An approach for analyzing time dilation in the TSR (v8. 2018-02-14)

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Abstract. We present an approach to analyze time dilation in the theory of special relativity, starting out from a variant of the Lorentz transformation. The concepts of symmetry and simultaneity are essential in these investigations. We also stress the importance of the observational principle, *i.e.*, the location of clocks used for the clock comparisons of the two reference frames (RFs) moving relative to each other. For a specific RF we may follow just a single clock (SC), or we can use multiple clocks (MC) to follow a single clock on the other RF. In addition to these standard cases, we consider an approach, utilizing an auxiliary RF, which – in combination with symmetry considerations – provides a consistent definition of 'simultaneity at a distance'. We use the overall approach to discuss the travelling twin paradox, (now providing corrections to previous versions of the paper).

Key words: Lorentz transformation, symmetry, simultaneity, auxiliary reference frame, travelling twin.

1 Introduction

The present work presents basic features of time dilation in special relativity. In particular, we explore the use of the Lorentz transformation (LT) for one spatial parameter. We start out by providing a reformulation of the LT, being suitable for a graphical presentation. It also facilitates the specification of which clocks to apply for the clock comparisons between the two inertial reference frames (RFs). We will refer to the specification of clocks as the observational principle.

Symmetry is important when we discuss time dilation within special relativity. It may appear paradoxical that- at the same time as we have complete symmetry between the two RFs – we will also 'take the perspective' of one of them, apparently destroying symmetry. For instance the common statement that the 'moving clock goes slower' represent such an apparent paradox, which is handled somewhat differently in the literature. Some authors apply the expression 'as seen' by the observer on the other reference system, perhaps indicating that it is an apparent effect, not a physical reality. Actually, Giulini, [1] in his Section 3.3 states: 'Moving clocks slow down' is 'potentially misleading and should not be taken too literally'. Others stress that 'everything goes slower' on the 'moving system', not only the clocks; truly stating that the time dilation represents a physical reality also under the conditions of special relativity, (*i.e.* no gravitation *etc*.) However, for instance Pössel [2] points out that it is the procedure related to clock comparison ('observational principle') that decides which reference system has the time which is 'moving faster', resp. 'slower'.

Simultaneity at a distance is another important issue in special relativity. Using synchronized clocks of a specific reference frame we can define simultaneity of events 'in the perspective' of any RF, but the various RFs will give different specifications of simultaneity. However, we will introduce an *auxiliary reference frame,* which – in combination with symmetry requirements – provides a useful tool for specifying simultaneity at a distance.

We present an approach to meet these challenges in the analysis of time dilation, and also apply the suggested approach to provide a discussion of the 'travelling twin' example (under the strict conditions 'of special relativity); demonstrating how various specifications regarding the 'reunion' of the twins will give different results.

Actually, some authors also question the validity of the theory of special relativity (TSR) and the LT, (*e.g.* see McCausland [3], Phipps [4], Robbins [5]); and perhaps we should include Serret [6]. In particular Ref. [3] reviews various controversies on the topic (related to H. Dingle) during several decades, and gives many references. Ref. [5] also treats the Bergson-Einstein controversy, dating back to 1922. The scope of the present work, however, is more restricted, accepting the validity of the TSR as a premise. Our objective is mainly to investigate the logical implications of the Lorentz transformation and thereby provide an approach for analyzing relative time and simultaneity within the framework of the TSR.

2 The Lorentz transformation and some special cases

We first specify some basic notation and assumptions. Then we provide a variant of the LT as the basis for our investigations.

2.1 Basic notation and assumptions

We start out from the standard theoretical experiment: Two co-ordinate systems (inertial reference frames), pointing in the same direction are moving relative to each other at speed, v. We consider just one space co-ordinate, (x-axis), and investigate the relation between space and time parameters, (x, t) on one RF (denoted K), and the corresponding parameters (x_v, t_v) on the other. Thus, we have the following *basic simultaneity*: At time (*i.e. clock reading*), t and position, x on the first system, we observe that time equals t_v and position equals x_v on the other. We will base the discussions on the LT, including the following specifications:

- There is a complete *symmetry* between the two co-ordinate systems.
- On both RFs there is an arbitrary number of identical, synchronized clocks, located at any positions where it is required.
- We choose the *perspective* of one of the RFs, which we will denote K. *Simultaneity in the perspective* of this RF, K, means that all clocks on this RF show the same value, t, (and both times are given as a function of the position on K). We will refer to K as the *primary* system, and the other RF as the *secondary* system.
- Throughout we let SC refer to a RF utilizing a 'single clock' (or the 'same clock'), for the time comparisons with other RFs. Similarly, MC will refer to a reference frame, which utilizes 'multiple clocks' (at various locations) for time comparisons.

2.2 The standard formulation of the Lorentz transformation

In the above notation the LT takes the form

$$t_v = \frac{t-(vx)/c^2}{\sqrt{1-(v/c)^2}} \tag{1}$$

$$x_v = \frac{x-vt}{\sqrt{1-(v/c)^2}} \tag{2}$$

These formulas include the length contraction along the x-axis (inverse Lorentz factor):

$$k_x = \sqrt{1-(v/c)^2} \tag{3}$$

2.3 An alternative formulation

At any time, t and position, x we now introduce w equal to $w = x/t$. By inserting $x = wt$ in (1) we directly get that time on the secondary RF at this position equals:

$$t_v = t_v(w) = \frac{1-vw/c^2}{\sqrt{1-(v/c)^2}}t \tag{4}$$

Note the change in notation. In *eq.* (1) we suppressed the dependence of t_v on x Now, however, we pinpoint its dependence on w, and will – when appropriate write $t_v(w)$ rather than t_v. The new time dilation formula (4) will – for a given time, t on the primary system, K – give the time, $t_v(w)$ on the secondary system, K_v, as a linear, decreasing function of w. The important thing is that we take out t as a separate factor. The relation $t_v(w)/t$ is given as the 'general time dilation factor':

