Explanation of the Spacecraft Flyby Anomaly

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Abstract/Background

Anomalous changes in velocity were measured during spacecraft-Earth flybys of spacecraft launched on deep space missions between1990 and 2006. The amount of the velocity change was small and varied with orbital parameters. Not all such flybys exhibited the effect. Numerous attempts have been made to explain the velocity changes without success. The flyby anomaly is considered a major unresolved problem in astrophysics. A semi-empirical description which is in agreement with results for the anomalous velocity changes was developed by Anderson et al. [1]. During the development of a theory of the gravitational co-field [2], it was noted that the semi-empirical formulation of the flyby data was consistent with that model. This paper provides a physical explanation of the semi-empirical result.

Key Words: General Physics, Space Physics (Spacecraft-Earth Flyby Anomalies), Gravitational Co-Field,

I. EARTH SPACECRAFT FLYBY ANOMALIES

A small, anomalous change in velocity occurred in a number of spacecraft flybys of the Earth. A semi-empirical description which is in agreement with results for the anomalous velocity changes was developed by Anderson et al. [1]. The new classical model of the gravitational co-field [2] provides an explanation of "Flyby" experimental data and the semi-empirical description. Figure1 is a schematic of the NEAR flyby which produced the largest measured deflection of all the flybys.

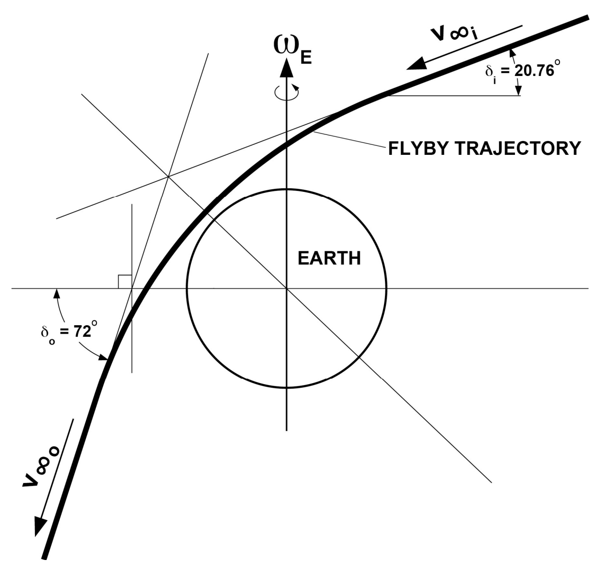

Figure 1. Schematic of the NEAR Spacecraft-Earth Flyby

Anderson, et al. found that this velocity change for the spacecraft flybys of the Earth can be described by

$$\Delta V_{\infty} = 2\omega_E v_{\infty}\left(\cos\delta_i - \cos\delta_0\right)\frac{R_E}{c} \tag{1}$$

where ω_E is the angular velocity of the Earth, R_E is the Earth's radius, and c is the speed of light. The remaining variables are as defined in Figure1. From the geometry of Figure 1, Equation (1) can be written in vector form as

$$\Delta V_\infty = (2\boldsymbol{\omega}_E x \mathbf{v}_{\infty_i} - 2\boldsymbol{\omega}_E x \mathbf{v}_{\infty_0})\frac{R_E}{c} \tag{2}$$

It is noted that the angles δ in Equation (1) are measured relative to the horizontal axis. If measured relative to the usual vertical axis, the cos δ becomes the sin of the complementary angle. In the formulation of Equation (2), the observed velocity changes are normal to the spacecraft orbit and to the Earth's angular velocity. The terms in parenthesis represent the difference in the accelerations of the spacecraft between the incoming and outgoing asymptotes. Given that the terms in the parenthesis represent accelerations, the R_E/c term represents a time interval of 2.124×10^{-2} sec. This implies that the measured change in velocity occurred in the vicinity of the closest approach, in the non-linear transition region between asymptotes.

