A motivic sterile neutrino

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Despite the resounding experimental success of the Standard Model, the mystery of neutrino mass and neutrino oscillations must be approached from a framework for quantum gravity. Using well established results in condensed matter physics and in motivic mathematics, we present a new view of the quantum vacuum based on neutrino braid diagrams in quantum computation. The prediction of an effective 1.29 eV non local sterile state from the Koide matrix for $\overline{\nu}$ masses fits known observational constraints.

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I. INTRODUCTION

As we know, three spin 1/2 Pauli operators define a quaternion basis $i, j, k \in \mathbb{H}$ for the spatial directions of Minkowski space. Thus the Pauli exclusion principle means that quantum information in a fermion generates spatial degrees of freedom. In contrast, bosons accumulate in the ground state of a Bose-Einstein condensate. Any theory with supersymmetry must explain how gravity distinguishes these regimes, and why Bose-Einstein condensates can exhibit antigravity and give photons mass.

Our approach[40–47] assumes that the categorical axioms of quantum logic, which are foundational to condensed matter physics, can greatly simplify the mathematical formulation of the Standard Model. This idea is justified by the success of polytope and operad methods in the computation of scattering amplitudes.

In 2010 it was observed[14,42] that a good candidate for a neutrino rest mass at 0.00117 eV exactly matches the present day CMB temperature, using only the relation

$$mc^2 = \frac{hc}{2\pi\lambda} = \beta kT, \tag{1}$$

for $\beta = 4.965$ Wien's constant. This coincidence was intriguing, because the most natural measure of cosmic time is the thermodynamic CMB temperature[41].

By the uncertainty principle, any knowledge of a precise mass must correspond to an indefinite time, forcing our models of mass generation to employ all possible scales. We considered[46] the information content of dyonic states for Standard Model particles, complementing the usual local states with a maximally non local mirror set of states, associated to a cosmological horizon. A good analogy is the dyon mirror pair[38] of topological surface states, or the superfluid picture[3]. This mirror is literally a mirror when particles are represented by ribbon diagrams[4], which is quite appropriate in the axioms of quantum computation.

Thinking about this pairing of local and cosmological horizons, related to natural UV and IR cutoffs, we can now elevate the inertial mass model of quantum inertia[23,24,33–35] to a foundational statement about the quantum vacuum: the localisation of mass in an electroweak creation vertex pairs local and cosmological information. Neutrino masses are foundational, suggesting a derived Higgs mass $m_H \simeq \sqrt{\mu m_P}$, for μ a neutrino mass scale and m_P the Planck scale. Quantum inertia recovers a MOND description for galactic rotation curves.

The prediction of a sterile neutrino mass requires precise rest masses for the active states, which we get from the Koide-Brannen scheme[6,31,32]. Quantum mechanics is axiomatised by symmetric monoidal categories[1,13], where the symmetry condition is imposed on a braided monoidal category. A non trivial braiding is required in pertinent ribbon categories, such as the category for Fibonacci anyons. The ribbon twist will denote charge. Pairs of Chern-Simons field theories for gravity[15] are often considered.

Once we accept ribbon categories as a diagnostic for any physical scale, we remember the knotted field lines in the intergalactic media of plasma cosmologies[2,30]. In some sense, gravity is merely a cosmological form of electromagnetism.

Section II covers the quantum information behind Standard Model states in this framework, and in section III we introduce our non local sterile neutrino candidate.

II. THE STANDARD MODEL

A. Lepton and quark braids

The chiral lepton and quark states of the Standard Model were given as ribbon diagrams[4] based on preon models. Let σ_1 and σ_2 be the generators of B_3, the braid group on three strands, with relation $\sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2$. The start of each row in Table I is the B_3 braid for the neutrino. Along each row, three electric anyon charges are added to the underlying braid. Standard Model singlets are also B_3 diagrams but with flipped braid crossings. Massless neutrinos have a fixed helicity, but both states are possible when neutrinos gain mass. Each row in Table I defines a parity cube with 8 vertices, listing states for three qubits.

