SunQM-3s7: Predict mass density r-distribution for gas/ice planets based on {N,n} QM probability distribution

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Abstract

Using the same method developed in paper SunQM-3s6, the (close to the true) mass density r-distribution for gas/ice planets has been estimated based on {N,n} QM probability distribution. Based on this calculation (as well as on other scientists' calculation), the center core mass density for Jupiter, Saturn, Uranus, Neptune, and the undiscovered {3,2} planet (if it does formed) are estimated to be: ~ 26000 kg/m^3, ~ 23000 kg/m^3, ~ 16500 kg/m^3, ~ 17000 kg/m^3, and ~ 16000 kg/m^3 respectively. The results from this paper (as well as from papers of SunQM-3s6 and SunQM-3s8) strongly suggest that all planets and stars were formed and evolved under planet's (or star's) QM.

Introduction

In a series of research articles, the quantum mechanics of Solar system has been established [1] ~ [11]. In previous paper SunQM-3s6 [10], from studying Earth's known internal structure and mass density distribution, I have developed a method which can be used to estimate any planet's internal structure and mass density. This method can be expressed as:

Planet mass = 4π ∫ (planet's QM probability density r-distribution) * W *D * r^2 dr

where mass density D = a * r + b, and W is a scaling factor. In paper SunQM-3s6, I applied this method to all four rocky planets, and obtained the internal structure and (close to the true) mass density r-distribution for these planets. In current paper, I will apply the same method to four gas/ice planets. Note: due to the size limitation, the same analysis for Sun has been spun-off from this paper, and moved to a new paper SunQM-3s8. Note: for {N,n} QM nomenclature as well as the general notes for {N,n} QM model, please see my paper SunQM-1 section VII. Note: Microsoft Excel's number format is often used in this paper, for example: x^2 = x^2, 3.4E+12 = 3.4×10^{12}, 5.6E-9 = 5.6×10^{-9}.

I. Predict Jupiter's internal structure and the mass density radial distribution by using p{N,n} QM probability function

So far, we do not have the experimental determined mass density radial distribution (like Earth's) for Jupiter. According to "https://sciencing.com/jupiters-core-vs-earths-core-21848.html", "*The core's density is estimated at 25,000 kg per cubic meter*". According to the method established from paper SunQM-3s6, let us first constitute the mass density linear equation D = a * r + b for Jupiter. After manual fitting, one good result is D = -0.000076 * r + 5310. It satisfies both conditions: 1) ∫ D dV = mass of Jupiter (see the integration equation below); 2) at surface r = 6.99E+7 m, D ≈ 0 kg/m^3. It is plotted in Figure 1a.

$$\int_0^{6.99\times10^7} 4\pi(-0.000076x+5310)x^2\,dx = 1896554786375158606732460032$$

From paper SunQM-1s3, we know that Jupiter (both its internal structure and its moon orbit system) can be described by a p{N,n//5} QM structure. If we define Jupiter's surface as p{0,1}, then p{0,1//5} = p{-1,5//5}. Let us set Jupiter's (Earth-sized) inner core as r_1, so $r_1 = r_{surface}$ / 5^2 = 6.99E+7 / 25 ≈ 2.80E+6 m. This means that when we set p{-1,1} as n=1 state, the radial probability density of Jupiter has function of r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2 + |R(5,*l*)|^2). Similar as that of Earth, Jupiter's mass radial distribution can also be (approximately, ignoring the p{-1,1] center region due to its small volume) described by a simple integration formula of QM probability

Mass (r, θ, φ) = ∭ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2 + |R(5,*l*)|^2) *W *D *sin(θ) * r^2 dr dθ dφ, [r=0, 6.99E+7 m; θ=0, π; φ=0, 2π]

or

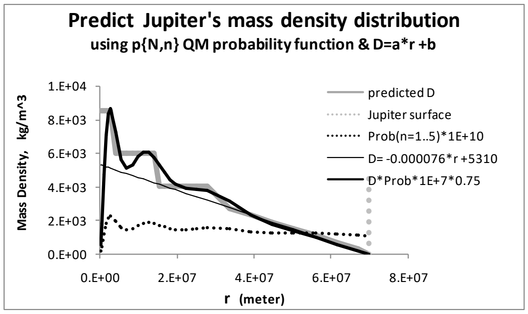
1.90E+27 kg = 4π ∫ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2 + |R(5,*l*)|^2) *W *(-0.000076*r + 5310) * r^2 dr, [r=0, 6.99E+7 m]

For the probability of r^2 * |R(5,*l*)|^2, among *l* = 0, 1, 2, 3, and 4, only r^2 * |R(5,4)|^2, makes significant contribution within r/r_1 =25. So the r^2 * |R(5,*l*)|^2 is simplified as r^2 * |R(5,4)|^2. The calculation of r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2 + |R(5,4)|^2) is the same as that in paper SunQM-3 Table 2, except now using r_1 = 2.80E+6 m. The table of r^2 *|R(n,l)|^2 calculation is not shown here. The resulted curve is shown in column 5 "Prob(n=1..5)" of Table 1, and it is plotted in Figure 1a as "Prob(n=1..5)*1E+10". Table 1 shows the calculation to predict Jupiter's internal structure and the mass density r-distribution using p{N,n} QM radial probability function, and Figure 1a and Figure 1b shows the result.

Table 1. Predict Jupiter's internal structure and the mass density r-distribution by using p{N,n} QM radial probability function.

