Freefall Through a Timelike Dimension

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Abstract

In the current paper, the Universe is modeled as a spherically symmetric isotropic collection of mass falling toward a gravitational center located in the time dimension. The interior black hole solution of the Schwarzschild metric in Kruskal-Szekeres coordinates is used to mathematically describe the freefall through the time dimension where worldlines emerge from the singularity at infinite proper speed through the cosmological time dimension, slow to zero proper velocity after a finite proper time, then accelerate to infinite proper velocity (through the time, not space dimension) once more where it reaches a future singularity. It is shown that during the first phase, where the velocity decreases, space appears to contract, whereas during the second phase, where the velocity increases, space appears to expand at an accelerated rate. With a simple coordinate change, we then see that the internal Schwarzschild metric takes the form of the FRW metric, which is the metric currently used to describe the expanding Universe.

Expansion and Collapse Along a Timelike Dimension

The current Big Bang model of the Universe says that the Universe expanded from an infinitely dense gravitational singularity at some time in the past. Current cosmological data suggests that this expansion was slowing down for some time, but is now continuing to expand at an accelerated rate. The Cosmological Principle suggests that from any reference frame in the Universe, the mass distribution is spherically symmetric and isotropic. In this paper, we will model the Universe as a spherically symmetric distribution of particles that are unaffected by each other's local gravitational fields (they are far enough apart that their local gravitational fields are negligible) that expand from and recollapse to gravitational singularities located at points in time rather than space.

In simple Newtonian mechanics, if we throw an apple straight up in the air with velocity $\frac{dx}{d\tau}$, the velocity will continuously decrease until at some height $\frac{dx}{d\tau} = 0$, at which point the apple will reverse direction and fall with increasingly negative $\frac{dx}{d\tau}$ until it returns to the ground. One important fact about this projectile motion example is that at maximum height, $\frac{d\tau}{dx} = \infty$, meaning that for the instant when the velocity is zero, points in space are infinitely far apart in time since at that moment, there is no motion through space. The relevance of this will be clear when we examine the mathematics of freefall in the cosmological time dimension.

To analyze the spherically symmetric Universe freefalling through the time dimension, we need the Schwarzschild solution where the radial coordinate is the timelike coordinate. Fortunately, the interior $(r < 1)$ solution of the Schwarzschild field (throughout the paper, we will work in units with Schwarzschild radius equal to 1) gives us precisely that. For $r < 1$, the signature of the Schwarzschild metric flips and the radial coordinate becomes a dimension measuring time while the t coordinate becomes a dimension measuring space. So let us take the center of our galaxy as the origin of an inertial reference frame. We can draw a line through the center of the reference frame that extends infinitely in both directions radially outward. This line will correspond to fixed angular coordinates (θ, ϕ) in the positive direction and $(\theta + \pi, \phi + \pi)$ in the negative direction. There are infinitely many such lines, but since we have an isotropic, spherically symmetric Universe, we only need to analyze this model along one of these lines, and the result will be the same for any line.

The radial distance in this frame is kind of a compound dimension. It is a distance in space as well as a distance in time. The farther away a galaxy is from us, the farther back in time the light we currently receive from it was emitted. Fortunately the $r < 1$ spacetime of the Schwarzschild solution plotted in Kruskal-Szekeres coordinates provides us with a method to understand this radial direction. Figure 1 shows the $r < 1$ solution on a Kruskal-Szekeres coordinate chart where, in this model, the hyperbolas of constant r represent spacelike slices of cosmological time and the rays of t represent radial distances from us, such that each galaxy will move inertially along a particular t ray, where the greater the magnitude of t, the greater the distance from us. Since we chose our galaxy as the origin of the frame, our inertial worldline corresponds to the $t = 0$ line down the center of the diagram.

