An approach for analysing time dilation in the TSR (v6, 2018-01-12)

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Abstract. We investigate time dilation in the theory of special relativity. The location of clocks used for time readings is essential for the magnitude of the observed time dilation. We focus on three observational principles: 1) A reference frame may apply a single clock (SC); 2) It may apply multiple clocks (MC); or 3) There is a completely symmetric situation between the two reference frames. Symmetry and simultaneity are the two essential concepts of our approach. We can define simultaneity 'in the perspective' of any reference frame. This may seem of limited interest, but by also using an auxiliary reference frame – in combination with symmetry considerations – we can also define 'simultaneity at a distance'; becoming essential for our investigations. We use the obtained framework to give a thorough discussion of the travelling twin paradox, and arrive at a conclusion regarding the twins' ages, which deviates from the prevalent view regarding this example.

Key words: Time dilation, Lorentz transformation; Dingle's question; observational principle; symmetry; simultaneity; auxiliary reference frame; travelling twin.

1 Introduction

The present work explores some basic concept within the theory of special relativity (TSR). The material of Chapter 2 is mainly rather well known, but we provide a somewhat modified version of the Lorentz transformation. We pinpoint the importance of the *observational principle*, that is, the specification of which clocks to apply for the required time comparisons. The chapter provides a unified framework for the various observational principles.

Chapter 3 follows up with a discussion on the time dilation formula and the common statement that the 'moving clock goes slower'. Some authors apply the expression 'as seen' by the observer on the other reference system, perhaps indicating that it is an apparent effect, not a physical reality. Actually, Giulini (2005; Section 3.3) states: 'Moving clocks slow down' is 'potentially misleading and should not be taken too literally'. Others stress that 'everything goes slower' on the 'moving system', not only the clocks; truly stating that the time dilation represents a physical reality also under the conditions of TSR (*i.e.* no gravitation *etc*.).

As pointed out *e.g.* on Pössel's web site (Special relativity) the phenomenon of time dilation stems from the fact that clocks of the two systems have to be compared at least twice, so it cannot be the same two clocks being compared. Thus, it is the procedure related to clock comparison that decides which reference system has the time which is 'moving faster', resp. 'slower'. It is essential that a single clock on *any* reference frame goes slower than the passing clocks on the other frame.

The main topic of Chapter 4 is 'simultaneity at a distance', which is problematic in the TSR. Using the synchronized clocks of a specific reference frame we can define simultaneity of events 'in the perspective' of this reference frame. Now, the various reference frames will give different specifications of simultaneity. However, the definition of an *auxiliary reference frame* will – in combination with symmetry requirements - provide a useful tool also to specify 'simultaneity at a distance'.

In Chapter 5 we apply our approach to give a lengthy discussion of the 'travelling twin' example; claiming to give a logical conclusion regarding the twins' ages. Our result will deviate from the solution usually given in the literature. The question of *symmetry* is essential. The TSR actually describes a symmetric situation for the two systems/observers moving relative to each other, but the literature does not seem to be completely consistent regarding this. For instance (Hamilton's Homepage), regarding the travelling win, describes a symmetric situation, while (Feynman 2008) does not. We restrict to consider symmetry and argue that a symmetric situation should also provide a symmetric definition of ‘simultaneity at a distance’, (and also a symmetric solution). A failure to do so seems to have caused some confusion in the past. Thus, the given approach will challenge the current narrative on time dilation and simultaneity in the TSR.

We finally note that some authors also question the validity of the TSR and the Lorentz transformation, (*e.g.* see McCausland 2011; Phipps 2013; Robbins 2013). In particular, McCausland (2011) reviews various controversies on the topic (related to H. Dingle) during several decades, and also gives a lot of references. Robbins (2013) also treats the Bergson-Einstein controversy, dating back to 1922. Further, Phipps (2013) presents a harsh critique of the TSR modelling from the standpoint of a physicist; for instance stating that the theory fails to include causes of the relative motion and thus will not capture the inherent asymmetry of the phenomenon. The scope of the present work, however, is more restricted, accepting the validity of the TSR as a premise. Our objective is to investigate the logical implications of the Lorentz transformation and thereby provide an approach for analyzing relative time and simultaneity within the framework of the TSR.

2 The Lorentz transformation and special cases

We here present the Lorentz transformation, and further investigate a variant of it.

2.1 Basic assumptions

We base the discussions on the standard theoretical experiment: Two co-ordinate systems (inertial reference frames), K and K_v (pointing in the same direction) are moving relative to each other at speed, v. We consider just one space co-ordinate, (x-axis), and investigate the relation between space and time parameters, (x, t) on system K and the corresponding parameters (x_v, t_v) on the system K_v. Thus, we have the following ‘basic simultaneity’: At time (*i.e. clock reading*) t and position x on K we observe that time equals t_v and position equals x_v on the system K_v. We base our discussions on the Lorentz transformation, including the following specifications:

- There is a complete *symmetry* between the two co-ordinate systems, K and K_v.
- On both reference frames there is an arbitrary number of identical, synchronized clocks, located at any positions where it is required.
- At time $t = t_v = 0$, clocks at the location $x = 0$ on K and location $x_v = 0$ on K_v are synchronized. This represents the defining starting point, from which all events are measured. We refer to this essential event as the ‘point of initiation’.
- We will choose the *perspective* of one of the RFs, (here usually K), and refer to this as the *primary* system. The events that the clock reading at any position, x give the same value, t, we say that these events are simultaneous in the perspective of this frame. In contrast, when the clock readings equal t all over the primary system, the observed readings, t_v on other, ‘secondary' system, (here K_v), will depend on the location of the clock reading.
- Throughout we let SC refer to a reference frame utilizing a ‘single clock’ (or ‘same clock’), for the time comparisons with other reference frames. Similarly MC will refer to a reference frame which utilizes ‘multiple clocks’ (at various locations) for time comparisons. Usually SC means having a clock located at the origin of its frame, and this we compare with multiple clocks along the other reference frame.