$$\gamma_v(w) = (1-\frac{vw}{c^2})\,/\sqrt{1-(\frac{v}{c})^2}$$

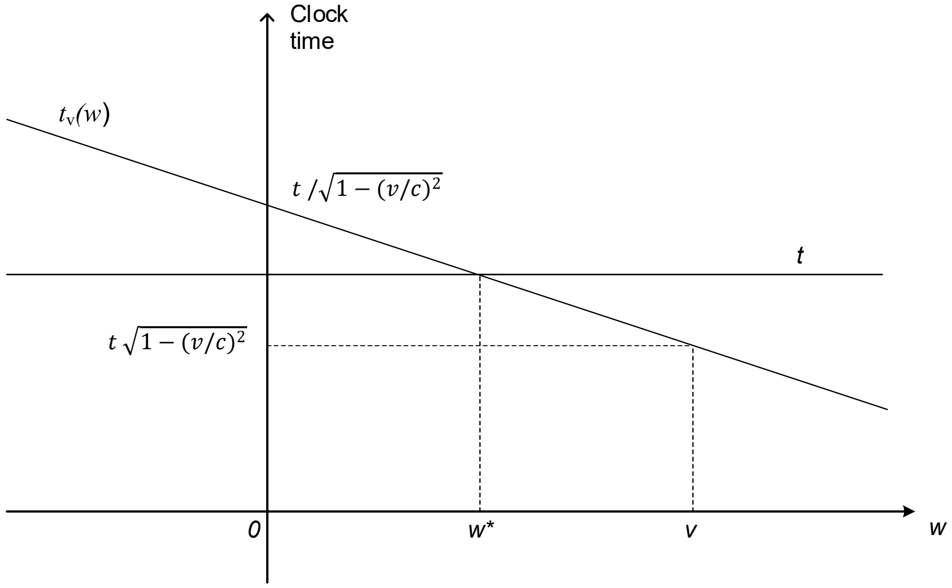

Figure 1. Clock readings in the perspective of *K*. Thus, 'time' all over *K* equals, *t.* while clock readings, $t_v(w)$ on the other RF is given as a function of *w*, *cf.* (4); where *w=x/t* provides the 'position' on *K*.

Thus, we can write (4) as

$$t_v = t_v(w) = \gamma_v(w)\, t.$$

Fig.1 provides an illustration of this time dilation formula. Here we give clock reading ('time') both on *K* and K_v *in the perspective of K*. So the figure illustrates an instant when time equals *t* all over this reference frame. The horizontal axis gives the 'position' $w = x/t$ on *K* at which the clock measurements are carried out. The vertical axis gives the actual clock readings. So as time on *K* equals *t* at any 'position', *w*, the clock readings on K_v at this instant, $t_v(w)$, depend on *w*; see decreasing straight line.

Now, in analogy to letting $x = wt$, we also define a w_v so that $x_v = w_v{\cdot}t_v = w_v{\cdot}t_v(w)$. By inserting both $x = wt$ and $x_v = w_v{\cdot}t_v$, in (2), we will obtain

$$w_v = \frac{x_v}{t_v(w)} = \frac{w-v}{1-\frac{w}{c}\cdot\frac{v}{c}} \tag{5}$$

Now equations (4), (5) represent an alternative version of the LT, here expressed by parameters (*t, w*) rather than (*t, x*). The most striking feature of this new version is that it is the single equation, (4), which involves the time parameters, *t* and $t_v(w)$; giving $t_v(w)$ as a factor independent of *t* multiplied with *t*.

The other equation (5) has a direct interpretation related to velocities. According to standard results of TSR, *e.g.* Refs. [7]-[9], the velocities v_1 and v_2 sums up to *v,* given by the formula

$$v = v_1 \oplus v_2 \overset{\text{def}}{=} \frac{v_1+v_2}{1+\frac{v_1}{c}\cdot\frac{v_2}{c}} \tag{6}$$

So now defining the operator $\oplus$ this way, eq. (5) actually says that $w_v = w\oplus(-v)$, implying $w_v \oplus v = w$; thus, clearly interpreting *w* and w_v as velocities along the *x* - axis. That is we have a moving position along the *x*-axis for clock comparisons. Therefore, this *w* specifies what we refer to as the *observational principle,* pinpointing that this is an essential factor for the resulting observed time dilation.

Note that we do not need to think of *w* as a velocity; rather as a way to specify a certain position $x = wt$ on the primary RF, *K*. However, we will later see that it can also be fruitful to interpret *w* as the velocity of a third observational RF relative to *K*.

A final comment. *Eq*. (4) gives our general formula for time dilation. Here we use the parameter *w* to specify the location of the relevant clock readings. This *w* provides the location on the *primary* RF, *i.e.*

K. Now we could of course reformulate the *eqs*. (4), (5) to give the time dilation and clock readings as a function of w_v, (and w as a function of w_v). In that case *K* would become the *secondary* RF; giving 'identical' expressions, just replacing v with $-v$.

2.4 Standard special cases (observational principles)

Now focusing on time dilation, *cf. eq.* (4), there are various interesting special cases (*observational principles*). First, if a specific clock located at the origin $x_v = 0$ on K_v is compared with the passing clocks on *K*. These clocks on *K* must have position $x = vt$, and thus we choose $w = v$ and directly get the relation

$$t_v(v) = t\sqrt{1-(v/c)^2} \tag{7}$$

which equals the 'standard' time dilation formula. Further, when a specific clock at the origin, $x = 0$, on *K* is used for comparisons with various passing clocks on K_v, we must choose $w = 0$ and thus get

$$t_v(0) = t\,/\sqrt{1-(v/c)^2} \tag{8}$$

as the relation between t and t_v. We specify the two special cases (7), (8) in Fig. 1 and will discuss these further in Ch. 3. Two other standard cases are obtained by inserting $w = \pm\, c$ in (4).

2.5 The symmetric case

There is another interesting special case of the LT, (4), (5). We can ask which value of w (and thus w_v) will result in $t_v(w) \equiv t$. We easily find that this equality is obtained by choosing $w = w^*$, where

$$w^* = \frac{c^2}{v}\left(1-\sqrt{1-(v/c)^2}\right) = \frac{v}{1+\sqrt{1-(v/c)^2}} \tag{9}$$

Further, by this choice of w we also get $w_v = -w^*$. This means that if we consistently consider the positions where simultaneously $x = w^*t$ and $x_v = -w^*t_v = -w^*t$, then no time dilation will be observed at these positions. In other words (*cf.* Fig. 1):

$$t_v(w^*) = t \tag{10}$$

At this position we find $x_v = -x$, and so we see this as the midpoint between the origins of the two reference frames; thus, providing a nice symmetry. Note that when we choose the observational principle, (9), then absolutely everything is symmetric, and it should be no surprise that we get $t_v = t$.

