Energy Of Light And Radius of Electron

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Based on conservation law of energy, Poynting vector describes the power per unit area in electromagnetic wave. The time-averaged power per unit area is independent of the wavelength and the frequency of the wave. One example is FM radio signal. In photoelectric effect, the incident light wave transfers energy to the electron. Light wave of higher frequency takes longer time to transfer more energy to the electron. The total energy absorbed by the electron is proportional to the area facing the incident light. From this area, the radius of the electron can be calculated.

I. INTRODUCTION

Electromagnetic waves carry energy as they travel through empty space. In 1884, John Henry Poynting derived an expression for the energy carried by electromagnetic wave[1], Poynting vector. It describes the instantaneous energy flux density and the direction of energy flux.

During the collision between the electron and the light wave, part of the energy of the light wave is absorbed by the electron. As manifested in photoelectric effect, more energy is absorbed by the electron if the frequency of the incident wave is higher.

This relationship between energy and frequency provides an expression for the incident area which is related to the radius of the electron.

II. PROOF

Consider electromagnetic waves in vaccum.

A. Poynting Vector

Let $\vec{E}$ be the electric field in the wave. Let $\vec{B}$ be the magnetic field in the wave. Let $\vec{S}$ be the Poynting vector for the wave.

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \tag{1}$$

μ_0 is the magnetic constant.

$\vec{S}$ describes the energy per unit time per unit area and the direction of energy flux in electromagnetic wave. It is the instantaneous power density vector.

B. Power Density

The homogeneous electromagnetic wave can be represented by plane wave solution from Maxwell's equations[2].

$$\vec{E} = \vec{E}(\vec{r}, t) \tag{2}$$

$$= \vec{E_0} * cos(\vec{k} * \vec{r} - \omega * t + \phi) \tag{3}$$

$$\vec{B} = \vec{B}(\vec{r}, t) \tag{4}$$

$$= \frac{1}{c} * (\hat{k} \times \vec{E_0}) cos(\vec{k} * \vec{r} - \omega * t + \phi) \tag{5}$$

E_0 is the amplitude of electric field.

From equations (1,3,5),

$$\vec{S}(\vec{r}, t) = \sqrt{\frac{\epsilon_0}{\mu_0}} * E_0^2 * cos(\vec{k} * \vec{r} - \omega * t + \phi)^2 * \hat{k} \tag{6}$$

ϵ_0 is the electric constant.

$\vec{S}$ is the instantaneous power density vector. The time-averaged power density vector over a period of $2\pi/\omega$ is $< \vec{S} >$.

$$< \vec{S} > = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \vec{S} * dt \tag{7}$$

$$= \sqrt{\frac{\epsilon_0}{\mu_0}} * \frac{E_0^2}{2} * \hat{k} \tag{8}$$

$< S >$ is the time-averaged power density.

$$< S > = < \vec{S} > * \hat{k} \tag{9}$$

$< S >$ is a function of amplitude, E_0, and is independent of wavelength and frequency of the wave.

The total energy transmitted by an electromagnetic wave over a period of time upon an area is E_T.

$$E_T = < S > * T * A \tag{10}$$

A is the area in the direction of incident wave.

T is the duration of the interaction between the electron and the light wave.

E_T depends on the amplitude of the electric field, the duration and the normal area that absorbs the energy. It is independent of the wavelength and the frequency of the electromagnetic wave.

C. FM Radio

FM radio is a good example that the electromagnetic waves of various frequencies carry identical time-averaged power density.

The transmitter modulates the information onto a carrier signal, amplifies the signal and broadcasts it. The modulation produces electromagnetic waves of various frequencies but of identical amplitude[3].

$V_m(t)$ is the signal of information.

$V_c(t)$ is the signal of carrier.

$$V_c(t) = V_{co} sin(2 * \pi * f_c * t + f) \tag{11}$$

The FM signal is modulated as $V_{FM}(t)$

$$V_{FM}(t) = V_{co} sin(2\pi * [f_c + \frac{Df}{V_{m0}} * V_m(t)] * t + f) \tag{12}$$

The average power output of the transmitter is proportional to V_{co}^2. It is independent of the modulated frequency. Therefore, all electromagnetic wave pulses in the same FM carrier wave carry an identical time-averaged power. A longer carrier wave pulse carries greater energy.

III. THEORY

A. Photoelectric Effect

The photoelectric effect was discovered by Heinrich Hertz in 1897[4]. In 1905, Einstein proposed an explanation of the photoelectric effect[5] based on Max Planck's radiation law.

$$K = h * f - \phi \tag{13}$$

K is the maximum kinectic energy for an ejected electron.

ϕ is the binding potential, the minimum energy required to remove an electron from the atom.

f is the frequency of the incident light wave.

h is the Planck constant.

The incident wave interacts with the electrons inside the atom. Part of the energy in the light wave is transferred to the electron. This partially absorbed energy exceeds the binding potential of the electron and becomes the kinetic energy of the electron when the electron is ejected out of the atom.

From equation (10,13),

$$s * E_T = K + \phi = h * f \tag{14}$$

s is the absorption coefficient.

$$0 < s < 1 \tag{15}$$

From equation (10,14),

$$\frac{f}{T} = \frac{s * < S > * A}{h} \tag{16}$$

f is proportional to T, the duration of interaction between the electron and the light wave. An incident light wave of higher frequency will transfer energy to the electron for a longer duration. The longer duration results in more energy absorbed by the electron.

B. Electron Radius

The experimental data from ACME EDM[6] indicates that an electron can be a sphere of radius r. Its maximum cross section can represent the incident area that absorbs the energy of the incident light wave.

$$A = \pi * r^2 \tag{17}$$

From equation (16,17),

$$r = \sqrt{\frac{h * f}{s * < S > * T * \pi}} \tag{18}$$

IV. CONCLUSION

The energy of electromagnetic wave depends on the amplitude of its electric field. Longer wave pulse carries more energy. Neither the frequency nor the wavelength determines the energy of the wave.

If the light wave can be represented by the homogeneous plane wave, the frequency of the light wave determines the duration of interaction between the light wave and the electron. The longer duration results in more energy absorption by the electron.

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