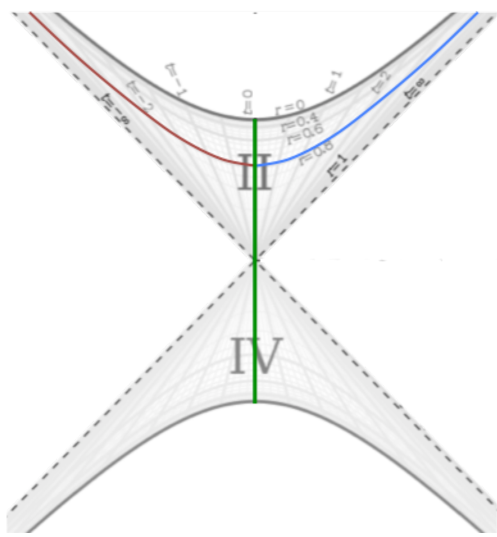

Figure 1 – Freefall Through Cosmological Time[1]

So we will use Figure 1 to describe the freefall of the galaxies through the cosmological time dimension. The worldlines will emerge from the uppermost hyperbola with some proper velocity in the cosmological time dimension, labeled by r, continue inertially

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[1] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

along lines of constant t to the center of the diagram where the proper velocity in the cosmological time dimension is zero, then continue to follow lines of constant t in the lower section until they reach the lower hyperbola where their proper velocity through the cosmological time dimension will increase to the same velocity they had when they emerged from the upper hyperbola.

To properly analyze the dynamics at play, we need to examine the Schwarzschild metric for $r < 1$. The complete metric is given by:

$$d\tau^2 = \frac{r}{1-r} dr^2 - \frac{1-r}{r} dt^2 - r^2 d\Omega^2 \tag{1}$$

Expressions for the proper time interval along lines of constant t and Ω and the proper distance interval along hyperbolas of constant r and Ω:

$$d\tau^2 = \frac{r}{1-r} dr^2 \tag{2}$$

$$ds^2 = \frac{1-r}{r} dt^2 \tag{3}$$

The forms of Equations 2 and 3 we would like to examine are given by:

$$\frac{dr}{d\tau} = \pm\sqrt{\frac{1-r}{r}} \tag{4}$$

$$\frac{ds}{dt} = \pm\sqrt{\frac{1-r}{r}} \tag{5}$$

First we should notice that neither Equation 4 nor 5 depend on the t coordinate. This is good because the t coordinate marks the position of other galaxies relative to ours. Since all galaxies are freefalling in time inertially, the particular position of any one galaxy should not matter. The proper velocity and proper distance only depends on the cosmological time r. At $r = 0$, corresponding to the beginning and end points of the freefall, the proper velocity is infinite and the proper distance to any point is infinite. So when the worldlines emerge from the upper hyperbola, they do so at the speed of light *through the time dimension*. Thus their proper velocity through time is infinite there. If something is travelling through space at the speed to light, the proper distance between points in space is zero. In this case, since we have infinite proper velocity in the time dimension, the proper distance between points in space will be infinite, because you would traverse an infinite amount of time in order to move through an infinitesimal amount of space. Thus, when the worldlines emerge from the singularity, they do so with infinite separation, as though space was infinitely expanded.

When $r = 1$, Equations 4 and 5 are both 0. Here, the proper velocity through time is now zero. This is like our previous example of throwing the apple in the air. In that case, it was noted that at maximum height, $\frac{d\tau}{dx} = \infty$ which reflected the fact that for the instant where the apple wasn't moving, the time between points in space would be infinite since at that instant, the apple's velocity through space was zero. In this case, it is our proper velocity in time that is zero. So at that instant, we are no longer moving through time and therefore all points in space are coincident. Picture a t vs. x graph with a line of constant t. All points x will be at the same time. This is analogous to being at rest in space where all points of time essentially converge to a single point in space. So this why the proper distance goes to zero there and why the lines of t in Figure 1 converge at that point; there is nothing singular there and physics does not break down, it is just an instant where our velocity through cosmological time goes to zero as our speed through cosmological time changes from negative to positive or vice versa.

So we see that during the first phase of this cycle, where the worldlines begin with an infinite proper velocity through time and ending with zero proper velocity through time, it appears as though space started out infinitely expanded and then contracted completely. The second half of the cycle will just be the reverse of this, where space will appear to expand out from a point in time and expand out infinitely.

