A New approach to Special Relativity

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Abstract: The success of Special Relativity (SR) comes from the requirement of Lorentz covariance to all physical equations. The explanation with regard to the Lorentz covariance is based on two hypotheses, namely the principle of special relativity and the constancy of the speed of light. However, the statements of the principle of special relativity are various and confusing. The covariance of physical equations and the equality of inertial frames of reference are mixed up. The equality of inertial frames of reference is obvious, but the covariance of the physical equations is a more advanced requirement. Additionally, the way that the propagation property of light is placed in a central position of SR has caused people misunderstandings towards space-time, and also there is a logical circularity between the measurement of speed of light and the synchronization of clocks. These have obstructed to correctly extend the theory of space-time from an inertial frame of reference to a non-inertial frame of reference. These are the main reasons why many people criticize SR. In present paper, the two hypotheses have been discussed in detail and a new requirement to the equations of Physics has been proposed. The requirement is the Requirement of Special Completeness, namely, the physical equations used to describe the dynamics of matter and/or fields should include the descriptions that not only the matter and/or fields are at rest relative to an inertial frame of reference, but also they move relative to this frame. Basing on this requirement and the equality of the inertial frames of reference, we can approach to SR. Thereby let the theory of Lorentz covariance has a clear and solid foundation. The constancy of the speed of light is just a deduction, not a premise. The Lorentz covariance is just a characteristic of the Special Complete equations. Maxwell equations automatically satisfy the Lorentz transformations without any modification, while Newton law of gravity does not, because Newton law of gravity is not Special Complete and Maxwell equations are. The new approach has paved a road leading towards the generalizing of the theory of space-time from the inertial frame of reference to non-inertial frame of reference without considering gravitation.

Résumé: Le succès de la relativité restreinte (RR) provient de l'exigence de la covariance de Lorentz à toutes les équations physiques. L'explication en ce qui concerne la covariance de Lorentz est fondée sur deux hypothèses, à sa-

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voir le principe de la relativité et la constance de la vitesse de la lumière. Cependant, les déclarations de principe de la relativité restreinte sont diverses et confus. La covariance des équations physiques et l'égalité des référentiels inertiels sont mélangées. L'égalité des référentiels inertiels est évidente, mais la covariance des équations physiques est une exigence plus avancée. En outre, la facon dont la propriété de propagation de la lumiére dans une position centrale dans la RR a provoqué des malentendus vers l'espace-temps, et il ya aussi une circularité logique entre la mesure de la vitesse de la lumière et de la synchronisation des horloges. Celles-ci ont obstrué d'étendre correctement la théorie de l'espace-temps à partir d'un repère inertiel à un cadre non-inertiel. Ce sont les principales raisons pour lesquelles beaucoup de gens critiquent RR. En présent document, les deux hypothèses ont été examinées en détail et une nouvelle exigence pour les équations de la physique a été proposée. L'exigence est l'exigence de complétude spécial, à savoir, les équations physiques utilisés pour décrire la dynamique de la matière et / ou les champs devraient inclure les descriptions que non seulement la matière et / ou les champs sont au repos par rapport à un référentiel inertiel, mais aussi ils se déplacent par rapport ce cadre. S'appuyant sur cette exigence et l'égalité des référentiels inertiels, nous pouvons nous approcher de RR. Ainsi la théorie de Lorentz covariance a une base claire et solide. La constance de la vitesse de la lumière est juste une déduction, pas une prémisse. La covariance de Lorentz est juste une caractéristique des équations spéciales complètes. Les équations de Maxwell satisfont automatiquement les transformations de Lorentz sans aucune modification, tandis que leloide Newton sur la gravité ne fait pas, parce que les équations decette loi ne sont pas spéciales complètes mais celles de Maxwell les sont. La nouvelle approche a ouvert une route menant vers la généralisation de la théorie de l'espace-temps du cadre de référence inertiel à cadre non inertiel sans tenir compte de la gravitation.

Keywords: Special Relativity; Principle of Relativity; Completeness; Constancy of the Speed of Light; Lorentz Transformations; Covariance; Minkowski Space-time

I. Introduction

Special Relativity (SR) has already been published for over 100 years. In the last 100 years, it has already permeated through the extensive realm of the modern physics, and has become one of the corner stone of the modern physics. However, the arguments with respect to SR have not ceased since its publishing[1-12].

