{2}\right)\right) \pi = \frac{1}{4} i \pi^2 \left[ \frac{\arg(21\,168 + 14\,976\sqrt{2} - x)}{2\pi} \right] + \frac{1}{8} \pi \log(x) - \frac{1}{8} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (21\,168 + 14\,976\sqrt{2} - x)^k x^{-k}}{k} \text{ for } x < 0$$



Open code

$$\frac{1}{8} \log\left(144\left(147 + 104\sqrt{2}\right)\right)\pi = \frac{1}{4} i \pi^2 \left[ \frac{\arg(144(147 + 104\sqrt{2}) - x)}{2\pi} \right] + \frac{1}{8} \pi \log(x) - \frac{1}{8} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (21\,168 + 14\,976\sqrt{2} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

Integral representations:

$$\frac{1}{8} \log\left(144\left(147 + 104\sqrt{2}\right)\right)\pi = \frac{\pi}{8} \int_1^{144(147+104\sqrt{2})} \frac{1}{t} dt$$

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$$\frac{1}{8} \log\left(144\left(147 + 104\sqrt{2}\right)\right)\pi = -\frac{i}{16} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(21\,167 + 14\,976\sqrt{2})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

The result 4,183682 is very near to is in the range of the mass of DM (dark matter) particle ( $\leq 4.2$ ). Indeed 4.183682 is very near to the upper bound value of mass of Dark Matter particle:

Using (22) this translates to an upper bound on the mass of the DM particle:

$$m \lesssim 4.2 \left( \frac{\sigma/m}{\text{cm}^2/\text{g}} \right)^{1/4} \text{ eV}. \quad (24)$$

Smaller and less massive galaxies result in a somewhat weaker bound.

The bound (24) on the DM particle mass is the main result of this Section. It shows that for values of  $\sigma/m$  satisfying the merging-cluster bound  $\sim 1 \text{ cm}^2/\text{g}$  [85–88],  $m$  must be somewhat below 4 eV. The dependence on the cross section is rather weak, however, scaling as the  $1/4$  power. It should be mentioned that the upper bound (24) would be somewhat tighter had we assumed a  $\rho \propto r^{-2}$  transition density profile outside the superfluid core, instead of  $\rho \propto r^{-3}$ .

(from: Phenomenological consequences of superfluid dark matter with baryon-phonon coupling- arXiv:1711.05748v1)

(7)

[22]

VII Theorems on approximate integrations and summation of series.

(1)  $1^2 \log 1 + 2^2 \log 2 + 3^2 \log 3 + \dots + x^2 \log x$   
 $= \frac{x(x+1)(2x+1)}{6} \log x - \frac{x^3}{9} + \frac{1}{2\pi^2} \left( \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \right)$   
 $+ \frac{x}{12} - \frac{1}{360x} + \dots$

(2)  $1 + \frac{x}{6} + \frac{x^2}{12} + \frac{x^3}{12} + \dots + \frac{x^k}{12} \theta = \frac{e^x}{2}$   
 where  $\theta = \frac{1}{3} + \frac{4}{135(x+k)}$  where  $k$  lies between  $\frac{8}{45}$  and  $\frac{9}{31}$ .

(3)  $1 + \left(\frac{x}{11}\right)^5 + \left(\frac{x^2}{11}\right)^5 + \left(\frac{x^3}{11}\right)^5 + \dots$   
 $= \frac{\sqrt{5}}{4\pi^2} \frac{e^{5x}}{5x^2 - x + \theta}$  where  $\theta$  vanishes when  $x = \infty$ .

(4)  $\frac{1^4}{e^{2x}} + \frac{2^4}{e^{4x}} + \frac{3^4}{e^{6x}} + \frac{4^4}{e^{8x}} + \dots$   
 $= \frac{2}{x^2} \left( \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \right) - \frac{1}{12x} + \frac{x}{1440} + \frac{x^3}{181440}$   
 $+ \frac{x^5}{7257600} + \frac{x^7}{159687200} + \dots$  when  $x$  is small  
 (Note:  $x$  may be given values from 0 to 2)

(5)  $\frac{1}{1001} + \frac{1}{1002^2} + \frac{3}{1003^3} + \frac{4^2}{1004^4} + \frac{5^3}{1005^5} + \dots$   
 $= \frac{1}{1000} - 10^{-440} \times 1.0125$  nearly

(6)  $\int_0^a e^{-x^2} dx = \frac{\sqrt{\pi}}{2} - \frac{e^{-a^2}}{2a} + \frac{1}{a} + \frac{2}{3a^3} + \frac{3}{5a^5} + \frac{4}{7a^7} + \dots$

(7) The coeff. of  $x^n$  in  $\frac{1}{1 - 2x + 2x^2 - 2x^3 + 2x^4 - \dots}$   
 $=$  The nearest integer to  $\frac{1}{2^n} \left\{ \cosh(n\sqrt{2}) - \frac{\sinh(n\sqrt{2})}{\sqrt{2}} \right\}$

Now, let's develop some expressions extracted from the following page of original Ramanujan's manuscript

From the (1) (**PAGE 1**), we have that:

$$1 \ln 1 + 2^2 \ln 2$$

Input:

$$1 \log(1) + 2^2 \log(2)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Exact result:

$$4 \log(2)$$

Decimal approximation:

More digits

$$2.772588722239781237668928485832706272302000537441021016482\dots$$

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Property:

$4 \log(2)$  is a transcendental number

Alternate form:

$$\log(16)$$

Alternative representations:

More

$$1 \log(1) + 2^2 \log(2) = \log_e(1) + 4 \log_e(2)$$

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$$1 \log(1) + 2^2 \log(2) = \log(a) \log_a(1) + 4 \log(a) \log_a(2)$$

[Open code](#)

$$1 \log(1) + 2^2 \log(2) = -4 \operatorname{Li}_1(-1) - \operatorname{Li}_1(0)$$

Series representations:

$$1 \log(1) + 2^2 \log(2) = 8 i \pi \left[ \frac{\arg(2-x)}{2\pi} \right] + 4 \log(x) - 4 \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \quad \text{for } x < 0$$

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$$1 \log(1) + 2^2 \log(2) =$$

$$4 \left[ \frac{\arg(2 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + 4 \log(z_0) + 4 \left[ \frac{\arg(2 - z_0)}{2\pi} \right] \log(z_0) - 4 \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

$$1 \log(1) + 2^2 \log(2) = 8i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + 4 \log(z_0) - 4 \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$1 \log(1) + 2^2 \log(2) = 4 \int_1^2 \frac{1}{t} dt$$

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$$1 \log(1) + 2^2 \log(2) = -\frac{2i}{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$((2(3)(5)/6))\ln 2 - (2^3)/9 + (1/(4\pi^2))(1 + 1/(2^3) + 1/(3^3)) + 2/12 - 1/(360*2)$$

Input:

$$\left(2 \times 3 \times \frac{5}{6}\right) \log(2) - \frac{2^3}{9} + \frac{1}{4\pi^2} \left(1 + \frac{1}{2^3} + \frac{1}{3^3}\right) + \frac{2}{12} - \frac{1}{360 \times 2}$$

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- $\log(x)$  is the natural logarithm

Exact result:

$$-\frac{521}{720} + \frac{251}{864\pi^2} + 5 \log(2)$$

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Decimal approximation:

More digits

2.771559533695822358003306145792782883624021565832218265696...

Alternate forms:

More

$$-\frac{521}{720} + \frac{251}{864\pi^2} + \log(32)$$

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$$5 \log(2) - \frac{3126\pi^2 - 1255}{4320\pi^2}$$

[Open code](#)

$$\frac{1255 + 6 \pi^2 (3600 \log(2) - 521)}{4320 \pi^2}$$

Alternative representations:

More

$$\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2} =$$

$$\frac{30 \log_e(2)}{6} - \frac{8}{9} + \frac{2}{12} - \frac{1}{720} + \frac{1 + \frac{1}{8} + \frac{1}{27}}{4 \pi^2}$$

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$$\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2} =$$

$$\frac{30}{6} \log(a) \log_a(2) - \frac{8}{9} + \frac{2}{12} - \frac{1}{720} + \frac{1 + \frac{1}{8} + \frac{1}{27}}{4 \pi^2}$$

Open code

$$\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2} =$$

$$\frac{60}{6} \coth^{-1}(3) - \frac{8}{9} + \frac{2}{12} - \frac{1}{720} + \frac{1 + \frac{1}{8} + \frac{1}{27}}{4 \pi^2}$$

Series representations:

$$\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2} =$$

$$-\frac{521}{720} + \frac{251}{864 \pi^2} + 10 i \pi \left[ \frac{\arg(2-x)}{2 \pi} \right] + 5 \log(x) - 5 \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \text{ for } x < 0$$

Open code

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$$\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2} = -\frac{521}{720} + \frac{251}{864 \pi^2} +$$

$$5 \left[ \frac{\arg(2-z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + 5 \log(z_0) + 5 \left[ \frac{\arg(2-z_0)}{2 \pi} \right] \log(z_0) - 5 \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}$$

Open code

$$\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2} =$$

$$-\frac{521}{720} + \frac{251}{864 \pi^2} + 10 i \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + 5 \log(z_0) - 5 \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2} = -\frac{521}{720} + \frac{251}{864 \pi^2} + 5 \int_1^2 \frac{1}{t} dt$$

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$$\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2} =$$

$$-\frac{521}{720} + \frac{251}{864 \pi^2} - \frac{5 i}{2 \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$\sqrt{\left(\left(\frac{2(3)(5)}{6}\right)\ln 2 - \frac{2^3}{9} + \frac{1}{4\pi^2}\left(1 + \frac{1}{2^3} + \frac{1}{3^3}\right) + \frac{2}{12} - \frac{1}{360 \times 2}\right)}$$

Input:

$$\sqrt{\left(2 \times 3 \times \frac{5}{6}\right) \log(2) - \frac{2^3}{9} + \frac{1}{4 \pi^2} \left(1 + \frac{1}{2^3} + \frac{1}{3^3}\right) + \frac{2}{12} - \frac{1}{360 \times 2}}$$

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- $\log(x)$  is the natural logarithm

Exact result:

$$\sqrt{-\frac{521}{720} + \frac{251}{864 \pi^2} + 5 \log(2)}$$

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Decimal approximation:

More digits

1.664800148274807896619608771727562731282540388459571038337...

Alternate forms:

$$\sqrt{-\frac{521}{720} + \frac{251}{864 \pi^2} + \log(32)}$$

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$$\frac{\sqrt{\frac{1}{30} (1255 + 6 \pi^2 (3600 \log(2) - 521))}}{12 \pi}$$

$$\frac{\sqrt{\frac{1}{30} (1255 - 3126 \pi^2 + 21600 \pi^2 \log(2))}}{12 \pi}$$

$$e^0 \sqrt{-\frac{521}{720} + \frac{251}{864 \pi^2} + 5 \log(2)} \approx 1.6648 \quad (\text{real, principal root})$$

$$e^{i \pi} \sqrt{-\frac{521}{720} + \frac{251}{864 \pi^2} + 5 \log(2)} \approx -1.6648 \quad (\text{real root})$$

Alternative representations:

More

$$\sqrt{\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2}} =$$

$$\sqrt{\frac{30 \log_e(2)}{6} - \frac{8}{9} + \frac{2}{12} - \frac{1}{720} + \frac{1 + \frac{1}{8} + \frac{1}{27}}{4 \pi^2}}$$

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$$\sqrt{\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2}} =$$

$$\sqrt{\frac{30}{6} \log(a) \log_a(2) - \frac{8}{9} + \frac{2}{12} - \frac{1}{720} + \frac{1 + \frac{1}{8} + \frac{1}{27}}{4 \pi^2}}$$

[Open code](#)

$$\sqrt{\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2}} =$$

$$\sqrt{\frac{60}{6} \coth^{-1}(3) - \frac{8}{9} + \frac{2}{12} - \frac{1}{720} + \frac{1 + \frac{1}{8} + \frac{1}{27}}{4 \pi^2}}$$

Series representations:

$$\sqrt{\frac{1}{6} \log(2) 2 (3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4 \pi^2} + \frac{2}{12} - \frac{1}{360 \times 2}} =$$

$$\sqrt{-\frac{521}{720} + \frac{251}{864 \pi^2} + 5 \left( 2 i \pi \left[ \frac{\arg(2-x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right)} \quad \text{for } x < 0$$

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$$\sqrt{\frac{1}{6} \log(2) 2(3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4\pi^2} + \frac{2}{12} - \frac{1}{360 \times 2}} = \sqrt{\left(-\frac{521}{720} + \frac{251}{864\pi^2} + 5 \left( \log(z_0) + \left[ \frac{\arg(2 - z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right)\right)}$$

Open code

$$\sqrt{\frac{1}{6} \log(2) 2(3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4\pi^2} + \frac{2}{12} - \frac{1}{360 \times 2}} = \sqrt{-\frac{521}{720} + \frac{251}{864\pi^2} + 5 \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right)}$$

Open code

$$\sqrt{\frac{1}{6} \log(2) 2(3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4\pi^2} + \frac{2}{12} - \frac{1}{360 \times 2}} = \sqrt{\left(-\frac{521}{720} + \frac{251}{864\pi^2} + 5 \left( 2i\pi \left[ -\frac{\pi + \arg\left(\frac{1}{z_0}\right) + \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right)\right)}$$

Integral representations:

$$\sqrt{\frac{1}{6} \log(2) 2(3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4\pi^2} + \frac{2}{12} - \frac{1}{360 \times 2}} = \sqrt{-\frac{521}{720} + \frac{251}{864\pi^2} + 5 \int_1^2 \frac{1}{t} dt}$$

Open code

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$$\sqrt{\frac{1}{6} \log(2) 2(3 \times 5) - \frac{2^3}{9} + \frac{1 + \frac{1}{2^3} + \frac{1}{3^3}}{4\pi^2} + \frac{2}{12} - \frac{1}{360 \times 2}} = \sqrt{-\frac{521}{720} + \frac{251}{864\pi^2} - \frac{5i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0}$$

sqrt((1(ln1)+4(ln2)))

Input:

$$\sqrt{1 \log(1) + 4 \log(2)}$$

Open code



- $\log(x)$  is the natural logarithm

Exact result:

$$2\sqrt{\log(2)}$$

Decimal approximation:

More digits

1.665109222315395512706329289790402095261177704528881458333...

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Property:

$2\sqrt{\log(2)}$  is a transcendental number

All 2nd roots of  $4\log(2)$ :

$$2e^0\sqrt{\log(2)} \approx 1.6651 \quad (\text{real, principal root})$$

$$2e^{i\pi}\sqrt{\log(2)} \approx -1.6651 \quad (\text{real root})$$

Alternative representations:

More

$$\sqrt{1\log(1) + 4\log(2)} = \sqrt{\log_e(1) + 4\log_e(2)}$$

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$$\sqrt{1\log(1) + 4\log(2)} = \sqrt{\log(a)\log_a(1) + 4\log(a)\log_a(2)}$$

[Open code](#)

$$\sqrt{1\log(1) + 4\log(2)} = \sqrt{-4\text{Li}_1(-1) - \text{Li}_1(0)}$$

Series representations:

$$\sqrt{1\log(1) + 4\log(2)} = 2\sqrt{2i\pi\left[\frac{\arg(2-x)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k(2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

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$$\sqrt{1\log(1) + 4\log(2)} = 2\sqrt{\log(z_0) + \left[\frac{\arg(2-z_0)}{2\pi}\right]\left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}$$

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$$\sqrt{1 \log(1) + 4 \log(2)} = \frac{2 \sqrt{2 i \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k}}{2}}$$

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$$\sqrt{1 \log(1) + 4 \log(2)} = \frac{2 \sqrt{2 i \pi \left[ -\frac{-\pi + \arg\left(\frac{1}{z_0}\right) + \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k}}{2}}$$

Integral representations:

$$\sqrt{1 \log(1) + 4 \log(2)} = 2 \sqrt{\int_1^2 \frac{1}{t} dt}$$

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$$\sqrt{1 \log(1) + 4 \log(2)} = \sqrt{\frac{2}{\pi}} \sqrt{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

$$1(\ln 1) + 4(\ln 2) + 9(\ln 3) + 16(\ln 4) + 25(\ln 5) + 36(\ln 6) + 49(\ln 7) + 64(\ln 8)$$

Input:

$$1 \log(1) + 4 \log(2) + 9 \log(3) + 16 \log(4) + 25 \log(5) + 36 \log(6) + 49 \log(7) + 64 \log(8)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Exact result:

$$4 \log(2) + 9 \log(3) + 16 \log(4) + 25 \log(5) + 36 \log(6) + 49 \log(7) + 64 \log(8)$$

Decimal approximation:

More digits

$$368.0139537724533591141165834858427684214262961793552536982\dots$$

[Open code](#)

Property:

$$4 \log(2) + 9 \log(3) + 16 \log(4) + 25 \log(5) + 36 \log(6) + 49 \log(7) + 64 \log(8)$$

is a transcendental number

[Open code](#)

Alternate forms:

$$264 \log(2) + 45 \log(3) + 25 \log(5) + 49 \log(7)$$

[Open code](#)



$$\begin{aligned}
& 1 \log(1) + 4 \log(2) + 9 \log(3) + 16 \log(4) + 25 \log(5) + 36 \log(6) + 49 \log(7) + 64 \log(8) = \\
& 406 i \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + 203 \log(z_0) + \\
& \sum_{k=1}^{\infty} \frac{1}{k} (-1)^{1+k} \left( 4(2 - z_0)^k + 9(3 - z_0)^k + 16(4 - z_0)^k + \right. \\
& \quad \left. 25(5 - z_0)^k + 36(6 - z_0)^k + 49(7 - z_0)^k + 64(8 - z_0)^k \right) z_0^{-k}
\end{aligned}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

$$\begin{aligned}
& 1 \log(1) + 4 \log(2) + 9 \log(3) + 16 \log(4) + 25 \log(5) + 36 \log(6) + 49 \log(7) + 64 \log(8) = \\
& 8 i \pi \left[ \frac{\arg(2 - x)}{2 \pi} \right] + 18 i \pi \left[ \frac{\arg(3 - x)}{2 \pi} \right] + 32 i \pi \left[ \frac{\arg(4 - x)}{2 \pi} \right] + 50 i \pi \left[ \frac{\arg(5 - x)}{2 \pi} \right] + \\
& 72 i \pi \left[ \frac{\arg(6 - x)}{2 \pi} \right] + 98 i \pi \left[ \frac{\arg(7 - x)}{2 \pi} \right] + 128 i \pi \left[ \frac{\arg(8 - x)}{2 \pi} \right] + \\
& 203 \log(x) + \sum_{k=1}^{\infty} \frac{1}{k} (-1)^{1+k} \left( 4(2 - x)^k + 9(3 - x)^k + 16(4 - x)^k + \right. \\
& \quad \left. 25(5 - x)^k + 36(6 - x)^k + 49(7 - x)^k + 64(8 - x)^k \right) x^{-k} \text{ for } x < 0
\end{aligned}$$

$$\begin{aligned}
& 1 \log(1) + 4 \log(2) + 9 \log(3) + 16 \log(4) + 25 \log(5) + 36 \log(6) + 49 \log(7) + 64 \log(8) = \\
& 4 \left[ \frac{\arg(2 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + 9 \left[ \frac{\arg(3 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + 16 \left[ \frac{\arg(4 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + \\
& 25 \left[ \frac{\arg(5 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + 36 \left[ \frac{\arg(6 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + \\
& 49 \left[ \frac{\arg(7 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + 64 \left[ \frac{\arg(8 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + 203 \log(z_0) + \\
& 4 \left[ \frac{\arg(2 - z_0)}{2 \pi} \right] \log(z_0) + 9 \left[ \frac{\arg(3 - z_0)}{2 \pi} \right] \log(z_0) + 16 \left[ \frac{\arg(4 - z_0)}{2 \pi} \right] \log(z_0) + \\
& 25 \left[ \frac{\arg(5 - z_0)}{2 \pi} \right] \log(z_0) + 36 \left[ \frac{\arg(6 - z_0)}{2 \pi} \right] \log(z_0) + 49 \left[ \frac{\arg(7 - z_0)}{2 \pi} \right] \log(z_0) + \\
& 64 \left[ \frac{\arg(8 - z_0)}{2 \pi} \right] \log(z_0) + \sum_{k=1}^{\infty} \frac{1}{k} (-1)^{1+k} \left( 4(2 - z_0)^k + 9(3 - z_0)^k + 16(4 - z_0)^k + \right. \\
& \quad \left. 25(5 - z_0)^k + 36(6 - z_0)^k + 49(7 - z_0)^k + 64(8 - z_0)^k \right) z_0^{-k}
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& 1 \log(1) + 4 \log(2) + 9 \log(3) + 16 \log(4) + 25 \log(5) + 36 \log(6) + 49 \log(7) + 64 \log(8) = \\
& \int_1^8 \left( \left( 12(3840 + 68832t + 320164t^2 + 568766t^3 + 426397t^4 + 129258t^5 + \right. \right. \\
& \quad \left. \left. 12180t^6) \right) / (t(6+t)(5+2t)(4+3t)(3+4t)(2+5t)(1+6t)) \right) dt
\end{aligned}$$

