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The Mössbauer experiment in a rotating system and the extraenergy shift between emission and absorption lines using Bimetric theory of gravitational-inertial field in Riemannian approximation explained

I.The Mössbauer experiment in a rotating system and the extraenergy shift between emission and absorption lines.

- In a series of papers published during the past decade with respect to Mössbauer experiments in a rotating system [71]-[75],it has been experimentally shown that the relative energy shift $\Delta E/E$ between the source of resonant radiation (situated at the center of the rotating system) and the resonant absorber (located on the rotor rim) is described by the relationship

$$\Delta E/E = -ku^2/c^2, \tag{1.1}$$

where u is the tangential velocity of the absorber, c the velocity of light in vacuum, and k some coefficient, which – contrary to what had been classically predicted equal $1/2$ (see for example [35]) – turns out to be substantially larger than $1/2$.

It cannot be stressed enough that the equality $k = 1/2$ had been predicted by general theory of relativity (GTR) on account of the special relativistic time dilation effect delineated by the tangential displacement of the rotating absorber, where the "clock hypothesis" by Einstein (i.e., the non-reliance of the time rate of any clock on its acceleration [35]) was straightly adopted. Hence, the revealed inequality $k > 1/2$ indicates the presence of some additional energy shift (next to the usual time dilation effect arising from tangential displacement alone) between the emitted and absorbed resonant radiation.

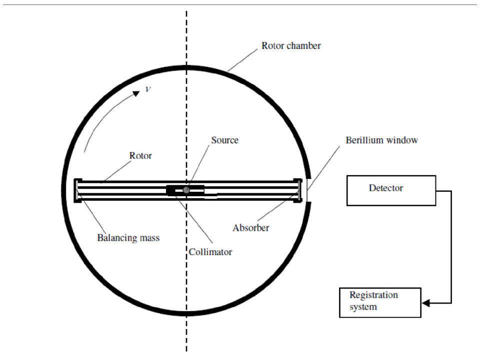

Fig.1.General scheme of Mössbauer experiment sinrotating systems.A source of resonant radiation is located on the rotational axis;an absorber is located on the rotor rim, while a detector of gamma-quanta is placed outside the rotor system,and it counts gamma-quanta at the time moment, when source,absorber and detector are aligned in a straight line. Adapted from [75].

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II.The inertional field equation in Riemannian approximation

We write the inertial field equations in Riemannian approximation of the form

$$R_i^{\mathbf{ac}\,k} = \frac{\eta^{\mathbf{ac}}}{c^4}\left(T_i^{\mathbf{ac}\,k} - \frac{1}{2}T^{\mathbf{ac}}\right), \tag{2.1}$$

where $\eta^{\mathbf{ac}}$ is dimensional constant with absolute value equal to 1.
We intoduce now 4-potential $U_\mu, \mu = 0,1,2,3$ in 4-D Minkovski space-time

$$U_\mu = (U_0, U_1, U_2, U_3) \tag{2.2}$$

We define a tensor of the accelerations by $T_{\nu\mu}^{\mathbf{ac}} = \partial_\nu U_\mu, \mu, \nu = 0,1,2,3$

$$T_{\nu\mu}^{\mathbf{ac}} = \begin{pmatrix} \frac{\partial U_0}{\partial x_0} & \frac{\partial U_0}{\partial x_1} & \frac{\partial U_0}{\partial x_2} & \frac{\partial U_0}{\partial x_3} \\ \frac{\partial U_1}{\partial x_0} & \frac{\partial U_1}{\partial x_1} & \frac{\partial U_1}{\partial x_2} & \frac{\partial U_1}{\partial x_3} \\ \frac{\partial U_2}{\partial x_0} & \frac{\partial U_2}{\partial x_1} & \frac{\partial U_2}{\partial x_2} & \frac{\partial U_2}{\partial x_3} \\ \frac{\partial U_3}{\partial x_0} & \frac{\partial U_3}{\partial x_1} & \frac{\partial U_3}{\partial x_2} & \frac{\partial U_3}{\partial x_3} \end{pmatrix} \tag{2.3}$$