2.2 The standard formulation of the Lorentz transformation

The Lorentz transformation represents the fundament for our discussion of time dilation. Note that we introduce a change of the standard notation. Rather than the usual t' and x' we will write t_v and x_v. Then the Lorentz transformation takes the form

$$t_v = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - (\frac{v}{c})^2}} \quad (1)$$

$$x_v = \frac{x - vt}{\sqrt{1-(\frac{v}{c})^2}} \tag{2}$$

Thus the position, x_v is identical to to (has the same location as) x when the clocks at this positions show time t and t_v, respectively. The formulas include the length contraction along the x-axis (inverse Lorentz factor):

$$k_x = \sqrt{1 - (\frac{v}{c})^2} \tag{3}$$

2.3 An alternative formulation

Taking the perspective of K, we may at any time t choose an observational position equal to $x = wt$, (for an arbitrarily chosen w). By inserting $x = wt$ in (1) we directly get that time on K_v at this position equals:

$$t_v = t_v(w) = \frac{1 - \frac{vw}{c^2}}{\sqrt{1-(\frac{v}{c})^2}} t \tag{4}$$

Thus to pinpoint the dependence on w we here -and when appropriate- write $t_v(w)$ rather than t_v. The new time dilation formula (4) will – for a given time, t on the primary system, K - give the time, $t_v(w)$ on the secondary system, K_v, as a linear, decreasing function of w. We can now also introduce

$$\gamma_v(w) = (1 - \frac{vw}{c^2}) / \sqrt{1 - (\frac{v}{c})^2}$$

as the 'generalized time dilation factor', valid for any location, (any $w=x/t$), *i.e.* any observational principle. Thus, we can write (4) as

$$t_v = t_v(w) = \gamma_v(w)\, t.$$

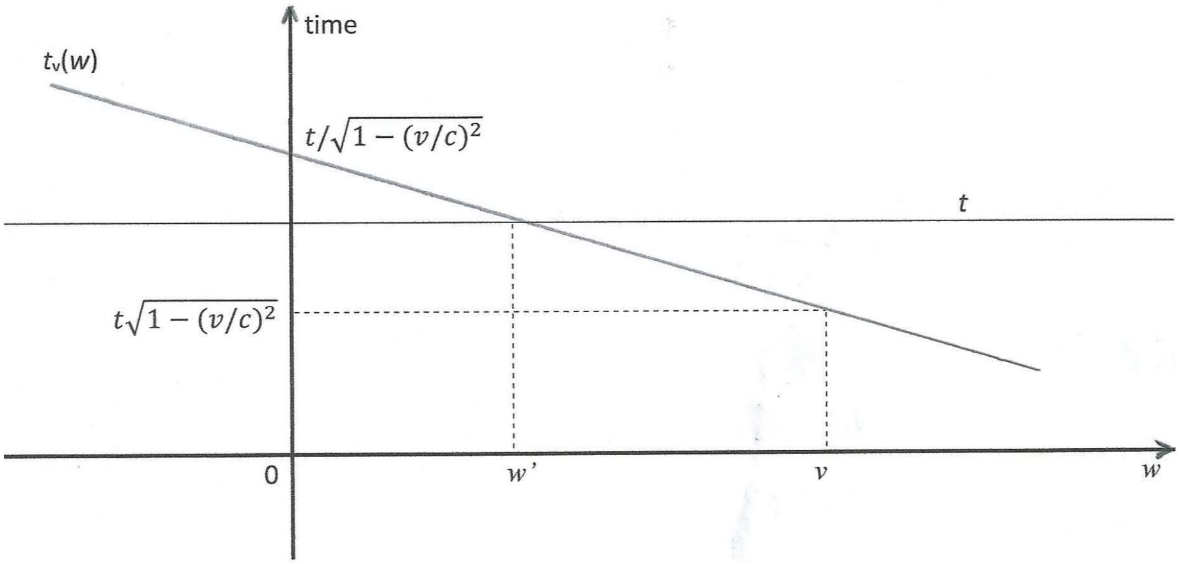

Figure 1. *Time dilation.* We have the perspective of K, and so time all over K equals, t. Time, $t_v(w)$, on K_v is given as a function of w, see *eq.* (4); (where $w=x/t$ gives the 'position' on K).

Fig.1 provides an illustration of this time dilation formula. Here we give time both on K and K_v in the perspective of K. So the figure illustrates an instant when time equals t all over this reference frame. The horizontal axis gives the 'position' $w = x/t$ at which the clock measurements are carried out. The vertical axis gives the actual clock readings. As stated, time on K equals t at any 'position', w; see horizontal line. Further, the clock readings on K_v at this instant, that is $t_v(w)$, depend on w; see decreasing straight line. Hokstad (2016) gives a more comprehensive figure.

Now, in analogy to letting $x = wt$, we also define a w_v so that $x_v = w_v \cdot t_v = w_v \cdot t_v(w)$. By inserting both $x = wt$ and $x_v = w_v \cdot t_v$, in (2), we will after some manipulations obtain

$$w_v = \frac{x_v}{t_v(w)} = \frac{w-v}{1-\frac{w}{c}\cdot\frac{v}{c}} \tag{5}$$

So equations (4), (5) represent an alternative version of Lorentz transformation, here expressed by parameters (t, w) rather than (t, x). The equation (5) has a direct interpretation. According to standard results of TSR *e.g.* (Einstein 2004; Feynman 2008; Giulini 2005; Mermin 2005) the velocities v_1 and v_2 sums up to v, given by the formula

$$v = v_1 \oplus v_2 \overset{\text{def}}{=} \frac{v_1+v_2}{1+\frac{v_1}{c}\cdot\frac{v_2}{c}} \tag{6}$$

So by defining the operator $\oplus$ this way, eq. (5) actually says that $w_v = w \oplus (-v)$, implying $w_v \oplus v = w$; thus, clearly interpreting w and w_v as velocities along the x - axis.