[Open code](#)

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$$1 \log(1) + 4 \log(2) + 9 \log(3) + 16 \log(4) + 25 \log(5) + 36 \log(6) + 49 \log(7) + 64 \log(8) = \int_{-i\infty+\gamma}^{i\infty+\gamma} \left( -\frac{2i\Gamma(-s)^2\Gamma(1+s)}{\pi\Gamma(1-s)} - \frac{25i2^{-1-2s}\Gamma(-s)^2\Gamma(1+s)}{\pi\Gamma(1-s)} - \frac{9i2^{-1-s}\Gamma(-s)^2\Gamma(1+s)}{\pi\Gamma(1-s)} - \frac{8i3^{-s}\Gamma(-s)^2\Gamma(1+s)}{\pi\Gamma(1-s)} - \frac{49i2^{-1-s}\times 3^{-s}\Gamma(-s)^2\Gamma(1+s)}{\pi\Gamma(1-s)} - \frac{18i5^{-s}\Gamma(-s)^2\Gamma(1+s)}{\pi\Gamma(1-s)} - \frac{32i7^{-s}\Gamma(-s)^2\Gamma(1+s)}{\pi\Gamma(1-s)} \right) ds \text{ for } -1 < \gamma < 0$$

From the (4) (PAGE 1) , we have that:

$$[2/8(1 + 1/8 + 1/27 + 1/64)-1/24+2/1440+8/181440+32/7257600+64/159667200]$$

Input:

$$\frac{2}{8} \left( 1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} \right) - \frac{1}{24} + \frac{2}{1440} + \frac{8}{181440} + \frac{32}{7257600} + \frac{64}{159667200}$$

Exact result:

$$\frac{10146317}{39916800}$$

Decimal approximation:

$$0.254186633197049863716530383197049863716530383197049863716...$$

Repeating decimal:

$$0.25418663319704986371653038\overline{319704986371653038} \text{ (period 18)}$$

Continued fraction:

Linear form

$$3 + \frac{1}{1 + \frac{1}{14 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{11 + \frac{1}{5 + \frac{1}{3 + \frac{1}{4 + \frac{1}{1 + \frac{1}{23}}}}}}}}}}}}}}}}$$



Open code

$$8^4 \exp\left(\frac{1}{8} \left(1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64}\right) 2 - \frac{1}{24} + \frac{2}{1440} + \frac{8}{181440} + \frac{32}{7257600} + \frac{64}{159667200}\right) = 4096 \left(\frac{\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}}{z}\right)^{10146317/39916800}$$

The result 5281,43 is very near to the rest mass of B meson  $5279.53 \pm 33$

$$\left[ \frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1} \right]$$

Input:

$$\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}$$

Decimal approximation:

0.258880492329823032789774701202053629630228289808126519734...

Property:

$$\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8} \text{ is a transcendental number}$$

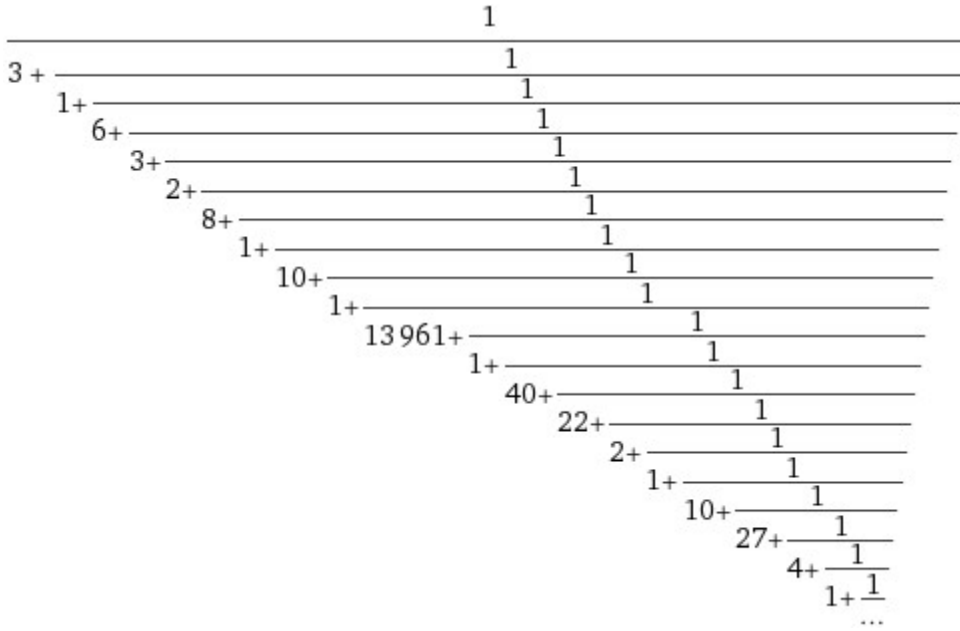
Alternate forms:

$$\frac{5}{e-1} - \frac{5}{1+e} - \frac{6}{1+e^2} + \frac{3(e-2)}{2(1-e+e^2)} - \frac{3(2+e)}{2(1+e+e^2)} - \frac{8}{1+e^4}$$

$$\frac{30 + 31e^2 + 36e^4 + 16e^6 + 6e^8 + e^{10}}{(e-1)(1+e)(1+e^2)(1-e+e^2)(1+e+e^2)(1+e^4)}$$

Continued fraction:

Linear form



Alternative representation:

$$\frac{1}{e^2 - 1} + \frac{4}{e^4 - 1} + \frac{9}{e^6 - 1} + \frac{16}{e^8 - 1} = \frac{1}{\exp^2(z) - 1} + \frac{4}{\exp^4(z) - 1} + \frac{9}{\exp^6(z) - 1} + \frac{16}{\exp^8(z) - 1} \text{ for } z = 1$$

Series representations:

$$\frac{1}{e^2 - 1} + \frac{4}{e^4 - 1} + \frac{9}{e^6 - 1} + \frac{16}{e^8 - 1} = \frac{1}{-1 + \sum_{k=0}^{\infty} \frac{2^k}{k!}} + \frac{4}{-1 + \sum_{k=0}^{\infty} \frac{4^k}{k!}} + \frac{9}{-1 + \sum_{k=0}^{\infty} \frac{6^k}{k!}} + \frac{16}{-1 + \sum_{k=0}^{\infty} \frac{8^k}{k!}}$$

$$\frac{1}{e^2 - 1} + \frac{4}{e^4 - 1} + \frac{9}{e^6 - 1} + \frac{16}{e^8 - 1} = \left( 30 + 31 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 + 36 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4 + 16 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^6 + 6 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^8 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{10} \right) / \left( \left( -1 + \sum_{k=0}^{\infty} \frac{1}{k!} \right) \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 - \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4 \right) \right)$$



$$\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1} =$$

$$\left( 4 \left( 30720 + 7936 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^2 + 2304 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^4 + 256 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^6 + \right. \right.$$

$$\left. \left. 24 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^8 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{10} \right) \right) /$$

$$\left( \left( -2 + \sum_{k=0}^{\infty} \frac{1+k}{k!} \right) \left( 2 + \sum_{k=0}^{\infty} \frac{1+k}{k!} \right) \left( 4 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^2 \right) \left( 4 - 2 \sum_{k=0}^{\infty} \frac{1+k}{k!} + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^2 \right) \right.$$

$$\left. \left( 4 + 2 \sum_{k=0}^{\infty} \frac{1+k}{k!} + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^2 \right) \left( 16 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^4 \right) \right)$$

Note that  $1 / 0.25888049 * \text{Pi} = 12,135300939788059109678923210009$

$$448 / [1/(e^2-1) + 4/(e^4-1) + 9/(e^6-1) + 16/(e^8-1)]$$

Input:

$$\frac{448}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}}$$

Decimal approximation:

1730.528229331516918909287075227063032112011948572305672243...

Property:

$$\frac{448}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}}$$

is a transcendental number

Alternate forms:

$$\frac{448 (e-1)(1+e)(1+e^2)(1-e+e^2)(1+e+e^2)(1+e^4)}{30 + 31 e^2 + 36 e^4 + 16 e^6 + 6 e^8 + e^{10}}$$

$$-2240 + 448 e^2 + \frac{448 (149 + 124 e^2 + 148 e^4 + 44 e^6 + 15 e^8)}{30 + 31 e^2 + 36 e^4 + 16 e^6 + 6 e^8 + e^{10}}$$

Continued fraction:

Linear form

$$\begin{array}{r}
1730 + \frac{1}{1 + \frac{1}{1 + \frac{1}{8 + \frac{1}{2 + \frac{1}{1 + \frac{1}{4 + \frac{1}{4 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{30 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{10 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}} \\
\hline
\frac{1}{e^{2-1}} + \frac{4}{e^{4-1}} + \frac{9}{e^{6-1}} + \frac{16}{e^{8-1}}
\end{array}$$

Alternative representation:

$$\frac{448}{\frac{1}{e^{2-1}} + \frac{4}{e^{4-1}} + \frac{9}{e^{6-1}} + \frac{16}{e^{8-1}}} = \frac{448}{\frac{1}{\exp^{2(z)-1}} + \frac{4}{\exp^{4(z)-1}} + \frac{9}{\exp^{6(z)-1}} + \frac{16}{\exp^{8(z)-1}}} \quad \text{for } z = 1$$

Series representations:

$$\frac{448}{\frac{1}{e^{2-1}} + \frac{4}{e^{4-1}} + \frac{9}{e^{6-1}} + \frac{16}{e^{8-1}}} = \frac{448}{-1 + \sum_{k=0}^{\infty} \frac{2^k}{k!} - 1 + \sum_{k=0}^{\infty} \frac{4^k}{k!} - 1 + \sum_{k=0}^{\infty} \frac{6^k}{k!} - 1 + \sum_{k=0}^{\infty} \frac{8^k}{k!}}$$

$$\begin{aligned}
&\frac{448}{\frac{1}{e^{2-1}} + \frac{4}{e^{4-1}} + \frac{9}{e^{6-1}} + \frac{16}{e^{8-1}}} = \\
&\left( 448 \left( -1 + \sum_{k=0}^{\infty} \frac{1}{k!} \right) \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 - \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \right. \\
&\quad \left. \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4 \right) \right) / \\
&\left( 30 + 31 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 + 36 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4 + 16 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^6 + 6 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^8 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{10} \right)
\end{aligned}$$

$$\frac{448}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} =$$

$$\left( 112 \left( -2 + \sum_{k=0}^{\infty} \frac{1+k}{k!} \right) \left( 2 + \sum_{k=0}^{\infty} \frac{1+k}{k!} \right) \left( 4 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^2 \right) \left( 4 - 2 \sum_{k=0}^{\infty} \frac{1+k}{k!} + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^2 \right) \right.$$

$$\left. \left( 4 + 2 \sum_{k=0}^{\infty} \frac{1+k}{k!} + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^2 \right) \left( 16 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^4 \right) \right) /$$

$$\left( 30720 + 7936 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^2 + 2304 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^4 + \right.$$

$$\left. 256 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^6 + 24 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^8 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{10} \right)$$

The result 1730.5282 is very near to the value of the mass of the candidate “glueball”  $f_0(1710)$  that is 1723 (+ 6 – 5).

$$2\pi / [1/(e^2-1) + 4/(e^4-1) + 9/(e^6-1) + 16/(e^8-1)]$$

Input:

$$2 \times \frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}}$$

Exact result:

$$\frac{2\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}}$$

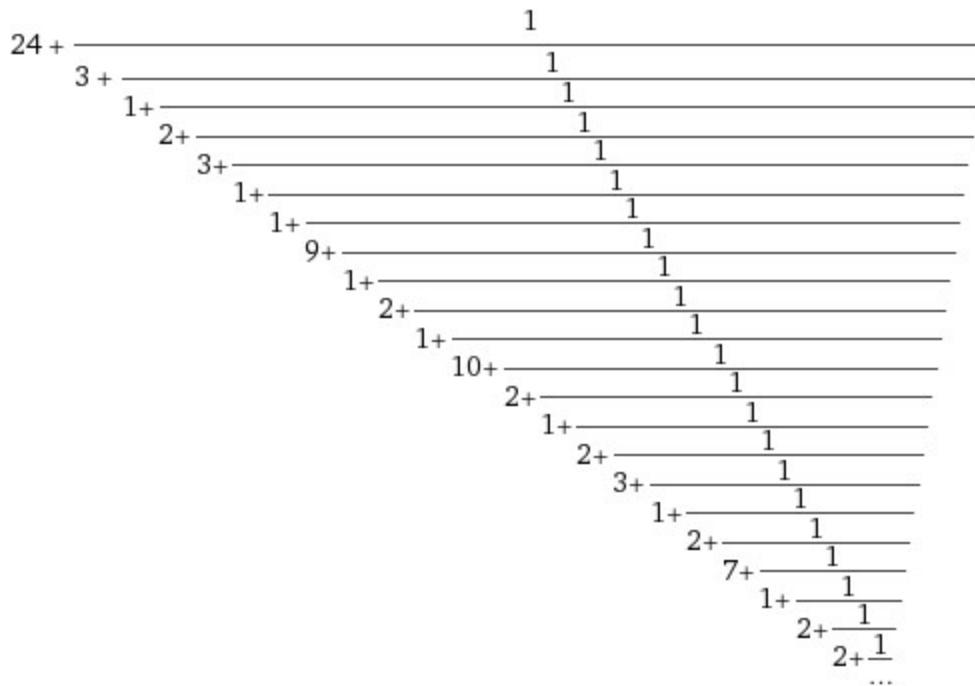
Decimal approximation:

24.27060166115020760008514474524603494638137378005596083088...

Alternate form:

$$\frac{2(e-1)(1+e)(1+e^2)(1-e+e^2)(1+e+e^2)(1+e^4)\pi}{30+31e^2+36e^4+16e^6+6e^8+e^{10}}$$

Continued fraction:  
Linear form



Alternative representations:

$$\frac{2\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{360^\circ}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}}$$

$$\frac{2\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = -\frac{2i \log(-1)}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}}$$

$$\frac{2\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{2\pi}{\frac{1}{\exp^2(z)-1} + \frac{4}{\exp^4(z)-1} + \frac{9}{\exp^6(z)-1} + \frac{16}{\exp^8(z)-1}} \quad \text{for } z = 1$$

Series representations:

$$\frac{2\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{8 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}}$$

$$\frac{2\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{8 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}{\frac{1}{-1 + \sum_{k=0}^{\infty} \frac{2^k}{k!}} + \frac{4}{-1 + \sum_{k=0}^{\infty} \frac{4^k}{k!}} + \frac{9}{-1 + \sum_{k=0}^{\infty} \frac{6^k}{k!}} + \frac{16}{-1 + \sum_{k=0}^{\infty} \frac{8^k}{k!}}}$$

$$\frac{2\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{2\pi \left( -1 + \sum_{k=0}^{\infty} \frac{1}{k!} \right) \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 - \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4 \right)}{\left( 30 + 31 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 + 36 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4 + 16 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^6 + 6 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^8 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{10} \right)}$$

Integral representations:

$$\frac{2\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{8}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{2\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{4}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{2\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{4}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}} \int_0^{\infty} \frac{1}{1+t^2} dt$$

The result  $24.2706 \approx 24$  represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates

Note that:

$$24.2706 \left[ \frac{1}{(e^2-1)} + \frac{4}{(e^4-1)} + \frac{9}{(e^6-1)} + \frac{16}{(e^8-1)} \right]$$

Input interpretation:

$$24.2706 \left( \frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1} \right)$$

Result:

6.28318...

Continued fraction:

Linear form

$$6 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{7 + \frac{1}{2 + \frac{1}{12 + \frac{1}{1 + \frac{1}{15 + \frac{1}{\dots}}}}}}}}}}$$

Practically equal to:

6.283185307179586476925286766559005768394338798750211641949...