Let us now examine redshifts during these two phases. A plot of $\frac{dr}{d\tau}$ vs. r during the 1st phase is given in Figure 2 below:

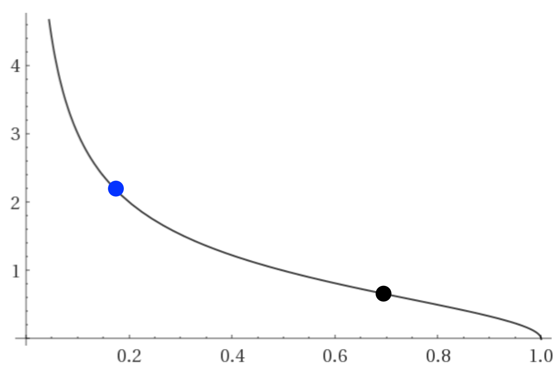

Figure 2 - $\frac{dr}{d\tau}$ vs. r During 1st Phase

In Figure 2, the right dot represents an inertial observer at some time during the expansion phase. The left dot represents the spot from which a distant signal was emitted which the black dot is receiving at its present time, so the left dot is in the right dot's past. Since $\frac{dr}{d\tau}$ represents proper velocity through time, we can use it to determine relative velocities and infer the trends of redshift from that. Notice that in this case $\frac{dr}{d\tau}$ of the left dot is greater than $\frac{dr}{d\tau}$ of the right dot. Thus, these two events have a negative relative velocity. This means that the signal the right dot receives will be blueshifted when she receives it. So during the 1st phase, past signals will be blueshifted in the frame of present inertial observers and therefore the Universe will effectively heat up as the proper velocity through time slows to a stop at $r = 1$. Also notice from Figure 1 that all past signals emitted during the expansion phase will reach the entire width of the Universe by the end of expansion (in Figure 1, light signals travel along 45-degree lines). Given this, we can logically conclude that what we think of as the Big Bang is actually the end of the first phase of the Universe and the Cosmic Microwave Background is all the unabsorbed light emitted during this phase.

A plot of $\frac{dr}{d\tau}$ vs. r during the 2nd phase is given in Figure 3 below:

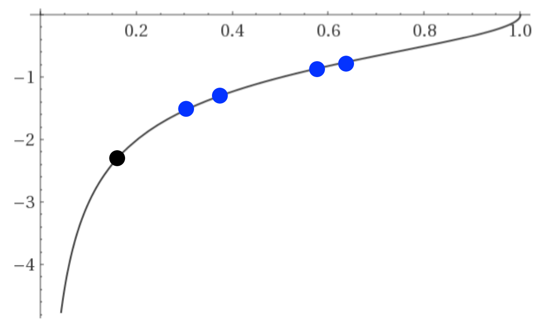

Figure 3 - $\frac{dr}{d\tau}$ vs. r During 2nd Phase

In Figure 3, time moves forward as we move right to left along the diagram. So the leftmost dot represents us at our current cosmological time in the Universe. The other dots represent galaxies at various distances from us (the farther they are to the right of the diagram, the greater the distance from us) whose signals we are currently receiving. Since the signals we receive now were emitted in the past, they were emitted from galaxies with a lower proper velocity than we have currently. Thus, the signals should all be redshifted since our proper velocity is currently greater than the velocities of the galaxies when they emitted the signals and the magnitude of the redshift should be proportional to the difference in velocity. Now note that in the region where the other dots are, the difference in velocity of the two more distant emitters is less than the difference in velocity of the closer emitters. This difference means that when we get the signals from the galaxies, the difference in redshift for the two closer galaxies will be greater than the difference in redshift from the two more distant galaxies, which looks like accelerated expansion. But on this graph, there is actually an inflection point at $r = 0.75$. That means that from $r = 1$ to $r = 0.75$, it would appear as if the Universe is expanding, but the expansion is slowing down. Then from $r = 0.75$ to $r = 0$, the Universe will look like it is expanding at an accelerated rate. This change from a negatively accelerating expansion to a positively accelerating expansion is consistent with current cosmological data. Our increasing proper velocity through cosmological time causes this spatial expansion as we accelerate through time just as our decreasing velocity through time caused a contraction of space during the 1st phase.