Why could such scene appear? The reasons that SR can obtain plenteous achievements are because it has inherited and extended the Galilean principle of relativity from single Mechanical phenomenon to all physical phenomena and it has also correctly adopted Lorentz Transformations (LT)[13]. However, Einstein's derivation and explanation of LT rely upon a procedure of clock of synchronization in which light signals are used. One easily thinks that the property of space-time is determined by the propagation property of light[14,15]. Also there is the logical circularity between the measurement of the speed of light and the synchronization of clocks. Other less well-known problem is the problem of statement regarding to the principle of special relativity or the problem how to comprehend the covariance of physical equations and the equality of inertial frames of reference. People always mix up the equality of inertial frames of reference with the covariance of physical equations. The equality of inertial frames of reference is obvious. The physical laws obtained by doing physical experiments in an inertial frame of reference will be identical with the laws obtained in another inertial frame of reference by doing the same experiment. But the covariance of physical equations is a more advanced requirement. The equality does not equal to the covariance! Actually, in order to make the physical equations covariant, we must bring some certain requirements to bear on the physical equations. We know that not all of physical equations automatically fulfill the covariance. For instance, Gauss law of Electrostatics does not satisfy Lorentz covariance. In those years, Einstein had found Newton law of gravity did not satisfy Lorentz covariance. We also know, if the physical laws can be formulated into tensor form, the physical laws must be covariant. Why? Since the equality is not equal to the covariance, we have no reason to regard the equality of inertial frames of reference as the requirement of the covariance to use. However, if not adopting the requirement of the covariance, only using the equality of inertial frames of reference, we cannot establish the theory of Lorentz covariance. Actually, we need to bring a requirement to bear on physical laws or equations to let them have the covariance. We will discuss these in details in present paper. All above problems make the foundations of SR is in fact no better than Thomson's[16] and Lorentz's[17] explanations through the use of the hypotheses of the "length contraction" and "time dilatation". These are the main reasons why many people criticize SR. These have also obstructed Einstein correctly to extend his theory of space-time from inertial frames of reference to noninertial frames of reference, because it is difficult to synchronize the clocks by using light signals in accelerated frames of reference.

In order to give the theory of Lorentz covariance or SR a solid foundation, a number of authors have paid their efforts.[4,7,8,15,18-31] Thereinto, the theory of "Special Relativity without the postulate of constancy of light" has achieved a great progress. They have obtained the Lorentz-like transformations without light. But they still realize that the equality of inertial frames of reference equals to the covariance of physical equations or laws. They believe they can propose the requirement of the covariance to the physical equations according to the equality of inertial frames of reference.

In present paper, we will discuss the two principles of SR, and put forward a new foundation to approach to the theory of Lorentz covariance or SR. It is not necessary to put the propagation property of light in the centre position of the theory, just adopt the equality of inertial frames of reference and a requirement of completeness to the physical equations or laws (see Chapter III, d). The paper is organized as follow: In Chapter II, we discuss two basic principles of SR. In Chapter III, we reiterate the definitions of space, time and inertial frame of reference, rename the principle of special relativity and give it the certain content, and propose a new requirement to the physical equations. In Chapter IV, we deduce that the physical equations that fulfill the new requirement have the universal applicability and the covariance. In Chapter V, we introduce the Lorentz-like transformations. In Chapter VI, we elaborate relationship of the tensor expression of the physical equations and the new requirement.

II. On Discussions of Two Basic Principles of SR

a. The Problems of the Statements on Principle of Special Relativity

By and large, there are two statements with respect to the principle of special relativity. The first, "all physical laws are identical in all inertial frames of reference", we will call this statement the physical statement. Einstein wrote in his book:[14] "If K is an inertial system, then every other system K' which moves uniformly and without rotation relatively to K is also an inertial system; the laws of nature are in concordance for all inertial systems. This statement we shall call the "principle of special relativity"". The second, "The physical equations are covariant under the transformations of space-time", we will call this statement the mathematical statement. Weinberg wrote in his well-known book[32] "Gravitation and Cosmology": "The new physics, consisting of Maxwell's electrodynamics and Einstein's mechanics, then satisfied a new principle of relativity, the Principle of Special Relativity, which says that all physical equations must be invariant under LT".