Conversion from radians to degrees:

360°

Property:

$2\pi$  is a transcendental number

Continued fraction:

Linear form

- $$6 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{7 + \frac{1}{2 + \frac{1}{146 + \frac{1}{3 + \frac{1}{6 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{7 + \frac{1}{5 + \frac{1}{5 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{2 + \frac{1}{42 + \frac{1}{5 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Alternative representation:

$$24.2706 \left( \frac{1}{e^2 - 1} + \frac{4}{e^4 - 1} + \frac{9}{e^6 - 1} + \frac{16}{e^8 - 1} \right) =$$

$$24.2706 \left( \frac{1}{\exp^2(z) - 1} + \frac{4}{\exp^4(z) - 1} + \frac{9}{\exp^6(z) - 1} + \frac{16}{\exp^8(z) - 1} \right) \text{ for } z = 1$$

Series representations:

$$24.2706 \left( \frac{1}{e^2 - 1} + \frac{4}{e^4 - 1} + \frac{9}{e^6 - 1} + \frac{16}{e^8 - 1} \right) =$$

$$\frac{24.2706}{-1 + \sum_{k=0}^{\infty} \frac{2^k}{k!}} + \frac{97.0824}{-1 + \sum_{k=0}^{\infty} \frac{4^k}{k!}} + \frac{218.435}{-1 + \sum_{k=0}^{\infty} \frac{6^k}{k!}} + \frac{388.33}{-1 + \sum_{k=0}^{\infty} \frac{8^k}{k!}}$$

$$24.2706 \left( \frac{1}{e^2 - 1} + \frac{4}{e^4 - 1} + \frac{9}{e^6 - 1} + \frac{16}{e^8 - 1} \right) =$$

$$\frac{24.2706}{-1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2} + \frac{97.0824}{-1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4} + \frac{218.435}{-1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^6} + \frac{388.33}{-1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^8}$$

$$24.2706 \left( \frac{1}{e^2 - 1} + \frac{4}{e^4 - 1} + \frac{9}{e^6 - 1} + \frac{16}{e^8 - 1} \right) =$$

$$\frac{388.33}{-1 + \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \right)^8}} + \frac{218.435}{-1 + \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \right)^6}} + \frac{97.0824}{-1 + \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \right)^4}} + \frac{24.2706}{-1 + \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \right)^2}}$$

$$\pi / [1/(e^2-1) + 4/(e^4-1) + 9/(e^6-1) + 16/(e^8-1)]$$

Input:

$$\frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}}$$

Decimal approximation:

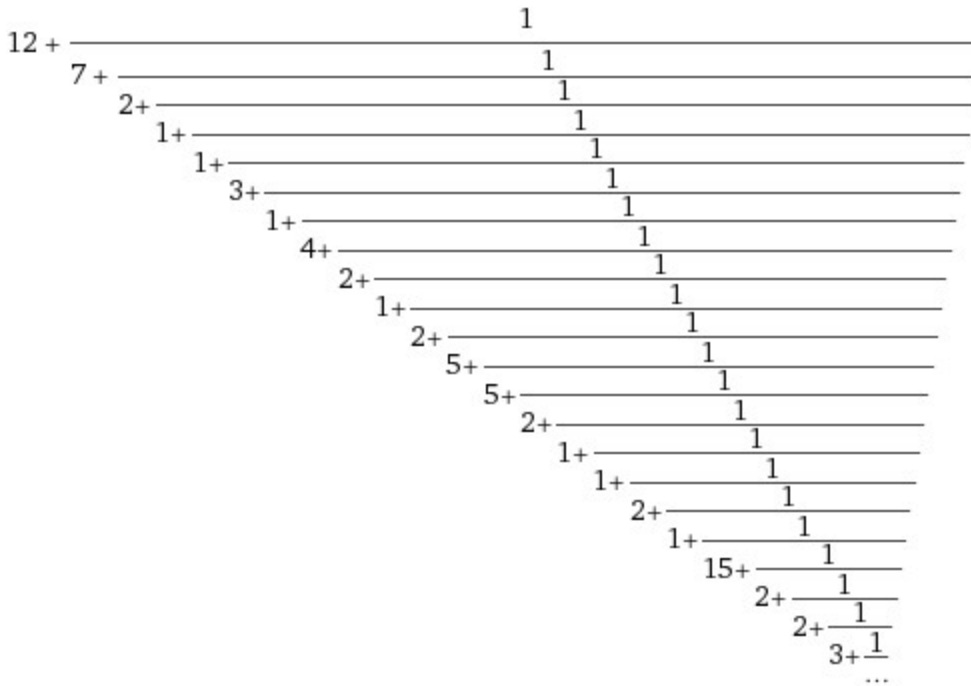
12.13530083057510380004257237262301747319068689002798041544...

Alternate form:

$$\frac{(e-1)(1+e)(1+e^2)(1-e+e^2)(1+e+e^2)(1+e^4)\pi}{30 + 31e^2 + 36e^4 + 16e^6 + 6e^8 + e^{10}}$$

Continued fraction:

Linear form



Alternative representations:

$$\frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{180^\circ}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}}$$

$$\frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = -\frac{i \log(-1)}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}}$$

$$\frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{\pi}{\frac{1}{\exp^2(z)-1} + \frac{4}{\exp^4(z)-1} + \frac{9}{\exp^6(z)-1} + \frac{16}{\exp^8(z)-1}} \quad \text{for } z = 1$$

Series representations:

$$\frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}}$$

$$\frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}{-1 + \sum_{k=0}^{\infty} \frac{2^k}{k!} + \frac{4}{-1 + \sum_{k=0}^{\infty} \frac{4^k}{k!}} + \frac{9}{-1 + \sum_{k=0}^{\infty} \frac{6^k}{k!}} + \frac{16}{-1 + \sum_{k=0}^{\infty} \frac{8^k}{k!}}}$$



$$\frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{\left( \pi \left( -1 + \sum_{k=0}^{\infty} \frac{1}{k!} \right) \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 - \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \right.}{\left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4 \right)} \cdot \left( 30 + 31 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 + 36 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4 + 16 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^6 + 6 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^8 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{10} \right)$$

Integral representations:

$$\frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{4}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{2}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{2}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}} \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$64\pi / [1/(e^2-1) + 4/(e^4-1) + 9/(e^6-1) + 16/(e^8-1)]$$

Input:

$$64 \times \frac{\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}}$$

Exact result:

$$\frac{64\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}}$$

Decimal approximation:

776.6592531568066432027246318478731182842039609617907465883...

Alternate form:

$$\frac{64(e-1)(1+e)(1+e^2)(1-e+e^2)(1+e+e^2)(1+e^4)\pi}{30+31e^2+36e^4+16e^6+6e^8+e^{10}}$$

Continued fraction:

Linear form

$$776 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{14 + \frac{1}{3 + \frac{1}{8 + \frac{1}{1 + \frac{1}{1 + \frac{1}{93 + \frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{14 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\dots}}$$

Alternative representations:

$$\frac{64\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{11520^\circ}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}}$$

$$\frac{64\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = -\frac{64i \log(-1)}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}}$$

$$\frac{64\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{64\pi}{\frac{1}{\exp^2(z)-1} + \frac{4}{\exp^4(z)-1} + \frac{9}{\exp^6(z)-1} + \frac{16}{\exp^8(z)-1}} \quad \text{for } z = 1$$

Series representations:

$$\frac{64\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{256 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}}$$

$$\frac{64\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{256 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}{\frac{1}{-1+\sum_{k=0}^{\infty} \frac{2^k}{k!}} + \frac{4}{-1+\sum_{k=0}^{\infty} \frac{4^k}{k!}} + \frac{9}{-1+\sum_{k=0}^{\infty} \frac{6^k}{k!}} + \frac{16}{-1+\sum_{k=0}^{\infty} \frac{8^k}{k!}}}$$

$$\begin{aligned} \frac{64\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} &= \\ &\left( 64\pi \left( -1 + \sum_{k=0}^{\infty} \frac{1}{k!} \right) \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 - \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \right. \\ &\quad \left. \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4 \right) \right) / \\ &\left( 30 + 31 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 + 36 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^4 + 16 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^6 + 6 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^8 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{10} \right) \end{aligned}$$

Integral representations:

$$\frac{64\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{256}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{64\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{128}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{64\pi}{\frac{1}{e^2-1} + \frac{4}{e^4-1} + \frac{9}{e^6-1} + \frac{16}{e^8-1}} = \frac{128}{\frac{1}{-1+e^2} + \frac{4}{-1+e^4} + \frac{9}{-1+e^6} + \frac{16}{-1+e^8}} \int_0^{\infty} \frac{1}{1+t^2} dt$$

From (3) (PAGE 1), we have that:

$$[\sqrt{5}/(4\pi^2)] [e^{10}] 1/[(5*4-2+1/3+((4/(135(2+0.165))))]$$

Input:

$$\frac{\sqrt{5}}{4\pi^2} e^{10} \times \frac{1}{5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)}}$$

Result:

67.99932036...

Continued fraction:

Linear form

$$67 + \frac{1}{1 + \frac{1}{1470 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{8 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}}}}}}}}}$$

Series representations:

$$\frac{e^{10} \sqrt{5}}{\left(5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)}\right) (4\pi^2)} = \frac{0.0136262 e^{10} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}{\pi^2}$$

$$\frac{e^{10} \sqrt{5}}{\left(5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)}\right) (4\pi^2)} = \frac{0.0136262 e^{10} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{\pi^2}$$

$$\frac{e^{10} \sqrt{5}}{\left(5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)}\right) (4\pi^2)} = \frac{0.0068131 e^{10} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\pi^2 \sqrt{\pi}}$$

$$41/((\sqrt{5}+1)/2)) * [\sqrt{5}/(4\pi^2)] [e^{10}] 1/[(5*4-2+1/3+((4/(135(2+0.165))))]$$

Input:

$$\left(\frac{41}{2(\sqrt{5}+1)} \times \frac{\sqrt{5}}{4\pi^2}\right) e^{10} \times \frac{1}{5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)}}$$

Result:

1723.061539...

Series representations:

$$\frac{e^{10} 41 \sqrt{5}}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) (\sqrt{5} + 1) (4 \pi^2)} = \frac{1.11735 e^{10} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}{\pi^2 \left( 1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)}$$

$$\frac{e^{10} 41 \sqrt{5}}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) (\sqrt{5} + 1) (4 \pi^2)} = \frac{1.11735 e^{10} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{\pi^2 \left( 1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)}$$

$$\frac{e^{10} 41 \sqrt{5}}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) (\sqrt{5} + 1) (4 \pi^2)} = \frac{1.11735 e^{10} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\pi^2 \left( 2 \sqrt{\pi} + \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) \right)}$$

$$1/72 * 41/((\text{sqrt}(5)+1)/2))*[\text{sqrt}(5)/(4\text{Pi}^2)][e^10]1/[(5*4-2+1/3+((4/(135(2+0.165))))]$$

Input:

$$\left( \frac{1}{72} \times \frac{41}{\frac{1}{2} (\sqrt{5} + 1)} \times \frac{\sqrt{5}}{4 \pi^2} \right) e^{10} \times \frac{1}{5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)}}$$

Result:

23.93141026...

Continued fraction:

Linear form

$$23 + \cfrac{1}{1 + \cfrac{1}{13 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{113 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Series representations:

$$\frac{e^{10} (41 \sqrt{5})}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) 72 ((\sqrt{5} + 1)(4\pi^2))} = \frac{0.0155187 e^{10} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}{\pi^2 \left( 1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)}$$

$$\frac{e^{10} (41 \sqrt{5})}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) 72 ((\sqrt{5} + 1)(4\pi^2))} = \frac{0.0155187 e^{10} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{\pi^2 \left( 1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)}$$

$$\frac{e^{10} (41 \sqrt{5})}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) 72 ((\sqrt{5} + 1)(4\pi^2))} = \frac{0.0155187 e^{10} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\pi^2 \left( 2 \sqrt{\pi} + \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) \right)}$$

$$18/((\sqrt{5}+1)/2)) * [\sqrt{5}/(4\pi^2)] [e^{10}] 1/[(5*4-2+1/3+((4/(135(2+0.165))))]$$

Input:

$$\left( \frac{18}{\frac{1}{2} (\sqrt{5} + 1)} \times \frac{\sqrt{5}}{4\pi^2} \right) e^{10} \times \frac{1}{5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)}}$$

Result:

756.4660414...

Continued fraction:

Linear form

$$756 + \frac{1}{2 + \frac{1}{6 + \frac{1}{1 + \frac{1}{6 + \frac{1}{4 + \frac{1}{7 + \frac{1}{36 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}}}}}}}}}$$

Series representations:

$$\frac{e^{10} 18 \sqrt{5}}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) (\sqrt{5} + 1) (4 \pi^2)} = \frac{0.490543 e^{10} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}{\pi^2 \left( 1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)}$$

$$\frac{e^{10} 18 \sqrt{5}}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) (\sqrt{5} + 1) (4 \pi^2)} = \frac{0.490543 e^{10} \sqrt{4} \sum_{k=0}^{\infty} \frac{(-\frac{1}{4})^k (-\frac{1}{2})_k}{k!}}{\pi^2 \left( 1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{(-\frac{1}{4})^k (-\frac{1}{2})_k}{k!} \right)}$$

$$\frac{e^{10} 18 \sqrt{5}}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) (\sqrt{5} + 1) (4 \pi^2)} = \frac{0.490543 e^{10} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma(-\frac{1}{2} - s) \Gamma(s)}{\pi^2 \left( 2 \sqrt{\pi} + \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma(-\frac{1}{2} - s) \Gamma(s) \right)}$$

$$1/142*41/((\text{sqrt}(5)+1)/2))*[\text{sqrt}(5)/(4\text{Pi}^2)][e^10]1/[(5*4-2+1/3+((4/(135(2+0.165))))]$$

Input:

$$\left( \frac{1}{142} \times \frac{41}{\frac{1}{2} (\sqrt{5} + 1)} \times \frac{\sqrt{5}}{4 \pi^2} \right) e^{10} \times \frac{1}{5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)}}$$

Result:

12.13423619...

Continued fraction:

Linear form

$$\begin{array}{l}
 12 + \frac{1}{7 + \frac{1}{2 + \frac{1}{4 + \frac{1}{2 + \frac{1}{5 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{25 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}} \\
 \end{array}$$

Series representations:

$$\frac{e^{10} (41 \sqrt{5})}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) 142 ((\sqrt{5} + 1)(4\pi^2))} = \frac{0.00786865 e^{10} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}{\pi^2 \left( 1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)}$$

$$\begin{aligned}
 & \frac{e^{10} (41 \sqrt{5})}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) 142 ((\sqrt{5} + 1)(4\pi^2))} = \\
 & \frac{0.00786865 e^{10} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{\pi^2 \left( 1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{e^{10} (41 \sqrt{5})}{\frac{1}{2} \left( 5 \times 4 - 2 + \frac{1}{3} + \frac{4}{135(2+0.165)} \right) 142 ((\sqrt{5} + 1)(4\pi^2))} = \\
 & \frac{0.00786865 e^{10} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\pi^2 \left( 2 \sqrt{\pi} + \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) \right)}
 \end{aligned}$$



(7)

[22]

VII Theorems on approximate integrations and summation of series.

$$(1) \quad 1^2 \log 1 + 2^2 \log 2 + 3^2 \log 3 + \dots + x^2 \log x$$

$$= \frac{x(x+1)(2x+1)}{6} \log x - \frac{x^3}{9} + \frac{1}{4\pi^2} \left( \frac{1}{7} + \frac{1}{2} + \frac{1}{5} + \dots \right)$$

$$+ \frac{x}{12} - \frac{1}{360x} + \dots$$

$$(2) \quad 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^k}{k!} \theta = \frac{e^x}{2}$$

where  $\theta = \frac{1}{3} + \frac{4}{135(x+k)}$  where  $k$  lies between  $\frac{8}{45}$  and  $\frac{9}{27}$ .

$$(3) \quad 1 + \left(\frac{x}{1!}\right)^5 + \left(\frac{x^2}{2!}\right)^5 + \left(\frac{x^3}{3!}\right)^5 + \dots$$

$$= \frac{\sqrt{5}}{4\pi^2} \frac{e^{5x}}{5x^2 - x + \theta} \quad \text{where } \theta \text{ vanishes when } x = \infty.$$

$$(4) \quad \frac{1^4}{e^{2 \cdot 1}} + \frac{2^4}{e^{2 \cdot 2}} + \frac{3^4}{e^{2 \cdot 3}} + \frac{4^4}{e^{2 \cdot 4}} + \dots$$

$$= \frac{3}{x^2} \left( \frac{1}{7} + \frac{1}{2} + \frac{1}{3} + \dots \right) - \frac{1}{12x} + \frac{x}{1440} + \frac{x^3}{181440}$$

$$+ \frac{x^5}{7257600} + \frac{x^7}{159667200} + \dots \quad \text{when } x \text{ is small}$$

(Note:  $x$  may be given values from 0 to 2)

$$(5) \quad \frac{1}{100^1} + \frac{1}{100^2} + \frac{3}{100^3} + \frac{4^2}{100^4} + \frac{5^3}{100^5} + \dots$$

$$= \frac{1}{1000} - 10^{-440} \times 1.0125 \text{ nearly}$$

$$(6) \quad \int_0^a e^{-x^2} dx = \frac{\sqrt{\pi}}{2} - \frac{e^{-a^2}}{2a} + \frac{1}{a} + \frac{2}{2a} + \frac{3}{a} + \frac{4}{2a} + \dots$$

$$(7) \quad \text{The coeff. of } x^n \text{ in } \frac{1}{1 - 2x + 2x^2 - 2x^3 + 2x^4 - \dots}$$

$$= \text{The nearest integer to } \frac{1}{2^n} \left\{ \cosh(n\sqrt{2}) - \frac{\sinh(n\sqrt{2})}{\sqrt{2}} \right\}$$

(7) [22]

VII Theorems on approximate integrations and summation of series.

(1)  $1^k \log 1 + 2^k \log 2 + 3^k \log 3 + \dots + x^k \log x$   
 $= \frac{x(x+1)(2x+1)}{6} \log x - \frac{x^3}{9} + \frac{1}{4\pi^2} (\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots)$   
 $+ \frac{x}{12} - \frac{1}{360x} + \dots$

(2)  $1 + \frac{x}{12} + \frac{x^2}{24} + \frac{x^3}{12} + \dots + \frac{x^x}{12} \theta = \frac{e^x}{2}$   
 where  $\theta = \frac{1}{3} + \frac{4}{135(x+k)}$  where  $k$  lies between  $\frac{8}{45}$  and  $\frac{9}{27}$ .

(3)  $1 + (\frac{x}{12})^5 + (\frac{x^2}{12})^5 + (\frac{x^3}{12})^5 + \dots$   
 $= \frac{\sqrt{5}}{4\pi^2} \frac{e^{5x}}{5x^2 - x + \theta}$  where  $\theta$  vanishes when  $x = \infty$ .

(4)  $\frac{1^k}{e^{2x}} + \frac{2^k}{e^{4x}} + \frac{3^k}{e^{6x}} + \frac{4^k}{e^{8x}} + \dots$   
 $= \frac{2}{x^2} (\frac{1}{12} + \frac{1}{24} + \frac{1}{36} + \dots) - \frac{1}{12x} + \frac{x}{1440} + \frac{x^3}{181440}$   
 $+ \frac{x^5}{7257600} + \frac{x^7}{159667200} + \dots$  when  $x$  is small  
 (Note:  $x$  may be given values from 0 to 2)

(5)  $\frac{1}{1001} + \frac{1}{1002^2} + \frac{1}{1003^3} + \frac{1}{1004^4} + \frac{1}{1005^5} + \dots$   
 $= \frac{1}{1000} - 10^{-440} \times 1.0125$  nearly

(6)  $\int_0^a e^{-x^2} dx = \frac{\sqrt{\pi}}{2} - \frac{e^{-a^2}}{2a} + \frac{1}{a} + \frac{2}{3a} + \frac{3}{5a} + \frac{4}{7a} + \dots$

(7) The coeff. of  $x^n$  in  $\frac{1}{1 - 2x + 2x^2 - 2x^3 + 2x^4 - \dots}$   
 $=$  The nearest integer to  $\frac{1}{2^n} \left\{ \cosh(n\sqrt{2}) - \frac{\sinh(n\sqrt{2})}{\sqrt{2}} \right\}$

(2)

[19]

(3) 
$$\int_0^{\infty} \frac{1}{\{1+(\frac{x}{a})^2\} \{1+(\frac{x}{2a})^2\} \{1+(\frac{x}{3a})^2\} \dots \{1+(\frac{x}{(n-1)a})^2\} \{1+(\frac{x}{na})^2\} \{1+(\frac{x}{(n+1)a})^2\} \dots} dx$$

$$= \frac{\sqrt{\pi}}{2} \cdot \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n)} \cdot \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n)} \cdot \frac{\Gamma(n+1)}{\Gamma(n+\frac{1}{2})}$$

(4) If  $\int_0^{\infty} \frac{\cos nx}{e^{2\pi\sqrt{x}} - 1} dx = \phi(n)$ , then

$$\int_0^{\infty} \frac{\sin nx}{e^{2\pi\sqrt{x}} - 1} dx = \phi(n) - \frac{1}{2n} + \phi(\frac{n^2}{4}) \sqrt{\frac{2n}{\pi}}$$

$\phi(n)$  is a complicated function. The following are certain special values.