II. ELEMENTS OF THE THEORY OF THE CO-FIELD USED IN FLYBY ANALYSIS

From the Anderson et al. semi-empirical description of the data, it is possible to present an interpretation of the data from the perspective of the theory of the gravitational co-field [2]. A brief description of the results of [2] applicable to the flyby problem follows. A gravitational-mechanical force field is defined as consisting of Newton's gravitational force equation and a rotational force term. The gravitational-mechanical field defined in terms of force/unit mass is

$$\boldsymbol{\Gamma} = \frac{M}{4\pi\epsilon_{0g} r^2}\hat{\mathbf{r}} + \beta r\ddot{\theta}\hat{\boldsymbol{\theta}} \tag{3}$$

where β is a constant. Whereas Newton and Heaviside described the equations of electricity and magnetism in terms of divergences and curls, the inverse approach in [2] was to apply curls to Equation (3) and its result to define the equations of the co-field. Taking the curl of $\boldsymbol{\Gamma}$ in cylindrical coordinates, and restricting the result to the axial direction, with $\omega = \dot{\theta}$, yields

$$\nabla x \boldsymbol{\Gamma} = \frac{\partial 2\boldsymbol{\omega}}{\partial t} \tag{4}$$

This result can be considered as defining an inertial induction of 2ω. The value of a gravitational co-field at a given point in the field is thus given by

$$\boldsymbol{\Omega} = 2\boldsymbol{\omega} \tag{5}$$

The acceleration of a mass in an Ω field resulting from the integral version of the curl is

$$\mathbf{a} = 2\boldsymbol{\omega} x \mathbf{v} \tag{6}$$

In cylindrical coordinates, using the value of Ω obtained from the differential curl version is

$$\nabla x \boldsymbol{\Omega} = \gamma\Omega_\theta \frac{\hat{\mathbf{r}}}{r} \tag{7}$$

where γ is a constant. Equation (7) is the inertial version of Amperes equation in E & M and can be put in similar form. However, the important conclusion from Equation (7) is that the rotating earth produces an angular velocity dipole:

$$\Omega = \left[\frac{R_o}{R}\right]^3 2\omega_E \tag{8}$$

In Equation (8), R_0 is the on-axis dipole radius defined by $R_0{}^2 = 0.33R_E{}^2$, Ro = 0.574R_E, R_E is the radius of the Earth, and ω_E is the angular velocity of the Earth.

III CO-FIELD ANALYSIS OF FLYBY RESULTS

In Equation (1), $\Delta V_\infty = 2\omega_E v_\infty\left(\cos\delta_i - \cos\delta_0\right)\frac{R_E}{c}$, and in Equation (2), the terms preceding R_E/c represent the difference between the asymptotic accelerations of a space vehicle in its orbit past the Earth. As noted, the R_E/c term semi-empirically represents a time interval required to produce the observed ΔV_∞. An alternate treatment of R_E/c is proposed. Since the terms preceding R_E/c represent an acceleration change between the spacecraft trajectory asymptotes, the time interval of the change must also occur along the trajectory, but in the non-linear part of the trajectory between asymptotes. The trajectory between the asymptotes can be approximated by $R_f\theta$, where R_f is the distance from the center of the Earth to the point of nearest approach of the orbit. The angle θ is the effective angle in radians, over which the change in acceleration occurs. Dividing by V_f, the spacecraft speed at closest approach, $R_f\theta/V_f$ is the corresponding time of acceleration for ΔV_∞. To maintain the Equation (1) value of ΔV_∞, it is required that

$$\frac{R_E\theta_E}{c} = \kappa\frac{R_f\theta}{V_f} \tag{9}$$

This assumes that in the semi-empirical model, R_E is actually an arc distance with an arc θ_E =1 rad for all the flybys. The left side of Equation (9) is a constant but the time interval is exceeding short (21.24 ms). Conversely, the term on the right yields an unrealistically large value of θ without a compensating constant κ. The terms on the right of Equation (9) must correspond to the value of R_E/c for agreement with Equations (1) and (2). The angle θ must thus vary with each flyby. The resulting equivalent equation to that of Equation (1) is:

$$\Delta V_\infty = \kappa 2\omega_E v_\infty\left(\cos\delta_i - \cos\delta_0\right)\frac{R_f\theta}{V_f} \tag{10}$$