Note that w^* has a simple interpretation. Recalling the definition of the operator $\oplus$ in eq. (6) for adding velocities in TSR, ($v = v_1 \oplus v_2$), it is easily verified that when w^* is given by (9), then we get $w^* \oplus w^* = v$. So this confirms that when our point of observation 'moves' with velocity w^* relative to *K* and $-w^*$, relative to K_v, it corresponds exactly to the case that the relative speed between *K* and K_v equals v.

3 "The moving clock": SC *vs.* MC

We now take a closer look at the observational principles given by (7) and (8). These relate clock readings at a location where one of the clocks are positioned *at the origin* of a RF. Thus, we can combine *eqs.* (7) and (8) into one single formula:

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2} \tag{11}$$

Here t^{SC} is the clock reading of the specific clock at the origin (of either *K* or K_v). Further t^{MC} is the clock reading of the clock at the same location, but on the other frame. So, for instance, t^{SC} replaces $t_v(v)$ in *eq.* (7) and t in *eq.* (8); while t^{MC} replaces t in *eq.* (7) and $t_v(0)$ in *eq.* (8), This is clearly demonstrated in Fig. 1. At both these locations it is the SC that gives the lower value.

Note: Extended notation

Note that the notation of *eq.* (11), using t^{SC} and t^{MC}, shall not replace the more general notation, $t_v = t_v(w)$ and t, as used in (4). The new terms t^{SC} and t^{MC} shall just help us to realize the symmetry

of two specific positions, also being marked out in Fig. 1. Thus, we will still use the general notation, but at the two locations, $w = 0$ and $w = v$, we may in addition apply the essential result, (11).
Also, observe that we in (11) have dropped the subscript, v in both time parameters. This just means that (11) is valid irrespective of which RF is chosen as the primary. However, we could (and will later) add a subscript v to either t^{SC} or t^{MC}, to indicate which of the systems we choose as the primary/secondary RF. Thus, using parameters (t_v^{SC} , t^{MC}) means that we 'follow' a fixed SC at the origin of the 'secondary' RF, and using (t^{SC}, t_v^{MC}) means that we 'follow' a SC on the primary RF.

We may consider this relation (11) as the 'essential Lorentz transformation', and is in my opinion more useful that the rather ambiguous (7). Further, (11) is much more than an efficient way to write the two *eqs.* (7) and (8). By *eq.* (11) we stress that (7) and (8) actually represent the same result, and is thus more informative than (7) and (8). Actually, this choice of which reference frame shall apply a single clock is crucial, and it introduces an asymmetry between the two RFs.

Before we leave (11) some further comments are relevant. First, observers on both reference frames will agree on this result (11). Thus, it is somewhat misleading to apply the phrase 'as seen' regarding any clock reading. All time readings are objective, and all observers (observational equipment) on the location in question will 'see' the same thing. The main point is rather that observers at *different reference frames* will not agree regarding simultaneity of events.

Secondly, we have the formulation 'moving clock goes slower'. It is true that an observer on one RF, observing a *specific clock* (on the other RF) passing by, will see this clock going slower, when it is compared to his own clocks. So in a way this confirms the standard phrase 'moving clock goes slower'. However, we could equally well take the perspective of the single clock, considering this to be at rest, implying that the clocks on the other RF are moving. The point is definitely not that clock(s) on one RF are moving and clocks on the other are not. It is the observational principle that decides which of the two clocks initially at the origin, which we observe to move slower. Therefore, the talk about the 'moving clock' could be rather misleading.

We should stress that relations like *eq.* (11) are well known. For instance, *eq.* (3-1) of Smith [10] equals our *eq.* (11), and our concepts SC and MC correspond to the concepts of'proper' and 'improper' time used in that book. Actually, 'coordinate time' is a more common term in the literature than 'improper time'. However, it seems that this relation (11) and the insight that it provides has not received the attention it deserves.

We further stress that it is not required to point at one reference frame to be SC (having 'proper' time), and the other to be MC (having 'improper' time). We may at the same time have clock(s) on *both* RFs observed to 'go slower'. The equation (11) just says that if we follow a specific clock (here located at the origin), we will observe that this goes slower than the passing clocks on the other RF.

This point in my opinion also gives an answer to 'Dingle's question'. Dingle [11] raises the question of symmetry regarding the travelling twin paradox: "Which of the two clocks in uniform motion does the special theory require to work more slowly? This is an important question, which according to the discussion in McCausland [12] so far has not been given a satisfactory answer.

However, it is clearly not the case that the clock(s) on *one* of the two reference frames go(es) slower than the clock(s) on the other. We could very well choose to follow *both* the two clocks being at the origin at time 0; which will give that both reference frames have a clock 'going slower'. So, the result on time dilation is actually fully symmetric with respect to the two reference frames! The question is not which reference frame has a clock that 'goes slower'; it is rather which observational principle we have chosen. This fully demonstrates that it is rather inappropriate to apply the statement 'moving clock goes slower'.

Actually, Professor Dingle in his later work claimed that the TSR itself was inconsistent; see thorough discussion by ref. [3]. However, according to [3], Dingle again seems to have focused on the apparent inconsistency of our *eqs.* (7), (8), rather than discussing the interpretation of the more relevant *eq.* (11).

4 Using an auxiliary reference frame of symmetry

We proceed to investigate the important question of simultaneity at a distance. We primarily elaborate on the fundamental result (11). However, we will now treat the two reference frames in a symmetric way, and denote them K_1 and K_2. In addition, we introduce an auxiliary RF, K. We chose this as our primary RF, and so we make our observations 'in the perspective' of this auxiliary K.

To get a completely symmetric situation we let K_1 having speed $-w^*$ with respect to K, and K_2 having speed w^* with respect to K. As the speed between K_1 and K_2 shall equal v, it follows that we define w^* by (9); (*cf.* discussion at end of Section 2.5.)