Figure 4 shows the past light cone of an inertial observer at a given time during the 2nd phase:

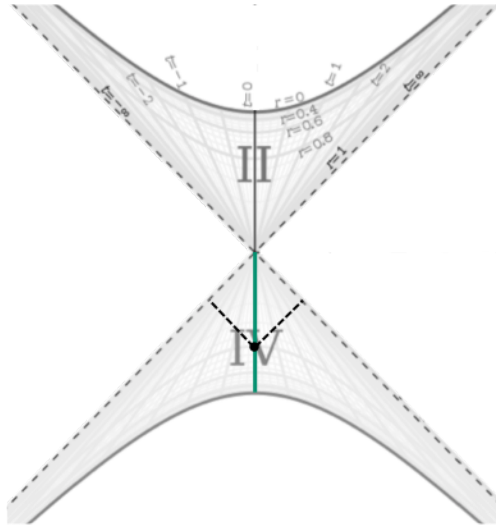

Figure 4 – Past Light Cone of Inertial Observer During the 2^{nd} Phase[2]

Notice that at all times during the second phase, the past light cone includes the entire width of the Universe. This means that the observable Universe is the entire Universe and therefore the Universe has a finite mass. From this we can conclude that the Schwarzschild radius of the entire Universe (r_s, which we have so far set to 1 in the current paper) is the Schwarzschild radius calculated from the mass of the observable Universe. We see however from Equation 5 that the proper distance to the edge of the observable Universe when $r < 1$ is infinite (because t ranges from negative infinity to infinity. This is also evident by looking at the future light cones of observers in the 2^{nd} phase). This is analogous to the distance to $r = 1$ for the external solution described in [1] where, for an observer at rest in a gravitational field, it appears as though the distance to $r = 1$ is finite even though an observer would need to traverse an infinite amount of time and space to reach $r = 1$.

We can calculate the duration of each phase of the Universe in the frame of an inertial observer by integrating Equation 4 from 0 to 1. This integral yields a value of $\frac{\pi}{2}$, but this is in units where the Schwarzschild radius is 1. The total time for each phase is therefore:

$$\tau = \frac{\pi}{2} r_s \tag{6}$$

Where r_s is measured in light-years and τ is in years (or equivalent units where the speed of light is 1). Therefore, the 1^{st} phase of the Universe (the time before what we currently call the Big Bang) must have lasted for billions of years during which the Universe heated up. So the total time, in natural units, for an inertial observer to get from the top hyperbola, down to the bottom hyperbola and back to the top is the circumference of a circle whose radius is the Schwarzschild radius of the observable Universe.

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[2] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

A plot of τ vs. r for both phases is given in Figure 5 below. It illustrates well the relationship to typical spatial projectile motion.

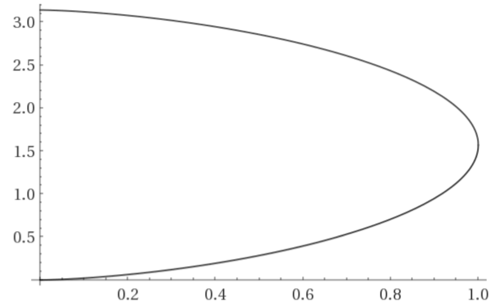

Figure 5 - τ vs. r for One Complete Cycle

As the Universe reaches the end of the 2nd phase, all worldlines will become light-like, at which point, it should be expected that the cycle would start anew. In fact, it is known that the curvature at $r = 0$ is infinite. We can explain this by saying that the worldlines reverse direction at $r = 0$ as if the spacetime is folded there and the worldlines go up one side and down the other (the infinite curvature is at the fold). So we should expect that in Figure 1, a cycle of the Universe will begin with worldlines emerging from $r = 0$, continue to $r = 1$, then continue down the bottom of the diagram (region IV in Figure 1) back to $r = 0$, then reverse and go back up, starting the next cycle. Thus, the Universe should perpetually cycle like an eternal gravitational spring.