The physical statement is the generalization of the Galilean Principle of Relativity. The sentence can be obviously understood in a way which states that the physical laws obtained by doing physical experiments in an inertial frame of reference will be identical with the laws obtained in another inertial frame of reference by doing the same experiment. According to this comprehension, we can deduce that one can obtain completely identical Gauss law of Electrostatics in all inertial frames of reference (if Gauss law is a precise law of experiment in one inertial frame of reference). Therefore, Gauss law of Electrostatics satisfies the physical statement of the principle of special relativity. We do not think that Gauss law is only suited for the frame of reference in which the ether exist, but in all other inertial frames of reference. However, Gauss law alone (suppose that we still do not know other laws of Electromagnetism) cannot satisfy the mathematical statement of the principle of special relativity, namely, does not satisfy the Lorentz covariance. But we cannot deny that Gauss law of Electrostatics is a correct physical law.

In the past, people always thought the two statements were identical and had not realized these two statements were in fact different. The physical statement emphasizes the equality of all inertial frames of reference. No any inertial frame of reference is special. The absolute space-time does not exist. The physical statement does not relate to the relationships of the physical quantities observed in different inertial frames of reference. In other words, it does not relate to the transformations of the physical quantities between different inertial frames of reference. The mathematical statement extends the meaning of the physical statement and emphasizes the transformations of physical quantities observed the same one series of motion of matter and/or fields from different inertial frames of reference. Thus this is so different with the original meaning of the physical statement. Just as the previous analysis about Gauss law of Electrostatics, the physical statement of the relativity principle can assure that the physical laws or equations are identical in different inertial frames of reference, but cannot ensure that the physical laws or equations are covariant under the transformations of space-time. In two relatively moving inertial frames of reference, the physical quantities measured separately by observers to the same one series of motion of matter and/or fields are different. For example, a charge, in an inertial frame of reference which is at rest relative to the charge, there is an electric field; but in another inertial frame of reference which is moving relative to the charge, there is an electric field and a magnetic field. In fact, when we talk about the covariance of physical equations, we must make clear what meaning the transformations of physical quantities (or equations) are and what meaning the transformations of space-time are! The transformations of physical quantities mean the relationships of the physical quantities measured the same one series of motion of matter and/or fields in different inertial frames of reference. The transformations of physical quantities can finally be come down to the transformations of space-time (we will discuss this in detail in Chapter IV, section c). The transformations of space-time actually are the transformations of the physical quantities measured the same one series of motion of matter and/or fields in different inertial frames of reference. However, the adoption of the mathematical statement is the important factor which allowed SR to obtain many correct results. We will see that the mathematical statement is a stronger requirement than the physical statement and is a deduction of more basic premise and requirement. But if the mathematical statement is used directly as the basic premise of SR, this premise is not obvious and cannot easily be intuitively understood. It is not appropriate to use an obscure requirement as a basic premise of a theory without any further explanation. In doing so, it will usually cover up many essential meanings and hinder our understanding of physical concepts. The hypotheses of the "length contraction" and the "time dilation" respectively proposed by Thomson[16] and Lorentz[17] are just like so. This is one of the reasons that lead SR encounters criticism.

b. Adopting the Constancy of the Speed of Light as a Basic Premise Is Inappropriate

Einstein put the propagation property of light in the centre position of his theory of relativity. His derivation and explanation of LT relies upon a procedure of clock of synchronization in which light signals are used. Many cannot help but ask why the speed of light is constant and easily thinks that the property of space-time is determined by the propagation property of light.[14,15] Also there is a logical circularity between the measurement of the speed of light and the synchronization of clocks.[33] In order to synchronizing the clocks, we need to know the speed of light; but for measuring the speed of light, we need to synchronize the clocks.

At the beginning Einstein faced the controversy of lots of different opinions, he said: "The theory of relativity is often criticized for giving, without justification, a central theoretical role to the propagation of light, in that it founds the concept of time upon the law of propagation of light".[14] To this, his answer was: the constancy of the speed of light actually "neither a supposition nor a hypothesis about the physical nature of light, but a stipulation which I can make of my own freewill in order to arrive at a definition of simultaneity". According to this kind of stipulation of freewill, Einstein deduced LT and appeared to have explained the significance of LT, but again sunk into new contradiction and perplexity. That is why a number of authors have tried to deduce LT without the postulate of constancy of the speed of light.[4,7,8,15,18-31]