In 3-D we obtain $T_{\nu\mu}^{\mathbf{ac}} = \partial_\nu U_\mu, \mu, \nu = 0,1,2$

$$T^{\mathbf{ac}}_{\nu\mu} = \begin{bmatrix} \frac{\partial U_0}{\partial x_0} & \frac{\partial U_0}{\partial x_1} & \frac{\partial U_0}{\partial x_2} \\ \frac{\partial U_1}{\partial x_0} & \frac{\partial U_1}{\partial x_1} & \frac{\partial U_1}{\partial x_2} \\ \frac{\partial U_2}{\partial x_0} & \frac{\partial U_2}{\partial x_1} & \frac{\partial U_2}{\partial x_2} \end{bmatrix} \tag{2.4}$$

In polar coordinates we obtain

$$T^{\mathbf{ac}}_{\nu\mu} = \begin{bmatrix} \frac{\partial U_0}{\partial x_0} & \frac{\partial U_0}{\partial r} & \frac{\partial U_0}{\partial \theta} \\ \frac{\partial U_1}{\partial x_0} & \frac{\partial U_1}{\partial r} & \frac{\partial U_1}{\partial \theta} \\ \frac{\partial U_2}{\partial x_0} & \frac{\partial U_2}{\partial r} & \frac{\partial U_2}{\partial \theta} \end{bmatrix} \tag{2.5}$$

We assume now that $U_0 = 0, U_1 = U_1(r), U_2 = U_2(\theta)$. From Eq.(2.5) we obtain

$$T^{\mathbf{ac}}_{\nu\mu} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\partial U_1}{\partial r} & 0 \\ 0 & 0 & \frac{\partial U_2}{\partial \theta} \end{bmatrix} \tag{2.6}$$

We assume now that $U_2(\theta) = const$,thus

$$T^{\mathbf{ac}}_{\nu\mu} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\partial U_1}{\partial r} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.7}$$

Motion of a particle in inartial field with a flat background metric $\mathbf{g}^b_{ik}$

Remind that the motion of a free material particle is determined in the special theory of relativity from the principle of least action,

$$\delta S = -mc\delta \int ds = 0, \tag{2.8}$$

according to which the particle moves so that its world line is an extremal between a given pair of world points, in our case a straight line (in ordinary three-dimensional space this corresponds to uniform rectilinear motion).

The motion of a particle in inartial field is determined by the principle of least action in this same form (2.8), since the inartial field is nothing but a change in the *flat background metric* g^b_{ik} of space-time, manifesting itself only in a change in the expression for ds in terms of the $dx^i, i = 0,1,2,3$. Thus, in inartial field the particle moves so that its world point moves along an extremal or, as it is called, a geodesic line in the four-space x^0, x^1, x^2, x^3; however, since in the presence of the inartial field space-time is not galilean, this line is not a "straight line", and the real spatial motion of the particle is neither uniform nor rectilinear.

Remark 2.1.Note that the full metric tensor $\breve{g} = g^{\mathbf{full}}_{ik}$ reads

$$\breve{g} = g_{ik}^{\mathbf{full}} = g_{ik}^{ac} + g_{ik}^{b}, \tag{2.9}$$

where (i) the non flat metric g_{ik}^{ac} determining the inartial field and where (ii) g_{ik}^{b} is the flat background metric related to the given coordinate transform.