Observe that mirror braids for charged leptons and quarks, with opposite charges for a given neutrino diagram, are not included in Table I. We may think of these states as magnetic charges. The dyon is associated to holographic surfaces, in analogy to a topological insulator[38], but our surfaces define the fundamental quantum vacuum. Braid composition of a particle and antiparticle annihilates to a neutral photon identity diagram.

These particle states correspond to an algebra of ideals for $\mathbb{C} \otimes \mathbb{O}$[21,22,26], where the complex factor introduces the ribbon twist for charge, so that $\mathbb{C} \otimes \mathbb{O}$ accounts for the charge $U(1)$ and $SU(3)$ color groups. An alternative but equivalent description uses the quantum group $SL_q(2)$[20]. The $U(1)$ is represented by the braid group B_2, and it is natural

TABLE I. Standard Model electric braid states

ν_L	e_L^-	$\overline{u}_L(1)$	$\overline{u}_L(2)$	$\overline{u}_L(3)$	$d_L(1)$	$d_L(2)$	$d_L(3)$
$\sigma_1\sigma_2^{-1}$	$---$	$0--$	$-0-$	$--0$	-00	$0-0$	$00-$
$\overline{\nu}_R$	e_R^+	$u_R(1)$	$u_R(2)$	$u_R(3)$	$\overline{d}_R(1)$	$\overline{d}_R(2)$	$\overline{d}_R(3)$
$\sigma_2\sigma_1^{-1}$	$+++$	$0++$	$+0+$	$++0$	$+00$	$0+0$	$00+$
$\overline{\nu}_L$	e_L^+	$u_L(1)$	$u_L(2)$	$u_L(3)$	$\overline{d}_L(1)$	$\overline{d}_L(2)$	$\overline{d}_L(3)$
$\sigma_1^{-1}\sigma_2$	$+++$	$0++$	$+0+$	$++0$	$+00$	$0+0$	$00+$
ν_R	e_R^-	$\overline{u}_R(1)$	$\overline{u}_R(2)$	$\overline{u}_R(3)$	$d_R(1)$	$d_R(2)$	$d_R(3)$
$\sigma_2^{-1}\sigma_1$	$---$	$0--$	$-0-$	$--0$	-00	$0-0$	$00-$

to use B_4 for $SU(3)$, since $SU(3)$ carries the B_4 representation for the Fibonacci anyon[19,29], which is universal for quantum computation. Then B_3 fills $SU(2)$, viewed as a compactified component of spacetime.

Braid groups are also represented by Majorana operators[28]. In particular, a cyclic B_3 group is generated by

$$\sigma_1 = \frac{1}{\sqrt{2}}(1+i), \;\; \sigma_2 = \frac{1}{\sqrt{2}}(1+j), \;\; \sigma_{12} = \frac{1}{\sqrt{2}}(1+k), \tag{2}$$

where i, j, k are the quaternion units. The Fibonacci anyon B_3 is a rotation of the $\pi/4$ phase defined by the $1/\sqrt{2}$ to the generators

$$\sigma_1 = e^{7\pi i/10}, \quad \sigma_2 = (\phi i + \sqrt{\phi}k)\sigma_1(\phi i + \sqrt{\phi}k)^{-1}, \tag{3}$$

where $\phi = (\sqrt{5}-1)/2$ is the inverse golden ratio.

Tracing the boundary of two linked loops[5], we form a Hopf link and evaluate the Jones polynomial at the quantum dimension $\phi + 1$, giving an estimate for the fine structure constant[40,45]

$$\sqrt{\alpha}^{-1} = 4\cosh\frac{2\pi}{\phi+3}, \tag{4}$$

with $\alpha^{-1} = 137.096$.