										A= 2.26E+05			
										B= 0.13			
r_1=2.80E+6 m			1.63E+27		1.00E+10	0.75	1.64E+27			C= 21600.0		1.63E+27	
r/r_1 =	r/r_1*r_1	D= -0.000076*r +5310	mass= D*ΔV	Prob(n=1..5)	Prob(n=1..5)*1E+10	D*Prob*1E+7*0.75	mass= (D*Prob*1E+7*0.75) *ΔV	r, p{-1,1}, p{0,1}	predicted D	D=A/r^B-C	(A/r^B-C)*Prob*1E+7	mass= ((A/r^B-C)*Prob*1E+7)*ΔV	predicted D
unit	m	kg/m^3	kg			kg/m^3	kg	m	kg/m^3		kg/m^3	kg	kg/m^3
0.1	2.80E+05	5.29E+03	4.84E+20	1.386E-08	139	5.50E+02	5.03E+19	2.80E+05	8.50E+03	2.27E+04	3.14E+03	2.88E+20	2.60E+04
0.2	5.59E+05	5.27E+03	3.38E+21	4.537E-08	454	1.79E+03	1.15E+21	5.59E+05	8.50E+03	1.89E+04	8.55E+03	5.48E+21	2.60E+04
0.4	1.12E+06	5.23E+03	2.68E+22	1.213E-07	1213	4.75E+03	2.44E+22	1.12E+06	8.50E+03	1.54E+04	1.86E+04	9.55E+22	2.60E+04
0.6	1.68E+06	5.18E+03	7.21E+22	1.82E-07	1820	7.08E+03	9.85E+22	1.68E+06	8.50E+03	1.35E+04	2.45E+04	3.41E+23	2.60E+04
0.8	2.24E+06	5.14E+03	1.39E+23	2.159E-07	2159	8.32E+03	2.26E+23	2.24E+06	8.50E+03	1.22E+04	2.63E+04	7.13E+23	2.60E+04
1	2.80E+06	5.10E+03	2.28E+23	2.257E-07	2257	8.63E+03	3.85E+23	2.80E+06	8.50E+03	1.12E+04	2.53E+04	1.13E+24	2.60E+04
1.5	4.19E+06	4.99E+03	1.09E+24	1.936E-07	1936	7.25E+03	1.58E+24	4.19E+06	6.00E+03	9.53E+03	1.85E+04	4.01E+24	1.10E+04
2	5.59E+06	4.89E+03	2.07E+24	1.541E-07	1541	5.64E+03	2.39E+24	5.59E+06	6.00E+03	8.39E+03	1.29E+04	5.47E+24	1.10E+04
2.5	6.99E+06	4.78E+03	3.34E+24	1.418E-07	1418	5.08E+03	3.55E+24	6.99E+06	6.00E+03	7.53E+03	1.07E+04	7.45E+24	1.10E+04
3	8.39E+06	4.67E+03	4.87E+24	1.518E-07	1518	5.32E+03	5.54E+24	8.39E+06	6.00E+03	6.85E+03	1.04E+04	1.08E+25	1.10E+04
3.5	9.79E+06	4.57E+03	6.64E+24	1.689E-07	1689	5.78E+03	8.41E+24	9.79E+06	6.00E+03	6.28E+03	1.06E+04	1.54E+25	1.10E+04
4	1.12E+07	4.46E+03	8.63E+24	1.816E-07	1816	6.07E+03	1.17E+25	1.12E+07	6.00E+03	5.80E+03	1.05E+04	2.04E+25	1.10E+04
4.5	1.26E+07	4.35E+03	1.08E+25	1.852E-07	1852	6.05E+03	1.50E+25	1.26E+07	6.00E+03	5.39E+03	9.98E+03	2.48E+25	1.10E+04
5	1.40E+07	4.25E+03	1.32E+25	1.805E-07	1805	5.75E+03	1.78E+25	1.40E+07	6.00E+03	5.02E+03	9.06E+03	2.81E+25	5.00E+03
5.5	1.54E+07	4.14E+03	1.57E+25	1.708E-07	1708	5.30E+03	2.01E+25	1.54E+07	4.00E+03	4.69E+03	8.01E+03	3.03E+25	5.00E+03
6	1.68E+07	4.04E+03	1.83E+25	1.597E-07	1597	4.83E+03	2.20E+25	1.68E+07	4.00E+03	4.40E+03	7.02E+03	3.19E+25	5.00E+03
6.5	1.82E+07	3.93E+03	2.11E+25	1.504E-07	1504	4.43E+03	2.38E+25	1.82E+07	4.00E+03	4.13E+03	6.21E+03	3.33E+25	5.00E+03
7	1.96E+07	3.82E+03	2.39E+25	1.445E-07	1445	4.14E+03	2.59E+25	1.96E+07	4.00E+03	3.88E+03	5.60E+03	3.51E+25	5.00E+03
8	2.24E+07	3.61E+03	5.59E+25	1.434E-07	1434	3.88E+03	6.01E+25	2.24E+07	4.00E+03	3.44E+03	4.93E+03	7.64E+25	5.00E+03
9	2.52E+07	3.40E+03	6.75E+25	1.507E-07	1507	3.84E+03	7.63E+25	2.52E+07	4.00E+03	3.06E+03	4.61E+03	9.17E+25	5.00E+03
10	2.80E+07	3.19E+03	7.90E+25	1.573E-07	1573	3.76E+03	9.32E+25	2.80E+07	4.00E+03	2.73E+03	4.29E+03	1.06E+26	5.00E+03
12	3.36E+07	2.76E+03	1.84E+26	1.514E-07	1514	3.13E+03	2.09E+26	3.36E+07	2.70E+03	2.16E+03	3.26E+03	2.18E+26	2.40E+03
14	3.91E+07	2.34E+03	2.17E+26	1.325E-07	1325	2.32E+03	2.16E+26	3.91E+07	2.30E+03	1.68E+03	2.23E+03	2.08E+26	2.00E+03
16	4.47E+07	1.91E+03	2.36E+26	1.23E-07	1230	1.76E+03	2.18E+26	4.47E+07	1.90E+03	1.28E+03	1.58E+03	1.95E+26	1.60E+03
18	5.03E+07	1.49E+03	2.36E+26	1.247E-07	1247	1.39E+03	2.21E+26	5.03E+07	1.50E+03	9.36E+02	1.17E+03	1.86E+26	1.20E+03
20	5.59E+07	1.06E+03	2.10E+26	1.265E-07	1265	1.01E+03	2.00E+26	5.59E+07	1.10E+03	6.29E+02	7.96E+02	1.58E+26	8.00E+02
22	6.15E+07	6.35E+02	1.54E+26	1.213E-07	1213	5.78E+02	1.40E+26	6.15E+07	7.00E+02	3.56E+02	4.31E+02	1.05E+26	4.00E+02
24	6.71E+07	2.10E+02	6.11E+25	1.101E-07	1101	1.74E+02	5.05E+25	6.71E+07	3.00E+02	1.09E+02	1.20E+02	3.48E+25	1.00E+02
25	6.99E+07	-2.40E+00	-3.96E+23	1.039E-07	1039	-1.87E+00	-3.09E+23	6.99E+07	0.00E+00	-6.33E+00	-6.58E+00	-1.09E+24	0.00E+00

Note: if using r_1 = 0.28 (with unit of E+7 meter), then max Prob = 2.26. If using r_1 = 2.8E+6 (with unit of meter), then max Prob = 2.26E-7. The E-7 probability is due to that this radial wave function R(n*l*) is normalized for hydrogen atom's a_0 = 5.29E-11 m. So when using this probability, I need to scale it up to ~1E+7 times to make it around to 1. We can avoid this trouble by deducing out the radial wave function R(n*l*) that specifically normalized to Jupiter's r_1 = 2.8E+6 m. But I am only a citizen scientist of QM, it is too much work for me to do it.



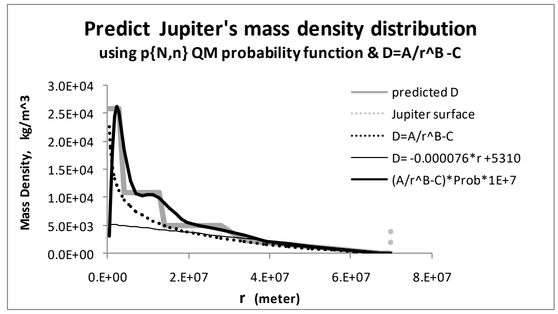


Figure 1a (left). Predict Jupiter's internal structure and the mass density r-distribution by using p{N,n} QM's radial probability function and a linear (D = a * r + b) scaling up.
Figure 1b (right). Predict Jupiter's internal structure and the mass density r-distribution by using p{N,n} QM's radial probability function and a curved (D = A / r^B - C) scaling up.

In Figure 1a, based on the D = -0.000076 * r +5310 linear curve, I manually adjust the factor W for curve of "D*Prob*1E+7*W", in a way that the "mass = (D*Prob*1E+7*W)*ΔV" value (in column 8 top line) equals to "mass = D*ΔV" value (in column 4 top line), the resulted W = 0.75. Then column 7 "D*Prob*1E+7*0.75" curve is plotted in Figure 1a. After that, I construct a stepped line (see the grey thick line in Figure 1a) according to the "D*Prob*1E+7*0.75" curve based on my eye judgment. According to this stepped line, I can predict that there are four (major) layers with three interfaces for Jupiter's internal structure: An obvious p{-1,1//5} core (or inner core, or Earth-sized core) with D ≈ 8500 kg/m^3, r ≈ 2.8E+6 m (same as p{-1,1} 's r = 6.99E+7 /25 = 2.8E+6 m), a p{-1,2//5} core with D ≈ 6000 kg/m^3, r ≈ 1.4E+7 m, (a little bit larger then p{-1,2//5} 's r =6.99E+7 /25*4 = 1.1E+7 m), and a unobvious p{-1,3//5} core (or a out core) with D ≈ 4000 kg/m^3, r ≈ 2.8E+7 m, (a little bit larger than p{-1,3//5} 's r =6.99E+7 /25*9 = 2.5E+7 m), and a most outer (atmosphere) layer with D decreasing from ≈2700 kg/m^3 at r ≈ 2.8E+7 m, to D ≈ 0 kg/m^3 at Jupiter surface.