Next we specify the observational principle. We choose to operate the auxiliary reference frame as MC, and so both K_1 and K_2 are SC.[1] Then single clocks at the origins of K_1 and K_2 are at any time compared with various clocks along K. Now we can apply the relation (11) between the auxiliary RF, K and the two RFs K_1 and K_2, giving, (also see the Note, *Alternative notation* in Chapter 3):

$$t_v^{SC} = t^{MC}\sqrt{1-(v/c)^2}, \quad \text{both for } v = -w^* \text{ and } v = w^* \tag{12}$$

It directly follows that

$$t_{-w^*}^{SC} = t_{w^*}^{SC} = t^{MC}\sqrt{1-(w^*/c)^2} \tag{13}$$

So here $t_{-w^*}^{SC}$ is the clock reading of the clock at the origin of K_1, and $t_{w^*}^{SC}$ is the reading of the clock at the origin of K_2. The essential result in (13) is that $t_{-w^*}^{SC} = t_{w^*}^{SC}$. So in the perspective of K these are now the simultaneous clock readings at the origins of the two 'main' reference frames, K_1 and K_2, moving relative to each other at speed, v.

We illustrate these results in Fig. 2, which provides an analogy to Fig. 1. While Fig. 1 presented time dilation between two RFs, taking the perspective of one of them, Fig. 2 gives a symmetric picture with respect to two RFs, also introducing a third RF, K, and taking the perspective of this new one. Fig. 2 gives a snapshot of the clock measurements at an instant when all clocks on K read time t; *cf.* horizontal line marked t.

The parameter, w (horizontal axis) refers to the 'positions' ($w = x/t$) on the auxiliary reference frame, K. The reference frames, K_1 and K_2 move relative to K at speed $-w^*$ and w^*, respectively. Thus, the lines $t_{-w^*}(w)$ and $t_{w^*}(w)$ give the time measured on clocks at K_1 and K_2, respectively; as a function of w (and time t) on K. We focus on three positions on K, *i.e.* w equal to $-w^*$, 0 and w^*, respectively. These three values correspond to the origins of the three reference frames, K_1, K and K_2, respectively.

First, the letter a in the figure indicates the simultaneous clock readings of reference frames K_1 and K_2, observed at the origin of K. At this position the clocks on K_1 and K_2 show the same time, and are simultaneously located at the same location, w = 0; so we are actually just referring to 'basic simultaneity'. For *these* measurements the reference frame K is a SC system, and its clock will appear slower than the corresponding clocks on K_1 and K_2: we observe the line t falling below the point a.

As stated, the two points marked with b correspond to the SC time readings at the origins of K_1 and K_2. Thus, using the *extended notation* introduced in Chapter 3, we have $t_{-w^*}^{SC} = t_{-w^*}(-w^*)$ and $t_{w^*}^{SC} = t_{w^*}(w^*)$. According to our result (13), these are identical. So the clock on K_1 at the position $-w^*$ and the clock on K_2 at the position w^* give identical time readings.

[1] Alternatively we could let the auxiliary reference frame, K operate as SC; but would then just obtain the same result as given in Section 2.5, and this is therefore of limited interest.

These origins have moved apart after time 0; and the evens that these two clock readings are equal are not simultaneous, neither in the perspective of K_1 nor in that of K_2, (clearly illustrated in Fig. 2). However, *eq.* (13) tells that *in the perspective of the auxiliary reference frame* we have two simultaneous events. Now simultaneity in the perspective of the auxiliary RF may seem a weak form of simultaneity. But, when we have this symmetry, the result becomes interesting, and not very surprising. Rather, I would postulate that this symmetric 'simultaneity at a distance represents a valid form of simultaneity. This is not a strong assumption, considering this a consequence of the complete symmetry we have here. Claiming that the one of the two events *b* would occur prior to the other would represent a contradiction.

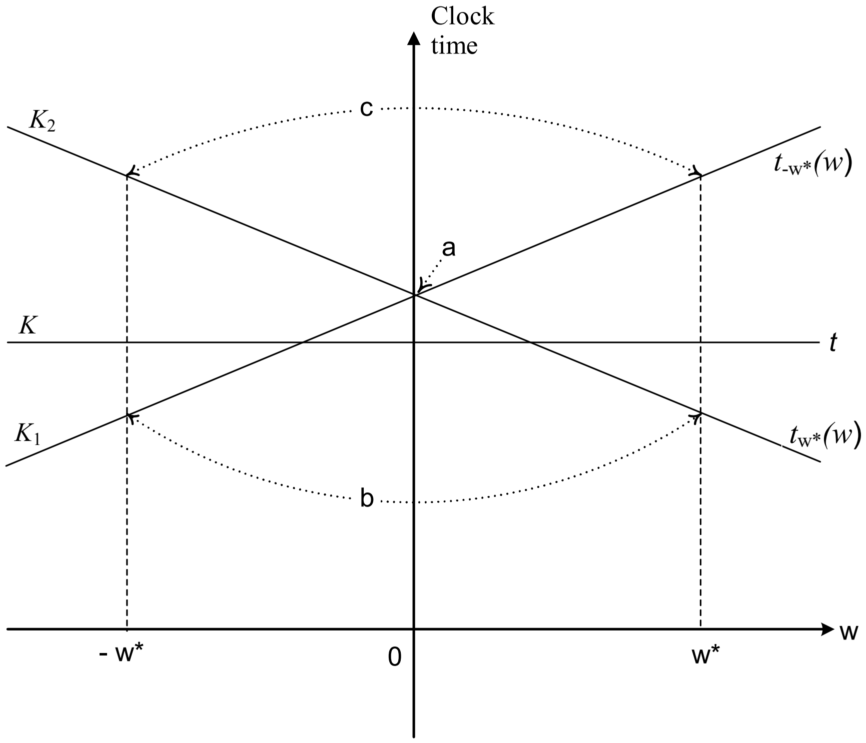

Figure 2 Clock measurements ('time') in the perspective of the auxiliary reference frame, *K*, where the reference frames K_1 and K_2 have velocity $-w^*$ and w^*, respectively, relative to *K*. (Here *w* represents 'position' on *K*.)