Note that we do not need to think of w as a velocity; rather as a way to specify a certain position $x = wt$ on K, representing the location of the clocks being applied at time t. However, we will later see that it can also be fruitful to interpret w as the velocity of a third observational reference frame.

2.4 Standard special cases

Focusing on time, *cf. eq.* (4), there are various interesting special cases. First, if a specific clock located at the origin $x_v = 0$ on K_v is compared with the passing clocks on K. These clocks on K must have position $x = vt$, and thus we choose $w=v$ and get the relation

$$t_v(v) = t\sqrt{1-(v/c)^2} \tag{7}$$

Which equals the 'standard' time dilation formula. Further, when a specific clock at the origin $x = 0$, on K is used for comparisons with various passing clocks on K_v, we must choose $w=0$ and get

$$t_v(0) = t\,/\sqrt{1-(v/c)^2} \tag{8}$$

as the relation between t and t_v. The two special cases (7), (8) are specified in Fig. 1, and we also return to these in Chapter 3. Two other special cases (*observational principles*) are obtained by inserting $w = \pm c$; *e.g.* see Hokstad (2016) and also previous ViXra versions of the present paper.

2.5 The symmetric case

Now consider another special case of the Lorentz transformation, (4), (5). We ask which value of w (and thus w_v) would results in $t_v(w) = t$. We easily find that this equality is obtained by choosing $w=w'$, where

$$w' = \frac{c^2}{v}\left(1-\sqrt{1-\left(\frac{v}{c}\right)^2}\right) = \frac{v}{1+\sqrt{1-(v/c)^2}} \tag{9}$$

By this choice of w we also get $w_v = -w'$. This means that if we consistently consider the positions where simultaneously $x = w't$ and $x_v = -w't_v = -w't$, then no time dilation will be observed at these positions. In other words (*cf.* Fig. 1):

$$t_v(w') = t \tag{10}$$

At this position $x_v = -x$, and so we see it as the midpoint between the origins of the two reference frames; thus, providing a nice symmetry. Note that when we choose the observational principle, (8), then absolutely everything is symmetric, and it should be no surprise that we get $t_v = t$.

Note that w' has a simple interpretation. Recalling the definition of the operator $\oplus$ in eq. (6) for adding velocities in TSR, $(v = v_1 \oplus v_2)$, it is easily verified that when w' is given by (9), then we get $w' \oplus w' = v$. So this confirms that when our point of observation moves with velocity w' relative to K and $-w'$, relative to K_v, it corresponds exactly to the case that the relative speed between K and K_v equals v.

3 "The moving clock": SC *vs.* MC

The special cases (7) and (8) are of particular interest, as they give the 'standard' time dilation result. Both relate the reading of the clock *at the origin* of one reference frame to the reading of a clock on the other frame at the same position. Thus, we can combine (7) and (8) into a single formula:

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2} \tag{11}$$

Here t^{SC} is the clock reading of the specific clock at the origin (of either K or K_v). Further t^{MC} is the clock reading of the clock at the same position, but on the other frame. (So, for instance, t^{SC} replaces $t_v(v)$ in *eq.* (7), and replaces $t_v(0)$ in *eq.* (8).) This is clearly demonstrated in Fig. 1. At one position the clock on K goes slower, at another it is the clock at K_v. But at both locations it is the SC that is the slower.

We may consider this relation (11) as the 'essential Lorentz transformation', and is in my opinion more useful that the rather ambiguous (7). Actually, it is much more than an effective way to write the two *eqs.* (7) and (8). By *eq.* (11) we stress that (7) and (8) actually represent the same result, and is thus more informative than (7) and (8). Actually, this choice of which reference frame shall apply a single clock is crucial, and it also introduces an asymmetry between the two reference frames.

Note that we in (11) have dropped the subscript, v in both time parameters. This just means that (11) is valid irrespective of which reference frame is chosen as SC. But, if needed, we could add a subscript v to either t^{SC} or t^{MC}, to indicate which of the systems we choose as the primary/secondary system. Thus, the pair (t_v^{SC} , t^{MC}) represents the case that the reference frame with a fixed clock at the origin is considered to be 'secondary', and (t^{SC}, t_v^{MC}) represents the other situation that it is considered the primary reference frame.

Actually *eq.* (11) illustrates the following notation: There is a SC reference frame, K^{SC}; so we utilizes just one clock on this system, giving time readings, t^{SC} on the clock located at its origin, $x^{SC} = 0$. Further, t^{MC} is the time on a MC reference frame, K^{MC}, at the position, x^{MC}.

Before we leave (11) some comments are relevant. First observers on both reference frames will agree on this result (11). Thus, it is somewhat misleading to apply the phrase 'as seen' regarding the clock reading on 'the other' system. The time readings are objective, and all observers (observational equipment) on the location in question will 'see' the same thing. The main point is rather that observers at *different reference frames* will not agree regarding simultaneity of events.

Secondly, we have the formulation 'moving clock goes slower'. It is true that an observer on a reference frame (K^{MC}), observing a *specific clock* (on K^{SC}) passing by, will see this clock going slower when it is compared to his own clocks. So in a certain sense this confirms the standard phrase 'moving clock goes slower'. However, we could equally well take the perspective of the single clock, considering this to be at rest, implying that the clocks on K^{MC} are moving. The point is definitely not that clock(s) on K^{SC} are moving and clocks on K^{MC} are not. Rather, we could look at the symmetry of the situation: We are starting out with two clocks at origin, moving relative to each other. Then the decision on which of the two clocks we will compare with a clock on the other system, (a decision that can be interchanged at random!), will decide which of the two clocks comes out as the slow one.

So, first of all, none of the clocks are more moving than the other. Further, it is the observational principle that decides which of the two clocks initially at the origin, which we observe to move slower. Therefore, I find the talk about the 'moving clock' rather misleading.