$\phi(1) = \frac{1}{12}$ ;  $\phi(\frac{\pi}{2}) = \frac{1}{4\pi}$ ;  $\phi(\pi) = \frac{2-\sqrt{2}}{8}$ ;  $\phi(2\pi) = \frac{1}{8}$

$\phi(\frac{3\pi}{2}) = \frac{2-3\sqrt{2}}{16}$ ;  $\phi(\frac{\pi}{4}) = \frac{6+\sqrt{2}}{4} - \frac{5\sqrt{2}}{8}$ ;  $\phi(\infty) = 0$

$\phi(\frac{3\pi}{4}) = \frac{1}{8} - \sqrt{2} (\frac{3}{8} - \frac{1}{4\pi})$

(5) 
$$\int_0^{\infty} \frac{dx}{(1+x^2)(1+a^2x^2)(1+b^2x^2)(1+c^2x^2) \dots}$$

$$= \frac{\pi}{2(1+a+a^3+a^5+a^7+\dots)}$$

where 1, 3, 5, 7, ... are sums of natural nos.

(6) 
$$\int_0^{\infty} \frac{\sin 2nx}{x(\cosh \pi x + \cos \pi x)} dx =$$

$$\frac{\pi}{2} - 2 \left( \frac{e^{-2n} \cos 2n}{\cosh \frac{\pi}{2}} - \frac{e^{-2n} \cos 2n}{2 \cosh \frac{\pi}{4}} + \frac{e^{-2n} \cos 2n}{4 \cosh \frac{\pi}{8}} - \dots \right)$$

(7) 
$$\int_0^{\infty} \frac{\tan^{-1} \frac{2xz}{x^2+x^2-2z^2}}{e^{2\pi z} - 1} dz$$
 can be exactly found if  $2x$  is any integer and  $x$  any quantity.

From the equation (5) (PAGE 2), we have, for  $n = 4.1833$  and  $x = 0.012$  :

$$\text{integrate } (\sin 0.1004) / ((0.012(\cosh \pi + \cos \pi)))$$

Assuming trigonometric arguments in radians, we obtain

Indefinite integral:

$$\int \frac{\sin(0.1004)}{0.012 (\cosh(\pi) + \cos(\pi))} dx = 0.788581 x + \text{constant}$$

$$\frac{\pi}{4} - 2 \left[ \frac{\exp(-4.1833) \cos 4.1833}{\cosh(\pi/2)} - \frac{\exp(-12.549) \cos 12.549}{3 \cosh(3\pi/2)} + \frac{\exp(-20.9165) \cos 20.9165}{5 \cosh(5\pi/2)} \right]$$

Input interpretation:

$$\frac{\pi}{4} - 2 \left( \exp(-4.1833) \times \frac{\cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \exp(-12.549) \times \frac{\cos(12.549)}{3 \cosh\left(3 \times \frac{\pi}{2}\right)} + \exp(-20.9165) \times \frac{\cos(20.9165)}{5 \cosh\left(5 \times \frac{\pi}{2}\right)} \right)$$

Result:

0.791533...

- $\cosh(x)$  is the hyperbolic cosine function

Alternative representations:

$$\frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$\frac{\pi}{4} - 2 \left( \frac{\cosh(-4.1833 i) \exp(-4.1833)}{\cos\left(\frac{i\pi}{2}\right)} - \frac{\cosh(-12.549 i) \exp(-12.549)}{3 \cos\left(\frac{3i\pi}{2}\right)} + \frac{\cosh(-20.9165 i) \exp(-20.9165)}{5 \cos\left(\frac{5i\pi}{2}\right)} \right)$$

$$\frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$\frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) (e^{-4.1833 i} + e^{4.1833 i})}{2 \cos\left(\frac{i\pi}{2}\right)} - \frac{\exp(-12.549) (e^{-12.549 i} + e^{12.549 i})}{2 (3 \cos\left(\frac{3i\pi}{2}\right))} + \frac{\exp(-20.9165) (e^{-20.9165 i} + e^{20.9165 i})}{2 (5 \cos\left(\frac{5i\pi}{2}\right))} \right)$$

$$\frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$\frac{\pi}{4} - 2 \left( \frac{\cosh(4.1833 i) \exp(-4.1833)}{\cos\left(-\frac{i\pi}{2}\right)} - \frac{\cosh(12.549 i) \exp(-12.549)}{3 \cos\left(-\frac{3i\pi}{2}\right)} + \frac{\cosh(20.9165 i) \exp(-20.9165)}{5 \cos\left(-\frac{5i\pi}{2}\right)} \right)$$

Series representations:

$$\frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$\frac{\pi}{4} - \frac{2 \exp(-4.1833) \sum_{k=0}^{\infty} \frac{(-1)^k e^{2.8622k}}{(2k)!}}{I_0\left(\frac{\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{\pi}{2}\right)} + \frac{2 \exp(-12.549) \sum_{k=0}^{\infty} \frac{(-1)^k e^{5.05928k}}{(2k)!}}{3 \left( I_0\left(\frac{3\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{3\pi}{2}\right) \right)} -$$

$$\frac{2 \exp(-20.9165) \sum_{k=0}^{\infty} \frac{(-1)^k e^{8.08108k}}{(2k)!}}{5 \left( I_0\left(\frac{5\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{5\pi}{2}\right) \right)}$$

$$\frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) = \left( -120 \exp(-4.1833) \right.$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 2^{-2k_2-2k_3} \times 3^{2k_2} \times 4.1833^{2k_1} \times 5^{2k_3} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} +$$

$$40 \exp(-12.549) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 2^{-2k_2-2k_3} \times 5^{2k_3} \times 12.549^{2k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} -$$

$$24 \exp(-20.9165) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 2^{-2k_2-2k_3} \times 3^{2k_2} \times 20.9165^{2k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} +$$

$$15 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{2^{-2k_1-2k_2-2k_3} \times 3^{2k_1} \times 5^{2k_3} \pi^{2k_1+2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} \Big/$$

$$\left( 60 \left( \sum_{k=0}^{\infty} \frac{\left(\frac{9}{4}\right)^k \pi^{2k}}{(2k)!} \right) \left( \sum_{k=0}^{\infty} \frac{4^{-k} \pi^{2k}}{(2k)!} \right) \sum_{k=0}^{\infty} \frac{\left(\frac{25}{4}\right)^k \pi^{2k}}{(2k)!} \right)$$

$$\begin{aligned}
& \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \right. \\
& \quad \left. \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) = \\
& \left( -120 \exp(-4.1833) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} \times 9^{k_2} \times 25^{k_3} e^{2.8622 k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} + \right. \\
& \quad 40 \exp(-12.549) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} \times 25^{k_3} e^{5.05928 k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} - \\
& \quad 24 \exp(-20.9165) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} \times 9^{k_2} e^{6.08108 k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} + \\
& \quad \left. 15 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{4^{-k_1-k_2-k_3} \times 9^{k_1} \times 25^{k_3} \pi^{2k_1+2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} \right) / \\
& \left( 60 \left( \sum_{k=0}^{\infty} \frac{\left(\frac{9}{4}\right)^k \pi^{2k}}{(2k)!} \right) \left( \sum_{k=0}^{\infty} \frac{4^{-k} \pi^{2k}}{(2k)!} \right) \sum_{k=0}^{\infty} \frac{\left(\frac{25}{4}\right)^k \pi^{2k}}{(2k)!} \right)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \right. \\
& \quad \left. \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) = \\
& \frac{1}{4} \left( \pi + 4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left( - \frac{e^{-4.375/s+s} \exp(-4.1833) \sqrt{\pi}}{i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{\pi}{2}} \sinh(t) dt} + \frac{e^{-39.3694/s+s} \exp(-12.549) \sqrt{\pi}}{3 i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{3\pi}{2}} \sinh(t) dt} - \right. \right. \\
& \quad \left. \left. \frac{e^{-109.375/s+s} \exp(-20.9165) \sqrt{\pi}}{5 i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{5\pi}{2}} \sinh(t) dt} \right) ds \right) \text{ for } \gamma > 0
\end{aligned}$$

$$\frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$\frac{\pi}{4} + \frac{2 \exp(-4.1833)}{1 + \frac{\pi}{2} \int_0^1 \sinh\left(\frac{\pi t}{2}\right) dt} \int_{\frac{\pi}{2}}^{4.1833} \sin(t) dt - \frac{2 \exp(-12.549)}{3 \left(1 + \frac{3\pi}{2} \int_0^1 \sinh\left(\frac{3\pi t}{2}\right) dt\right)} \int_{\frac{\pi}{2}}^{12.549} \sin(t) dt + \frac{2 \exp(-20.9165)}{5 \left(1 + \frac{5\pi}{2} \int_0^1 \sinh\left(\frac{5\pi t}{2}\right) dt\right)} \int_{\frac{\pi}{2}}^{20.9165} \sin(t) dt$$

$$\frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$\frac{1}{4} \left( \pi + 4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left( -\frac{2.09165^{-2s} \exp(-4.1833) \Gamma(s) \sqrt{\pi}}{i \pi \Gamma\left(\frac{1}{2} - s\right) \int_{\frac{i\pi}{2}}^{\frac{\pi}{2}} \sinh(t) dt} + \frac{6.2745^{-2s} \exp(-12.549) \Gamma(s) \sqrt{\pi}}{3 i \pi \Gamma\left(\frac{1}{2} - s\right) \int_{\frac{i\pi}{2}}^{\frac{3\pi}{2}} \sinh(t) dt} - \frac{10.4583^{-2s} \exp(-20.9165) \Gamma(s) \sqrt{\pi}}{5 i \pi \Gamma\left(\frac{1}{2} - s\right) \int_{\frac{i\pi}{2}}^{\frac{5\pi}{2}} \sinh(t) dt} \right) ds \right) \text{ for } 0 < \gamma < \frac{1}{2}$$

We have that:

$$-20 + 47^2 * (((\pi/4 - 2[(\exp(-4.1833)\cos(4.1833)/(\cosh(\pi/2)) - \exp(-12.549)\cos(12.549)/(3\cosh(3\pi/2)) + \exp(-20.9165)\cos(20.9165)/(5\cosh(5\pi/2))]))))$$

Input interpretation:

$$-20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \exp(-4.1833) \times \frac{\cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \exp(-12.549) \times \frac{\cos(12.549)}{3 \cosh\left(3 \times \frac{\pi}{2}\right)} + \exp(-20.9165) \times \frac{\cos(20.9165)}{5 \cosh\left(5 \times \frac{\pi}{2}\right)} \right) \right)$$

[Open code](#)

- $\cosh(x)$  is the hyperbolic cosine function



Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1728.50...

Alternative representations:

More

$$\begin{aligned}
 & -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = \\
 & -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\cosh(-4.1833 i) \exp(-4.1833)}{\cos\left(\frac{i\pi}{2}\right)} - \frac{\cosh(-12.549 i) \exp(-12.549)}{3 \cos\left(\frac{3i\pi}{2}\right)} + \frac{\cosh(-20.9165 i) \exp(-20.9165)}{5 \cos\left(\frac{5i\pi}{2}\right)} \right) \right)
 \end{aligned}$$

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$$\begin{aligned}
 & -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = -20 + 47^2 \\
 & \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) (e^{-4.1833 i} + e^{4.1833 i})}{2 \cos\left(\frac{i\pi}{2}\right)} - \frac{\exp(-12.549) (e^{-12.549 i} + e^{12.549 i})}{2 (3 \cos\left(\frac{3i\pi}{2}\right))} + \frac{\exp(-20.9165) (e^{-20.9165 i} + e^{20.9165 i})}{2 (5 \cos\left(\frac{5i\pi}{2}\right))} \right) \right)
 \end{aligned}$$

[Open code](#)

$$\begin{aligned}
 & -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = \\
 & -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\cosh(4.1833 i) \exp(-4.1833)}{\cos\left(-\frac{i\pi}{2}\right)} - \frac{\cosh(12.549 i) \exp(-12.549)}{3 \cos\left(-\frac{3i\pi}{2}\right)} + \frac{\cosh(20.9165 i) \exp(-20.9165)}{5 \cos\left(-\frac{5i\pi}{2}\right)} \right) \right)
 \end{aligned}$$

Series representations:

More

$$\begin{aligned}
 & -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = \\
 & -20 + \frac{2209 \pi}{4} - \frac{4418 \exp(-4.1833) \sum_{k=0}^{\infty} \frac{(-1)^k e^{2.8622 k}}{(2k)!}}{I_0\left(\frac{\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{\pi}{2}\right)} + \\
 & \frac{4418 \exp(-12.549) \sum_{k=0}^{\infty} \frac{(-1)^k e^{5.05928 k}}{(2k)!}}{3 \left( I_0\left(\frac{3\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{3\pi}{2}\right) \right)} - \\
 & \frac{4418 \exp(-20.9165) \sum_{k=0}^{\infty} \frac{(-1)^k e^{6.08108 k}}{(2k)!}}{5 \left( I_0\left(\frac{5\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{5\pi}{2}\right) \right)}
 \end{aligned}$$

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$$\begin{aligned}
 & -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \right. \right. \\
 & \left. \left. \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = \left( -265\,080 \exp(-4.1833) \right. \\
 & \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 2^{-2k_2-2k_3} \times 3^{2k_2} \times 4.1833^{2k_1} \times 5^{2k_3} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} + 88\,360 \\
 & \exp(-12.549) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 2^{-2k_2-2k_3} \times 5^{2k_3} \times 12.549^{2k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} - \\
 & 53\,016 \exp(-20.9165) \\
 & \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 2^{-2k_2-2k_3} \times 3^{2k_2} \times 20.9165^{2k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} - \\
 & 1200 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{2^{-2k_1-2k_2-2k_3} \times 3^{2k_1} \times 5^{2k_3} \pi^{2k_1+2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} + \\
 & \left. 33\,135 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{2^{-2k_1-2k_2-2k_3} \times 3^{2k_1} \times 5^{2k_3} \pi^{2k_1+2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} \right) / \\
 & \left( 60 \left( \sum_{k=0}^{\infty} \frac{\left(\frac{9}{4}\right)^k \pi^{2k}}{(2k)!} \right) \left( \sum_{k=0}^{\infty} \frac{4^{-k} \pi^{2k}}{(2k)!} \right) \sum_{k=0}^{\infty} \frac{\left(\frac{25}{4}\right)^k \pi^{2k}}{(2k)!} \right)
 \end{aligned}$$

$$\begin{aligned}
& -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = \\
& -20 + \frac{2209 \pi}{4} - \frac{4418 J_0(4.1833) \exp(-4.1833)}{I_0\left(\frac{\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{\pi}{2}\right)} + \frac{4418 J_0(12.549) \exp(-12.549)}{3 \left( I_0\left(\frac{3\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{3\pi}{2}\right) \right)} - \\
& \frac{4418 J_0(20.9165) \exp(-20.9165)}{5 \left( I_0\left(\frac{5\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{5\pi}{2}\right) \right)} - \\
& \frac{8836 \exp(-4.1833) \sum_{k=1}^{\infty} (-1)^k J_{2k}(4.1833)}{I_0\left(\frac{\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{\pi}{2}\right)} + \\
& \frac{8836 \exp(-12.549) \sum_{k=1}^{\infty} (-1)^k J_{2k}(12.549)}{3 \left( I_0\left(\frac{3\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{3\pi}{2}\right) \right)} - \\
& \frac{8836 \exp(-20.9165) \sum_{k=1}^{\infty} (-1)^k J_{2k}(20.9165)}{5 \left( I_0\left(\frac{5\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{5\pi}{2}\right) \right)}
\end{aligned}$$

Integral representations:

More

$$\begin{aligned}
& -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = \\
& \frac{1}{4} \left( -80 + 2209 \pi + 4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left( - \frac{2209 e^{-4.375/s+s} \exp(-4.1833) \sqrt{\pi}}{i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{\pi}{2}} \sinh(t) dt} + \right. \right. \\
& \left. \frac{2209 e^{-39.3694/s+s} \exp(-12.549) \sqrt{\pi}}{3 i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{3\pi}{2}} \sinh(t) dt} - \right. \\
& \left. \left. \frac{2209 e^{-109.375/s+s} \exp(-20.9165) \sqrt{\pi}}{5 i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{5\pi}{2}} \sinh(t) dt} \right) ds \right) \text{ for } \gamma > 0
\end{aligned}$$

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$$\begin{aligned}
& -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh(\frac{\pi}{2})} - \right. \right. \\
& \quad \left. \left. \frac{\exp(-12.549) \cos(12.549)}{3 \cosh(\frac{3\pi}{2})} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh(\frac{5\pi}{2})} \right) \right) = \\
& -20 + \frac{2209 \pi}{4} + \frac{4418 \exp(-4.1833)}{1 + \frac{\pi}{2} \int_0^1 \sinh(\frac{\pi t}{2}) dt} \int_{\frac{\pi}{2}}^{4.1833} \sin(t) dt - \\
& \frac{4418 \exp(-12.549)}{3 \left( 1 + \frac{3\pi}{2} \int_0^1 \sinh(\frac{3\pi t}{2}) dt \right)} \int_{\frac{\pi}{2}}^{12.549} \sin(t) dt + \\
& \frac{4418 \exp(-20.9165)}{5 \left( 1 + \frac{5\pi}{2} \int_0^1 \sinh(\frac{5\pi t}{2}) dt \right)} \int_{\frac{\pi}{2}}^{20.9165} \sin(t) dt
\end{aligned}$$

$$\begin{aligned}
& -20 + 47^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh(\frac{\pi}{2})} - \right. \right. \\
& \quad \left. \left. \frac{\exp(-12.549) \cos(12.549)}{3 \cosh(\frac{3\pi}{2})} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh(\frac{5\pi}{2})} \right) \right) = \\
& \frac{1}{4} \left( -80 + 2209 \pi + 4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left( - \frac{2209 \times 2.09165^{-2s} \exp(-4.1833) \Gamma(s) \sqrt{\pi}}{i \pi \Gamma(\frac{1}{2} - s) \int_{\frac{i\pi}{2}}^{\frac{\pi}{2}} \sinh(t) dt} + \right. \right. \\
& \quad \left. \left. \frac{2209 \times 6.2745^{-2s} \exp(-12.549) \Gamma(s) \sqrt{\pi}}{3 i \pi \Gamma(\frac{1}{2} - s) \int_{\frac{i\pi}{2}}^{\frac{3\pi}{2}} \sinh(t) dt} - \right. \right. \\
& \quad \left. \left. \frac{2209 \times 10.4583^{-2s} \exp(-20.9165) \Gamma(s) \sqrt{\pi}}{5 i \pi \Gamma(\frac{1}{2} - s) \int_{\frac{i\pi}{2}}^{\frac{5\pi}{2}} \sinh(t) dt} \right) ds \right) \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

31^2 \* (((Pi/4-2[(exp(-4.1833)cos4.1833/(cosh(Pi/2))-exp(-12.549)cos12.549/(3cosh(3Pi/2))+exp(-20.9165)cos20.9165/(5cosh(5Pi/2))]))))

Input interpretation:

$$\begin{aligned}
& 31^2 \left( \frac{\pi}{4} - 2 \left( \exp(-4.1833) \times \frac{\cos(4.1833)}{\cosh(\frac{\pi}{2})} - \right. \right. \\
& \quad \left. \left. \exp(-12.549) \times \frac{\cos(12.549)}{3 \cosh(3 \times \frac{\pi}{2})} + \exp(-20.9165) \times \frac{\cos(20.9165)}{5 \cosh(5 \times \frac{\pi}{2})} \right) \right)
\end{aligned}$$

[Open code](#)

- $\cosh(x)$  is the hyperbolic cosine function

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Result:

- More digits  
760.663...