In conclusion, this is the most significant result obtained by using the auxiliary RF: We manage to establish a simultaneity of events at K_1 and K_2 'at a distance'. This is a key question in a proper handling of time dilation to achieve this. Also see the further discussion on simultaneity in Hokstad [13].

Finally, in Fig. 2 we also see two clock readings corresponding to the letter *c*. These exhibit the same type of symmetric simultaneity as the points *b*, and the only difference is that the time readings at *c* will *not* correspond to the origins of the two main RFs themselves, but rather to the location on the other frame at the same position. We give a numerical example related to Fig. 2 in Appendix.

This concludes our discussion on the interpretation/handling of time dilation in TSR. The observant reader might realize that the experimental set-up given here is well suited for handling the travelling twin paradox; which we discuss in the next chapter.

5 Example: The travelling twin

We now utilize the framework provided in the previous chapters to analyze the so-called travelling twin example, which goes back to Langevin [14]. As stated for instance in Mermin [9] the travelling twin paradox shall illustrate that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together.

5.1 A numerical example

Ref. [9] (in Chapter 10) gives the following numerical example: "If one twin goes to a star 3 light years away in a super rocket that travels at 3/5 the speed of light, the journeys out and back each takes 5 years in the frame of the earth. Since the slowing-down factor is $\sqrt{1-(3/5)^2} = 4/5$ the twin on the rocket will age only 4 years on the outward journey, and another 4 years on the return journey. When she gets back home, she will be 2 years younger than her stay-at-home sister, who has aged the full 10 years." So the claim is that the referred difference in ageing occurs during the periods of the journey with a constant speed; *i.e.* under the conditions of the TSR, (ignoring the acceleration/deceleration periods). After all the whole argument relies on the Lorentz transformation! Thus, our discussion will restrict to the periods of constant velocity.

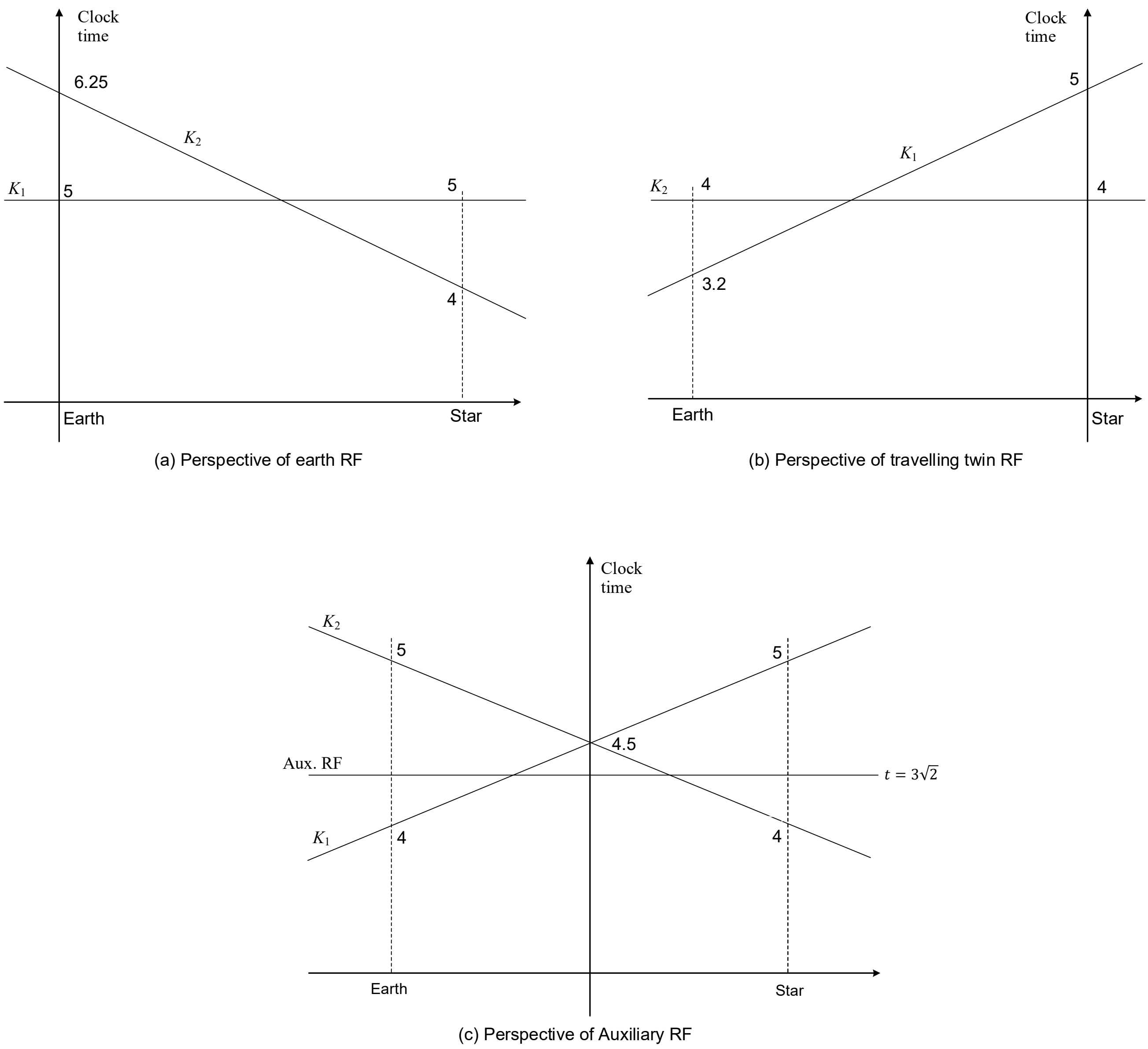

Figure 3 Clock readings by the arrival at the star, from different perspectives.

First looking at the travel *to* the star, I fully agree with the above claim; which is also illustrated in Fig. 3(a). Here K_1 is the RF of the earth/star (earthbound twin), and we have also introduced an 'imaginary' RF, K_2 of the travelling twin; (We could equip him with additional rockets at appropriate (constant) distances and synchronized clocks.) The SC of the travelling twin will – when compared with passing clocks of the eart/star– observe that his goes slower than these, and at arrival clocks show 4 and 5 years respectively. Similarly, the earthbound twin will observe that his clock goes slower than the passing clocks of K_2. In particular, when his own clock shows 5 years, the passing clock of his travelling twins RF show 5/0.8 = 6.25 years. This is just an example of Fig. 1.