This predication of internal structure closely matches the {N,n} QM analysis result for Jupiter in paper SunQM-1s3 section I-b. However, comparing to Jupiter's inner core mass density = 25000 kg/m^3 mentioned before, Figure 1a 's D = 8500 kg/m^3 is too low. It is obvious that this is caused by using the linear D = a * r + b for scaling up the probability curve. It is well known that due to the gravity compression, the radial distribution of mass density of a planet (or Sun) is curved line (not linear line). In paper SunQM-3, I used D = A / r^B to mimic this curved radial distribution of mass density for Sun. In current paper, after many tries, I find that I have to use D = A / r^B - C (instead of D = A / r^B) to mimic this curved radial distribution of mass density for Jupiter. The conditions for the right D curve are:

1) The total mass integration of "D*Porb*1E+7" from r = 0 to 6.99E+7 m has to equal to Jupiter's mass;
2) At Jupiter surface, "D*Porb*1E+7" ≈ 0 kg/m^3;
3) At Jupiter inner core (r < 2.8E+6 m), "D*Porb*1E+7" ≈ 25000 kg/m^3.

After fitting manually, one good result is D = 2.26E+5 / r^0.13 -21600 (see Table 1. columns 11-14). It satisfies all three conditions within reasonable error range.

Then I construct a stepped line (see column 15 of Table 1, and see the grey thick line in Figure 1b) according to the "(A / r^B - C)*Prob*1E+7" curve based on my eye judgment. According to this stepped line, I can predict that there are four (major) layers with three interfaces for Jupiter's internal structure: an obvious p{-1,1//5} core (or inner core, or Earth-sized core) with D ≈ 26000 kg/m^3, r ≈ 2.8E+6 m (same as p{-1,1//5} 's r = 6.99E+7 /25 = 2.8E+6 m); a p{-1,2//5} core with D ≈

11000 kg/m^3, r ≈ 1.26E+7 m, (a little bit larger then p{-1,2//5} 's r = 6.99E+7 /25*4 = 1.1E+7 m); and a unobvious p{-1,3//5} core (or a out core) with D ≈ 5000 kg/m^3, r ≈ 2.8E+7 m, (a little bit larger than p{-1,3//5} 's r = 6.99E+7 /25*9 = 2.5E+7 m); and a most outer (atmosphere) layer with D decreasing from ≈ 2400 kg/m^3 at r ≈ 2.8E+7 m, to D ≈ 0 kg/m^3 at Jupiter surface. This is the internal structure and the mass density distribution I predicted for Jupiter. I believe it is very close to the true value of Jupiter's.

Comparing results between Figure 1b and Figure 1a, we see that they have the same internal (core) structure, but different mass density distribution. In this model, it is obvious that the Jupiter's {N,n} QM radial probability density curve determines Jupiter's internal core structure, and D curve (which is used to scale-up the QM probability distribution) determines the mass density of each core.

II. Predict Neptune's internal structure and the mass density r-distribution by using {N,n} QM probability function

So far no experimental determined mass density radial distribution (like Earth's) has been found for Neptune. Now let's constitute the mass density linear equation D = a * r + b for Neptune. After manual fitting, one good result is D = -0.000258 * r +6400. It satisfy both conditions 1) ∫ D dV = mass of Neptune (see the integration equation below); 2) at surface r = 2.48E+7 m, D ≈ 0 kg/m^3. It is plotted in Figure 2a.

$$\int_0^{2.48\times10^7} 4\pi(-0.000258\,x+6400)\,x^2\,dx = 102\,303\,203\,473\,436\,279\,213\,916\,160$$

From my analysis in paper SunQM-1s3 section IV and section X, the current Neptune has a p{N,n//2} QM structure. It includes a (Earth sized) core (let us define it as) p{0,1//2}, a p{-1,1//2} sized inner core, and a p{1,1//2} sized atmosphere. All of them have ~ 100% mass occupancy. A p{N,n//2} QM can be naturally described as a p{N,n//4} QM. If I choose to use p{N,n//2} QM to predict Neptune's internal structure, then I have to use the same method as that for Saturn (see section V). It is relatively complicated. If I choose to use p{N,n//4} QM to predict Neptune's internal structure, then I am able to use the same method as that for Jupiter (see section I), and it is relatively easier. So I choose to use p{N,n//4} QM to predict Neptune's internal structure. In p{N,n//4} QM, let's choose Neptune's inner core as p{0,1//4}, and its r as r_1 (= 2.48E+7 / 16 = 1.55E+6 m). Then its Earth-sized core is at p{0,2//4}, and its surface is at p{0,4//4}. The rest calculations are almost same as that for Jupiter's, except that Jupiter has a p{N,n//5} QM structure, while Neptune has a p{N,n//4} QM structure.

Similar as that of Earth and Jupiter, Neptune's mass radial distribution can also be described by a simple integration formula of QM probability:

Mass (r, θ, φ) = ∭ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2) *W *D *sin(θ) * r^2 dr dθ dφ, [r=0, 2.48E+7 m; θ=0, π; φ=0, 2π]
or
1.02E+26 kg = 4π ∫ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2) *W * (-0.000258*r +6400) * r^2 dr, [r=0, 2.48E+7 m]

Again, the table of r^2 *|R(n,*l*)|^2 calculation is not shown here. The resulted curve is shown in column 5 "Prob(n=1..4)" of Table 2, and it is plotted in Figure 2a as "Prob(n=1..4)*4E+9". Table 2 shows the calculation to predict Neptune's internal structure and the mass density r-distribution using p{N,n} QM radial probability function, and Figure 2a and Figure 2b shows the result. In column 7 "D*Prob*1E+7*0.4" of Table 2, instead of integration, I manually scaled-up the Neptune's probability curve based on D = -0.000258 * r + 6400, with W = 1E+7 * 0.4, and this scaled-up curve is plotted in Figure 2a as "D*Prob*1E+7*0.4".

Again the linear D = a * r + b scaling up gives too low D (≈ 9500 kg/m^3) value at the inner core of Neptune. A curved scaling up with D = A / r^B - C has been tested (see columns 11-15 in Table 2). After many hours manual fitting, one possible curve is D = 169000 / r^0.014 - 133157. It gives:

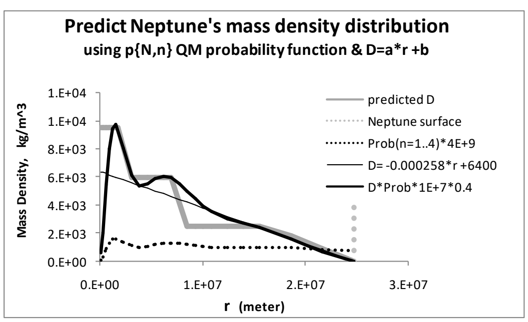
1) The total mass integration of "D*Porb*1E+7" from r = 0 to 2.48E+7 m equals to Neptune's mass;

2) At Neptune surface, “D*Porb*1E+7” ≈ 0 kg/m^3 (see Figure 2b);
3) At Neptune's p{0,1//4} inner core (r < 1.55E+6 m), “D*Porb*1E+7” ≈ 21000 kg/m^3;

Both Figure 2a and 2b predict that Neptune has a p{0,1//4} inner core with r ≈ 1.55 ~ 1.8E+6 m, an Earth-sized core p{0,2//4} at r ≈ 6.98E+6 m, and an unobvious (liquid) atmosphere core p{0,3//4} at r ≈ 1.55E+7 m. However, the fittings in Figure 2a and 2b only give the range of the mass density for each core (9500~21000 kg/m^3, 6000~8000 kg/m^3, 2500~2500 kg/m^3, respectively). My best guess is 17000 kg/m^3 for p{0,1//4} inner core, 7000 kg/m^3 for p{0,2//4} core, 2500 kg/m^ for p{0,3//4} (liquid) atmosphere layer, and 2500 → 0 kg/m^3 for the outer atmosphere layer.