Alternative representations:

- More

$$31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) =$$

$$31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\cosh(-4.1833 i) \exp(-4.1833)}{\cos\left(\frac{i\pi}{2}\right)} - \frac{\cosh(-12.549 i) \exp(-12.549)}{3 \cos\left(\frac{3i\pi}{2}\right)} + \frac{\cosh(-20.9165 i) \exp(-20.9165)}{5 \cos\left(\frac{5i\pi}{2}\right)} \right) \right)$$

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$$31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) =$$

$$31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) (e^{-4.1833 i} + e^{4.1833 i})}{2 \cos\left(\frac{i\pi}{2}\right)} - \frac{\exp(-12.549) (e^{-12.549 i} + e^{12.549 i})}{2 (3 \cos\left(\frac{3i\pi}{2}\right))} + \frac{\exp(-20.9165) (e^{-20.9165 i} + e^{20.9165 i})}{2 (5 \cos\left(\frac{5i\pi}{2}\right))} \right) \right)$$

[Open code](#)

$$31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) =$$

$$31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\cosh(4.1833 i) \exp(-4.1833)}{\cos\left(-\frac{i\pi}{2}\right)} - \frac{\cosh(12.549 i) \exp(-12.549)}{3 \cos\left(-\frac{3i\pi}{2}\right)} + \frac{\cosh(20.9165 i) \exp(-20.9165)}{5 \cos\left(-\frac{5i\pi}{2}\right)} \right) \right)$$

Series representations:

- More

$$\begin{aligned}
& 31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = \\
& \frac{961 \pi}{4} - \frac{1922 \exp(-4.1833) \sum_{k=0}^{\infty} \frac{(-1)^k e^{2.8622k}}{(2k)!}}{I_0\left(\frac{\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{\pi}{2}\right)} + \\
& \frac{1922 \exp(-12.549) \sum_{k=0}^{\infty} \frac{(-1)^k e^{5.05928k}}{(2k)!}}{3 \left( I_0\left(\frac{3\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{3\pi}{2}\right) \right)} - \\
& \frac{1922 \exp(-20.9165) \sum_{k=0}^{\infty} \frac{(-1)^k e^{6.08108k}}{(2k)!}}{5 \left( I_0\left(\frac{5\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{5\pi}{2}\right) \right)}
\end{aligned}$$

$$\begin{aligned}
& 31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = \left( 961 \left( -120 \exp(-4.1833) \right. \right. \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 2^{-2k_2-2k_3} \times 3^{2k_2} \times 4.1833^{2k_1} \times 5^{2k_3} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} + 40 \\
& \frac{\exp(-12.549)}{\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 2^{-2k_2-2k_3} \times 5^{2k_3} \times 12.549^{2k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} - 24 \\
& \frac{\exp(-20.9165)}{\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 2^{-2k_2-2k_3} \times 3^{2k_2} \times 20.9165^{2k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} + \\
& \left. \left. 15 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{2^{-2k_1-2k_2-2k_3} \times 3^{2k_1} \times 5^{2k_3} \pi^{2k_1+2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} \right) \right) / \\
& \left( 60 \left( \sum_{k=0}^{\infty} \frac{\left(\frac{\pi}{4}\right)^k \pi^{2k}}{(2k)!} \right) \left( \sum_{k=0}^{\infty} \frac{4^{-k} \pi^{2k}}{(2k)!} \right) \sum_{k=0}^{\infty} \frac{\left(\frac{25}{4}\right)^k \pi^{2k}}{(2k)!} \right)
\end{aligned}$$

$$\begin{aligned}
& 31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \right. \right. \\
& \quad \left. \left. \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = \left( 961 \left( -120 \exp(-4.1833) \right. \right. \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} \times 9^{k_2} \times 25^{k_3} e^{2.8622 k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} + \\
& \quad 40 \exp(-12.549) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} \times 25^{k_3} e^{5.05928 k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} - \\
& \quad 24 \exp(-20.9165) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} \times 9^{k_2} e^{6.08108 k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} + \\
& \quad \left. \left. 15 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{4^{-k_1-k_2-k_3} \times 9^{k_1} \times 25^{k_3} \pi^{2k_1+2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} \right) \right) / \\
& \left( 60 \left( \sum_{k=0}^{\infty} \frac{\left(\frac{9}{4}\right)^k \pi^{2k}}{(2k)!} \right) \left( \sum_{k=0}^{\infty} \frac{4^{-k} \pi^{2k}}{(2k)!} \right) \sum_{k=0}^{\infty} \frac{\left(\frac{25}{4}\right)^k \pi^{2k}}{(2k)!} \right)
\end{aligned}$$

Integral representations:

More

$$\begin{aligned}
& 31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \right. \right. \\
& \quad \left. \left. \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) = \\
& \frac{1}{4} \left( 961 \pi + 4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left( - \frac{961 e^{-4.375/s+s} \exp(-4.1833) \sqrt{\pi}}{i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{\pi}{2}} \sinh(t) dt} + \right. \right. \\
& \quad \left. \frac{961 e^{-39.3694/s+s} \exp(-12.549) \sqrt{\pi}}{3 i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{3\pi}{2}} \sinh(t) dt} - \right. \\
& \quad \left. \left. \frac{961 e^{-109.375/s+s} \exp(-20.9165) \sqrt{\pi}}{5 i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{5\pi}{2}} \sinh(t) dt} \right) ds \right) \text{ for } \gamma > 0
\end{aligned}$$

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$$31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) =$$

$$\frac{961 \pi}{4} + \frac{1922 \exp(-4.1833)}{1 + \frac{\pi}{2} \int_0^1 \sinh\left(\frac{\pi t}{2}\right) dt} \int_{\frac{\pi}{2}}^{4.1833} \sin(t) dt -$$

$$\frac{1922 \exp(-12.549)}{3 \left(1 + \frac{3\pi}{2} \int_0^1 \sinh\left(\frac{3\pi t}{2}\right) dt\right)} \int_{\frac{\pi}{2}}^{12.549} \sin(t) dt +$$

$$\frac{1922 \exp(-20.9165)}{5 \left(1 + \frac{5\pi}{2} \int_0^1 \sinh\left(\frac{5\pi t}{2}\right) dt\right)} \int_{\frac{\pi}{2}}^{20.9165} \sin(t) dt$$

$$31^2 \left( \frac{\pi}{4} - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) \right) =$$

$$\frac{1}{4} \left( 961 \pi + 4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left( - \frac{961 \times 2.09165^{-2s} \exp(-4.1833) \Gamma(s) \sqrt{\pi}}{i \pi \Gamma\left(\frac{1}{2} - s\right) \int_{\frac{i\pi}{2}}^{\frac{\pi}{2}} \sinh(t) dt} + \right. \right.$$

$$\left. - \frac{961 \times 6.2745^{-2s} \exp(-12.549) \Gamma(s) \sqrt{\pi}}{3 i \pi \Gamma\left(\frac{1}{2} - s\right) \int_{\frac{i\pi}{2}}^{\frac{3\pi}{2}} \sinh(t) dt} - \right.$$

$$\left. \frac{961 \times 10.4583^{-2s} \exp(-20.9165) \Gamma(s) \sqrt{\pi}}{5 i \pi \Gamma\left(\frac{1}{2} - s\right) \int_{\frac{i\pi}{2}}^{\frac{5\pi}{2}} \sinh(t) dt} \right) ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

We have that:

integrate  $-13 + 47^2 * (\sin(0.1003992))/((0.012(\cosh(\pi)+\cos(\pi)))$

Indefinite integral:



$$\int \left( -13 + \frac{47^2 \sin(0.1003992)}{0.012 (\cosh(\pi) + \cos(\pi))} \right) dx = 1728.96 x + \text{constant}$$

integrate 27\*36 \* (sin0.1003992)/((0.012(coshPi+cosPi))

Indefinite integral:

$$\int \frac{27 \times 36 \sin(0.1003992)}{0.012 (\cosh(\pi) + \cos(\pi))} dx = 766.495 x + \text{constant}$$

integrate (1728+26) \* (sin0.1003992)/((0.012(coshPi+cosPi))

Indefinite integral:

$$\int \frac{(1728 + 26) \sin(0.1003992)}{0.012 (\cosh(\pi) + \cos(\pi))} dx = 1383.16 x + \text{constant}$$

integrate (728-32) \* (sin0.1003992)/((0.012(coshPi+cosPi))

Indefinite integral:

$$\int \frac{(728 - 32) \sin(0.1003992)}{0.012 (\cosh(\pi) + \cos(\pi))} dx = 548.848 x + \text{constant}$$

We note that the two results is very near. Indeed:

$$\int \frac{\sin(0.1004)}{0.012 (\cosh(\pi) + \cos(\pi))} dx = 0.788581 x + \text{constant}$$

$$\frac{d}{dx}(0.788581 x) = 0.788581$$

$$\frac{\pi}{4} - 2 \left( \exp(-4.1833) \times \frac{\cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \exp(-12.549) \times \frac{\cos(12.549)}{3 \cosh\left(3 \times \frac{\pi}{2}\right)} + \exp(-20.9165) \times \frac{\cos(20.9165)}{5 \cosh\left(5 \times \frac{\pi}{2}\right)} \right)$$

Result:

0.791533...

Result:

0.791532832946769999594787530224069367341696536965481718929...

0.788581  $\approx$  0.791532832946769

Note that:

$$(1 / ((0.788581 + 0.791532832946769) / 2))^2$$

$$\frac{1}{\left(\frac{0.788581 + 0.791532832946769}{2}\right)^2}$$

1.602076467285328191581633308763561913765863520248234848613

Continued fraction:

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{18 + \frac{1}{1 + \frac{1}{1 + \frac{1}{34 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{19 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}$$

And

$$144 \times 6 \times \frac{1}{\left(\frac{0.788581 + 0.791532832946769}{2}\right)^2}$$

1384.194067734523557526531178771717493493706081494474909201

Continued fraction:

Linear form

$$1384 + \frac{1}{5 + \frac{1}{6 + \frac{1}{1 + \frac{1}{1 + \frac{1}{5 + \frac{1}{2 + \frac{1}{1 + \frac{1}{10 + \frac{1}{1 + \frac{1}{7 + \frac{1}{2 + \frac{1}{6 + \frac{1}{1 + \frac{1}{1 + \frac{1}{7 + \frac{1}{1 + \frac{1}{12 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

More

$$1570 + \frac{2319}{\pi} - \frac{350}{\sqrt{\pi}} + 2995 \sqrt{\pi} - 1921 \pi \approx 1384.194067734523557535528$$

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$$\frac{1}{2} (-153 C - 38 + 339 \pi - 80 \pi^2 - 119 \pi \log(2) + 849 \pi \log(3)) \approx$$

1384.1940677345235575282270

root of  $7x^4 - 9691x^3 + 2269x^2 + 4415x + 1482$  near  $x = 1384.19$   $\approx$

1384.19406773452355760274

The result 1384,194 is practically equal to the rest mass of Sigma baryon  $1383.7 \pm 1.0$

Furthermore:

$$2 * (1.63074)^2 \pi / 4 - 2 [(\exp(-4.1833) \cos 4.1833 / (\cosh(\pi/2)) - \exp(-12.549) \cos 12.549 / (3 \cosh(3\pi/2))) + \exp(-20.9165) \cos 20.9165 / (5 \cosh(5\pi/2))]$$

$$2 \times 1.63074^2 \times \frac{\pi}{4} - 2 \left( \exp(-4.1833) \times \frac{\cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \exp(-12.549) \times \frac{\cos(12.549)}{3 \cosh\left(3 \times \frac{\pi}{2}\right)} + \exp(-20.9165) \times \frac{\cos(20.9165)}{5 \cosh\left(5 \times \frac{\pi}{2}\right)} \right)$$

Result:

More digits

4.18337...

Alternative representations:

More

$$\frac{1}{4} (2 \times 1.63074^2) \pi - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$\frac{2 \pi 1.63074^2}{4} - 2 \left( \frac{\cosh(-4.1833 i) \exp(-4.1833)}{\cos\left(\frac{i\pi}{2}\right)} - \frac{\cosh(-12.549 i) \exp(-12.549)}{3 \cos\left(\frac{3i\pi}{2}\right)} + \frac{\cosh(-20.9165 i) \exp(-20.9165)}{5 \cos\left(\frac{5i\pi}{2}\right)} \right)$$

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$$\frac{1}{4} (2 \times 1.63074^2) \pi - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$\frac{2 \pi 1.63074^2}{4} - 2 \left( \frac{\exp(-4.1833) (e^{-4.1833 i} + e^{4.1833 i})}{2 \cos\left(\frac{i\pi}{2}\right)} - \frac{\exp(-12.549) (e^{-12.549 i} + e^{12.549 i})}{2 \left(3 \cos\left(\frac{3i\pi}{2}\right)\right)} + \frac{\exp(-20.9165) (e^{-20.9165 i} + e^{20.9165 i})}{2 \left(5 \cos\left(\frac{5i\pi}{2}\right)\right)} \right)$$

Open code

$$\frac{1}{4} (2 \times 1.63074^2) \pi - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$\frac{2 \pi 1.63074^2}{4} - 2 \left( \frac{\cosh(4.1833 i) \exp(-4.1833)}{\cos\left(-\frac{i\pi}{2}\right)} - \frac{\cosh(12.549 i) \exp(-12.549)}{3 \cos\left(-\frac{3i\pi}{2}\right)} + \frac{\cosh(20.9165 i) \exp(-20.9165)}{5 \cos\left(-\frac{5i\pi}{2}\right)} \right)$$

Series representations:

More

$$\frac{1}{4} (2 \times 1.63074^2) \pi - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$1.32966 \pi - \frac{2 \exp(-4.1833) \sum_{k=0}^{\infty} \frac{(-1)^k e^{2.8622 k}}{(2k)!}}{I_0\left(\frac{\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{\pi}{2}\right)} + \frac{2 \exp(-12.549) \sum_{k=0}^{\infty} \frac{(-1)^k e^{5.05928 k}}{(2k)!}}{3 \left( I_0\left(\frac{3\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{3\pi}{2}\right) \right)} -$$

$$\frac{2 \exp(-20.9165) \sum_{k=0}^{\infty} \frac{(-1)^k e^{6.08108 k}}{(2k)!}}{5 \left( I_0\left(\frac{5\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{5\pi}{2}\right) \right)}$$

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$$\begin{aligned}
& \frac{1}{4} (2 \times 1.63074^2) \pi - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \right. \\
& \left. \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) = \left( 1.32966 \left( -1.50415 \exp(-4.1833) \right. \right. \\
& \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} \times 9^{k_2} \times 25^{k_3} e^{2.8622 k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} + \right. \\
& \left. 0.501383 \exp(-12.549) \right. \\
& \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} \times 25^{k_3} e^{5.05928 k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} - 0.30083 \right. \\
& \left. \exp(-20.9165) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} \times 9^{k_2} e^{6.08108 k_1} \pi^{2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} + \right. \\
& \left. \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{4^{-k_1-k_2-k_3} \times 9^{k_1} \times 25^{k_3} \pi^{2k_1+2k_2+2k_3}}{(2k_1)! (2k_2)! (2k_3)!} \right) / \\
& \left( \left( \sum_{k=0}^{\infty} \frac{\left(\frac{9}{4}\right)^k \pi^{2k}}{(2k)!} \right) \left( \sum_{k=0}^{\infty} \frac{4^{-k} \pi^{2k}}{(2k)!} \right) \sum_{k=0}^{\infty} \frac{\left(\frac{25}{4}\right)^k \pi^{2k}}{(2k)!} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} (2 \times 1.63074^2) \pi - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \right. \\
& \left. \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) = \\
& 1.32966 \pi - \frac{2 J_0(4.1833) \exp(-4.1833)}{I_0\left(\frac{\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{\pi}{2}\right)} + \frac{2 J_0(12.549) \exp(-12.549)}{3 \left( I_0\left(\frac{3\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{3\pi}{2}\right) \right)} - \\
& \frac{2 J_0(20.9165) \exp(-20.9165)}{5 \left( I_0\left(\frac{5\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{5\pi}{2}\right) \right)} - \frac{4 \exp(-4.1833) \sum_{k=1}^{\infty} (-1)^k J_{2k}(4.1833)}{I_0\left(\frac{\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{\pi}{2}\right)} + \\
& \frac{4 \exp(-12.549) \sum_{k=1}^{\infty} (-1)^k J_{2k}(12.549)}{3 \left( I_0\left(\frac{3\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{3\pi}{2}\right) \right)} - \frac{4 \exp(-20.9165) \sum_{k=1}^{\infty} (-1)^k J_{2k}(20.9165)}{5 \left( I_0\left(\frac{5\pi}{2}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{5\pi}{2}\right) \right)}
\end{aligned}$$

Integral representations:  
More

$$\frac{1}{4} (2 \times 1.63074^2) \pi - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) = 1.32966 \left( \pi + 0.752074 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left[ -\frac{e^{-4.375/s+s} \exp(-4.1833) \sqrt{\pi}}{i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{\pi}{2}} \sinh(t) dt} + \frac{e^{-39.3694/s+s} \exp(-12.549) \sqrt{\pi}}{3 i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{3\pi}{2}} \sinh(t) dt} - \frac{e^{-109.375/s+s} \exp(-20.9165) \sqrt{\pi}}{5 i \pi \sqrt{s} \int_{i\frac{\pi}{2}}^{\frac{5\pi}{2}} \sinh(t) dt} \right] ds \right) \text{ for } \gamma > 0$$

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$$\frac{1}{4} (2 \times 1.63074^2) \pi - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) = 1.32966 \pi + \frac{2 \exp(-4.1833)}{1 + \frac{\pi}{2} \int_0^1 \sinh\left(\frac{\pi t}{2}\right) dt} \int_{\frac{\pi}{2}}^{4.1833} \sin(t) dt - \frac{2 \exp(-12.549)}{3 \left(1 + \frac{3\pi}{2} \int_0^1 \sinh\left(\frac{3\pi t}{2}\right) dt\right)} \int_{\frac{\pi}{2}}^{12.549} \sin(t) dt + \frac{2 \exp(-20.9165)}{5 \left(1 + \frac{5\pi}{2} \int_0^1 \sinh\left(\frac{5\pi t}{2}\right) dt\right)} \int_{\frac{\pi}{2}}^{20.9165} \sin(t) dt$$

$$\frac{1}{4} (2 \times 1.63074^2) \pi - 2 \left( \frac{\exp(-4.1833) \cos(4.1833)}{\cosh\left(\frac{\pi}{2}\right)} - \frac{\exp(-12.549) \cos(12.549)}{3 \cosh\left(\frac{3\pi}{2}\right)} + \frac{\exp(-20.9165) \cos(20.9165)}{5 \cosh\left(\frac{5\pi}{2}\right)} \right) =$$

$$1.32966 \left( \pi + 0.752074 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left[ - \frac{2.09165^{-2s} \exp(-4.1833) \Gamma(s) \sqrt{\pi}}{i \pi \Gamma\left(\frac{1}{2} - s\right) \int_{i\frac{\pi}{2}}^{\frac{\pi}{2}} \sinh(t) dt} + \frac{6.2745^{-2s} \exp(-12.549) \Gamma(s) \sqrt{\pi}}{3 i \pi \Gamma\left(\frac{1}{2} - s\right) \int_{i\frac{\pi}{2}}^{\frac{3\pi}{2}} \sinh(t) dt} - \frac{10.4583^{-2s} \exp(-20.9165) \Gamma(s) \sqrt{\pi}}{5 i \pi \Gamma\left(\frac{1}{2} - s\right) \int_{i\frac{\pi}{2}}^{\frac{5\pi}{2}} \sinh(t) dt} \right] ds \right) \text{ for } 0 < \gamma < \frac{1}{2}$$

Continued fraction:

Linear form

$$4 + \frac{1}{5 + \frac{1}{2 + \frac{1}{4 + \frac{1}{1 + \frac{1}{6 + \frac{1}{1 + \frac{1}{3 + \frac{1}{6 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}$$

The result 4.18337 is very near to the upper bound value of mass of Dark Matter particle:

Using (22) this translates to an upper bound on the mass of the DM particle:

$$m \lesssim 4.2 \left( \frac{\sigma/m}{\text{cm}^2/\text{g}} \right)^{1/4} \text{ eV}. \quad (24)$$

Smaller and less massive galaxies result in a somewhat weaker bound.