Fig 3(b) demonstrates that if we take the perspective of the travelling twin – and his earthbound twin moves relative to him at velocity ($-v$) – then his arrival at the star is 'simultaneous with the event that the clock of his earthbound twin shows 4·0.8 = 3.2 years. So they agree on the degree of time dilation at both locations (factor 0.8), but they dramatically disagree about simultaneity.

Therefore, in Fig. 3(c) we include the perspective of the auxiliary RF of symmetry introduced in Ch. 4. The detailed calculations are given in Appendix A.1, but we mention that this implies that both twins travels away (in opposite directions) from the origin of the auxiliary RF at speed $w^* = c/3$. Quite obviously, also here the clocks of K_1 and K_2 at the point of arrival at the star read 5 and 4 years, respectively. These are actual clock readings, which will not change whatever perspective we choose, as long as we carry out the observation 'on location'. So the difference here is the chosen symmetry, obviously giving that the clock readings of 5 years for the earthbound twin and 4 years for the clock of the RF to the travelling twin passing the earth are simultaneous with the arrival at the star. This is a fully symmetric situation, and we follow both twins as SC; *cf.* discussion in Ch. 4. From Fig 3(c) we also see that at the origin of the auxiliary RF, both K_1 and K_2 show time 4.5 years at this moment; while we can show that the clock time on the auxiliary RF equals $3\sqrt{2} \approx 4.24$ years (see Appendix A.1).

So we have two essential facts. First; whatever perspective we chose, the clock of the travelling twin shows just 4 years when he arrives, while the clock of the earth/star system on this location shows 5 years. No choice of perspective can affect this fact. Secondly, there is a problem specifying the event on earth being simultaneous with this event. We could say that it is undetermined, but I would say that the result of Fig. 3(c) is the most relevant. After all, if we mark out a distance on K_2 equal to the distance between the earth and the star, then of course the earthbound twin will see this location exactly when his own clock shows 4 years (and the passing clock shows 5 years). So this is anyway the symmetric solution, and what could the reason be to choose a nonsymmetrical one? We further elaborate on the question of simultaneity in Appendix A.2.

Here I think we have the essence of the so-called Dingle's question: Is it actually a contradiction that both twins have a slower clock than the clocks passing on the other RF? I think not. Following the LT, Fig 3(c) just illustrates that the SC of *both* twins goes slower than the passing clocks on the other twins RF. This is actually symmetric. The intriguing question is what the clocks show if they meet again.

5.2 The 'reunion' of the twins

As seen, it is quite straightforward to handle the travel *to* the star. This line of events is fully specified and give one unique result, independent both of perspective and of simultaneity at a distance. However, there are various ways to bring the two twins together again, and the result will depend on the decisions of how to carry this out. We find it enlightening to look at a few of these options. We recall the starting point: The travelling twin is at the star and the earthbound twin at the earth. The clocks at the star reads 4 and 5 years respectively, whilst we might question the clock readings at the earth at this instant; (simultaneity at a distance depends on the perspective).

Our first option is to let the travelling twin immediately follow the same way back along the RF of the earth/star (at the same speed). Thus, again there will be a SC moving along another RF at speed v, giving the same result as the travel *to* the star: the travelling twin ages 4 more years on the return travel, whilst the earthbound ages 5 more years. Therefore, this option gives the standard result referred in [9], the returning traveling twin has in total aged 8 years, whilst the earthbound has aged 10 years. The main point here is that the travelling twin has during the entire travel acted as a SC relative to the earth/star RF, and so his clock has throughout been slower than the passing clocks of the earth/star RF.

As another option, we let the earthbound twin set out from the earth at same speed, *v,* in order for the twins to reunite; while the travelling twin after his arrival remains on the star. Now we must also decide at which time the earthbound twin shall start his travel. Ideally we want the earthbound twin to set out on his travel simultaneously with the arrival of his twin at the star, but as we know, there is no unique definition of simultaneity at a distance. Taking a holistic view, we claim that the most 'natural' choice is to let the earthbound twin set out when his own clock shows 4 years, and consequently the passing clock of the traveling twin's RF shows 5 years; (also the earthbound twin's clock has been acting as SC). On the return the earthbound twin's clock acts as a SC, and so his clock on the arrival at the star shows 4+4= 8 years. When he compares with the passing clocks of the travelling twin RF, he will observe that these all the time goes faster than his, and by the arrival at star shows 5+5=10 years. So in this experimental set-up it is the earthbound twin that throughout acts as the SC, and he is the youngest by the reunion.

There are alternative instants for starting the travel of the earthbound twin. We could take *his* perspective and let him start on his travel to the star when his own clock shows 5 years, and thus, the passing clock of his twin's RF shows 6.25 years. Then the result at his arrival at the star becomes 5+4= 9 years and 6.25+5 = 11.25 years, respectively. However, this result is based on an interpretation of 'simultaneity at a distance', which seems rather flawed.

As a last option, we may decide that *both* twins shall move away from the earth, respectively star, in order to meet. Now considering the perspective of the auxiliary RF (*cf.* Fig. 3(c)), which at the point of turning has its origin at the midpoint in between the earth and the star. Thus, we decide that when the clocks of both twins show 4 years, both twins just turn towards each other with the same speed, $w^* = c/3$ relative to the auxiliary RF. Thus, also on the return both twins act as SC and experience the same time dilation relative to the auxiliary RF. Accordingly also the return travel takes 4 years on their own clocks. Thus, when the two twins meet again at the origin of the auxiliary RF, they have both aged 4+4=8 years. So by this fully symmetric approach; (first departing from each other at speed $w^* = c/3$ relative to the auxiliary RF, and then moving together again at the same speed), they actually age the same amount of years.

It is of course reassuring that when we specify a fully symmetric line of events for the two twins – both acting as SC systems – they will have the same age when they meet again. In summary, the decision taken on when/how to reunite them fully decides the twins' age at the reunion.