Example 2.1.Let us consider a transformation from an inertial frame, in which the space-time is Minkowskian, to a rotating frame of reference. Using cylindrical coordinates, the line element in the starting inertial frame is

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dz^2. \tag{2.10}$$

The transformation to a frame of reference $\{t', r', \phi' z'\}$ rotating at the uniform angular rate ω with respect to the starting inertial frame is given by

$$t = t', r = r',\ \ \phi = \phi' + \omega t', z = z' \tag{2.11}$$

Thus, Eq.(2.10) becomes the following canonical line element (Langevin metric) of the rotating frame

$$ds^2 = \left(1 - \frac{r'^2\omega^2}{c^2}\right) c^2 dt'^2 - 2\omega r'^2 d\phi' dt' - dr'^2 - r'^2 d\phi'^2 - dz'^2. \tag{2.12}$$

Remark 2.2.As we consider light propagating in the radial direction ($d\phi' = dz' = 0$), by using a flat metric (2.12) the line element reads

$$ds^2 = \left(1 - \frac{r'^2\omega^2}{c^2}\right) c^2 dt'^2 - dr'^2. \tag{2.13}$$

Setting the origin of the rotating frame in the source of the emitting radiation, we get a first canonical contribution in relative energy shift $\Delta E/E$ between emission and absorption lines, which arises from the inartial blueshift, that can be directly computed using Eq. (8.2.13) and which reads

$$\frac{\Delta E}{E} = \frac{E_{\mathbf{received}} - E_{\mathbf{emitted}}}{E_{\mathbf{emitted}}} = |g_{00}(r')|^{\frac{1}{2}} - 1 = -\frac{1}{2} u^2/c^2, \tag{2.14}$$

where $u = \omega r'$.

Instead of starting once again directly from the principle of least action, it is simpler to obtain the equations of motion of a particle in the inartial field by an appropriate generalization of the differential equations for the free motion of a particle in the special theory of relativity, i.e. in a galilean four-dimensional coordinate system. These equations are $du^i/ds = 0, i = 0,1,2,3.$ or $du^i = 0, i = 0,1,2,3.$, where $u^i = dx^i/ds$ is the four velocity. Clearly, in curvilinear coordinates this equation is generalized to the equation of the form

$$\breve{D} u^i = 0, \tag{2.15}$$

where the $\breve{D}$ is a covariant derivative corresponding to the full metric tensor $\breve{g} = g_{ik}^{\mathbf{full}}$. From the expression for the covariant differential of a vector, we have

$$du^i + \breve{\Gamma}_{kl}^i u^k dx^l = 0, \tag{2.16}$$

where the $\breve{\Gamma}_{kl}^i$ is a Christoffel symbols corresponding to the full metric tensor $\breve{g} = g_{ik}^{\mathbf{full}}$. Dividing the equation (2.16) by ds, we obtain

$$\frac{d^2x^i}{ds^2} + \breve{\Gamma}_{kl}^i \frac{dx^k}{ds} \frac{dx^l}{ds} = 0. \tag{2.17}$$

This is the required equation of motion. We see that the motion of a particle in inartial field is determined by the Christoffel symbols $\breve{\Gamma}_{kl}^i$ and the 2-derivative

$$\frac{d^2x^i}{ds^2} \tag{2.18}$$

is the four-acceleration of the particle. Thus we may call the quantity

$$-m\breve{\Gamma}^i_{kl}u^k u^l \tag{2.19}$$

the four-force,acting on the particle in the inartial field. Here, the full metric tensor $g^{\mathbf{full}}_{ik}$ plays the role of the potential of the inartial field and its derivatives determine the field intensity $\breve{\Gamma}^i_{kl}$.

Remark 2.3.Remind that by a suitable choice of the coordinate system one can always make all the $\breve{\Gamma}^i_{kl}$ zero at an arbitrary point of space-time.

As before, we define the four-momentum of a particle in inartial field as

$$p^i = mcu^i. \tag{2.20}$$

Its square reads

$$p_i p^i = m^2c^2. \tag{2.21}$$

We write now

$$\frac{\partial S}{\partial x^i} \tag{2.22}$$

for p_i we find the Hamilton-Jacobi equation for a particle in inartial field:

$$\breve{g}^{ik}\frac{\partial S}{\partial x^i}\frac{\partial S}{\partial x^k} = 0, \tag{2.23}$$

where $\breve{g}^{ik} = \left(g^{\mathbf{full}}_{ik}\right)^{-1}$.