The bound (24) on the DM particle mass is the main result of this Section. It shows that for values of  $\sigma/m$  satisfying the merging-cluster bound  $\sim 1 \text{ cm}^2/\text{g}$  [85–88],  $m$  must be somewhat below 4 eV. The dependence on the cross section is rather weak, however, scaling as the  $1/4$  power. It should be mentioned that the upper bound (24) would be somewhat tighter had we assumed a  $\rho \propto r^{-2}$  transition density profile outside the superfluid core, instead of  $\rho \propto r^{-3}$ .

(from: Phenomenological consequences of superfluid dark matter

with baryon-phonon coupling- arXiv:1711.05748v1)

We have from the (3) (PAGE 2) for  $\phi\left(\frac{\pi^2}{n}\right) = 16\pi^2$ ;  $\phi(n) = \frac{1}{16\pi^2}$  and  $m = 2$

$$[(1/(4\pi))^2 - 1/4 + 16(\pi^2) \sqrt{2 \times \pi^3/8}]$$

Input:

$$\left(\frac{1}{4\pi}\right)^2 - \frac{1}{4} + 16\pi^2 \sqrt{2 \times \frac{\pi^3}{8}}$$

[Open code](#)

Exact result:

$$-\frac{1}{4} + \frac{1}{16\pi^2} + 8\pi^{7/2}$$

Decimal approximation:

More digits

439.4138886078921733521838508196735432715459491145854783061...

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Property:

$-\frac{1}{4} + \frac{1}{16\pi^2} + 8\pi^{7/2}$  is a transcendental number

Alternate form:

$$\frac{1 - 4\pi^2 + 128\pi^{11/2}}{16\pi^2}$$

[Open code](#)

Continued fraction:

Linear form



$$439 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{12 + \frac{1}{27 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{1 + \frac{1}{3 + \frac{1}{4 + \frac{1}{1 + \frac{1}{8 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Open code

Series representations:

More

$$\left(\frac{1}{4\pi}\right)^2 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{1 - 64\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^2 + 262144\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^{11/2}}{256\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^2}$$

Open code

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$$\left(\frac{1}{4\pi}\right)^2 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{1 - 4\left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^2 + 128\left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^{11/2}}{16\left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^2}$$

Open code

$$\left(\frac{1}{4\pi}\right)^2 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \left(1 - 4\left(\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}\right)\right)^2 + 128\left(\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}\right)^{11/2} \Big/ \left(16\left(\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}\right)^2\right)$$

Open code

Integral representations:

More

$$\left(\frac{1}{4\pi}\right)^2 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{1 - 16 \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^2 + 4096 \sqrt{2} \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^{11/2}}{64 \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^2}$$

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$$\left(\frac{1}{4\pi}\right)^2 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{1 - 16 \left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^2 + 4096 \sqrt{2} \left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^{11/2}}{64 \left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^2}$$

[Open code](#)

$$\left(\frac{1}{4\pi}\right)^2 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{1 - 64 \left(\int_0^1 \sqrt{1-t^2} dt\right)^2 + 262144 \left(\int_0^1 \sqrt{1-t^2} dt\right)^{11/2}}{256 \left(\int_0^1 \sqrt{1-t^2} dt\right)^2}$$

[Open code](#)

Now:

$$\text{sqrt}\left[\frac{1}{\left(\frac{1}{4\pi}\right)^2} - \frac{1}{4} + 16(\pi^2) \text{sqrt}\left(\frac{2\pi^3}{8}\right)\right]$$

Input:

$$\sqrt{\frac{1}{\left(\frac{1}{4\pi}\right)^2} - \frac{1}{4} + 16\pi^2 \sqrt{2 \times \frac{\pi^3}{8}}}$$

[Open code](#)

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Exact result:

$$\sqrt{-\frac{1}{4} + 16\pi^2 + 8\pi^{7/2}}$$

Decimal approximation:

More digits

24.44015602346564941304351252863058667478011965367720672453...

[Open code](#)

Property:

$$\sqrt{-\frac{1}{4} + 16\pi^2 + 8\pi^{7/2}} \text{ is a transcendental number}$$

Alternate forms:

$$\sqrt{8\pi^2(2+\pi^{3/2})-\frac{1}{4}}$$

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$$\frac{1}{2}\sqrt{-1+64\pi^2+32\pi^{7/2}}$$

[Open code](#)

$$\sqrt{-\frac{1}{4}+16\pi^2+8\pi^{7/2}} e^0 \approx 24.440 \quad (\text{real, principal root})$$

$$\sqrt{-\frac{1}{4}+16\pi^2+8\pi^{7/2}} e^{i\pi} \approx -24.440 \quad (\text{real root})$$

Series representations:

More

$$\sqrt{\frac{1}{\left(\frac{1}{4\pi}\right)^2}-\frac{1}{4}+16\pi^2}\sqrt{\frac{2\pi^3}{8}} = \sqrt{-\frac{5}{4}+16\pi^2+16\pi^2}\sqrt{\frac{\pi^3}{4}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-\frac{5}{4}+16\pi^2+16\pi^2\sqrt{\frac{\pi^3}{4}}\right)^{-k}$$

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$$\sqrt{\frac{1}{\left(\frac{1}{4\pi}\right)^2}-\frac{1}{4}+16\pi^2}\sqrt{\frac{2\pi^3}{8}} = \sqrt{-\frac{5}{4}+16\pi^2+16\pi^2}\sqrt{\frac{\pi^3}{4}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{5}{4}+16\pi^2+16\pi^2\sqrt{\frac{\pi^3}{4}}\right)^{-k}}{k!}$$

[Open code](#)

$$\sqrt{\frac{1}{\left(\frac{1}{4\pi}\right)^2}-\frac{1}{4}+16\pi^2}\sqrt{\frac{2\pi^3}{8}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{4}+16\pi^2+16\pi^2\sqrt{\frac{\pi^3}{4}}-z_0\right)^k}{k!} z_0^{-k}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{1}{2} \sqrt{\left(\frac{1}{(4\pi)^2} - \frac{1}{4} + 16\pi^2\right) \sqrt{2\pi^3/8}}$$

Input:

$$\frac{1}{2} \sqrt{\left(\frac{1}{(4\pi)^2} - \frac{1}{4} + 16\pi^2\right) \sqrt{2 \times \frac{\pi^3}{8}}}$$

[Open code](#)

Exact result:

$$\frac{1}{2} \sqrt{-\frac{1}{4} + 16\pi^2 + 8\pi^{7/2}}$$

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Decimal approximation:

More digits

12.22007801173282470652175626431529333739005982683860336226...

[Open code](#)

Property:

$$\frac{1}{2} \sqrt{-\frac{1}{4} + 16\pi^2 + 8\pi^{7/2}} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{4} \sqrt{-1 + 64\pi^2 + 32\pi^{7/2}}$$

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$$\frac{1}{4} \sqrt{32\pi^2(2 + \pi^{3/2}) - 1}$$

Series representations:

More

$$\frac{1}{2} \sqrt{\left(\frac{1}{(4\pi)^2} - \frac{1}{4} + 16\pi^2\right) \sqrt{\frac{2\pi^3}{8}}} = \frac{1}{2} \sqrt{-\frac{5}{4} + 16\pi^2 + 16\pi^2 \sqrt{\frac{\pi^3}{4}}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-\frac{5}{4} + 16\pi^2 + 16\pi^2 \sqrt{\frac{\pi^3}{4}}\right)^{-k}$$

[Open code](#)

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$$\frac{1}{2} \sqrt{\frac{1}{\left(\frac{1}{4\pi}\right)^2} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}}} =$$

$$\frac{1}{2} \sqrt{-\frac{5}{4} + 16\pi^2 + 16\pi^2 \sqrt{\frac{\pi^3}{4}}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{5}{4} + 16\pi^2 + 16\pi^2 \sqrt{\frac{\pi^3}{4}}\right)^{-k}}{k!}$$

Open code

$$\frac{1}{2} \sqrt{\frac{1}{\left(\frac{1}{4\pi}\right)^2} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}}} =$$

$$\frac{1}{2} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{4} + 16\pi^2 + 16\pi^2 \sqrt{\frac{\pi^3}{4}} - z_0\right)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

We note that the sum of the results of the two expressions, multiplied by 47:

$24.440156 + 12.220078 = 36,660234$ ;  $36,660234 * 47 = 1723,030998$  is practically equal to the mass of meson  $f_0(1710)$  candidate glueball.

The expression (3) is, for  $n = 2$ , and  $x = 1$ :

integrate  $[(\sin 2)/((e^{2\pi}-1))]$

$$\int \frac{\sin(2)}{e^{2\pi} - 1} dx \approx \text{constant} + 0.00170124x$$

and the inverse function is:

integrate  $1/[(\sin 2)/((e^{2\pi}-1))]$

$$\int \frac{1}{\frac{\sin(2)}{e^{2\pi} - 1}} dx \approx \text{constant} + 587.807x$$

That for  $n = 1$ , is:

integrate  $1/[(\sin 1)/((e^{2\pi}-1))]$

$$\int \frac{1}{\frac{\sin(1)}{e^{2\pi-1}}} dx \approx \text{constant} + 635.187x$$

The sum of the two results is:  $587.807 + 635.187 = 1222,994$  result that is very near to the value of the rest mass of the Delta baryon  $1232 \pm 2$

From the right hand-side of (3) (**PAGE 2**) for  $\phi(n) = \phi(2\pi) = \frac{1}{16}$ ;  $\phi\left(\frac{1}{n}\right) = 16$  and  $m = 2$ , we have that:

$$\left[ \frac{1}{16} - \frac{1}{4} + 16(\pi^2) \sqrt{2\pi^3/8} \right]$$

Input:

$$\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{2 \times \frac{\pi^3}{8}}$$

[Open code](#)

Exact result:

$$8\pi^{7/2} - \frac{3}{16}$$

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Decimal approximation:

More digits

439.4700560339145272414686083532229352941144266911684628968...

[Open code](#)

Property:

$-\frac{3}{16} + 8\pi^{7/2}$  is a transcendental number

Alternate form:

$$\frac{1}{16} (128\pi^{7/2} - 3)$$

Series representations:

More

$$\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = -\frac{3}{16} + 1024 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^{7/2}$$

[Open code](#)

$$\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = -\frac{3}{16} + 8 \left( \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^{7/2}$$

[Open code](#)

$$\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = -\frac{3}{16} + 8 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^{7/2}$$

[Open code](#)

Integral representations:

More

• [More information](#)

$$\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = -\frac{3}{16} + 1024 \left( \int_0^1 \sqrt{1-t^2} dt \right)^{7/2}$$

[Open code](#)

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$$\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = -\frac{3}{16} + 64\sqrt{2} \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^{7/2}$$

[Open code](#)

$$\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = -\frac{3}{16} + 64\sqrt{2} \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^{7/2}$$

For  $\phi(n) = \phi\left(\frac{\pi}{2}\right) = \frac{1}{4\pi}$ ;  $\phi\left(\frac{1}{n}\right) = 4\pi$  and  $n = 2$ , we have that:

$$\left[ \frac{1}{4\pi} - \frac{1}{4} + 4(\pi^3) \sqrt{2\pi^3/8} \right]$$

Input:

$$\frac{1}{4\pi} - \frac{1}{4} + 4\pi^3 \sqrt{2 \times \frac{\pi^3}{8}}$$

[Open code](#)

Exact result:

$$-\frac{1}{4} + \frac{1}{4\pi} + 2\pi^{9/2}$$

Decimal approximation:

More digits

345.1358145043931355177793626260693610243877055882502759013...

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Property:

$-\frac{1}{4} + \frac{1}{4\pi} + 2\pi^{9/2}$  is a transcendental number

Alternate forms:

$$\frac{1}{4} \left( -1 + \frac{1}{\pi} + 8 \pi^{9/2} \right)$$

[Open code](#)

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$$\frac{1 - \pi + 8 \pi^{11/2}}{4 \pi}$$

Series representations:

More

$$\frac{1}{4\pi} - \frac{1}{4} + 4\pi^3 \sqrt{\frac{2\pi^3}{8}} = \frac{1 - 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} + 16384 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^{11/2}}{16 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

[Open code](#)

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$$\frac{\frac{1}{4\pi} - \frac{1}{4} + 4\pi^3 \sqrt{\frac{2\pi^3}{8}}}{1 - \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) + 8 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^{11/2}}{4 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}$$

[Open code](#)

$$\frac{\frac{1}{4\pi} - \frac{1}{4} + 4\pi^3 \sqrt{\frac{2\pi^3}{8}}}{1 - \sum_{k=0}^{\infty} 16^{-k} \left(\frac{1}{-5-8k} - \frac{1}{2+4k} + \frac{4}{1+8k} - \frac{1}{6+8k}\right) + 8 \left(\sum_{k=0}^{\infty} 16^{-k} \left(\frac{1}{-5-8k} - \frac{1}{2+4k} + \frac{4}{1+8k} - \frac{1}{6+8k}\right)\right)^{11/2}}{\left(4 \times \sum_{k=0}^{\infty} 16^{-k} \left(\frac{1}{-5-8k} - \frac{1}{2+4k} + \frac{4}{1+8k} - \frac{1}{6+8k}\right)\right)}$$

Integral representations:

More

$$\frac{\frac{1}{4\pi} - \frac{1}{4} + 4\pi^3 \sqrt{\frac{2\pi^3}{8}}}{1 - 4 \int_0^1 \sqrt{1-t^2} dt + 16384 \left(\int_0^1 \sqrt{1-t^2} dt\right)^{11/2}}{16 \int_0^1 \sqrt{1-t^2} dt}$$

[Open code](#)

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$$\frac{1}{4\pi} - \frac{1}{4} + 4\pi^3 \sqrt{\frac{2\pi^3}{8}} = \frac{1 - 2 \int_0^\infty \frac{1}{1+t^2} dt + 256\sqrt{2} \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^{11/2}}{8 \int_0^\infty \frac{1}{1+t^2} dt}$$

[Open code](#)

$$\frac{1}{4\pi} - \frac{1}{4} + 4\pi^3 \sqrt{\frac{2\pi^3}{8}} = \frac{1 - 2 \int_0^\infty \frac{\sin(t)}{t} dt + 256\sqrt{2} \left( \int_0^\infty \frac{\sin(t)}{t} dt \right)^{11/2}}{8 \int_0^\infty \frac{\sin(t)}{t} dt}$$

We note that the sum of the two results is:

$439.470056 + 345,1358145 = 784,6058705$  result that is very near to the value of the rest mass of the Omega meson:  $782.65 \pm 0.12$

For  $n = 1$  and  $x = 1$ , we obtain:

$$\left[ \frac{1}{16} - \frac{1}{2} + 16(\pi^2) \sqrt{2\pi^3} \right]$$

Input:

$$\frac{1}{16} - \frac{1}{2} + 16\pi^2 \sqrt{2\pi^3}$$

[Open code](#)

Exact result:

$$16\sqrt{2} \pi^{7/2} - \frac{7}{16}$$

Decimal approximation:

More digits

1243.101857085941827396415780170860364649299850339630216559...

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Property:

$-\frac{7}{16} + 16\sqrt{2} \pi^{7/2}$  is a transcendental number

Alternate form:

$$\frac{1}{16} \left( 256\sqrt{2} \pi^{7/2} - 7 \right)$$

Series representations:

More

$$\frac{1}{16} - \frac{1}{2} + 16\pi^2 \sqrt{2\pi^3} = -\frac{7}{16} + 16\pi^2 \sqrt{-1 + 2\pi^3} \sum_{k=0}^{\infty} (-1 + 2\pi^3)^{-k} \binom{\frac{1}{2}}{k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{16} - \frac{1}{2} + 16\pi^2 \sqrt{2\pi^3} = -\frac{7}{16} + 16\pi^2 \sqrt{-1 + 2\pi^3} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 2\pi^3)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

[Open code](#)

$$\frac{1}{16} - \frac{1}{2} + 16\pi^2 \sqrt{2\pi^3} = -\frac{7}{16} + 16\pi^2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2\pi^3 - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$[1/(4\pi)-1/2+4(\pi^3) \text{ sqrt}(2\pi^3)]$

Input:

$$\frac{1}{4\pi} - \frac{1}{2} + 4\pi^3 \sqrt{2\pi^3}$$

[Open code](#)

Exact result:

$$-\frac{1}{2} + \frac{1}{4\pi} + 4\sqrt{2} \pi^{9/2}$$

Decimal approximation:

More digits

976.2531046392883074442457077770025483156431354467835052489...