5.3 Concluding comments on the travelling twin

This so-called paradox is indeed thoroughly discussed in the literature. Shuler [15] informs that about 200 per reviewed academic papers with *clock paradox* or *twin*(*s*) *paradox* in their title can be identified since 1911, most of them since 1955. He comments: "An outside observer might reasonably conclude there is deep conviction that matter *should have been* settled, along with a nagging suspicion that *it is not.* Further: "Though the correct answer has never been in doubt the matter of *how to explain* the travelling twins appears be far from settled". He also refers to the following statement: "On the one hand, I think that the situation is well understood, and adequately explained in plenty of textbook. On the other hand… there are complementary explanations which take different points of view on the same underlying space-time geometry (though, alas, the authors don't always seem to realize this, which rather undermines my assertion that the effect is well enough understood)". It is my hope that the above discussion can contribute to a more thorough understanding.

Also Debs and Redhead [16] give a thorough discussion on this case. They refer to the two asymmetries that have been the basis for most of the standard explanations. The first group of arguments focuses on the effect of different standards of simultaneity, and secondly one can designate the acceleration as the main reason for the differential aging. However, regarding the last group of arguments they write "… since we are dealing with flat space-time, we regard the reference to general relativity in this context as decidedly misleading"; a statement in which I agree.

Thus, [16] follows up on discussing the simultaneity, and in particular argue for the *conventionality of simultaneity.* This implies that when establishing simultaneity at a distance by the use of light signals, the definition of simultaneity is essentially a matter on convention; it is not a single point but a full interval of time instants, which we can define to be simultaneous with a specific distant event.

This seems somehow related to the above approach of specifying various options for the return ('reunion'). If we want also the earthbound twin to move (relative to the earth), and to start the movement exactly at the arrival of the twin to the star, the choice of RF decides *when* to start (see Fig. 3). However, we should of course not interpret this to mean that all choices/results are equally valid. I would rather say that we should choose the time instant for start, which corresponds to the situation we want to model. Even if there are several possible definitions for simultaneity at a distance, this does not mean that all are equally valid.

To sum up our findings of this discussion. When we specify the outward travel to go from the earth to the star, the clock readings at the travelling twins arrival are unequivocally given; choice of RF being irrelevant. However, we may dispute what is the 'simultaneous' event on the earth. (Thus, we should not - as some authors do - simply assume that the earthbound twin's clock will show 5 years 'simultaneously' with the arrival of the travelling twin.) For the return travel there are numerous options in case we also allow the earthbound twin to move from the earth. Then we have to decide both the times for start of the movements and the magnitudes of the velocities. Again the choice of RF is in principle irrelevant. However, we can motivate our choice *e.g.* of the starting time for the earthbound twin, by considering a specific RF; (three being exemplified in Fig 3).

6 Summary and conclusions

Starting out from the Lorentz transformation (LT) we present some aspects of the approach for analyzing time dilation in the theory of special relativity.

First, we *reformulate the LT*, in a way that facilitates a *graphical presentation* of the clock time of both reference frames (RFs) in the same diagram. From this, we can also directly read out the effect of the *observational principle* on the observed time dilation. Here the specification of an observational principle, (being a result of the experimental set-up), means we state which clocks we use for time comparisons between the RFs.

We further suggest writing out the main observational principle as

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2}$$

Here t^{SC} is the clock reading of a fixed 'single clock' (SC), and t^{MC} the clock reading of the various 'multiple clocks' (MC) passing on the other RF. We find this formulation more informative than the potentially misleading phrase 'moving clock goes slower'. Actually multiple clocks on any RFs will observe that a single clock on the 'other system*'* goes slower, and a search for the one RF where time goes slower is indeed in vain. Although this is a well-known phenomenon, (and t^{SC} is often referred to as proper time), we like to demonstrate the usefulness of this formulation

Another observational principle is to permanently perform the clock comparisons at the midpoint between the origins of the two reference frames. This will give identical clock readings at the two frames. Therefore, when we apply this observational principle - being symmetric with respect to the two RFs - we also get a symmetric result! Thus, we can say that time dilation is caused by an asymmetry in the observational principle.

This midpoint can also be relevant if we consider the inclusion of an *auxiliary RF*. The main objective of introducing this could be to provide a sensible definition of simultaneity at a distance. For instance, we can then follow a SC on each of the main RFs, in the perspective of the auxiliary RF. Taking the perspective of an RF means that having the same clock readings at different locations corresponds to simultaneity.

We apply the given framework to discuss the challenges of the well-known travelling twin paradox; here seen as a thought experiment, (disregarding the acceleration periods). We find it enlightening to consider various experimental set-ups, that is, specifications on how the twins shall 'reunite', (implying a specific observational principle). A discussion of *single clock* (SC) *vs. multiple clocks* (MC) - as indicated - is the key to provide the solutions.

The outward travel becomes rather trivial, although simultaneity 'at a distance' (earth and star) is not straightforward. The return is more challenging if we allow also the earthbound twin to move relative to the earth. The result clearly depends on these choices. If we choose to let the travelling twin act as a SC system both ways, the standard solution comes out as the result rather directly. A purely symmetric set-up would require that they meet again half way between the earth and the star; actually giving that both have aged the same number of years.

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Appendix A Some additional material regarding the travelling twin example

We here present some additional material regarding the travelling twin example, thoroughly discussed in Ch. 5. In particular, we detail some of the numerical calculations. In this Appendix we let

t_1 = time on the clock of the earthbound twin

t_2 = time on the clock of the travelling twin

Similarly, the distance between earth and the 'star' is denoted x_1 = 3 light years, and since the rocket has speed, $v = (3/5)c$, we get $\sqrt{1-(v/c)^2} = 4/5$

A.1 The numerical calculation

It directly follows that in the RF of the earth/star, the rocket reaches the star at time, $t_1 = x_1/v = 5$ years. Further, the Lorentz transformation gives that at the arrival at the star this clock reads $t_2 = t_1\sqrt{1-(v/c)^2} = 4$ years; so obviously, $t_2/t_1 = \sqrt{1-(3/5)^2} = 0.8$ at the star when the travelling twin arrives. So it does follow from the LT that the two clocks at the star shows 4 and 5 years at the arrival. Further, the above presentation describes the travelling twin as a 'SC system', and so in this description the earthbound twin is located on a 'MC system'. Therefore, we can just look at *eq.* (11) and Fig. 1 to obtain the above result. These results are illustrated in Fig. 3(a) and 3(b); also showing the 'simultaneous' clock readings on the earth; *cf.* Apendix A.2 below.