Note that the insight provided by *eq.* (11) is in no way new. Our concepts SC and MC correspond to the concepts 'proper' and 'improper' time used by (Smith 2015). In particular, *eq.* (11) equals *eq.* (3-1) of that book. However, the literature does not seem to have utilized the full potential of this relation.

We should further stress one fact. It is not required to point at one reference frame to be SC (having 'proper' time), and the other to be MC (having 'improper' time). Actually, we can specify any clock at one of the two reference frames; and if we decide to follow this clock, we will find that it goes slower than the passing clocks on the other reference frame. Thus, we may at the same time have clock(s) on *both* reference frames observed to 'go slower'. The equations just say that if we follow a specific clock, we will observe that this goes slower than the passing clocks on the other reference frame.

This point is essential, and in my opinion gives an answer to 'Dingle's question'. Dingle (1972) raises the important question of symmetry regarding the travelling twin paradox: "Which of the two clocks in uniform motion does the special theory require to work more slowly? This is an important question, which according to the discussion in McCausland (2008) has so far not been given a satisfactory answer.

However, from the above discussion it is not the case that the clock(s) on *one* of the two reference frames go(es) slower than the clock(s) on the other, (as indicated by the Dingle's question). We could very well choose to follow *both* the two clocks being at the origin at time 0; which will give that both reference frames have a clock 'going slower'. So, the result on time dilation is actually fully symmetric with respect to the two reference frames! The question is not which reference frame has a clock that 'goes slower'; it is rather which observational principle we have chosen. This fully demonstrates that it is rather inappropriate to apply the statement 'moving clock goes slower'.

Actually, Professor Dingle in his later work claimed that the TSR itself was inconsistent; see thorough discussion by McCausland (2011). However, according to McCausland (2011), Dingle again seems to have focused on the apparent inconsistency of our *eqs.* (7), (8), rather than discussing the interpretation of the more relevant *eq*. (11).

A final comment: There might be a drawback with the presentation given in Fig. 1 that it does not provide a symmetry between the two reference frames. We see everything in the perspective of one of them (K). However, we solve this in the next chapter by introducing an auxiliary reference frame.

4 Using an auxiliary reference frame of symmetry

We proceed to investigate simultaneity at a distance; still with two reference frames moving relative to each other. Primarily we elaborate on the fundamental result (11). However, we will now treat the two reference frames in a symmetric way, and so denote them K_1 and K_2, rather than K and K_v. In addition, we introduce an auxiliary reference frame. We chose this as our primary reference frame and denote it K. Thus, now we make our observations 'in the perspective' of this auxiliary K.

To get a completely symmetric situation we let K_1 having speed $-w'$ with respect to K, and K_2 having speed w' with respect to K. As the speed between K_1 and K_2 shall equal v, it follows that we define w' by (9); (*ref.* discussion at end of Section 2.5.)

Next we specify the observational principle. We choose the auxiliary reference frame to operate as MC, and so both K_1 and K_2 are SC.[1] Then single clocks at the origins of K_1 and K_2 are at any time compared with various clocks along K. Now we can apply the relation (11) between the auxiliary reference frame, K and the two reference frames K_1 and K_2, giving

$$t_v^{SC}/\sqrt{1-(v/c)^2} \ = \ t^{MC}, \quad \text{both for } v = -w' \text{ and } v = w' \tag{12}$$

It directly follows that

$$t_{-w'}^{SC} = t_{w'}^{SC} = t^{MC}\sqrt{1-(w'/c)^2} \tag{13}$$

So here $t_{-w'}^{SC}$ is the clock reading of the clock at the origin of K_1, and $t_{w'}^{SC}$ is the reading of the clock at the origin of K_2. The essential result in (13) is that $t_{-w'}^{SC} = t_{w'}^{SC}$. In the perspective of K these are now the

[1] Alternatively we could let the auxiliary reference frame, K operate as SC; but would then just obtain the same result as given in Section 2.5, and is therefore of limited interest.

simultaneous clock readings at the origins of our two 'main' reference frames K_1 and K_2, moving relative to each other at speed, v. (Of course all observers agree on this result.)

We illustrate these results in Fig. 2, which provides an analogy to Fig. 1. While Fig. 1 presented time dilation between two reference frames, taking the perspective of one of them, Fig. 2 gives a symmetric picture with respect to these two reference frames. We obtain this by introducing a third reference frame, (K), and take the perspective of this K. Therefore, Fig. 2 gives a snapshot of the clock measurements at an instant when all clocks on K read time t; *cf.* the horizontal line marked t.