B. Fourier supersymmetry

Standard Model bosons and fermions are related by Fourier supersymmetry[43,47]. Each neutrino braid state in the last section is reduced to a 3×3 matrix representation of the underlying permutation in $C_3 \subset S_3$. The identity I_3 is the photon matrix γ. The two neutrinos are

$$\nu = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \overline{\nu} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{5}$$

Electric charge on each anyon strand is represented by one of three symbols: 1 for neutral, ω for $+1/3$, or $\overline{\omega}$ for $-1/3$. Then the charged leptons are

$$e_L^- = \overline{\omega} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad e_R^+ = \omega \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \tag{6}$$

which compose to the identity. Similarly,

$$e_L^+ = \omega \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad e_R^- = \overline{\omega} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{7}$$

Quarks use different charges on individual strands, as in

$$u_L(1) \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & \omega \\ 1 & 0 & 0 \end{pmatrix}, \quad u_L(2) \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega \\ \omega & 0 & 0 \end{pmatrix}, \quad u_L(3) \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & 1 \\ \omega & 0 & 0 \end{pmatrix}. \tag{8}$$

The $W^\pm$ bosons are represented by

$$W^- = \overline{\omega} I_3, \quad W^+ = \omega I_3. \tag{9}$$

For the Z boson there are six remaining neutral boson matrices, which are

$$Z_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \overline{\omega} \end{pmatrix}, \quad \overline{\omega} Z_1 = \begin{pmatrix} \overline{\omega} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad \omega Z_1 = \begin{pmatrix} \omega & 0 & 0 \\ 0 & \overline{\omega} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{10}$$

and their three conjugates. Altogether, when $\omega = 2\pi/3$ is the cubed root of unity, there are 27 matrices of the form $(\omega)^a (Z_1)^b (\nu)^c$ for $a, b, c \in \{0, 1, 2\}$. This algebra defines a basis for the 27 dimensional exceptional Jordan algebra over $\mathbb{O}^{27}$, showing that the $\nu\gamma$ copy of C_3 is a baby representation of triality.

The twisted Fourier transform $\mathbf{F}$ is defined on e_L^- by

$$\mathbf{F}(e_L^-) \equiv \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \overline{\omega} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \overline{\omega} \\ 1 & \overline{\omega} & \omega \end{pmatrix} \begin{pmatrix} 0 & \overline{\omega} & 0 \\ 0 & 0 & \overline{\omega} \\ \overline{\omega} & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \overline{\omega} & \omega \\ 1 & \omega & \overline{\omega} \end{pmatrix} = W^- \tag{11}$$

and on right handed states by

$$\mathbf{F}(e_R^-) \equiv \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \overline{\omega} & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \overline{\omega} \\ 1 & \overline{\omega} & \omega \end{pmatrix} \begin{pmatrix} 0 & 0 & \overline{\omega} \\ \overline{\omega} & 0 & 0 \\ 0 & \overline{\omega} & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \overline{\omega} & \omega \\ 1 & \omega & \overline{\omega} \end{pmatrix} = W^-. \tag{12}$$

Thus the full lepton states map to electroweak bosons

$$e^\pm \mapsto W^\pm, \quad \nu, \overline{\nu} \mapsto \gamma. \tag{13}$$

Particle braid groups are truncated by relations[26] of the form $\sigma_i^8 = I$, satisfied by (2). Taking a 27 dimensional three qutrit state space, labeled by the matrices (a, b, c), we can define the 24 off-diagonal elements of a Jordan matrix as a basis for the 24 dimensional Leech lattice[48], accounting for extra dimensions in bosonic M theory.

C. Mass matrices

The Pauli matrices i, j, k for qubits may be replaced by their mutually unbiased bases[9,39,49]. Such bases exist in any prime power dimension d, and are generated by a circulant $d \times d$ matrix, which for qubits and qutrits are

$$R_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}, \quad R_3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & 1 \\ 1 & 1 & \omega \\ \omega & 1 & 1 \end{pmatrix}. \tag{14}$$

Since $R_2^8 = I$ and $R_3^{12} = I$, the common geometric phase is $\pi/12$, well known to number theorists as the phase in the Dedekind eta function.