Table 2. Predict Neptune's mass density distribution by using p{N,n} QM probability function.

										A= 169000			
										B= 0.014			
r_1=	1.55E+06		8.15E+25			0.4	8.16E+25			C= 133157		8.15E+25	
r/r_1=	r/r_1*r_1	D= -0.000258*r +6400	mass= D*ΔV	Prob(n=1..4)	Prob(n=1..4)*4E+9	D*Prob*1E+7*0.4	mass= (D*Prob*1E+7*0.4)*ΔV	r, p{-1,1}, p{0,1}	predicted D	D=A/r^B-C	(A/r^B-C)*Prob*1E+7	mass= ((A/r^B-C)*Prob*1E+7)*ΔV	predicted D
	m	kg/m^3				kg/m^3		m	kg/m^3	kg/m^3	kg/m^3	kg	kg/m^3
0.1	1.55E+05	6360	9.92E+19	2.488E-08	100	633	9.87E+18	1.55E+05	9500	9805	2439	3.80E+19	21500
0.2	3.10E+05	6320	6.90E+20	8.14E-08	326	2058	2.25E+20	3.10E+05	9500	8425	6858	7.49E+20	21500
0.4	6.20E+05	6240	5.45E+21	2.176E-07	870	5431	4.74E+21	6.20E+05	9500	7058	15358	1.34E+22	21500
0.6	9.30E+05	6160	1.46E+22	3.268E-07	1307	8052	1.91E+22	9.30E+05	9500	6264	20469	4.85E+22	21500
0.8	1.24E+06	6080	2.81E+22	3.877E-07	1551	9429	4.35E+22	1.24E+06	9500	5704	22112	1.02E+23	21500
1	1.55E+06	6000	4.57E+22	4.052E-07	1621	9725	7.40E+22	1.55E+06	9500	5270	21355	1.63E+23	21500
1.5	2.33E+06	5800	2.15E+23	3.471E-07	1388	8053	2.98E+23	1.80E+06	9500	4487	15575	5.77E+23	8000
2	3.10E+06	5600	4.04E+23	2.741E-07	1096	6140	4.43E+23	3.10E+06	6000	3934	10782	7.78E+23	8000
2.5	3.88E+06	5400	6.42E+23	2.493E-07	997	5385	6.40E+23	3.88E+06	6000	3506	8740	1.04E+24	8000
3	4.65E+06	5200	9.23E+23	2.647E-07	1059	5506	9.77E+23	4.65E+06	6000	3158	8359	1.48E+24	8000
3.5	5.43E+06	5000	1.24E+24	2.938E-07	1175	5877	1.46E+24	5.43E+06	6000	2864	8415	2.08E+24	8000
4	6.20E+06	4800	1.58E+24	3.162E-07	1265	6072	2.00E+24	6.20E+06	6000	2610	8252	2.72E+24	8000
4.5	6.98E+06	4600	1.95E+24	3.231E-07	1293	5946	2.52E+24	6.98E+06	6000	2386	7710	3.26E+24	8000
5	7.75E+06	4401	2.33E+24	3.151E-07	1261	5547	2.93E+24	8.50E+06	2500	2186	6889	3.64E+24	2500
5.5	8.53E+06	4201	2.71E+24	2.975E-07	1190	4998	3.23E+24	8.53E+06	2500	2006	5967	3.85E+24	2500
6	9.30E+06	4001	3.10E+24	2.766E-07	1106	4426	3.43E+24	9.30E+06	2500	1841	5093	3.94E+24	2500
6.5	1.01E+07	3801	3.48E+24	2.577E-07	1031	3918	3.58E+24	1.01E+07	2500	1690	4356	3.98E+24	2500
7	1.09E+07	3601	3.84E+24	2.441E-07	977	3516	3.75E+24	1.09E+07	2500	1550	3785	4.04E+24	2500
8	1.24E+07	3201	8.44E+24	2.356E-07	942	3016	7.95E+24	1.24E+07	2500	1299	3059	8.06E+24	2500
9	1.40E+07	2801	9.48E+24	2.439E-07	976	2732	9.25E+24	1.40E+07	2500	1077	2627	8.89E+24	2500
10	1.55E+07	2401	1.01E+25	2.543E-07	1017	2442	1.03E+25	1.55E+07	2500	879	2236	9.45E+24	2500
12	1.86E+07	1601	1.82E+25	2.472E-07	989	1583	1.80E+25	1.86E+07	1800	537	1329	1.51E+25	1800
14	2.17E+07	801	1.27E+25	2.101E-07	840	674	1.07E+25	2.17E+07	900	249	524	8.30E+24	900
16	2.48E+07	2	3.37E+22	1.793E-07	717	1	2.42E+22	2.48E+07	0	0	0	4.39E+21	0



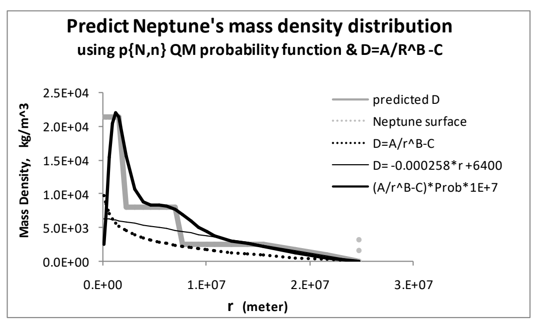


Figure 2a. Predict Neptune's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a linear (D=a*r +b) scaling up.
Figure 2b. Predict Neptune's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a curve (D=A/r^B -C) scaling up.

III. Predict Uranus' internal structure and the mass density r-distribution by using {N,n} QM probability function

So far no experimental determined mass density radial distribution (like Earth's) has been found for Uranus. The calculation for predicting Uranus internal structure and mass density is exactly the same as that for Neptune. First, let us constitute the mass density linear equation D= a * r + b for Uranus. After manual fitting, one possible result is D = -0.000193 * x + 4941. It satisfies both two conditions: 1) ∫ D dV = mass of Uranus (see the integration equation below); 2) at surface r = 2.56E+7 m, D ≈ 0 kg/m^3. It is plotted in Figure 3a.

$$\int_0^{2.56\times10^7} 4\pi(-0.000193\,x + 4941)\,x^2\,dx = 86\,819\,264\,480\,033\,892\,070\,326\,272$$

In p{N,n//4} QM, let's choose Uranus's inner core as p{0,1//4}, and its r as r_1 (= 2.56E+7 /16 = 1.6E+6 m). Then its Earth-sized core is at p{0,2//4}, and Uranus surface is at p{0,4//4}. Similar as that of Neptune, Uranus' mass radial distribution can also be described by a simple integration formula of QM probability:

Mass (r, θ, φ) = ∭ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2) *W *D *sin(θ) * r^2 dr dθ dφ, [r=0, 2.56E+7 m; θ=0, π; φ=0, 2π]
or
8.68E+25 kg = 4π ∫ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2) *W * (-0.000193*r +4941) * r^2 dr, [r=0, 2.56E+7 m]

Again, the table of r^2 *|R(n,l)|^2 calculation is not shown here. The resulted curve is shown in column 5 "Prob(n=1..4)" of Table 3, and it is plotted in Figure 3a as "Prob(n=1..4)*1E+10". Table 3 shows the calculation to predict Uranus' internal structure and the mass density r-distribution using p{N,n} QM radial probability function, and Figure 3a and Figure 3b shows the result. In column 7 "D*Prob*1E+7*0.412" of Table 3, instead of integration, I manually scaled-up the Uranus' probability curve based on D = -0.000193 * r + 4941, with W=1E+7*0.412, and this scaled-up curve is plotted in Figure 3a as "D*Prob*1E+7*0.412".