Remark 2.4.The equation of a geodesic in the form (2.17) is not valid to the propagation of a light signal, since along the world line of the propagation of a light ray the interval ds, as we know, is zero, so that all the terms in equation (2.17) become infinite. To get the equations of motion in the form needed for this case, we use the fact that the direction of propagation of a light ray in geometrical optics is determined by the wave vector tangent to the ray.

We can therefore write the four-dimensional wave vector in the form

$$k^i = dx^i/d\lambda, \tag{2.24}$$

where λ is some parameter varying along the ray. In the special theory of relativity, in the propagation of light in vacuum the wave vector does not vary along the path, that is, $dk^i = 0$. In inartial field this equation clearly goes over into $\breve{D}k^i = 0$ or

$$\frac{dk^i}{d\lambda} = \breve{\Gamma}^i_{kl}k^k k^l. \tag{2.25}$$

The absolute square of the wave four-vector is zero, that is, $k_i k^i = 0$. Substituting $\partial\psi/\partial x^i$ in place of k_i, where ψ is the eikonal, we find the eikonal equation in inartial field

$$\breve{g}^{ik}\frac{\partial\psi}{\partial x^i}\frac{\partial\psi}{\partial x^k} = 0. \tag{2.26}$$

In the limiting case of small velocities, the relativistic equations of motion of a particle in inartial field must go over into the corresponding non-relativistic equations. In this we must keep in mind that the assumption of small velocity implies the requirement that the inartial field itself be weak; if this were not so a particle located in it would acquire a high velocity.

Let us examine how, in this limiting case, the full metric tensor $g_{ik}^{\mathbf{full}} = g_{ik}^{\mathbf{ac}} + g_{ik}^{\mathbf{b}}$, where (i) the metric tensor $g_{ik}^{\mathbf{ac}}$ determining the inartial field is related to the nonrelativistic potential $\Phi^{\mathbf{ac}}$ of the inartial field and where (ii) $g_{ik}^{\mathbf{b}}$ is the flat background metric related to the given nonrelativistic potential $\Phi^{\mathbf{b}}$.

In nonrelativistic limit the motion of a particle in inartial field is determined by the Lagrangian

$$\mathcal{L} = \frac{mv^2}{2} - m\Phi^{\mathbf{b}} - m\Phi^{\mathbf{ac}}. \tag{2.27}$$

We now rewrite Eq.(2.27) it in the following form

$$\mathcal{L} = -mc^2 + \frac{mv^2}{2} - m\Phi^{\mathbf{b}} - m\Phi^{\mathbf{ac}}. \tag{2.29}$$

adding the constant $-mc^2$. This must be done so that the nonrelativistic Lagrangian in the absence of the field, $\mathcal{L} = -mc^2 + \frac{mv^2}{2}$, must be the same exactly as that to which the corresponding relativistic function $\mathcal{L} = -mc^2\sqrt{1 - v^2/c^2}$ reduces in the limit $v^2/c^2 \to 0$. Therefore the nonrelativistic action function S for a particle in inartial field has the form

$$S = -mc\int dt\left(c - \frac{v^2}{2c} + \frac{\Phi^{\mathbf{b}}}{c} + \frac{\Phi^{\mathbf{ac}}}{c}\right). \tag{2.30}$$

Comparing the Eq.(2.30) with the expression $S = -mc\int ds$ in the limit $v^2/c^2 \to 0$ we obtain

$$ds = \left(c - \frac{v^2}{2c} + \frac{\Phi^{\mathbf{b}}}{c} + \frac{\Phi^{\mathbf{ac}}}{c}\right)dt. \tag{8.2.31}$$

Squaring the Eq.(2.31) and dropping terms which vanish for $c \to \infty$, we obtain

$$ds^2 = \left(1 + \frac{2\Phi^{\mathbf{b}}}{c^2} + \frac{2\Phi^{\mathbf{ac}}}{c^2}\right)c^2dt^2 - d\mathbf{r}^2, \tag{2.32}$$

where $d\mathbf{r} = \mathbf{v}dt$.