Assuming that each local particle state defines a mass triplet, the double set of neutrino helicities in Table I allows for two distinct triplets of mass states for the neutrinos. We assign the $+\pi/12$ phase to the correct helicity neutrinos (case A) and the $-\pi/12$ phase to

TABLE II. neutrino masses (eV)

$\nu(A)$	0.0507 0.0089 0.0004
$\nu(B)$	0.0582 0.00117 0.0006

the wrong helicity ones (case B), noting that there is no local CPT violation and the masses agree with initial results from MINOS[8] in 2010. Both triplets add to a scale of 0.06 eV.

The 3×3 Fourier transform of the diagonal triplet $(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$ of square root lepton masses is defined by the Koide matrix

$$\sqrt{M} = \frac{\sqrt{\mu}}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & \delta & \overline{\delta} \\ \overline{\delta} & \sqrt{2} & \delta \\ \delta & \overline{\delta} & \sqrt{2} \end{pmatrix} \tag{15}$$

for a dimensionful scale μ and complex phase δ. Koide's relation[31,32] from the 1980s, which correctly predicted the τ mass, fixes the $\sqrt{2}$ parameter. In 2006, Brannen[6] found that $\sqrt{2}$ also accounts for the neutrino masses, as fitted by oscillation data. One is able to select the neutrino phases $\delta + \pi/12$ and $\delta - \pi/12$ relative to the charged lepton δ, which is close to $2/9$. The neutrino masses are listed in Table II.

We interpret the double ν phases as the interaction of two kinds of clock. A Keplerian clock employs the arithmetic mean of t_1 and t_2 in special relativity. A second cosmic clock is associated to a geometric mean $\sqrt{T_1 T_2}$, and Kepler's law differs by a factor of 2π from the cosmic equivalence principle in Sciama's law $GU = c^2 R$, where U is the mass of the observable universe and R its characteristic Hubble radius.

The PMNS and CKM mixing matrices are estimated by circulant operators[47], and also modeled by three dimensional representations of modular form symmetries[10,12,18], which include a golden ratio mixing for the group A_5.

III. THE NON LOCAL STERILE

Although null results for sterile neutrinos appear to exclude large parts of the oscillation parameter space, good arguments for eV range mass states remain[11,25]. For instance, upward moving showers observed in 2018 by the ANITA experiment[17], which must pass through the Earth, cannot be explained by heavier neutrinos[7].

Our non local vacuum scenario evades the usual Lagrangian formalism, and can provide a sterile candidate without adding any further local particle states to the Standard Model. We assume a standard $3+3$ scenario for the six neutrino states.

The central $\overline{\nu}$ mass in Table II is 0.00117 eV, precisely the peak present day CMB temperature by (1). Redshifting this temperature back to the CMB creation time at $z = 1100$ we obtain a wrong helicity mass at 1.29 eV, in good agreement with global fits[11] to oscillation data. The local neutrinos tend to keep their normal helicity, and when they flip it may be to an early universe state rather than a local one. In further work we hope to analyse neutrino anomalies in detail.

A possible objection to this conclusion is the use of only one $\nu(B)$ mass state rather than three. But CMB temperatures distinguish the past, present and future, and we have simply chosen to observe the present. Other CMB temperatures are also worth considering, such as the 20 K corresponding to a redshifted 0.0089 eV.

The empirical coincidences here are impressively consistent within the axiomatic framework. Pandey[37] has considered ν oscillations using a broken equivalence principle, and in breaking this principle quantum inertia can also explain[46] discrepancies in the Hubble parameter over cosmic scales[16]. An analysis[36] of extra dark energy components for ΛCDM favours a model that selects the critical CMB creation time.

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