What we see from this paper as well as [1] is that the Schwarzschild solution describes two different scenarios, neither of which is the so-called black hole. The solution for $r > 1$ describes the gravitational field when the gravitational source is a location in space for all time whereas the solution for $r < 1$ describes the gravitational field when the gravitational source is a location in time for all space, where the gravitational center is at $r = 1$ in both cases. This is why the metric signature flips at $r = 1$.

Radial Coordinate Change and the FRW Metric

We can make a coordinate change to make the metric resemble the FRW metric, which is the currently accepted metric for the Universe at-large. Basically, we want a radial coordinate whose interval is equal to the proper time interval of the inertial observer at rest ($t = const$). Thus, we can use Equation 4 to define T such that $\frac{dT}{dr} = \pm\sqrt{\frac{r}{1-r}}$. (+ on one the top half of Figure 1, - on the bottom or vice versa) Substituting this into Equation 1, we get the following:

$$d\tau^2 = dT^2 - \frac{1-r}{r} dt^2 - r^2 d\Omega^2 \tag{7}$$

In these coordinates, the proper time interval of the inertial observer at rest is just dR. The t and Ω intervals are multiplied by time-dependent functions (the r coordinate is a timelike coordinate) that play the role of the scale factors in the FRW metric for flat space. The T coordinate ranges from 0 at $r = 1$ to $\pm\frac{\pi}{2}$ at $r = 0$. As we can see, the scale factor squared in front of the dt^2 is just $\left(\frac{dr}{d\tau}\right)^2$ from Equation 4, which we have found can be interpreted as a proper velocity through time for an inertial observer.

Figure 6 is a graph of T vs. τ for an observer at rest, which shows the infinite curvature at $T = \pm\frac{\pi}{2}$:

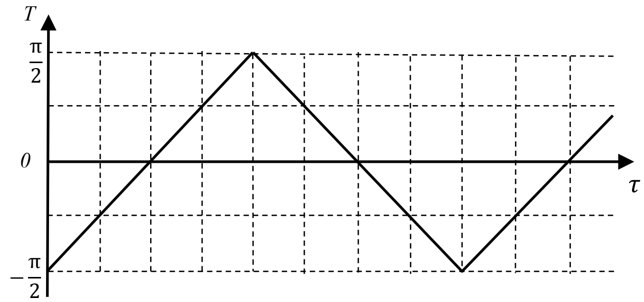

Figure 6 - T vs. τ

A Note on Galaxy Rotation

It is well known that the measured rotational velocities of the stars in the galaxies do not follow the Newtonian $\frac{1}{r^2}$ law. It has been found that the stars at the edge of the galaxy are revolving faster than is expected as though the gravity of the galaxy is stronger than what has been calculated based on the visible matter. The leading hypothesis used to explain this is that there exist unknown particles that interact with 'normal' matter only gravitationally. Direct evidence for these particles has not been found as of the writing of this paper. Consider Figure 7 as a possible explanation for this phenomenon in light of the arguments given in this paper regarding the divergence of inertial worldlines during the 2nd phase:

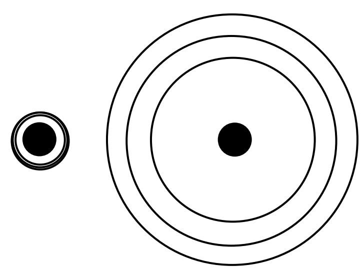

Figure 7 – Star Geodesics in a Galaxy At Different Cosmological Times

If when the galaxies formed, the stars were clustered in a tight ring around the center of the galaxy (left side of Figure 7), they would all have comparable orbital velocities. Then as cosmological time progressed, the space was stretched out, causing the geodesics themselves to stretch, such that the radii of their orbits become much larger, giving us the view of the galaxies we see today (right side of Figure 7). In this case, their orbital velocities would remain unchanged because it is the underlying space that has stretched their radii and therefore we would find that the drop off in velocity with distance could be very small. The orbital geodesics therefore tend to spiral outward over time while maintaining their orbital velocities. This hypothesis clearly needs more detailed analysis and may be flawed in its reasoning, but it is presented here to point out a possible connection.