Thus in the limiting case the component $g_{00}^{\mathbf{full}}$ of the full metric tensor is

$$g_{00}^{\mathbf{full}} = 1 + \frac{2\Phi^{\mathbf{b}}}{c^2} + \frac{2\Phi^{\mathbf{ac}}}{c^2}. \tag{2.33}$$

We choose now the background metric $g_{ik}^{\mathbf{b}}$ of the form is given by Eq.(8.2.12)

Remark 2.5.Note that

$$g_{00}^{\mathbf{b}} = 1 + \omega^2r^2. \tag{2.34}$$

III.The Mössbauer experiment in a rotating system explained

In the inertional field equations (8.2.1) we now carry out the transition to the limit of nonrelativistic mechanics.This is, for instance, the case in the nonrelativistic rotating system considered above in subsection VIII.1.Thus the acceleration of a particle of zero velocity lies in the direction of increasing r and is equal to

$$a = \omega^2r. \tag{3.1}$$

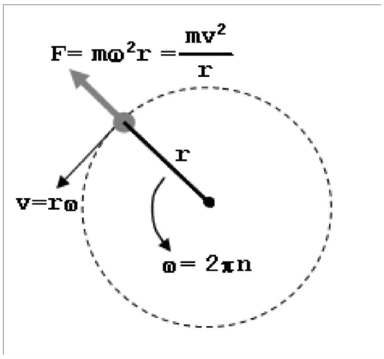

Fig.2.

$$U_1 = \omega^2 r^2/2. \tag{8.3.2}$$

This formula (3.2) is in accordance with the usual expression for the centrifugal force. We remind that the expression for the component $g_{00}^{\mathbf{ac}}$ of the metric tensor (the only one which we need) was found, for the limiting case which we are considering, in section II

$$g_{00}^{\mathbf{full}}(r) = g_{00}^{\mathbf{b}}(r) + g_{00}^{\mathbf{ac}}(r) = 1 + \frac{2\Phi^{\mathbf{b}}(r)}{c^2} + \frac{2\Phi^{\mathbf{ac}}(r)}{c^2}. \tag{3.3}$$

Further, we can use for the components of the inertian tensor the expression (2.7), where $U_1 = \omega^2 r^2/2$. Of all the components $T_i^{\mathbf{ac}\,k}$, there thus remains only

$$T_1^{\mathbf{ac}\,1} = \partial U_1/\partial r = \omega^2 r. \tag{3.4}$$

The scalar $T^{\mathbf{ac}} = T_i^{\mathbf{ac}\,i}$ will be equal to the value $T_1^{\mathbf{ac}\,1} = \partial U_1/\partial r = \omega^2 r$.
We write the field equations in the form (2.1). For $i = k = 0$ we get

$$R_0^{\mathbf{ac}\,0} = -\frac{1}{c^4}\omega^2 r \tag{3.5}$$

and $i = k = 1$ we get

$$R_1^{\mathbf{ac}\,1} = \frac{1}{c^4}\omega^2 r. \tag{3.6}$$

Remark 3.**1**.Note that in the approximation we are considering all the other equations vanish identically.