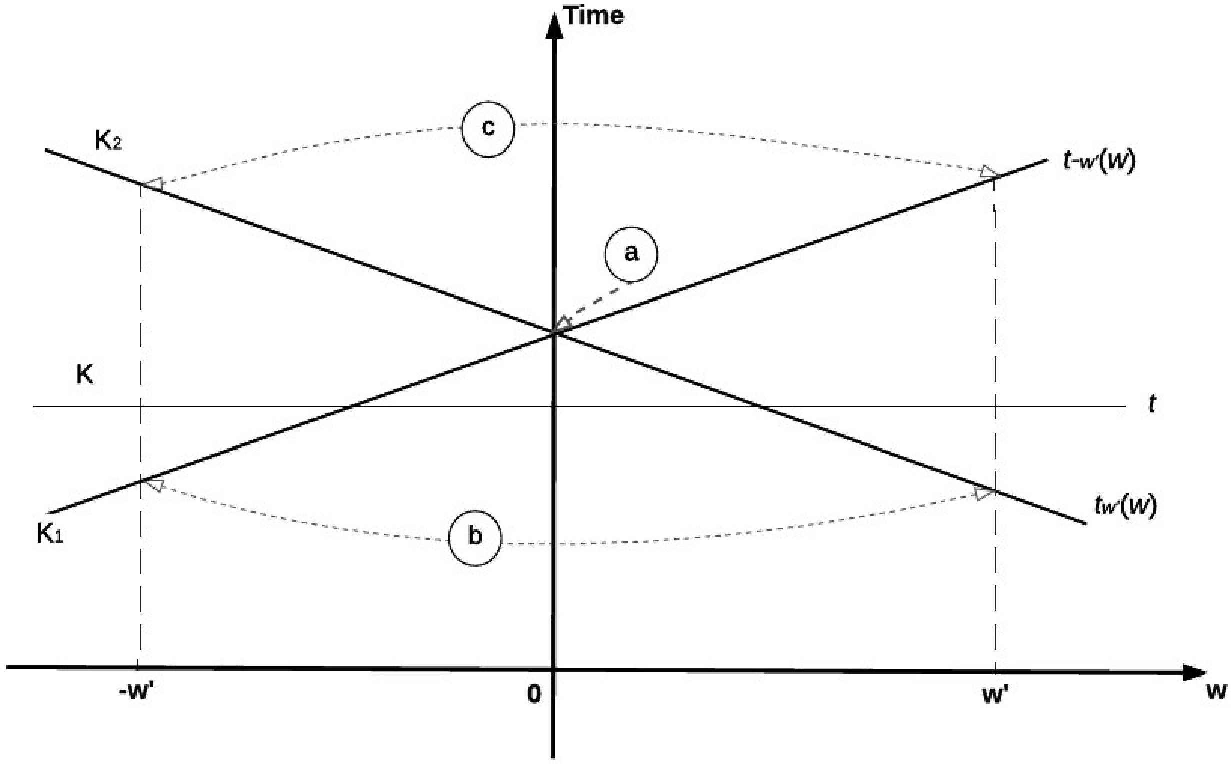

Figure 2. Clock measurements ('time') in the perspective of the auxiliary reference frame, K, where the reference frames K_1 and K_2 have speed -w' and w', respectively, relative to K.

The parameter, w (horizontal axis) refers to the 'positions' ($w = x/t$) on the auxiliary reference frame, K. The reference frames, K_1 and K_2 move relative to K at speed $-w'$ and w', respectively. Thus, the lines $t_{-w'}(w)$ and $t_{w'}(w)$ give the time measured on clocks at K_1 and K_2, respectively; as a function of w (and time t) on K. We focus on three positions on K, *i.e.* w equal to -w', 0 and w', respectively. These three values correspond to the origins of the three reference frames, K_1, K and K_2, respectively.

First, the letter a in the figure indicates the simultaneous clock readings of reference frames K_1 and K_2, observed at the origin of K. At this position the clocks on K_1 and K_2 show the same time, and are simultaneously located at the same location, w=0; so we are actually just referring to 'basic simultaneity'. For *these* measurements the reference frame K is a SC system, and its clock will appear slower than the corresponding clocks on K_1 and K_2: we observe the line t falling below the point a.

Next, the two points marked with b give identical time readings on K_1 at the position $-w'$ and on K_2 at the position w'. So in this situation the SC time readings at the origins of K_1 and K_2, that is, $t^{SC}_{-w'} = t_{-w'}(-w')$ and $t^{SC}_{w'} = t_{w'}(w')$, are identical according to (13). These origins have moved apart after time 0; so the events of these two clock readings are not simultaneous, neither in the perspective of K_1 nor in that of K_2.

However, *eq.* (13) tells that *in the perspective of the auxiliary reference frame* we have two simultaneous events. Now simultaneity in the perspective of the auxiliary reference frame may seem quite a weak form of simultaneity. However, when we also have this complete symmetry, the result becomes interesting, (and not very surprising). Actually, we postulate that this symmetric 'simultaneity at a distance represents a valid form of simultaneity. This is actually not a strong assumption. I consider this to be a consequence of the complete symmetry we have here. Claiming that the one of the events *b* occur prior to the other would represent a contradiction.

In conclusion, this seems to be the most significant result obtained by using the auxiliary reference frame: We manage to establish a (kind of) simultaneity of events at K_1 and K_2 'at a distance'. This is is a key question in a proper handling of time dilation to achieve this. Also see the discussion on simultaneity in Hokstad (2018).

Finally, in Fig. 2 we also see clock readings corresponding to the letter *c*. These exhibit the same type of symmetry/simultaneity as the points *b*, and the only difference is that the time readings at *c* will *not* correspond to the origins of the two main reference frames themselves, (but rather to the location on the other frame at the same position). A numerical example related to Fig. 2 is given in Appendix B.

The observant reader might realize that the experimental set-up given here is well suited for handling the travelling twin paradox; which we discuss in the next chapter.

5 The travelling twin

We now utilize the framework of the previous chapters to analyse the so-called travelling twin example, which goes back to Langevin (1911). As stated for instance in Mermin (2005) the travelling twin paradox shall illustrate that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together.

5.1 The problem

This paradox is indeed thoroughly discussed in the literature. Shuler (2014) informs that about 200 per reviewed academic papers with *clock paradox* or *twin*(*s*) *paradox* in their title can be identified since 1911, most of them since 1955. He comments: "An outside observer might reasonably conclude there is deep conviction that matter *should have been* settled, along with a nagging suspicion that *it is not.* He further states that "Though the correct answer has never been in doubt the matter of *how to explain* the travelling twins appears be far from settled". He also refers to the following statement: "On the one hand, I think that the situation is well understood, and adequately explained in plenty of textbook. On the other hand… there are complementary explanations which take different points of view on the same underlying space-time geometry (though, alas, the authors don't always seem to realize this, which rather undermines my assertion that the effect is well enough understood)". The last part of this statement is close to my own impression. However – as may be evident from the discussion of the previous chapter – I seriously challenge the standard answer to the paradox.