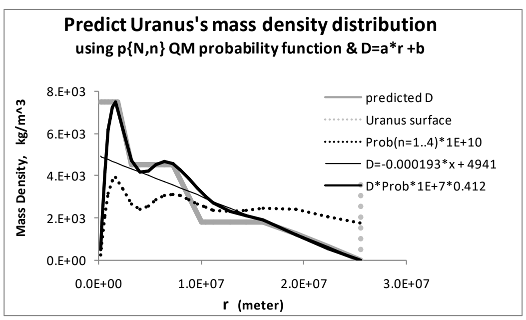
Again the linear D = a * r + b scaling up gives too low D (≈ 7500 kg/m^3) at the inner core of Uranus. A curved scaling up with D = A / r^B - C has been tested (see columns 11-15 in Table 3). After manual fitting, one possible curve is D = 143000 / r^0.013 -114559. It gives

1) The total mass integration of "D*Porb*1E+7" from r = 0 to 2.56E+7 m equals to Uranus' mass;
2) At Uranus surface, "D*Porb*1E+7" ≈ 0 kg/m^3;
3) At Uranus's p{0,1//4} inner core (r < 1.6E+6 m), "D*Porb*1E+7" ≈ 17000 kg/m^3;

Both Figure 3a and 3b predict that Uranus has a p{0,1//4} inner core with r ≈ 1.6 ~ 1.8E+6 m, an Earth-sized core p{0,2//4} at r ≈ 7.2E+6 m, and an unobvious (liquid) atmosphere core p{0,3//4} at r ≈ 1.6E+7 m. However, fittings in Figure 3a and 3b only give the range of the mass density for each core (7500~17000 kg/m^3, 4500~6500 kg/m^3, 1800~1800 kg/m^3, respectively). My best guess is 16500 kg/m^3 for p{0,1//4} inner core, 6000 kg/m^3 for p{0,2//4} core, 1800 kg/m^ for (liquid) atmosphere layer up to p{0,3//4}, and 1800 → 0 kg/m^3 for the outer atmosphere layer up to p{-1,4//4}.

Table 3. Predict Uranus' mass density r-distribution by using p{N,n} QM probability function.

										A=	143000			
										B=	0.013			
r_1=	1.60E+06		6.91E+25			0.412	6.91E+25			C=	114559		6.93E+25	
r/r_1=	r/r_1*r_1	D=-0.000193*x + 4941	mass= D*ΔV	Prob(n=1..4)	Prob(n=1..4)*1E+10	D*Prob*1E+7*0.412	mass=(D*Prob*1E+7*0.412)*ΔV	r, p{-1,1}, p{0,1}	predicted D		D=A/r^B-C	(A/r^B-C)*Prob*1E+7	mass=((A/r^B-C)*Prob*1E+7)*ΔV	predicted D
	m	kg/m^3	kg			kg/m^3	kg	m	kg/m^3		kg/m^3	kg/m^3	kg	kg/m^3
0.1	1.60E+05	4910	8.42E+19	2.41E-08	241	488	8.36E+18	1.60E+05	7500		7813	1883	3.23E+19	17000
0.2	3.20E+05	4879	5.86E+20	7.886E-08	789	1585	1.90E+20	3.20E+05	7500		6715	5296	6.36E+20	17000
0.4	6.40E+05	4817	4.63E+21	2.108E-07	2108	4184	4.02E+21	6.40E+05	7500		5628	11863	1.14E+22	17000
0.6	9.60E+05	4756	1.24E+22	3.166E-07	3166	6203	1.62E+22	9.60E+05	7500		4996	15814	4.12E+22	17000
0.8	1.28E+06	4694	2.38E+22	3.756E-07	3756	7263	3.69E+22	1.28E+06	7500		4549	17086	8.68E+22	17000
1	1.60E+06	4632	3.88E+22	3.925E-07	3925	7491	6.27E+22	1.60E+06	7500		4204	16503	1.38E+23	17000
1.5	2.40E+06	4478	1.82E+23	3.363E-07	3363	6204	2.53E+23	1.80E+06	7500		3580	12039	4.91E+23	6500
2	3.20E+06	4323	3.43E+23	2.655E-07	2655	4730	3.75E+23	3.20E+06	4500		3139	8335	6.61E+23	6500
2.5	4.00E+06	4169	5.45E+23	2.415E-07	2415	4148	5.43E+23	4.00E+06	4500		2798	6757	8.84E+23	6500
3	4.80E+06	4015	7.84E+23	2.564E-07	2564	4242	8.28E+23	4.80E+06	4500		2520	6463	1.26E+24	6500
3.5	5.60E+06	3860	1.05E+24	2.846E-07	2846	4527	1.23E+24	5.60E+06	4500		2286	6507	1.77E+24	6500
4	6.40E+06	3706	1.34E+24	3.063E-07	3063	4677	1.70E+24	6.40E+06	4500		2083	6381	2.31E+24	6500
4.5	7.20E+06	3551	1.65E+24	3.13E-07	3130	4580	2.13E+24	7.20E+06	4500		1905	5963	2.77E+24	6500
5	8.00E+06	3397	1.97E+24	3.053E-07	3053	4273	2.48E+24	1.00E+07	1800		1745	5328	3.10E+24	1800
5.5	8.80E+06	3243	2.30E+24	2.882E-07	2882	3850	2.73E+24	1.00E+07	1800		1601	4615	3.28E+24	1800
6	9.60E+06	3088	2.63E+24	2.68E-07	2680	3409	2.90E+24	1.00E+07	1800		1470	3939	3.35E+24	1800
6.5	1.04E+07	2934	2.95E+24	2.497E-07	2497	3018	3.04E+24	1.04E+07	1800		1349	3369	3.39E+24	1800
7	1.12E+07	2779	3.26E+24	2.365E-07	2365	2708	3.18E+24	1.12E+07	1800		1238	2927	3.43E+24	1800
8	1.28E+07	2471	7.16E+24	2.282E-07	2282	2323	6.74E+24	1.28E+07	1800		1037	2366	6.86E+24	1800
9	1.44E+07	2162	8.05E+24	2.363E-07	2363	2104	7.83E+24	1.44E+07	1800		860	2032	7.57E+24	1800
10	1.60E+07	1853	8.62E+24	2.464E-07	2464	1881	8.74E+24	1.60E+07	1800		702	1729	8.04E+24	1800
12	1.92E+07	1235	1.54E+25	2.394E-07	2394	1219	1.52E+25	1.92E+07	1235		429	1028	1.28E+25	1235
14	2.24E+07	618	1.08E+25	2.035E-07	2035	518	9.03E+24	2.24E+07	618		199	405	7.06E+24	618
16	2.56E+07	0	4.64E+21	1.737E-07	1737	0	3.32E+21	2.56E+07	0		0	0	-3.15E+21	0



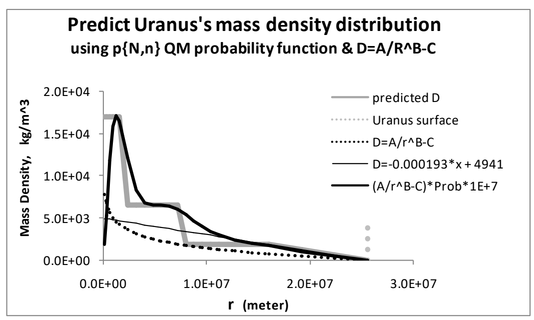


Figure 3a. Predict Uranus's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a linear (D = a * r + b) scaling up.