If, in Equation (10), the actual time of acceleration to yield the observed anomalous velocity is $\frac{R_f\theta}{V_f}$, then κ must roughly account for neglected orbital properties. These include factors such as radius of closest approach and inclination differences amongst the flyby orbits. To account for the effect of different radii of closest approach, the field dependence on radius from Equation (8) should be included in Equation (10) with a new attenuation constant K_0.

$$\Delta V_\infty = K_0\left[\frac{R_0}{R_f}\right]^3 2\omega_E v_\infty\left(\cos\delta_i - \cos\delta_0\right)\frac{R_f\theta}{V_f} \tag{11}$$

It is assumed for the subsequent analysis that K_0 is the same for all the flybys. This constant is determined by normalizing to the results of the NEAR flyby. In Figure (2), a generic dipole is imposed on the NEAR flyby trajectory. It is seen that the most intense field lines occupy about 1 radian of the non-linear region between asymptotes. Using this value of 1 radian for θ and the data from the NEAR flyby, K_0 = 2.67E-4. It is assumed for the subsequent analysis that K_0 is the same for all the flybys. Using this value of K_0 and the orbital parameters of the remaining flybys, one can calculate their respective values of θ. The constant K_0 represents a very large attenuation, and it seems unlikely that neglected orbital factors can account for more than a small part of it. Thus, the major part of K_0 is probably a co-field associated attenuation factor. Equation (12) is the resulting revised version of Equation (1). Equation 12 could also be presented in its basic vector form, but does not explicitly show the Δ cos δ dependence.

$$\Delta V_\infty = (2.67E-4)\left[\frac{R_0}{R_f}\right]^3 2\omega_E V_\infty\left(\cos\delta_i - \cos\delta_0\right)\frac{R_f\theta}{V_f} \tag{12}$$

IV SUMMARY

The trajectory of the NEAR spacecraft depicted in Figures 1 and 2, is Newtonian with a small perturbation induced by motion in the Earth's gravitational dipole co-field. Equation (12) defines the perturbation for all flybys. Table 1 summarizes values of their orbital data. It is noted that the values of θ, relative to NEAR, are all also about 1 radian except for Cassini and Messenger.

Table 1. Numerical Data and Results Relevant to Equation (12)

FLYBY	ΔV_∞ mm/s	$2\omega_E(R_o/R_f)^3$ rad/s	V_∞ m/s	Δ Cos δ	R_f/V_f sec	θ rad	R_f m	V_f m/s	DA Deg	φ Deg
NEAR	13.46	2.17E-05	6851	0.6254	542.4	1.00	6.91E+06	12739	66.9	33.0
GLL-1	3.92	1.82E-05	8949	0.1487	533.6	1.14	7.33E+06	13740	47.7	25.2
GLL-2	-4.6	2.41E-05	8877	-0.1699	474.0	1.00	6.67E+06	14080	51.1	-33.8
Cassini	-2	1.66E-05	16010	-0.0215	396.7	3.31	7.55E+06	19026	19.7	-23.5
Rosetta	1.80	1.24E-05	3863	0.1796	791.8	0.99	8.33E+06	10517	99.3	20.2
M'GER	0.02	1.08E-05	4056	0.0044	839.9	0.46	8.72E+06	10389	94.7	47.0

Notes: Earth's angular velocity ω_E = 7.292E-5 rad/s, Earths radius R_E = 6.371E+6 m, R_0 =3.660E+6m;
Δ cos δ is the difference between cosines of the incoming and outgoing asymptotes. DA is the deflection angle between asymptotes. The angle φ is the latitude of the point of closest approach.