Regarding the travelling twin example, Mermin (2005, Chapter 10) gives the following numerical example: "If one twin goes to a star 3 light years away in a super rocket that travels at 3/5 the speed of light, the journeys out and back each takes 5 years in the frame of the earth. Since the slowing-down factor is $\sqrt{1-(3/5)^2} = 4/5$ the twin on the rocket will age only 4 years on the outward journey, and another 4 years on the return journey. When she gets back home, she will be 2 years younger than her stay-at-home sister, who has aged the full 10 years." So the claim is that the referred difference in ageing occurs during the periods of the journey with a constant speed; *i.e.* under the conditions of the TSR, (ignoring the acceleration/deceleration periods). After all the whole argument relies on the Lorentz transformation! Thus, our discussion will restrict to the periods of constant velocity. Now throughout this chapter, we let K_1 be the reference frame of the earthbound twin and K_2 be the reference frame of the travelling twin. Thus

t_1 = time on the clock of the earthbound twin

t_2 = time on the clock of the travelling twin

Further, the distance between earth and the 'star' equals $x_1 = 3$ light years, and the rocket has speed, $v = (3/5)c$, giving $\sqrt{1-(v/c)^2} = 4/5$. It follows that in the reference frame of the earth/star, the rocket reaches the star at time, $t_1 = x_1/v = 5$ years. The clock on the rocket is then located at origin, $x_2 = 0$ on K_2, corresponding to $x_1 = vt_1$ on K_1. Thus, the Lorentz transformation gives that at the arrival at the star this clock reads $t_2 = t_1 \cdot \sqrt{1-(v/c)^2} = 4$ years; so obviously, $t_2/t_1 = \sqrt{1-(3/5)^2} = 0.8$ at the star (and the travelling twin); and the argument is valid also for the return travel.

This is a rather convincing argument. It does follow from the Lorentz transformation that the returning clock shows 8 years when he/she returns. However, recalling the discussion of Chapters 3-4 the case is not that straightforward, and since we have made no assumption of asymmetry regarding the periods of constant velocity, we have a true paradox.

Thus, we will not question the clock of the travelling twin, but take an overall look at the total situation. First, we observe - following the notation of Chapter 3 - that the above presentation describes the travelling twin as a 'SC system', and so in this description the earthbound twin is located on a 'MC system'. Therefore, we could just look at *eq.* (11) to obtain the above result. Of course this is closely related to the length contraction (*cf.* Appendix A): The travelling twin will observe that the distance between earth and the star does not equal $x_1 = 3$ light years but just $x_2 = 3 \cdot 0.8 = 2.4$ light years; fully 'explaining' the reduction in travelling time.

However, we have the common question (*e.g.* Dingle); here rephrased using our notation: Could we not similarly describe the situation as the travelling twin being located on a MC system, and the earthbound twin on a SC system (with his single clock located on the earth). If we insist on the symmetry of the situation, the answer must be 'yes'. Thus, we now simply assume that there is also a reference frame of the travelling twin with the required number of clocks. Say, he is equipped with rockets at appropriate and fixed distances from his own rocket, all moving with constant speed in the same direction as himself, and all equipped with a synchronized clock showing the same time, t_2. Whether this is practically feasible is not relevant here. We are referring to the model of the TSR, and point out what this theory tells about clock readings, *if* we provide such an arrangement.

Now we could alternatively consider the earthbound twin as travelling back and forth along the reference frame of the travelling twin. This will now give that the one way travelling time of the earthbound twin is 4 years; while arguing as above the time passed for the travelling twin equals 5 years.

We want the earthbound twin to make an observation of *his* clocks exactly 'at the instant' when his traveling twin arrives at the star. However, the core of the problem is that we actually do not fully know how to define this moment on the earth. The Lorentz transformation does not seem to give a definite answer regarding the simultaneity of events 'at a distance'. So to discuss this further, we consider various options regarding the moment at which one should choose for observing the clocks at/on the earth.

To proceed we introduce a symmetric auxiliary reference frame, K, with velocity $\pm w'$ relative to K_1 and K_2 (with w' given in (9)). Then we can consider simultaneity in the perspective of each of these three reference frames. Starting with the arrival at the star, where $t_2 = 4$, $t_1 = 5$, we now identify the corresponding simultaneous events on the earth from each of these three perspectives.

We present the result in Table 1. Note that on the earth it is the earthbound twins clock that acts as SC, that is here $t_1 = t^{SC}$ and $t_2 = t^{MC}$, and *eq.* (11) gives the result, $t_2/t_1 = 1/0.8 = 1.25$, for all observations on the earth, whatever instant after departure we consider. So, at this location it is the clock on the earth that always 'goes slower'.

These three perspectives correspond to the arrival of the travelling twin at the star, and thus, demonstrate the problem we have to define the ‘simultaneous’ event on the earth. First, in the perspective of the travelling twin, the clock reading (‘time’) equals 4 years. But when the earthbound twin observes a passing clock showing 4 years, the reading of his own clock is just $0.8 \cdot 4 = 3.2$ years; (perspective 1).

Next, In the perspective of the earthbound twin, his clock located at the star shows, as we know 5 years by the arrival of his twin. But when his own clock on earth shows 5 years, the passing clock of the travelling twin’s reference frame shows 6.25 years; (perspective 2). Should he consider this to be the relevant answer, he would expect the return of his twin brother after 12.5 years.

Table 1. Various clock readings (light years) at/on the earth, *potentially* ‘simultaneous to’ the arrival of the travelling twin at the star; (so, at the star we have $t_2 = 4$, $t_1 = 5$).

Clock reading at earth	Perspective of		
	1.Travelling twin	2.Earthbound twin	3.Auxiliary (symmetric)
Travelling twin system (t_2)	4	6.25	5
Earthbound twin system (t_1)	3.2	5	4

The third possibility is to apply the perspective of the symmetric auxiliary reference frame. In this perspective we treat both clocks belonging to the twins as SC. Then we get the following symmetric result regarding simultaneity: The arrival at the star occurs when both twins observe that their own clock shows 4 years, and the adjacent clock on the other reference frame shows 5 years. So by these direct measurements, they observe that the other twin at this moment apparently has aged more than himself by a factor 1.25. This gives a completely symmetric and consistent answer to the paradox. In addition to the symmetry, it is an important point here that in options 1 and 2 of Table 1 we directly follow the clock of just *one* twin, while we in option 3 follow *both* these clocks.

