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# A MATHEMATICAL EXPRESSION FOR INVERSE FINE-STRUCTURE CONSTANT

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## ABSTRACT

From the golden angle and the Pell constant, a simple, elegant and concise mathematical expression of the inverse fine-structure constant is given.

**Keywords** Fine structure constant · Golden angle · Pell constant

## 1 Introduction

The idea of finding mathematical expressions for dimensionless physic constants is an important topic that goes back to several physicists as Sir Arthur Eddington, Albert Einstein and Richard Feynman.

Recent years an extra effort have been done by mathematician Simon Plouffe, with their set of reasonable expressions based on simplicity and length for the mass ratios of fundamental particles.[1]

## 2 The fine-structure constant calculation

Following the same reasoning proposed by Plouffe, we begin a research for mathematical expression that satisfies a complete known value of the  $\alpha^{-1}$  constant.

The fine-structure constant has a dimensionless value experimentally determined by the most recent methods from atomic, condensed-matter [2], and have measured  $\alpha = 1/137.035999046(27)$ , at  $2.0 \times 10^{-10}$  accuracy, via the recoil frequency of cesium-133 atoms in a matter-wave interferometer.

Here is presented a concise and possibly fundamental mathematical expression for the inverse fine-structure constant:

$$\alpha^{-1} \approx [g] + \frac{\sqrt{\frac{P_{Pell}}{7}}}{8} \quad (1)$$

Where:

- $[g]$  is floor of the golden angle in degrees, defined as  $[306(2 - \varphi)] = 137$
- $P_{Pell}$  is the Pell Constant [3], defined as  $1 - \prod_{k=0}^{\infty} (1 - \frac{1}{2^{2k+1}})$  or  $1 - (\frac{1}{2}; \frac{1}{4})_{\infty} \approx 0.58057755820489240229$

Then  $\alpha^{-1} \approx 137.035999056$ , approximately the exact value as experimentally determined [2] even at the relative standard uncertainty; this formula shows an unprecedentedly level of accuracy because with only 4 mathematical symbols give 10 (or even more) exact digits of this physical constant.

### 3 Conclusions

The length of expression presented in this paper is simple, short, elegant, in contra-position of other speculative formulas intended before that the length is clearly longer than the output. This could be a novel approach to understand what Fine-Structure constant really is in the sense of future decimal agreement with incoming measurements, or at least an important mathematical curiosity to take in count for further investigations in the field.

### References

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