Figure 3b. Predict Uranus's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a curve (D = A / r^B - C) scaling up.

IV. Predict undiscovered planets {3,n=2..5}'s internal structure and the mass density r-distribution by using {N,n} QM probability function

In Table 2 of paper SunQM-3s6, we see that the undiscovered {3,2} planet (if the belt did have accreted into a planet) has mass of 7.12E+25 kg, and it is almost the same as that of Uranus (7.52E+25 kg). So I assume that it should have almost the same size and QM structure as that of Uranus. Therefore, based on the calculation of SunQM-3s6 Table 2, I predict that the undiscovered {3,2} planet has a p{0,1//4} inner core with r ≈ 1.36E+6 m, an Earth-sized core p{0,2//4} at r ≈

5.45E+6 m, an unobvious (liquid) atmosphere core p{0,3//4} up to r = 2.18E+7 * (3/4)^2 ≈1.23E+7 m, and a surface at p{-1,4//4} at r = 2.18E+7 m. Based on section III's result, my best guess is D ≈ 16000 kg/m^3 for p{0,1//4} inner core, 5500 kg/m^3 for p{0,2//4} core, 1750 kg/m^ for (liquid) atmosphere layer up to p{0,3//4}, and 1750 → 0 kg/m^3 for the outer atmosphere layer up to p{-1,4//4}.

For the rest three undiscovered planets {3,n=3..5} (if they did have accreted into planets), since their masses (from 3.99E+25 kg to 1.98E+25 kg) are close to {3,2}'s mass (7.12E+25 kg), I believe they all have the same {N,n//2} or {N,n//4} QM structure as that of Uranus'. The estimated sizes of atmosphere at p{-1,4//4}, the Earth-sized core at p{-1,2//4}, and the inner core at p{-1,1//4} for all {3,n=3..5} planets have been listed in Table 2 of paper SunQM-3s6. The mass density radial distribution should also be similar as that of Uranus too, only need to be scaled down a little bit. Therefore, I predict that their inner cores p{-1,1//4} 's mass density are between 15000 kg/m^3 to 12000 kg/m^3.

V. Predict Saturn's internal structure and the mass density radial distribution by using {N,n} QM probability function

So far, no experimental determined mass density radial distribution (like Earth's) for Saturn has been found. First, let us constitute the mass density linear equation D = a * r + b for Saturn. After manual fitting, one good result is D = -0.0000469 * r + 2730. It satisfies both conditions: 1) ∫ D dV = mass of Saturn (see the integration equation below); 2) at surface r = 5.82E+7 m, D ≈ 0 kg/m^3. It is plotted in Figure 4a.

$$\int_0^{5.82\times 10^7} 4\pi(-0.0000469\,x + 2730)\,x^2\,dx = 563\,846\,146\,050\,677\,159\,885\,799\,424$$

From my analysis in paper SunQM-1s3 section X, the current Saturn has an Earth-sized core (let us define it as) p{0,1//2}, and a inner core p{-1,1//2}, both with 100% mass occupancy, and both have p{N,n//2} QM structure. However, it has a p{0,3//2} sized atmosphere. The atmosphere from p{0,1//2} to p{0,2//2} has 100% mass occupancy. But the atmosphere from p{0,2//2} to p{0,3//2} has probably only ~50% mass occupancy. I believe that if Saturn has had 100% mass occupancy in atmosphere shell between p{0,2//2} and p{0,3//2}, it would have generated enough G-forced compression to transform its core from {N,1//2} QM to {N,1//3} QM, so that the whole Saturn would become a {N,n//3} QM structure. However, current Saturn's ~50% mass occupancy in n=3 shell is not enough to make this transformation to happen, so it is stuck at this hybridized (core is base-2, atmosphere is base-3) QM state. For this reason, it is better to choose Saturn's Earth-sized core (not the Saturn's surface) as the p{0,1}.

Based on the previous {N,n} QM structure model calculation of Earth and Jupiter, for Saturn I need to first analyze p{0,1//2} Earth-sized core, p{0,2//2} sized inner atmosphere, and p{0,3//2} sized outer atmosphere. This is equivalent to a p{N,n//3} QM, with p{0,1//3} Earth-sized core, p{0,2//3} sized inner atmosphere, and p{0,3//3} sized outer atmosphere. Since we choose the Earth-sized core as p{0,1//3}, and also choose it as r_1, so r_1 = 5.82E+7 /3^2 = 6.47E+6 m. Hence Saturn can be described by its QM radial probability density function of r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2), as plotted in Figure 4a. Similar as that of Earth and Jupiter, Saturn's mass radial distribution can also be (approximately, ignoring the p{-1,1] center region due to its small volume) described by a simple integration formula of QM probability:

Mass (r, θ, φ) = ∭ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2) *W *D *sin(θ) * r^2 dr dθ dφ, [r=0, 5.82E+7 m; θ=0, π; φ=0, 2π]

or

5.68E+26 kg = 4π ∫ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2) *W * (-0.0000469*r +2730) * r^2 dr, [r=0, 5.82E+7 m]

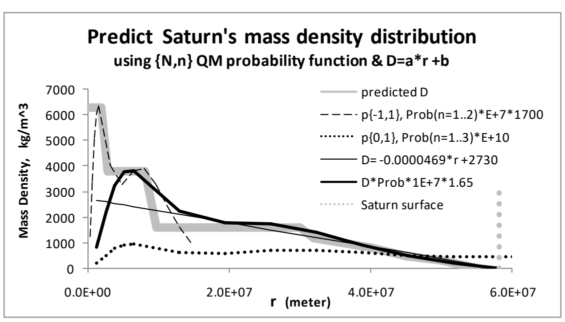
Table 4 shows the calculation to predict Saturn's internal structure and the mass density r-distribution using p{N,n} QM radial probability function, and Figure 4a and Figure 4b shows the result. The calculation of r^2 *|R(n,l)|^2 is shown in columns 1 through 8 in Table 4. The resulted curve is shown in column 8 "Prob(n=1..3)" of Table 4, and it is plotted in Figure 4a as "Prob(n=1..3)*E+10". In Table 4, instead of integration, I manually scaled-up the Saturn's probability curve based on D = -0.0000469 * r + 2730, with W = 1E+7*1.65 (see column 13 "D*Prob*1E+7*1.65" in Table 4), and this scaled-up curve is plotted in Figure 4a as "D*Prob*1E+7*1.65".

As mentioned before, Saturn has a inner core p{-1,1//2} inside the Earth-sized core p{0,1//2}. The probability density of this p[-1,1//2} core is not shown in the r^2 *(|R(1,0)|^2 + |R(2,l)|^2 + |R(3,l)|^2) curve with r_1 = 6.47E+6 m. To know the mass density of the inner core p{-1,1//2} inside the Earth-sized core p{0,1//2}, we need to use the probability density distribution of r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2) with r_1 = 6.47E+6 /4 m, or r_1 = 1.62E+6 m (see dashed line in Figure 4a). This Prob(n=1..2) curve is (manually) scaled up to make its n=2 peak (at ~ 6.47E+6 m) to match Prob(n=1..3) curve n=1 peak (also at ~ 6.47E+6 m, see columns 15-19 in Table 4, and also see Figure 4a). So now this scaled up Prob(n=1..2) curve also reflects the (close to) true mass density of p{-1,1//2} core and p{0,1//2} core. Then I construct a stepped line (see column 20-21 of Table 4, and see the grey thick line in Figure 4a) according to the "D*Prob*1E+7*1.65" curve based on my eye judgment. According to this stepped line, I can predict that there are four (major) layers with three interfaces for Saturn's internal structure:

The p{-1,1} inner core (0 m < r < ~ 2.0E+6 m) with D ≈ 6300 kg/m^3.
The (Earth-sized) p{0,1} core (~2.0E+6 m < r < ~8.08E+6 m) with D ≈ 3800 kg/m^3.
The inner (liquid) atmosphere layer (~8.08E+6 m < r < ~3E+7 m) with D ≈ 1600 kg/m^3.
The outer atmosphere layer (~3E+7 m < r < ~5.82E+7 m) with D ≈ 1600 → 0 kg/m^3.