Now we elaborate on this numerical example when we introduce an auxiliary symmetric RF, see illustration in Fig. 3(c) and Fig. 2, which represents the situation when the travelling twin has reached his point of destination. The above numerical values give $w^* = c/3$ (*eq.* (9)), being the speed between any twin and the auxiliary RF. Further, $\sqrt{1-(w^*/c)^2} = \frac{2}{3}\sqrt{2} \approx 0.94$.

Now apply *eq.* (4), here inserting w^* for *v,* and we obtain the following clock readings of the three RFs; observe that we utilize the notation of Fig. 2; numerical values inserted in Fig. 3(c):

i. The auxiliary reference frame (primary). Time is constant, t, (see horizontal line in figures).
ii. Earthbound twin. Time as function of w: $t_{-w^*}(w) = (\sqrt{2}/4)\cdot(3 + w/c)t$.
iii. Travelling twin. Time as function of w: $t_{w^*}(w) = (\sqrt{2}/4)\cdot(3 - w/c)t$.

In Fig. 2 we now let the observational point *b,* correspond to $w^* = c/3$; Further let time for the travelling twin equal 4 years, (*i.e.* his arrival). Thus $t_{w^*}(w^*) = 4$, which gives $t = 3\sqrt{2} \approx 4.24$ years, and then we have completely specified the clock readings by the arrival, in the perspective of the auxiliary RF.

So using this $t = 3\sqrt{2}$ in the expressions $t_{w^*}(w)$ and $t_{-w^*}(w)$ above, we will – referring to Fig.2 - get that the point *a* corresponds to 4.5 years and the point *c* corresponds to 5 years; in full agreement with the given example.

In summary, the clock reading of the auxiliary RF at this instant equals $t = 3\sqrt{2} \approx 4.24$ years. Further we found $\sqrt{1-(w^*/c)^2} = \frac{2}{3}\sqrt{2}$. So the time at point *a* equals $3\sqrt{2}/\sqrt{1-(w^*/c)^2} = 4.5$, and the time at *b* equals $3\sqrt{2}\cdot\sqrt{1-\left(\frac{w^*}{c}\right)^2} = 4$; following the standard time dilation expressions. A main objective of the auxiliary RF could be to allow us to treat the RFs of *both* twins as SC, and thus to establish a symmetric 'simultaneity at a distance'. We provide some additional comments on this in Appendix A.2.

A.2 Simultaneity at a distance

A rather relevant question should be: What event on earth is simultaneous with the arrival of the travelling twin of the star; (or simply: What does the clock on the earth show 'at this instant'). The LT does not give a definite answer regarding the simultaneity of events 'at a distance'. As this may affect how we treat the problem, we now elaborate a bit further on main options regarding this simultaneity.

First, it is to be understood that we now assume that there is also a RF of the travelling twin with the required number of clocks. Say, he is equipped with rockets at appropriate and fixed distances from his own rocket, all moving with constant speed in the same direction as himself, and all equipped with a synchronized clock showing the same time, t_2. Whether this is practically feasible is not relevant here. We are referring to the model of the TSR, and point out what this theory tells about clock readings, *if* we provide such an arrangement. Now we have established two RFs as required by the LT.

To proceed, we also introduce a symmetric auxiliary RF, *K*, with velocities $\pm w^*$, respectively, relative to the RFs of the two twins; (with w^* given in (9)). Then we can consider simultaneity in the perspective of each of these three RFs. Starting with the arrival at the star, where $t_2 = 4$, $t_1 = 5$, we identify the simultaneous event on the earth from these three perspectives; also illustrated in Fig. 3.

We present the result in Table 1. Note that on the earth it is the earthbound twins clock that acts as SC, that is here t_1 being a SC time reading, and t_2 a MC time reading. Thus, *eq.* (11) gives the result, $t_2 / t_1 = 1/0.8 = 1.25$, for all observations on the earth, whatever instant we consider after departure. Therefore, at this location it is the clock on the earth that always 'goes slower'!

First, in the perspective of the travelling twin, the clock reading of his clock equals 4 years. So when the clock of his RF (showing 4 years) passes the earth, the clock on the earth just reads $0.8{\cdot}4 = 3.2$ years; see perspective 1 in Table 1.

Next, in the perspective of the earthbound twin, we have calculated that his clock located at the star, reads 5 years by the arrival of his twin. But when his own clock on earth shows 5 years, the passing clock of the travelling twin's reference frame then shows $5{\cdot}1.25 = 6.25$ years; see perspective 2. (If this is the relevant answer, we should expect the return of the twin brother after 12.5 years.)

Table 1. Various clock readings (light years) at/on the earth, *potentially* 'simultaneous to' the arrival of the travelling twin at the star; (so, at the star we have $t_1 = 5$, $t_2 = 4$).

Clock reading at earth	Perspective of		
	1. Travelling twin	2. Earthbound twin	3. Auxiliary RF (symmetric)
Earthbound twin system (***t₁***)	3.2	5	4
Travelling twin system (***t₂***)	4	6.25	5

The third possibility is to apply the perspective of the symmetric auxiliary RF. In this perspective, we treat both clocks belonging to the twins as SC. Then we get the following symmetric result regarding simultaneity: The arrival at the star occurs when both twins observe that their own clock shows 4 years, and the adjacent clock on the other RF shows 5 years; (closely related to length contraction). By these direct measurements, they observe that the other twin at this moment apparently has aged more than himself by a factor 1.25. This gives a completely symmetric and consistent answer to the paradox. In addition to the symmetry, it is an important point here that in options 1 and 2 of Table 1 we directly follow the clock of just *one* twin, while we in option 3 follow *both* these clocks.

However, as concluded in Ch. 5, there is no unique answer here, without specifying further, the details of the process of 'reunion'.