5.2 Conclusion and further discussion

Above we presented a rather lengthy discussion about simultaneity, which is the essential question in this paradox. However, we could directly apply the approach of Chapter 4 to provide our solution to the paradox. Fig. 2 illustrates clock readings of the three reference frames, choosing the auxiliary frame, K as the primary one. In the perspective of this frame, the event ($t_2 = 4$ years, $t_1 = 5$ years) at the star is simultaneous to the event ($t_1 = 4$ years, $t_2 = 5$ years) on the earth. So by following both twins, and thus considering *both* their systems as SC, the conclusion is that both twins have aged 4+4=8 years when they meet again; (both twins are described as SC systems also on the return). Thus, we follow both clocks in a symmetric way, from the moment when they depart (having basic simultaneity) to the moment when they are again united (again basic simultaneity). So when it is often seen as paradoxical that we apparently have to ‘choose’ *one* twin to age more slowly, our solution is that they both age more slowly, as compared to ‘passing clocks’ on the other reference frame.

The reduction in the clock time, compared to the result of 10 years, is related to the length contraction, experienced by SC systems, *cf.* Appendix A. We, further refer to Appendix B for a numerical calculation related to Fig. 2, based on the above example.

So what does the literature say about this? Obviously quite a lot; as seen *e.g.* from the referred paper by Shuler (2014). Also Debs and Redhead (1995) give a thorough discussion on this case. They refer to the two asymmetries that have been the basis for most of the standard explanations. The first group of arguments focuses on the effect of different standards of simultaneity in different frames, and secondly one can designate the acceleration as the main reason for the differential aging. However, they state “… since we are dealing with flat space-time, we regard the reference to general relativity in this context as decidedly misleading”; a statement in which we agree.

Thus, Debs and Redhead (1995) follow up on discussing simultaneity, and in particular argue for the *conventionality of simultaneity.* This implies that when establishing simultaneity at a distance by the use of light signals, the definition of simultaneity is essentially a matter on convention, (any time in a certain interval can be said to be simultaneous with a distant event).

I am uncertain about the interpretation of this. We might apply a similar argument to the above approach of using auxiliary reference frames: Various reference frames at different speeds would give different results concerning simultaneity at the two 'main' frames. This might correspond to various degrees of asymmetry, (in addition to the *symmetric* solution discussed above). However, we should not interpret this to mean that all solutions are equally valid. I would rather say that we should choose the auxiliary reference frame, which corresponds to the situation we want to model. If we want to model a symmetric situation, there should also be a symmetric reference frame, as we have chosen here. Even if there are several possible definitions for simultaneity at a distance, this does not mean that all are equally valid.

So our argument regarding simultaneity is closely linked to arguments concerning symmetry. After all, we calculate the claimed magnitude of the difference in ageing by use of the Lorentz transformation, which truly exhibits symmetry. I find it hard to defend an asymmetric solution to this symmetric mathematical framework; apparently by adding some ad hoc assumptions outside the scope of the mathematical framework.

Thus, several standard arguments regarding the twins' ageing seems to have a problem in handling simultaneity. Often the standard narrative implicitly seems to assume that the arrival of the twin at the star occurs 'simultaneously' with the earthbound twin having aged 5 years (in the present example). It is true that the Lorentz transformation tells that the clock of the earthbound system, *which is located at the 'star'*, shows 5 years by the arrival of the traveling twin (and his clock reads 4 years). However, that does not imply that we can say that the clock on the earth reads 5 years 'at the same time'. In my understanding, one cannot infer such simultaneity from the TSR.

This objection applies for instance to the argumentation given in Smith (2015, Chapter 6), which also claims that the returning twin has aged less. However, the various arguments given there starts out with statements like, 'He [the earthbound twin] thinks the whole trip took *T* seconds', (where *T* corresponds to 5 years in our case), 'the earth twin knows the outbound trip took *T*/2 seconds', and 'the whole trip takes a time *T*'. I would say that a crucial question is the totality of information in principle available for the earthbound twin, as discussed in the previous section; *cf.* Table 1. Then we should not take for granted 'what he thinks', and he will not necessarily 'know' that the whole trip takes 10 years (in the earths frame). We should take a more holistic view, considering the various, apparently, contradictory observations we might have, and so the argumentation *e.g.* of Smith (2015) appears too simplistic.

Now one could say that also the solution presented here is somewhat paradoxical as it involves apparently contradictory observations for events 'at a distance'. However, this paradox is seemingly inherent in the Lorentz transformation. By accepting this as a model for how the world is 'working', I find the solution presented here consistent and logical.

6 Summary and conclusions

Starting out from the Lorentz transformation, we present an approach for analysing time dilation and simultaneity in the special theory of relativity. As a basis for the discussions, we have that

- There is a complete *symmetry* between the two reference frames.
- At time 0 the clocks at the origin of the reference frames are at the same location; then being synchronized. This represents the defining starting point of all events, and we refer to this as the 'point of initiation'.
- We may take the perspective of one reference frame and specify this to be the *primary* one. Thus, simultaneity in the perspective of this frame, means having the same clock reading at any location on this frame.
- An investigation of time dilation includes a specification of the *observational principle;* that is, we specify the location of the clocks that are used for time comparisons between the reference frames.

- We do not use the expression '*as seen*' (from the other reference frame). Observers (observational equipment) on both reference frames agree on the time readings; as these are carried out 'on location'.

Our first result is not new. However, we prefer to write the standard time dilation formula as

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2}$$

This gives a mutual relation between the two reference frames, and tells that when we follow a 'single clock' (SC) on one of the reference frames, this clock goes slower than the passing 'multiple clocks' (MC) on the other frame by a factor $\sqrt{1-(v/c)^2}$. We find this formulation more informative than the potentially misleading phrase 'moving clock goes slower'. There is no reason to see the single clock to be moving and the other clocks not. It is the observational principle that matters. In general the specification of which system(s) act as SC and which act as MC is crucial for how to comprehend this phenomenon. In more complex situations, we may consider both types of observations on both frames.