Table 4. Predict Saturn's internal structure and the mass density radial distribution by using p{N,n} QM probability function.

																					A= 1.63E+05					
															r1=						B= 0.05		p{0,1}		p{-1,1}	
Saturn r	6.47E+06 meters							p{0,1}			4.46E+26	1.65	4.43E+26	p{-1,1}	1.62E+06 meters			1700		linear	C= 66672.0		4.42E+26	0.475	4.58E+24	curved
r/r_1=	$r^2 * \|R_{1,0}\|^2$	$r^2 * \|R_{2,0}\|^2$	$r^2 * \|R_{2,1}\|^2$	$r^2 * \|R_{3,0}\|^2$	$r^2 * \|R_{3,1}\|^2$	$r^2 * \|R_{3,2}\|^2$	Prob(n=1..3)	r/r_1*6.47E+6	p{0,1}, Prob(n=1..3)*E+10	D= -0.0000469*r +2730	mass= D*ΔV	D*Prob*1E+7*1.65	mass= (D*Prob*1E+7*1.65)*ΔV	r/r_1*1.62E+6	$r^2 * \|R_{1,0}\|^2$	$r^2 * \|R_{2,0}\|^2$	$r^2 * \|R_{2,1}\|^2$	p{-1,1}, Prob(n=1..2)*E+7*1700	r, p{-1,1}, p{0,1}	predicted D	D=A/r^B-C	(A/r^B-C)*Prob*1E+7	((A/r^B-C)*Prob(1..3)*1E+7)*ΔV	(A/r^B-C)*Prob(1..2)*1E+7*0.475	mass= ((A/r^B-C)*Prob*1E+7)*ΔV	predicted D
								m		kg/m^3	kg	kg/m^3	kg	m				kg/m^3	m	kg/m^3		kg/m^3	kg	kg/m^3	kg	kg/m^3
0.2	1.66E-08	2.05E-09	8.44E-12	6.07E-10	2.96E-12	7.04E-16	1.93E-08	1.29E+06	1.93E+02	2.67E+03	2.42E+22	848	7.68E+21	3.23E+05	6.63E-08	8.20E-09	3.38E-11	1.27E+03	3.23E+05	6.30E+03	13978	2.69E+03	2.44E+22	7.00E+03	9.91E+20	2.30E+04
0.4	4.45E-08	5.31E-09	1.11E-10	1.56E-09	3.86E-11	3.94E-14	5.15E-08	2.59E+06	5.15E+02	2.61E+03	1.65E+23	2216	1.41E+23	6.47E+05	1.78E-07	2.12E-08	4.42E-10	3.39E+03	6.47E+05	6.30E+03	11230	5.78E+03	3.67E+23	1.59E+04	1.58E+22	2.30E+04
0.6	6.71E-08	7.49E-09	4.58E-10	2.17E-09	1.59E-10	3.93E-13	7.73E-08	3.88E+06	7.73E+02	2.55E+03	4.39E+23	3252	5.60E+23	9.70E+05	2.68E-07	2.99E-08	1.83E-09	5.10E+03	9.70E+05	6.30E+03	9667	7.48E+03	1.29E+24	2.16E+04	5.81E+22	2.30E+04
0.8	7.99E-08	8.00E-09	1.19E-09	2.27E-09	4.08E-10	1.93E-12	9.18E-08	5.17E+06	9.18E+02	2.49E+03	8.34E+23	3768	1.26E+24	1.29E+06	3.20E-07	3.20E-08	4.74E-09	6.06E+03	1.29E+06	6.30E+03	8577	7.87E+03	2.64E+24	2.37E+04	1.24E+23	2.30E+04
1	8.37E-08	7.11E-09	2.37E-09	1.95E-09	8.07E-10	6.45E-12	9.60E-08	6.47E+06	9.60E+02	2.43E+03	1.34E+24	3842	2.12E+24	1.62E+06	3.35E-07	2.84E-08	9.48E-09	6.34E+03	2.00E+06	6.30E+03	7742	7.43E+03	4.11E+24	2.32E+04	2.00E+23	2.30E+04
2	4.53E-08	0.00E+00	1.40E-08	3.31E-11	4.24E-09	2.12E-10	6.38E-08	1.29E+07	6.38E+02	2.12E+03	1.68E+25	2234	1.77E+25	3.23E+06	1.81E-07	0.00E+00	5.58E-08	4.03E+03	2.80E+06	3.80E+03	5207	3.32E+03	2.63E+25	1.17E+04	1.45E+24	7.80E+03
3	1.38E-08	8.66E-09	2.60E-08	3.10E-09	6.20E-09	1.24E-09	5.90E-08	1.94E+07	5.90E+02	1.82E+03	3.92E+25	1771	3.81E+25	4.85E+06	5.52E-08	3.46E-08	1.04E-07	3.29E+03	4.85E+06	3.80E+03	3764	2.22E+03	4.78E+25	8.12E+03	2.73E+24	7.80E+03
4	3.32E-09	2.27E-08	3.02E-08	5.90E-09	4.47E-09	3.58E-09	7.01E-08	2.59E+07	7.01E+02	1.52E+03	6.36E+25	1756	7.36E+25	6.47E+06	1.33E-08	9.06E-08	1.21E-07	3.82E+03	6.47E+06	3.80E+03	2759	1.93E+03	8.11E+25	8.27E+03		7.80E+03
5	7.02E-10	2.93E-08	2.71E-08	4.74E-09	1.40E-09	7.01E-09	7.03E-08	3.23E+07	7.03E+02	1.21E+03	8.39E+25	1407	9.72E+25	8.08E+06	2.81E-09	1.17E-07	1.09E-07	3.89E+03	8.08E+06	3.80E+03	1988	1.40E+03	9.66E+25	7.51E+03		7.80E+03
6	1.37E-10	2.76E-08	2.07E-08	1.68E-09	0.00E+00	1.07E-08	6.09E-08	3.88E+07	6.09E+02	9.10E+02	9.38E+25	914	9.42E+25	9.70E+06	5.47E-10	1.10E-07	8.28E-08	3.29E+03	9.70E+06	1.60E+03	1365	8.31E+02	8.56E+25	5.75E+03		1.60E+03
7	2.52E-11	2.16E-08	1.41E-08	1.45E-11	1.42E-09	1.39E-08	5.11E-08	4.53E+07	5.11E+02	6.07E+02	8.73E+25	511	7.36E+25	1.13E+07	1.01E-10	8.64E-08	5.64E-08	2.43E+03	1.13E+07	1.60E+03	843	4.30E+02	6.19E+25	3.86E+03		1.60E+03
8	4.46E-12	1.49E-08	8.85E-09	1.17E-09	4.97E-09	1.59E-08	4.59E-08	5.17E+07	4.59E+02	3.04E+02	5.81E+25	230	4.40E+25	1.29E+07	1.78E-11	5.98E-08	3.54E-08	1.62E+03	1.29E+07	1.60E+03	393	1.80E+02	3.45E+25	2.35E+03		1.60E+03
9	7.63E-13	9.47E-09	5.22E-09	4.60E-09	9.20E-09	1.66E-08	4.50E-08	5.82E+07	4.50E+02	4.20E-01	1.03E+23	0	7.67E+22	1.46E+07	3.05E-12	3.79E-08	2.09E-08	9.99E+02	1.46E+07	1.60E+03	0	-1.40E-01	-3.45E+22	1.34E+03		1.60E+03
10	1.27E-13	5.62E-09	2.93E-09	8.83E-09	1.28E-08	1.60E-08	4.62E-08	6.47E+07	4.62E+02										2.00E+07	1.60E+03						1.60E+03
12	3.36E-15	1.71E-09	8.21E-10	1.49E-08	1.57E-08	1.26E-08	4.57E-08	7.76E+07	4.57E+02										2.50E+07	1.60E+03						1.60E+03
14	8.38E-17	4.54E-10	2.06E-10	1.52E-08	1.37E-08	8.37E-09	3.79E-08	9.05E+07	3.79E+02										3.00E+07	1.60E+03						1.60E+03
16	2.01E-18	1.09E-10	4.75E-11	1.18E-08	9.60E-09	4.92E-09	2.65E-08	1.03E+08	2.65E+02										3.20E+07	1.20E+03						1.20E+03
18	4.65E-20	2.44E-11	1.03E-11	7.71E-09	5.84E-09	2.63E-09	1.62E-08	1.16E+08	1.62E+02										4.00E+07	8.00E+02						8.00E+02
20	1.05E-21	5.16E-12	2.12E-12	4.44E-09	3.19E-09	1.30E-09	8.94E-09	1.29E+08	8.94E+01										4.50E+07	5.00E+02						5.00E+02
22	2.33E-23	1.04E-12	4.21E-13	2.33E-09	1.61E-09	6.08E-10	4.55E-09	1.42E+08	4.55E+01										5.00E+07	3.00E+02						3.00E+02
24	5.08E-25	2.03E-13	8.07E-14	1.14E-09	7.60E-10	2.70E-10	2.17E-09	1.55E+08	2.17E+01										5.50E+07	3.00E+01						3.00E+01
27	1.59E-27	1.66E-14	6.44E-15	3.48E-10	2.24E-10	7.42E-11	6.47E-10	1.75E+08	6.47E+00										5.82E+07	0.00E+00						0.00E+00