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Property:

$$-\frac{1}{2} + \frac{1}{4\pi} + 4\sqrt{2} \pi^{9/2} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{4} \left( -2 + \frac{1}{\pi} + 16\sqrt{2} \pi^{9/2} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1 - 2\pi + 16\sqrt{2} \pi^{11/2}}{4\pi}$$

Series representations:

More

$$\frac{1}{4\pi} - \frac{1}{2} + 4\pi^3 \sqrt{2\pi^3} = -\frac{1}{2} + \frac{1}{4\pi} + 4\pi^3 \sqrt{-1 + 2\pi^3} \sum_{k=0}^{\infty} (-1 + 2\pi^3)^{-k} \left(\frac{1}{2}\right)_k$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{4\pi} - \frac{1}{2} + 4\pi^3 \sqrt{2\pi^3} = -\frac{1}{2} + \frac{1}{4\pi} + 4\pi^3 \sqrt{-1 + 2\pi^3} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 2\pi^3)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

[Open code](#)

$$\frac{1}{4\pi} - \frac{1}{2} + 4\pi^3 \sqrt{2\pi^3} = -\frac{1}{2} + \frac{1}{4\pi} + 4\pi^3 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2\pi^3 - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

The sum of the two results is:  $1243.101857 + 976.2531046 = 2219,3549616$ . We observe that 1243.10 is very near to the value of the rest mass of the Delta baryon  $1232 \pm 2$  and that 976.2531046 is very near to the value of the rest mass of Eta prime meson  $957.66 \pm 0.24$

We observe also that:

$$[(148) - 1/4 + 16(\pi^2) \text{sqrt}(2\pi^3/8)]$$

Input:

$$148 - \frac{1}{4} + 16\pi^2 \sqrt{2 \times \frac{\pi^3}{8}}$$

[Open code](#)

Exact result:

$$\frac{591}{4} + 8\pi^{7/2}$$

Decimal approximation:

More digits

587.4075560339145272414686083532229352941144266911684628968...

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Property:

$\frac{591}{4} + 8\pi^{7/2}$  is a transcendental number

Alternate form:

$$\frac{1}{4} (591 + 32\pi^{7/2})$$

Series representations:

More

$$148 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{591}{4} + 1024 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^{7/2}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$148 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{591}{4} + 8 \left( \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^{7/2}$$

[Open code](#)

$$148 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{591}{4} + 8 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right)^{7/2}$$

[Integral representations:](#)

• [More](#)

$$148 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{591}{4} + 1024 \left( \int_0^1 \sqrt{1-t^2} dt \right)^{7/2}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$148 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{591}{4} + 64\sqrt{2} \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^{7/2}$$

[Open code](#)

$$148 - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}} = \frac{591}{4} + 64\sqrt{2} \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^{7/2}$$

We have also that:

$$(((4 * [1/16-1/4+16(Pi^2) sqrt(2Pi^3/8)])))-27$$

[Input:](#)

$$4 \left( \frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{2 \times \frac{\pi^3}{8}} \right) - 27$$

[Open code](#)

[Exact result:](#)

$$4 \left( 8\pi^{7/2} - \frac{3}{16} \right) - 27$$

[Decimal approximation:](#)

• [More digits](#)

1730.880224135658108965874433412891741176457706764673851587...

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Property:

$-27 + 4\left(-\frac{3}{16} + 8\pi^{7/2}\right)$  is a transcendental number

Alternate forms:

$$32\pi^{7/2} - \frac{111}{4}$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{4}(128\pi^{7/2} - 111)$$

Series representations:

More

$$4\left(\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}}\right) - 27 = -\frac{111}{4} + 4096\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^{7/2}$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$4\left(\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}}\right) - 27 = -27 + 4\left(-\frac{3}{16} + 8\left(\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}\right)^{7/2}\right)$$

[Open code](#)

$$4\left(\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}}\right) - 27 = -\frac{111}{4} + 32\left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^{7/2}$$

Integral representations:

More

$$4\left(\frac{1}{16} - \frac{1}{4} + 16\pi^2 \sqrt{\frac{2\pi^3}{8}}\right) - 27 = -\frac{111}{4} + 4096\left(\int_0^1 \sqrt{1-t^2} dt\right)^{7/2}$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$4 \left( \frac{1}{16} - \frac{1}{4} + 16 \pi^2 \sqrt{\frac{2 \pi^3}{8}} \right) - 27 = -\frac{111}{4} + 256 \sqrt{2} \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^{7/2}$$

[Open code](#)

$$4 \left( \frac{1}{16} - \frac{1}{4} + 16 \pi^2 \sqrt{\frac{2 \pi^3}{8}} \right) - 27 = -\frac{111}{4} + 256 \sqrt{2} \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^{7/2}$$

$$(((1.65578 * [1/16-1/4+16(\text{Pi}^2) \text{sqrt}(2\text{Pi}^3/8)]))) + 36$$

Input interpretation:

$$1.65578 \left( \frac{1}{16} - \frac{1}{4} + 16 \pi^2 \sqrt{2 \times \frac{\pi^3}{8}} \right) + 36$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Result:

More digits

763.666...

Percent increase:

$$1.65578 \left( \frac{1}{16} - \frac{1}{4} + 16 \pi^2 \sqrt{\frac{2 \pi^3}{8}} \right) + 36 = 763.666 \text{ is } 4.94733$$

$$\% \text{ larger than } 1.65578 \left( \frac{1}{16} - \frac{1}{4} + 16 \pi^2 \sqrt{\frac{2 \pi^3}{8}} \right) = 727.666.$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Series representations:

More

$$1.65578 \left( \frac{1}{16} - \frac{1}{4} + 16 \pi^2 \sqrt{\frac{2 \pi^3}{8}} \right) + 36 =$$

$$35.6895 + 26.4925 \pi^2 \sqrt{-1 + \frac{\pi^3}{4}} \sum_{k=0}^{\infty} 4^k (-4 + \pi^3)^{-k} \binom{\frac{1}{2}}{k}$$

[Open code](#)

$$1.65578 \left( \frac{1}{16} - \frac{1}{4} + 16 \pi^2 \sqrt{\frac{2 \pi^3}{8}} \right) + 36 =$$

$$35.6895 + 26.4925 \pi^2 \sqrt{-1 + \frac{\pi^3}{4}} \sum_{k=0}^{\infty} \frac{(-4)^k (-4 + \pi^3)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

Open code

$$1.65578 \left( \frac{1}{16} - \frac{1}{4} + 16 \pi^2 \sqrt{\frac{2 \pi^3}{8}} \right) + 36 =$$

$$35.6895 + 26.4925 \pi^2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)_k \left(-\frac{1}{2}\right)_k (\pi^3 - 4 z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

“These are the expressions that I have obtained for the number of primes less than a give number”. (Queste sono le espressioni che ho ottenuto per il numero di numeri primi minori di un dato numero)

(3)

have them published if my results are recognised [7] 3  
 by eminent men like you. you ask me to give you  
 the expression I have got for the number of prime num-  
 bers within a given number. These are the expressions  
 that I have obtained for the number of primes less than  
 a give number.

The no. of prime numbers less than  $e^a = \int_0^{\infty} \frac{a^x dx}{x^{S_{x+1}} \Gamma(x+1)}$   
 where  $S_{x+1} = \frac{1}{1^{x+1}} + \frac{1}{2^{x+1}} + \frac{1}{3^{x+1}} + \frac{1}{4^{x+1}} + \dots$

The no of prime numbers less than  $n =$   
 $\frac{2}{\pi} \left\{ \frac{2}{B_2} \left(\frac{\log n}{2\pi}\right) + \frac{4}{3 B_4} \left(\frac{\log n}{2\pi}\right)^3 + \frac{6}{5 B_6} \left(\frac{\log n}{2\pi}\right)^5 + \frac{8}{7 B_8} \left(\frac{\log n}{2\pi}\right)^7 + \dots \right\}$   
 where  $B_2 = \frac{1}{6}$ ;  $B_4 = \frac{1}{30}$ ;  $B_6 = \frac{1}{42}$  &c, the Bernoullian nos.

The no. of prime numbers less than  $n =$   
 $\int_u^n \frac{dx}{\log x} - \frac{1}{2} \int_u^{\sqrt{n}} \frac{dx}{\log x} - \frac{1}{3} \int_u^{\sqrt[3]{n}} \frac{dx}{\log x} - \frac{1}{5} \int_u^{\sqrt[5]{n}} \frac{dx}{\log x} + \frac{1}{6} \int_u^{\sqrt[6]{n}} \frac{dx}{\log x}$   
 $- \frac{1}{7} \int_u^{\sqrt[7]{n}} \frac{dx}{\log x} + \frac{1}{10} \int_u^{\sqrt[10]{n}} \frac{dx}{\log x} - \frac{1}{11} \int_u^{\sqrt[11]{n}} \frac{dx}{\log x} - \frac{1}{13} \int_u^{\sqrt[13]{n}} \frac{dx}{\log x}$   
 $+ \frac{1}{14} \int_u^{\sqrt[14]{n}} \frac{dx}{\log x} + \frac{1}{15} \int_u^{\sqrt[15]{n}} \frac{dx}{\log x} - \frac{1}{17} \int_u^{\sqrt[17]{n}} \frac{dx}{\log x} - \frac{1}{19} \int_u^{\sqrt[19]{n}} \frac{dx}{\log x} + \dots$

where  $\mu = 1.45136380$  nearly.

The numbers 1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19 &c above are  
 numbers containing dissimilar prime divisors; hence  
 4, 8, 9, 12, 16, 18, 20 &c are excluded. (Plus sign for even no. of prime  
 divisors and minus sign for odd no. of prime divisors).  
 As soon as a term becomes less than unity in practical  
 Calculations we should stop at the term before any red  
 line marked above and not any where; hence the first  
 four terms are necessary even when  $n$  is very small.

From the expressions in the above page (PAGE 3), we have that for  $x = 2$ , and  $a = \frac{1}{2} = 0.5$  :

$e^{0.5}$





$$\sqrt{e} = \frac{\sqrt{\sum_{k=0}^{\infty} \frac{1+k}{k!}}}{\sqrt{2}}$$

- $n!$  is the factorial function
  - [More information](#)

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

Input:

$$1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64}$$

[Open code](#)

Exact result:

Step-by-step solution

**2035**

**1728**

Decimal approximation:

More digits

1.177662037037037037037037037037037037037037037037037037037037037...

Repeating decimal:

1.177662037 (period 3)

[Open code](#)

Mixed fraction:

Step-by-step solution

**1  $\frac{307}{1728}$**

Continued fraction:

Linear form

$$1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{8}}}}}}}}}$$

integrate [0.25/(4\*1.177662037)] [0, 31]

Definite integral:

Step-by-step solution

$$\int_0^{31} \frac{0.25}{4 \times 1.177662037} dx = 1.64521$$

that is equal to:

$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6} \approx 1,645$$

We remember that:

The zeta function has simple zeros in the negative even integers, called **trivial zeros**, while all the other zeros are arranged symmetrically with respect to the straight line  $\text{Re}(s) = \frac{1}{2}$  called the **critical line**, and are all contained in the strip  $0 < \text{Re}(s) < 1$ , called the **critical strip**.

$\zeta(1.97968)$

1.66440...

while

$\zeta(2)$

$$\frac{\pi^2}{6}$$

1.644934066848226436472415166646025189218949901206798437735...

where  $1,97968 \leq 2$

A) Now, we have that:

$$2/\pi * [12(\ln 2/(2\pi)) + 40(\ln 2/(2\pi))^3 + 50.4(\ln 2/(2\pi))^5 + 61.71428(\ln 2/(2\pi))^7]$$

Input interpretation:

$$\frac{2}{\pi} \left( 12 \times \frac{\log(2)}{2\pi} + 40 \left( \frac{\log(2)}{2\pi} \right)^3 + 50.4 \left( \frac{\log(2)}{2\pi} \right)^5 + 61.71428 \left( \frac{\log(2)}{2\pi} \right)^7 \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

0.877486228...

Result:

0.877486227579269069358962423991793215423882990476885562782...

Alternative representations:

• [More](#)

$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 2}{\pi} =$$

$$\frac{2 \left(\frac{24 \coth^{-1}(3)}{2\pi} + 40 \left(\frac{2 \coth^{-1}(3)}{2\pi}\right)^3 + 50.4 \left(\frac{2 \coth^{-1}(3)}{2\pi}\right)^5 + 61.7143 \left(\frac{2 \coth^{-1}(3)}{2\pi}\right)^7\right)}{\pi}$$

[Open code](#)

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$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 2}{\pi} =$$

$$\frac{2 \left(\frac{12 \log(a) \log_a(2)}{2\pi} + 40 \left(\frac{\log(a) \log_a(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(a) \log_a(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(a) \log_a(2)}{2\pi}\right)^7\right)}{\pi}$$

[Open code](#)

$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 2}{\pi} =$$

$$\frac{2 \left(\frac{12 \log_e(2)}{2\pi} + 40 \left(\frac{\log_e(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log_e(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log_e(2)}{2\pi}\right)^7\right)}{\pi}$$

Series representations:

$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 2}{\pi} =$$

$$\frac{1}{\pi} 2 \left( \frac{6 \left( 2 i \pi \left\lfloor \frac{\operatorname{ar}g(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right)}{\pi} + \right.$$

$$\frac{5 \left( 2 i \pi \left\lfloor \frac{\operatorname{ar}g(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right)^3}{\pi^3} +$$

$$\frac{1.575 \left( 2 i \pi \left\lfloor \frac{\operatorname{ar}g(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right)^5}{\pi^5} +$$

$$\left. \frac{0.482143 \left( 2 i \pi \left\lfloor \frac{\operatorname{ar}g(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right)^7}{\pi^7} \right) \text{ for } x < 0$$

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$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 2}{\pi} = \frac{1}{\pi}$$

$$2 \left( \frac{6 \left( \log(z_0) + \left\lfloor \frac{\operatorname{arg}(2-z_0)}{2\pi} \right\rfloor \right) \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{\pi} + \right.$$

$$\frac{5 \left( \log(z_0) + \left\lfloor \frac{\operatorname{arg}(2-z_0)}{2\pi} \right\rfloor \right) \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{\pi^3} +$$

$$\frac{1.575 \left( \log(z_0) + \left\lfloor \frac{\operatorname{arg}(2-z_0)}{2\pi} \right\rfloor \right) \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{\pi^5} +$$

$$\left. \frac{0.482143 \left( \log(z_0) + \left\lfloor \frac{\operatorname{arg}(2-z_0)}{2\pi} \right\rfloor \right) \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{\pi^7} \right)$$

$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 2}{\pi} =$$

$$\frac{1}{\pi} 2 \left( \frac{6 \left( 2i\pi \left\lfloor \frac{\pi - \operatorname{arg}\left(\frac{2}{z_0}\right) - \operatorname{arg}(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right)}{\pi} + \right.$$

$$\frac{5 \left( 2i\pi \left\lfloor \frac{\pi - \operatorname{arg}\left(\frac{2}{z_0}\right) - \operatorname{arg}(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right)}{\pi^3} +$$

$$\frac{1.575 \left( 2i\pi \left\lfloor \frac{\pi - \operatorname{arg}\left(\frac{2}{z_0}\right) - \operatorname{arg}(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right)}{\pi^5} +$$

$$\left. \frac{0.482143 \left( 2i\pi \left\lfloor \frac{\pi - \operatorname{arg}\left(\frac{2}{z_0}\right) - \operatorname{arg}(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right)}{\pi^7} \right)$$

Integral representations:

$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 2}{\pi} =$$

$$\frac{12 \left(\pi^6 \int_1^2 \frac{1}{t} dt + 0.833333 \pi^4 \left(\int_1^2 \frac{1}{t} dt\right)^3 + 0.2625 \pi^2 \left(\int_1^2 \frac{1}{t} dt\right)^5 + 0.0803571 \left(\int_1^2 \frac{1}{t} dt\right)^7\right)}{\pi^8}$$

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$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 2}{i^7 \pi^{15} 6} = \frac{1}{i^7 \pi^{15} 6}$$

$$\left( i^6 \pi^{12} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + 0.208333 i^4 \pi^8 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^3 + \right.$$

$$0.0164063 i^2 \pi^4 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^5 +$$

$$\left. 0.00125558 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^7 \right) \text{ for } -1 < \gamma < 0$$

Continued fraction:  
 Linear form  
 Continued fraction:  
 Linear form

$$\frac{1}{1 + \frac{1}{7 + \frac{1}{6 + \frac{1}{6 + \frac{1}{3 + \frac{1}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{7 + \frac{1}{1 + \frac{1}{14 + \frac{1}{1 + \frac{1}{2 + \frac{1}{6 + \frac{1}{1 + \frac{1}{4 + \frac{1}{28 + \frac{1}{3 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

The result is 0.8774862275792690693589624239917932154238829904768855

Now:

3.79357/Pi \*  
 $[12(\ln 2/(2\pi)) + 40(\ln 2/(2\pi))^3 + 50.4(\ln 2/(2\pi))^5 + 61.71428(\ln 2/(2\pi))^7]$

Input interpretation:  
 $\frac{3.79357}{\pi} \left( 12 \times \frac{\log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.71428 \left(\frac{\log(2)}{2\pi}\right)^7 \right)$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:  
 More digits  
 1.66440...