Remark 3.**2**.For the calculation of $R_0^{\mathbf{ac}\,0}$ from the canonical general formula, we note that terms containing derivatives of the quantities Γ_{ki}^i are in every case quantities of the second order. Terms containing derivatives with respect to $x^0 = ct$ are small (compared with terms with derivatives with respect to the coordinates $x^\alpha, \alpha = 1,2,3$) since they contain extra powers of $1/c$. As a result, there remains

$$R_{00}^{\mathbf{ac}} = R_0^{\mathbf{ac}\,0} = \partial\Gamma_{00}^\alpha/\partial x^\alpha, \tag{8.3.7}$$

where

$$\Gamma_{00}^\alpha \simeq -\frac{1}{2}g^{\mathbf{ac}\,\alpha\beta}\frac{\partial g_{00}^{\mathbf{ac}}}{\partial x^\beta}. \tag{8.3.8}$$

Substituting (3.8) into (3.7) we get

$$R_0^{\mathbf{ac}\,0} \simeq \frac{1}{c^2}\Delta\Phi^{\mathbf{ac}}(r) = -\frac{1}{c^4}\omega^2 r. \tag{8.3.9}$$

Finally we obtain radial Poisson equation

$$\frac{rd^2\Phi^{\mathbf{ac}}(r)}{dr^2} + \frac{d\Phi^{\mathbf{ac}}(r)}{dr} = \frac{d}{dr}\left(\frac{rd\Phi^{\mathbf{ac}}(r)}{dr}\right) = -\frac{1}{c^2}\omega^2 r. \tag{3.10}$$

By integration one obtains

$$\Phi^{\mathbf{ac}}(r) = -\frac{1}{4c^2}\omega^2 r^2. \tag{3.11}$$

Substituting (3.11) into (3.3) we get

$$\begin{gathered} g_{00}^{\mathbf{full}}(r) = g_{00}^{\mathbf{b}}(r) + g_{00}^{\mathbf{ac}}(r) = 1 + \frac{2\Phi^{\mathbf{b}}(r)}{c^2} + \frac{2\Phi^{\mathbf{ac}}(r)}{c^2} = \\ 1 - \frac{1}{c^2}\omega^2 r^2 - \frac{1}{2c^2}\omega^2 r^2 = 1 - \frac{3}{2c^2}\omega^2 r^2 \end{gathered} \tag{8.3.12}$$

Suppose that light flashes are emitted from a point $r = r_1$ at an interval Δt. The field being static, the flashes will reach the observer at $r = r_2$ after the same interval Δt. The ratio of the proper time intervals at these two points is

$$\frac{\Delta\tau_1}{\Delta\tau_2} = \sqrt{\frac{g_{00}^{\mathbf{b}}(r_1) + g_{00}^{\mathbf{ac}}(r_1)}{g_{00}^{\mathbf{b}}(r_2) + g_{00}^{\mathbf{ac}}(r_2)}}\,. \tag{3.13}$$

Hence, the ratio of frequences is

$$\frac{\omega_1}{\omega_2} = \frac{\Delta\tau_2}{\Delta\tau_1} = \sqrt{\frac{g_{00}^{\mathbf{b}}(r_2) + g_{00}^{\mathbf{ac}}(r_2)}{g_{00}^{\mathbf{b}}(r_1) + g_{00}^{\mathbf{ac}}(r_1)}} = \sqrt{\frac{1 - \frac{3}{2c^2}\omega^2 r_2^2}{1 - \frac{3}{2c^2}\omega^2 r_1^2}}\,. \tag{8.3.14}$$

Substituting $r_1 = 0$ into (3.14) we get

$$\frac{\omega_1}{\omega_2} = \frac{\Delta\tau_2}{\Delta\tau_1} = \sqrt{\frac{g_{00}^{\mathbf{full}}(r_2)}{g_{00}^{\mathbf{full}}(0)}} = \sqrt{1 - \frac{3}{2c^2}\omega^2 r_2^2} = 1 - \frac{3}{4c^2}\omega^2 r_2^2 = 1 - \frac{3u^2}{4c^2}. \tag{3.15}$$

Therefore

$$\Delta E/E = -\frac{3u^2}{4c^2}.= -0.75\frac{u^2}{c^2}\,. \tag{3.16}$$

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