We get another observational principle by permanently performing clock comparisons at the midpoint between the origins of the two reference frames. This position gives identical clock readings at the two frames. We note that this observational principle is symmetric with respect to the two reference frames. So when we otherwise get different clock readings (in the otherwise symmetric situation), this is caused by an asymmetry in the chosen observational principle.

Some see it as a paradox that the clock on one reference frame shall go slower in a symmetric situation (*cf.* Dingle's question: 'which of the two clocks goes more slowly'). Here we stress that multiple clocks on *both* reference frames will observe that a single clock on the 'other system' goes slower. Thus, a search for the one reference frame where time goes slower is indeed in vain.

Perhaps the main element of the approach presented here is the method for (in some cases) to decide simultaneity at a distance. It is well known that simultaneity will differ *in the perspective* of the various reference frames. However, we introduce an *auxiliary reference frame,* which position at any instant is completely symmetric with respect to the two 'main' reference frames. When we consider a SC on each of the main reference frames moving relative to the auxiliary system, it follows by symmetry that identical clock reading on these correspond to simultaneity (at least in some sense). This solution has the advantage that it maintains the symmetry of the situation. Here an asymmetric solution must be seen as a contradiction.

We apply this framework to analyse the travelling twin paradox; (disregarding the acceleration periods), and present a symmetric experimental set-up, where we follow the single clocks of *both* twins. Then we arrive at the conclution that the age of *both* twins is reduced by a factor $\sqrt{1-(v/c)^2}$, as compared to the travelling time calculated at a stationary reference frame. From any SC perspective one will actually observe a length contraction, and thus also a time reduction. Further, there is a symmetry! By specifying a reference frame for each twin, we see that they will both observe this phenomenon.

Actually, under the conditions of complete symmetry it would be rather meaningless to claim a time dilation, causing different ageing at the two reference frames. Therefore, it would be required to identify conditions (departures from symmetry) that could cause time dilation to represent a physical reality. I cannot see a proper identification of such conditions in the travelling twin example. In order to observe a difference in ageing, we probably have to introduce *e.g.* gravitation/acceleration effects, and then cannot argue by the STR.

Thus, the problem with standard arguments on the travelling twin paradox seems to be a failure to handle properly the 'simultaneity at a distance'. In particular, if we want to specify a symmetric situation, also the definition of simultaneity (at a distance) should be symmetric. If the situation is not to be symmetric, the modelling should reflect this, without introducing some ad hoc assumptions to arrive at an asymmetric solution. The present paper insists on symmetry, and we introduce a special auxiliary reference frame to obtain simultaneity at a distance (in a symmetric way).

Another comment regarding our findings: An observer moving relative to a reference frame where the event takes place could be a rather unreliable observer. Various observational principles will provide him with different results. Thus, one should be careful to let such an 'outside' observer define the phenomenon, without taking an overall view and properly consider his own position.

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Appendix A   A comment on length contraction

The interpretation of x and x_v in (1) and (2) is as follows: The position x_v on K_v corresponds to the position x on K at an instant where the clock located at x_v shows time t_v and the clock at x shows time t. However, we can further give x and x_v a slightly different interpretation. Again consider a 'SC system', K_v moving at relative speed along a system, K. Now let a distance ('measure stick'), x be marked out on K in the same direction as this movement. As known, the time difference, t_v^{SC} measured on K_v for its single clock to pass this distance will imply – when we already have observed the relative velocity, v – that the length of the distance x equals:

$$x_v^{SC} = v \cdot t_v^{SC} = x\sqrt{1-(\frac{v}{c})^2} \tag{A.1}$$

(we could of course equally well say that the end points of the measure stick, x, pass the SC on K_v). So here x_v^{SC} equals the length of x on K as calculated from the clock readings on K_v. Therefore, the observed length contraction (A.1) corresponds exactly to the time dilation observed for the single clock on K_v moving relative to the fixed distance, x on K. Thus, we conclude that the travelled distance, x is shorter because of the observed time difference on the SC at K_v. Thus, the length contraction and time dilation are indeed two aspects of the same phenomenon.

Appendix B   Example of a numerical calculation

Now to familiarize a little further with the argument regarding the 'simultaneity at a distance' in the perspective of the auxiliary reference frame, we elaborate on the numerical example in Chapter 6. We use Fig. 2 as an illustration, and let this to represent the situation when the travelling twin has reached his point of destination. Using the numerical values, of Chapter 6 we will from (9) get $w' = c/3$ ($= 5v/9$), being the speed between any twin and the auxiliary system. Further, we get that the 'slowing down factor' related to this speed equals $\sqrt{1-(w'/c)^2} = \frac{2}{3}\sqrt{2} \approx 0.94$. Using this, the clock readings ('time') of the three reference frames are as follows:

i. The auxiliary reference frame (primary). Time is constant, t, (see horizontal line in Fig. 2).
ii. Travelling twin. Time a function of w: $t_{-w'}(w) = (\sqrt{2}/4) \cdot (3 - w/c)t$.
iii. Earthbound twin. Time a function of w: $t_{w'}(w) = (\sqrt{2}/4) \cdot (3 + w/c)t$.

In Fig. 2 we now let the observational point b have value 4 years, (*i.e.* the clock readings of the travelling twin at his arrival), corresponding to $w = w' = c/3$. This gives $t = 3\sqrt{2} \approx 4.24$ years. Inserting this in expressions above, we will – referring to Fig.2 - get that point a corresponds to 4.5 years and point c corresponds to 5 years; in full agreement with the given example.

The clock reading, $t = 3\sqrt{2} \approx 4.24$ years of the auxiliary (primary) system seems less relevant. The main role of the auxiliary reference frame is to allow us to treat the reference frames of *both* twins as SC, and thus to establish 'simultaneity at a distance' with respect to these.