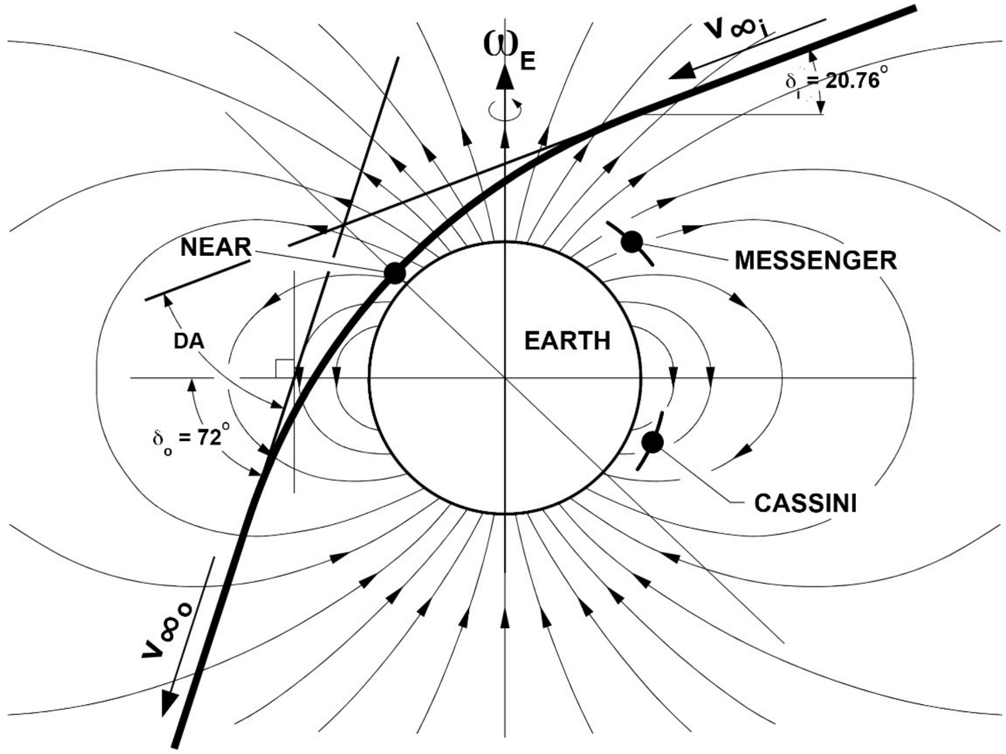

Figure 2. NEAR Flyby with Generic Ω Dipole Field Superimposed

A comparison of the NEAR orbital parameters and its ΔV_∞ with other flybys illustrates the factors which determine the magnitude of the flyby anomaly. The NEAR, Cassini, and Messenger flybys represent extremes amongst the flybys. The relative positions of the distance of closest approach for NEAR, Cassini, and Messenger and their latitudes, irrespective of longitude, are shown in Figure 2. NEAR has the largest ΔV_∞ value of 13.6 mm/s, with Cassini at 2 and Messenger at 0.02. In Equation (12), the value of ΔV_∞ is increased by large values of V_∞, $\Delta \cos \delta$, θ and by small values of R_f and V_f. While all of these orbital factors contribute to the observed ΔV_∞, the $\Delta \cos \delta$ term appears to predominate in determining whether a ΔV_∞ is observed and its magnitude. Except for the Rosetta flyby, the values of ΔV_∞, in descending order, correspond with those of $\Delta \cos \delta$. Finally, the interaction of the spacecraft velocity with the Ω dipole field varies with the position of closest approach. Maximum contribution to ΔV_∞ occurs when **ω** is normal to **v**. For NEAR, the field lines having a large angle to the orbit are most intense roughly from the Earth's axis to the point of closest approach as seen in Figure 2. They subtend about 1 radian of arc of the non-linear portion of the trajectory. Normalization to the NEAR trajectory produced θ values of 3.31 and 0.46 radians for Cassini and Messenger, respectively. The lines of Ω to the trajectory are less inclined to the trajectory for Cassini, but the interaction angle θ is larger. The orbital inclination for Messenger is less favorable. In summary, the Newtonian trajectory provides the conditions for the size of the flyby anomaly. The Ω field-velocity interaction yields the perturbation.

V CONCLUSION

The anomalous velocity changes observed during spacecraft flybys of the Earth are explained in terms of the flyby's orbital parameters interacting with the Earth's gravitational co-field [2]. The experimental foundation for the present work was the semi-empirical equation developed by Anderson et al. [1]. The semi-empirical equation provides a good description of the observed anomalous velocities in terms of orbital parameters. The present analysis partially re-formulates the Anderson et al. equation, and identifies a probable new physical constant which attenuates the effect of the co-field. The gravitational co-field model provides a physical explanation of the flyby anomaly through its prediction of the $\mathbf{2\omega}\times\mathbf{v}$ interaction and of the gravitational dipole co-field.

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