On the Quantum Entanglement of Cycles

The following is a rough hypothesis regarding quantum entanglement between cycles of the Universe. The Everett interpretation of Quantum mechanics asserts that every possible outcome of wavefunction measurement is realized in parallel Universes. If we consider a wavefunction of two entangled particles with two possible states, it has been found experimentally that when measuring the states of both entangled particles, one particle will be found in one of the states while the other will necessarily be found in the other state. In other words, with the entangled particles, both possible states of the wavefunction will be realized when the particles are measured, even though the measurement outcome of a single particle will appear random. It is perhaps the case that the infinite cycles of the Universe are quantumly entangled such that a given cycle corresponds to one set of possible outcomes of all quantum measurements of all the wave functions. When combining all of the infinite cycles, we might find that all possible outcomes of all wavefunction measurements are realized where each set of outcomes corresponds to a particular cycle of the Universe. Figure 8 depicts the cycles in a way to visualize this entanglement hypothesis:

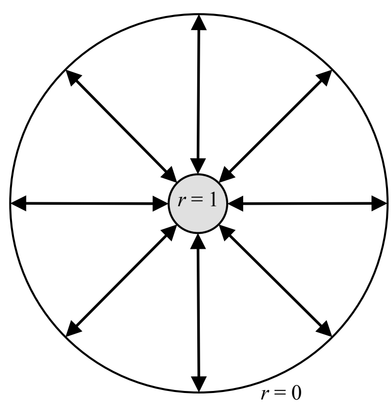

Figure 8 – Quantum Entanglement Between Cycles

For each cycle, when $r = 1$, the Universe is infinitely dense and infinitely hot. It may be here that the cycles become entangled. In Figure 7, the central circle represents the $r = 1$ state of the Universe. From there, each cycle is represented by the spokes extending out to $r = 0$ and back. So we can imagine that all cycles are common at $r = 1$, where they become entangled, and then each individual cycle will represent one set of possible outcomes of the wavefunctions. Thus, the outcomes will appear random during any one particular cycle, but in fact the outcomes are determined, where a given set of outcomes labels a particular cycle and all possible outcomes are realized when one looks at all the cycles combined.

Perhaps a better way to look at it is with Figure 9:

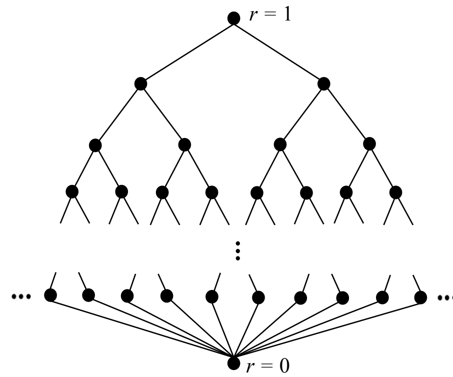

Figure 9 – Quantum Branching

At the top of Figure 9, we have the Big Bang at $r = 1$, where all cycles are identical (infinitely dense, infinitely hot). As we move toward $r = 0$, various quantum measurements are made, where the outcome of each measurement corresponds to a branch on the diagram. When we get to $r = 0$, all cycles will again be identical, where they are all infinitely sparse and infinitely cold. As we can see, if we choose any one of the lines connecting to $r = 0$, we can deterministically trace back a unique path to $r = 1$. Thus, each line connecting to $r = 0$ would represent one of the possible cycles. We see that Figure 9 appears non-deterministic when moving from $r = 1$ to $r = 0$, yet still allows for each cycle to represent a unique, determined set of outcomes.

References

[1] Laforet, C.: Integration Over Manifolds with Variable Coordinate Density.