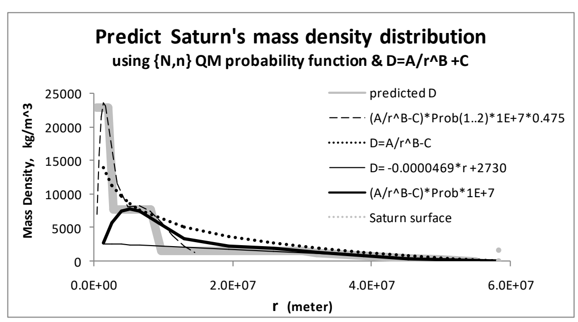


Figure 4a. Predict Saturn's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a linear (D = a * r + b) scaling up.
Figure 4b. Predict Saturn's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a curve (D = A / r^B - C) scaling up.

Due to by using the linear D = a * r + b for scaling up, D ≈ 6300 kg/m^3 is too low for Saturn's inner core. According to Jupiter, Earth, and Sun's inner core mass density information (obtained from the wiki), after many fittings, I give the best estimated mass density for other planets' inner core (calculation is not shown here because the method is too rough). Saturn's inner core is estimated to have D ≈ 21000 kg/m^3. Then, same as that for other gas/ice planets, D = A / r^B - C model is used with following four conditions:

1) The total mass integration of "D*Porb*1E+7" from r = 0 to 5.82E+7 m has to equal to Saturn's mass;
2) At Saturn surface, "D*Porb*1E+7" ≈ 0 kg/m^3.
3) At surface r = 6.47E+6 m, p{-1,1//2} 's probability should match to p{0,1//3} 's probability.
4) The inner core has mass density not too different from 21000 kg/m^3.

After over 10 hours of manual fitting, the closest result is D = 1.63E+5 / r^0.05 - 66672 (see Table 4. columns 22-24, and Figure 4b). It satisfies conditions from 1) to 3), but has "D*Porb*1E+7" ≈ 23000 kg/m^3 at Saturn's p{-1,1} inner core (r < 2.0E+6 m). Both Figure 4a and 4b predict that Saturn has a p{-1,1//2} inner core with r ≈ 2.0E+6 m, an Earth-sized core p{0,1//2} at r ≈ 8.08E+6 m, and a (liquid) atmosphere core p{0,2//2} up to r ≈ 3E+7 m. However, fittings in Figure 4a and 4b only give the range of the mass density for each core (6300~23000 kg/m^3, 3800~7800 kg/m^3, 1600~1600 kg/m^3, respectively). My best guess is 23000 kg/m^3 for inner core p{-1,1//2}, 7800 kg/m^3 for p{0,1//2} core, 1600 kg/m^ for p{0,2//2} (liquid) atmosphere layer, and 1200 → 0 kg/m^3 for the outer atmosphere layer up to p{0,3//2}.

Conclusions

Based on the hypothesis of "internal structure of all planets following their {N,n} QM probability density curve", the mass density for all gas/ice planets' internal structures are summarized in table below:

	Jupiter D (kg/m^3)	Saturn D (kg/m^3)	Uranus D (kg/m^3)	Neptune D (kg/m^3)	{3,2} planet D (kg/m^3)
inner core	26000	23000	~16500	~17000	~16000
	11000	7800	~6000	~7000	~5500
	5000	1600	1800	2500	~1750
	2400→0	1600→0	1800→0	2500→0	~1750→0

References

[1] Yi Cao, SunQM-1: Quantum mechanics of the Solar system in a {N,n//6} QM structure. http://vixra.org/pdf/1805.0102v2.pdf （original submitted on 2018-05-03）

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[11] Yi Cao, SunQM-3s8: Using {N,n} QM to study Sun's internal structure, convective zone formation, planetary differentiation and temperature r-distribution. http://vixra.org/pdf/1808.0637v1.pdf (submitted on 2018-08-29)

[12] A series of my papers that to be published (together with current paper):
SunQM-3s7: Predict mass density r-distribution for gas/ice planets based on {N,n} QM probability distribution.
SunQM-3s9: Using {N,n} QM to explain the sunspot drift, the continental drift, and Sun's and Earth's magnetic dynamo.
SunQM-3s10: Updates and Q/A for SunQM series papers.
SunQM-3s4: Using {N,n} QM structure and multiplier n' to analyze Saturn's ring structure.
SunQM-3s5: Using {N,n} QM structure and n*l*0 effect to analyze the Bipolar outflow.
SunQM-5: A new version of QM based on interior {N,n}, multiplier n', |R(n,l)|^2 |Y(l,m)|^2 guided mass occupancy, and RF, and its application from string to universe.
SunQM-5s1: White dwarf, neutron star, and black hole re-analyzed by using the internal {N,n} QM.

[13] Major QM books, data sources, software I used for this study are:
Douglas C. Giancoli, Physics for Scientists & Engineers with Modern Physics, 4th ed. 2009.
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Stephen T. Thornton & Andrew Rex, Modern Physics for scientists and engineers, 3rd ed. 2006.
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Free software: R
Microsoft Excel.
Public TV's space science related programs: PBS-NOVA, BBC-documentary, National Geographic-documentary, etc.
Journal: Scientific American.