Continued fraction:

- Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{48 + \frac{1}{2 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}$$

Alternative representations:

- More

$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 3.79357}{\pi} = \frac{3.79357 \left(\frac{24 \coth^{-1}(3)}{2\pi} + 40 \left(\frac{2 \coth^{-1}(3)}{2\pi}\right)^3 + 50.4 \left(\frac{2 \coth^{-1}(3)}{2\pi}\right)^5 + 61.7143 \left(\frac{2 \coth^{-1}(3)}{2\pi}\right)^7\right)}{\pi}$$

[Open code](#)

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$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 3.79357}{\pi} = \frac{1}{\pi} 3.79357 \left(\frac{12 \log(a) \log_a(2)}{2\pi} + 40 \left(\frac{\log(a) \log_a(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(a) \log_a(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(a) \log_a(2)}{2\pi}\right)^7\right)$$

[Open code](#)

$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 3.79357}{\pi} = \frac{3.79357 \left(\frac{12 \log_e(2)}{2\pi} + 40 \left(\frac{\log_e(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log_e(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log_e(2)}{2\pi}\right)^7\right)}{\pi}$$

[Open code](#)

Series representations:

$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 3.79357}{\pi} =$$

$$\frac{1}{\pi} 3.79357 \left( \frac{6 \left( 2 i \pi \left\lfloor \frac{\text{arg}(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right)}{\pi} + \right.$$

$$\frac{5 \left( 2 i \pi \left\lfloor \frac{\text{arg}(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right)^3}{\pi^3} +$$

$$\frac{1.575 \left( 2 i \pi \left\lfloor \frac{\text{arg}(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right)^5}{\pi^5} +$$

$$\left. \frac{0.482143 \left( 2 i \pi \left\lfloor \frac{\text{arg}(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right)^7}{\pi^7} \right) \text{ for } x < 0$$

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$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 3.79357}{\pi} = \frac{1}{\pi}$$

$$3.79357 \left( \frac{6 \left( \log(z_0) + \left\lfloor \frac{\text{arg}(2-z_0)}{2\pi} \right\rfloor \right) \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{\pi} + \right.$$

$$\frac{5 \left( \log(z_0) + \left\lfloor \frac{\text{arg}(2-z_0)}{2\pi} \right\rfloor \right) \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{\pi^3} +$$

$$\frac{1.575 \left( \log(z_0) + \left\lfloor \frac{\text{arg}(2-z_0)}{2\pi} \right\rfloor \right) \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{\pi^5} +$$

$$\left. \frac{0.482143 \left( \log(z_0) + \left\lfloor \frac{\text{arg}(2-z_0)}{2\pi} \right\rfloor \right) \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{\pi^7} \right)$$



$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 3.79357}{\pi} =$$

$$\frac{1}{\pi} 3.79357 \left( \frac{6 \left( 2 i \pi \left[ \frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right)}{\pi} + \right.$$

$$5 \left( 2 i \pi \left[ \frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right)^3 +$$

$$1.575 \left( 2 i \pi \left[ \frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right)^5 +$$

$$\left. 0.482143 \left( 2 i \pi \left[ \frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right)^7 \right)$$

Integral representations:

$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 3.79357}{\pi} =$$

$$\frac{1}{\pi^8} 22.7614 \left( \pi^6 \int_1^2 \frac{1}{t} dt + 0.833333 \pi^4 \left( \int_1^2 \frac{1}{t} dt \right)^3 + \right.$$

$$\left. 0.2625 \pi^2 \left( \int_1^2 \frac{1}{t} dt \right)^5 + 0.0803571 \left( \int_1^2 \frac{1}{t} dt \right)^7 \right)$$

[Open code](#)

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$$\frac{\left(\frac{12 \log(2)}{2\pi} + 40 \left(\frac{\log(2)}{2\pi}\right)^3 + 50.4 \left(\frac{\log(2)}{2\pi}\right)^5 + 61.7143 \left(\frac{\log(2)}{2\pi}\right)^7\right) 3.79357}{\pi} = \frac{1}{i^7 \pi^{15}} 11.3807$$

$$\left( i^6 \pi^{12} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + 0.208333 i^4 \pi^8 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^3 + \right.$$

$$0.0164063 i^2 \pi^4 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^5 +$$

$$\left. 0.00125558 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^7 \right) \text{ for } -1 < \gamma < 0$$

B) Now, for  $u = 1.45136380$  and  $n = 2$ , we have the following integrals:

1/1 integrate  $[1/\ln(2)] x, [1.45136380, (2)]$

Definite integral:

Step-by-step solution

$$\int_{1.45136}^2 \frac{x}{\log(2)} dx = 1.3659$$

1/2 integrate  $[1/\ln(2)] x, [1.45136380, (\text{sqrt}(2))]$

Input interpretation:

$$\frac{1}{2} \int_{1.45136380}^{\sqrt{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

Result:

-0.0383962

- $\log(x)$  is the natural logarithm

1/3 integrate  $[1/\ln(2)] x, [1.45136380, ((2)^{1/3})]$

Input interpretation:

$$\frac{1}{3} \int_{1.45136380}^{\sqrt[3]{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

-0.124807

1/5 integrate  $[1/\ln(2)] x, [1.45136380, ((2)^{1/5})]$

Input interpretation:

$$\frac{1}{5} \int_{1.45136380}^{\sqrt[5]{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

-0.113533

1/6 integrate  $[1/\ln(2)] x, [1.45136380, ((2)^{1/6})]$

Input interpretation:

$$\frac{1}{6} \int_{1.45136380}^{\sqrt[6]{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

Result:

-0.101774

1/7 integrate [1/ln(2)] x,[1.45136380, ((2)^1/7]

Input interpretation:

$$\frac{1}{7} \int_{1.45136380}^{\sqrt[7]{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

Result:

-0.0914507

1/10 integrate [1/ln(2)] x,[1.45136380, ((2)^1/10]

Input interpretation:

$$\frac{1}{10} \int_{1.45136380}^{\sqrt[10]{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

Result:

-0.0690877

1/11 integrate [1/ln(2)] x,[1.45136380, ((2)^1/11]

Input interpretation:

$$\frac{1}{11} \int_{1.45136380}^{\sqrt[11]{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

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Result:

-0.0637504

1/13 integrate [1/ln(2)] x,[1.45136380, ((2)^1/13]

Input interpretation:

- $\log(x)$  is the natural logarithm

- $\log(x)$  is the natural logarithm

- $\log(x)$  is the natural logarithm

- $\log(x)$  is the natural logarithm

$$\frac{1}{13} \int_{1.45136380}^{\sqrt[13]{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

Result:

-0.0551512

1/14 integrate [1/ln(2)] x,[1.45136380, ((2)^1/14]

Input interpretation:

$$\frac{1}{14} \int_{1.45136380}^{\sqrt[14]{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

Result:

-0.0516468

1/15 integrate [1/ln(2)] x,[1.45136380, ((2)^1/15]

Input interpretation:

$$\frac{1}{15} \int_{1.45136380}^{\sqrt[15]{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

-0.048553

1/17 integrate [1/ln(2)] x,[1.45136380, ((2)^1/17]

Input interpretation:

$$\frac{1}{17} \int_{1.45136380}^{\sqrt[17]{2}} \frac{1}{\log(2)} x dx$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

-0.0433442

1/19 integrate [1/ln(2)] x,[1.45136380, ((2)^1/19]

Input interpretation:

- $\log(x)$  is the natural logarithm

- $\log(x)$  is the natural logarithm

- $\log(x)$  is the natural logarithm

- $\log(x)$  is the natural logarithm



$$\frac{\zeta\left(\frac{3}{2}\right)}{3}$$

0.870791782895162781116189189308023876856933550800021135857...

and  $2,97712 \leq 3$

Note that:

$1,6644049 * 0,877486228 = 1,4604923775657172$  that is a very good approximation to absolute value of

$$\zeta\left(\frac{1}{2}\right) \approx -1.46035450880958681289$$

The sum of A and B

1.6644049+0.8774862275792690693589624239917932154238829904768855

Input interpretation:

1.6644049 + 0.8774862275792690693589624239917932154238829904768855

[Open code](#)

Result:

2.5418911275792690693589624239917932154238829904768855

Note that:

$(1.6644049+0.8774862275792690693589624239917932154238829904768855) * 10^3$

Input interpretation:

$(1.6644049 + 0.8774862275792690693589624239917932154238829904768855) \times 10^3$

[Open code](#)

Result:

2541.8911275792690693589624239917932154238829904768855

Continued fraction:

Linear form

$$\begin{array}{r}
 2541 + \frac{1}{1 + \frac{1}{8 + \frac{1}{5 + \frac{1}{2 + \frac{1}{2 + \frac{1}{10 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{7 + \frac{1}{5 + \frac{1}{18 + \frac{1}{1 + \frac{1}{973 + \frac{1}{30 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}} \\
 \end{array}$$

The result is very near to the value of the rest mass of charmed Sigma baryon  $2518.8 \pm 0.6$

We have also that:

$$(1.6644049 - 0.8774862275792690693589624239917932154238829904768855) \times 10^3$$

Input interpretation:

$$(1.6644049 - 0.8774862275792690693589624239917932154238829904768855) \times 10^3$$

[Open code](#)

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Result:

786.9186724207309306410375760082067845761170095231145

Continued fraction:

Linear form

$$786 + \cfrac{1}{1 + \cfrac{1}{11 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{57 + \cfrac{1}{49 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

The result 786,9186 is very near to the value of the rest mass of the rest mass of Omega meson  $782.65 \pm 0.12$

$$(1.6644049 \times 0.8774862275792690693589624239917932154238829904768855) \times 10^3$$

Input interpretation:  
 $(1.6644049 \times 0.8774862275792690693589624239917932154238829904768855) \times 10^3$   
[Open code](#)

Result:  
 • More digits  
 1460.492376865450577459496917407818187538266426376381562938...

Continued fraction:  
 • Linear form

$$1460 + \cfrac{1}{2 + \cfrac{1}{32 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{12 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$





## while

Input interpretation:

$$\frac{1}{3} \exp(1.6644049 + 0.8774862275792690693589624239917932154238829904768855)$$

[Open code](#)

## Enlarge Data Customize A Plaintext Interactive

Result:

More digits

4.2345575...

Continued fraction:

Linear form

$$4 + \frac{1}{4 + \frac{1}{3 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{13 + \frac{1}{1 + \frac{1}{6 + \frac{1}{3 + \frac{1}{\dots}}}}}}}}}}}}$$

The value 12,703 is very near to the value of the black hole entropy 12,57, while the result 4,2345 is in the range of the mass of DM (dark matter) particle.

$$2\pi [(1.6644049 + 0.8774862275792690693589624239917932154238829904768855) * 108]$$

Input interpretation:

$$2\pi ((1.6644049 + 0.8774862275792690693589624239917932154238829904768855) \times 108)$$

[Open code](#)

Result:

More digits

1724.8867...

Continued fraction:

Linear form

$$1724 + \frac{1}{1 + \frac{1}{7 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{3 + \frac{1}{\dots}}}}}}}}$$

The result is practically equal to the value of the mass of meson  $f_0(1710)$ , candidate glueball. Note that the result is a length of a circle  $C = 2\pi r$  with  $r = 274,5242417\dots$

$$(1.6644049+0.8774862275792690693589624239917932154238829904768855)^e$$

Input interpretation:

$$(1.6644049 + 0.8774862275792690693589624239917932154238829904768855)^e$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Result:

More digits

12.62789...

Continued fraction:

Linear form

$$12 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{5 + \frac{1}{31 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}}}$$

The result 12,627 is very near to the value of the black hole entropy 12,57.

integrate

$$(1.6644049+0.8774862275792690693589624239917932154238829904768855) [0, 3\pi]$$

Definite integral:

Step-by-step solution

$$\int_0^{3\pi} (1.6644049 + 0.8774862275792690693589624239917932154238829904768855) dx = 23.9568$$

The value  $23,9568 \approx 24$ , represent the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

$10^2$  integrate

$$(1.6644049+0.8774862275792690693589624239917932154238829904768855) [0, 2\pi]$$

Input interpretation:

$$10^2 \int_0^{2\pi} (1.6644049 + 0.8774862275792690693589624239917932154238829904768855) dx$$

[Open code](#)

Result:

1597.12

Enlarge Data Customize A Plaintext Interactive

Indefinite integral assuming all variables are real:

1597.12 x + constant

The result 1597,12 is very near to the range of Lambda meson that is  $\approx 1600$ :

### $\Lambda(1600)$ MASS

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
<b>1560 to 1700 (<math>\approx 1600</math>) OUR ESTIMATE</b>			
1592 $\pm$ 10	ZHANG	13A	DPWA Multichannel
1568 $\pm$ 20	GOPAL	80	DPWA $\bar{K}N \rightarrow \bar{K}N$
1703 $\pm$ 100	ALSTON-...	78	DPWA $\bar{K}N \rightarrow \bar{K}N$
1573 $\pm$ 25	GOPAL	77	DPWA $\bar{K}N$ multichannel
1596 $\pm$ 6	KANE	74	DPWA $K^- p \rightarrow \Sigma \pi$
1620 $\pm$ 10	LANGBEIN	72	IPWA $\bar{K}N$ multichannel
• • • We do not use the following data for averages, fits, limits, etc. • • •			
1572 or 1617	<sup>1</sup> MARTIN	77	DPWA $\bar{K}N$ multichannel
1646 $\pm$ 7	<sup>2</sup> CARROLL	76	DPWA Isospin-0 total $\sigma$
1570	KIM	71	DPWA K-matrix analysis

<sup>1</sup> The two MARTIN 77 values are from a T-matrix pole and from a Breit-Wigner fit.

<sup>2</sup> A total cross-section bump with  $(J+1/2) \Gamma_{el} / \Gamma_{total} = 0.04$ .

integrate

(1.6644049+0.8774862275792690693589624239917932154238829904768855) [0, 48]

$$\int_0^{48} (1.6644049 + 0.8774862275792690693589624239917932154238829904768855) dx = 122.011$$

The result 122,011 is very near to the value of mass of Higgs boson (122-126.8)

integrate

(1.6644049\*0.8774862275792690693589624239917932154238829904768855) [0, 1728]

$$\int_0^{1728} 1.6644049 \times 0.8774862275792690693589624239917932154238829904768855 dx = 2523.73$$

Continued fraction:

Linear form

$$2523 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}$$

The result 2523.73 is very near to the value of rest mass of the charmed Sigma baryon  $2518.8 \pm 0.6$

integrate

$$[1.6644049 + 0.8774862275792690693589624239917932154238829904768855] [0, 1728/\pi]$$

Definite integral:

Step-by-step solution

$$\int_0^{1728/\pi} (1.6644049 + 0.8774862275792690693589624239917932154238829904768855) dx = 1398.14$$

The result 1398,14 is very near to the rest mass of Sigma baryon  $1387.2 \pm 0.5$

integrate

$$[1.6644049 + 0.8774862275792690693589624239917932154238829904768855] [0, 729]$$

$$\int_0^{729} (1.6644049 + 0.8774862275792690693589624239917932154238829904768855) dx = 1853.04$$

The result 1853,04 is very near to the rest mass of D meson  $1864.84 \pm 0.17$

integrate

$$[1.6644049 * 0.8774862275792690693589624239917932154238829904768855] [0, 729]$$

$$\int_0^{729} 1.6644049 \times 0.8774862275792690693589624239917932154238829904768855 dx = 1064.7$$

The result 1064.7 is a good approximation to the rest mass of Phi meson  $1019.445 \pm 0.020$ .





Alternate form:

$$2 \log(2) + 3 \log(3) + \log(1823)$$

Continued fraction:

Linear form

- $$12 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{20 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Alternative representations:

$$\log(196 884) = \log_e(196 884)$$

$$\log(196 884) = \log(a) \log_a(196 884)$$

$$\log(196 884) = -\text{Li}_1(-196 883)$$

Integral representations:

$$\log(196 884) = \int_1^{196 884} \frac{1}{t} dt$$

$$\log(196 884) = -\frac{i}{2 \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{196 883^{-s} \Gamma(-s)^2 \Gamma(1 + s)}{\Gamma(1 - s)} ds \quad \text{for } -1 < \gamma < 0$$



## Appendix B

From:

### Non-Perturbative Effects on a Fractional D3-Brane

Gabriele Ferretti and Christoffer Petersson

<https://arxiv.org/abs/0901.1182v3>

We have that:

We begin by considering the case where we have placed  $N_1$  D3<sub>1</sub>-branes at node 1, together with the D(-1)<sub>1</sub>-instanton. The gaugino has tadpoles on mixed disks with either  $\omega_{\dot{\alpha}}$  and  $\bar{\mu}$  moduli insertions or with  $\bar{\omega}_{\dot{\alpha}}$  and  $\mu$  insertions. In addition to the profile contribution these amplitudes give rise to we also get a contribution from when we act with the supersymmetry generators that were broken by the D(-1)<sub>1</sub>-instanton [5]. This shifts the zero modes that correspond to the broken supersymmetries and thereby introduces an extra term in the gaugino profile that depends explicitly on  $\theta^\alpha$ . From this analysis we

obtain an expression for a pair of gauginos with the following structure [54],

$$\text{tr} [\Lambda^\alpha \Lambda_\alpha] = \frac{\rho^4 \theta^\alpha \theta_\alpha}{[(X-x)^2 + \rho^2]^4} + \dots, \quad (2.20)$$

where  $X^\mu$  is the space-time coordinate while  $x^\mu$  still denotes the position and  $\rho$  the size of the instanton. The ellipses denote terms with less powers of  $\theta^\alpha$  that will not be important for our purposes.

The expression (2.20) for the pair of gauginos in terms of the unconstrained moduli fields can now be inserted into the moduli space integral yielding

$$\langle \text{tr} [\Lambda^\alpha \Lambda_\alpha] \rangle = \Lambda^{b_1} \int d\{x, \theta, \lambda, D, \omega, \bar{\omega}, \mu, \bar{\mu}\} \text{tr} [\Lambda^\alpha \Lambda_\alpha] e^{-S_{\text{moduli}}^{0-d}}. \quad (2.21)$$

As usual,  $x^\mu$  and  $\theta^\alpha$  correspond to the supertranslations broken by the D(-1)<sub>1</sub>-instanton and do not appear explicitly in the instanton action  $S_{\text{moduli}}^{0-d}$ . They do however appear in the expression for the gaugino pair and we can use (2.20) when performing the integrals over these two variables,

$$\int d^4x d^2\theta \text{tr} [\Lambda^\alpha \Lambda_\alpha] = \int d^4x \frac{\rho^4}{[(X-x)^2 + \rho^2]^4} = \frac{\pi^2}{6}, \quad (2.22)$$

where we see that the factors of  $\rho$  cancel off and we simply get a dimensionless constant which we can absorb in the prefactor  $\Lambda^{b_1}$  of the remaining integral

$$\langle \text{tr} [\Lambda^\alpha \Lambda_\alpha] \rangle = \Lambda^{b_1} \int d\{\lambda, D, \omega, \bar{\omega}, \mu, \bar{\mu}\} e^{-S_{\text{moduli}}^{0-d}}. \quad (2.23)$$

Now, the crucial point is that the integral that remains to be calculated in (2.23) is precisely the integral one evaluates when computing the superpotential correction generated by the instanton configuration.

Practically, the expression:

$$\int d^4x d^2\theta \operatorname{tr} [\Lambda^\alpha \Lambda_\alpha] = \int d^4x \frac{\rho^4}{[(X-x)^2 + \rho^2]^4} = \frac{\pi^2}{6},$$

represent an expression for a pair of gaugini with the following structure where  $X^\mu$  is the space-time coordinate while  $x^\mu$  still indicates the position and  $\rho$  the size of the instanton. Note that  $x^\mu$  and  $\theta^\alpha$  correspond to the breaks of supertranslations from instanton  $D(-1)_1$  and appear in the expression of the pair of gaugini and we can use (2.20) when we execute the integrals on these two variables. We observe that the factors of  $\rho$  cancel each other out and we simply obtain a dimensionless constant that we can incorporate in the gaugino of the multiplet vector of the remaining integral (2.23).

Note that  $\frac{\pi^2}{6}$  is  $\zeta(2) = 1,644934 \dots$

$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6} \approx 1.64493406684822643647;$$

That is:

$\frac{\pi^2}{6}$  is a transcendental number

and

Continued fraction:

• Linear form

$$\begin{aligned} & 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}} \end{aligned}$$

that is connected with the already analyzed Ramanujan expression

## Conclusion

Since the name "hypothesis of the continuous" derives from the straight line of the real numbers, called "the continuum" and which concerns the possible dimensions for infinite sets, it is possible to propose the following comparison. The straight line of real numbers could be identified with the supersymmetric vacuum, where the information is "stored" which, in our case, is represented by the set of transcendental numbers which is an uncountable infinity and by the various Fundamental Physical Constant and the various Fundamental Mathematical Constants. The Cantor set, also an uncountable infinity, is fractal and could be identified with the toroidal infinite-dimensional Hilbert space, (see Clifford's torus) having "non-integer dimension", intermediate between the dimensions 0 and 1 respectively of the point and the straight line. Indeed its Hausdorff dimension is equal to  $\ln(2) / \ln(3) = 0.630929\dots$ . This could mean that though the universe is not a fractal, in the informal phase it would have been born anyway from a "fractal structure" (Hilbert space-Clifford's torus with Hausdorff dimension) (Antonio Nardelli)

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