Another point which has been overlooked by many is that the Einstein's stipulation about the constancy of the speed of light seriously impeded him from extending his theory of space-time from the inertial frame of reference to non-inertial frame of reference. This is because the method of synchronizing clocks (in which he gave the speed of light the central role of the theory) has met a serious difficulty in the non-inertial frame of reference. In addition, the consideration that Newton's theory of gravity does not satisfy the LT, together led Einstein to establish the General Relativity which has presently met with many difficulties.[34]

III. The Basic Concepts and Principles

In order to put forward a new foundation to approach to the theory of Lorentz covariance, we are necessary to reiterate some basic concepts and principle and propose a new requirement to the physical equations or laws.

a. Inertial Frame of Reference

To cognize and describe the motion of matter and/or fields, we must first choose a frame of reference. Without a frame of reference, the motion of matter and/or fields cannot be described. The frame of reference should be a solid entity that is free from the influence of the phenomena being observed. In present paper, we will only discuss these frames of reference that the object remains at rest or in rectilinear motion of uniform speed when the object is free from any external force including the gravity. We call these frames as inertial frame of reference and others as non-inertial frame of reference. According to the definition of the inertial frame of reference, we know that all frames of reference that an object maintains at rest or in rectilinear motion of uniform speed when the object is free from any external force including the gravity are the inertial frame of reference.

There are the forces acting on all the matters in the universe, therefore, generally speaking, an absolutely ideal inertial frame of reference does not exist, but we can always find a comparatively ideal inertial frame of reference. For instance, the center of mass of Sun is a better inertial frame of reference than the center of mass of Earth. The inertial frames of reference can be seen as the ideal frames of reference which are refined from nature. To do so is advantageous for simplifying the description of the motion of matter and/or fields.

It is worthwhile to point out that many often do not distinguish the difference between the frame of reference and the coordinate system. The frame of reference is an entity which is stable and solid, but the coordinate system is an abstract frame with a scale that is established for describing and measuring the motion of the matter or fields. One can choose different coordinate systems to describe the motion of matter and/or fields in one inertial frame of reference.

b. Space and Time

After selecting a frame of reference, to observe and describe the motion of matter and/or fields, we must adopt a way to make certain the state, the extensibility and the continuity of the motion of matter or field. We call the state and the extensibility of moving matter or field as the spatial character. We call the continuity of moving matter or field as the temporal character. We notice that we cannot confirm the motion of matter or field without a frame of reference and the concepts of space and time are meaningless without matter or field and the motions of matter or field relative to a frame of reference. Space and time are the most essential properties of the motions of matter or field. From the definition of an inertial frame of reference, we know that, in an inertial frame of reference, a body without any external force acting on it will remain in rectilinear motion of uniform speed in any direction. Therefore, in an inertial frame of reference, the extension of a body in any direction is homogeneous and isotropic and all standard clocks in every point of space will flow uniformly. Namely, in all inertial frames of reference, the space-time is homogeneous and space is isotropic. Thus we can establish an isotropic and homogeneous coordinate system and describe the motions of matter and/or fields by using mathematical methods. At this point, we will not consider the problems of measuring the coordinate system and the synchronization of the standard clock in every point of space. We will discuss this later. Here we suppose that the coordinate systems have been established and the clocks everywhere have been synchronized. We know that the geometric relationship of space in an inertial frame of reference is Euclidean, isotropic and homogeneous.

c. The Principle of Equality of Inertial Frames of Reference (PEIFR)

In all inertial frames of reference, if we do the same physical experiment, we can obtain a completely identical conclusion. We call this the Principle of Equality of Inertial Frames of Reference (PEIFR). If we can obtain a physical law through a physical experiment and can express this law by the equations, we can obtain essentially identical physical equations in all inertial frames of reference. For example, in an inertial frame of reference we do the experiment in which two charges rest and act on each other, we can obtain Coulomb's Law. According to the PEIFR, if we do the same experiment in another inertial frame of reference which is moving relative to the previous inertial frame of reference, we can obtain the same Coulomb's law. The PEIFR is not relative to the transformations of physical quantities and space-time.

In fact, we can see that the PEIFR is coincident with the physical statement of Einstein's principle of special relativity. However, we give this principle a different name to avoid the confusion with the mathematical statement of the principle of special relativity.

d. The Special Completeness (SC)

In an inertial frame of reference, if we do one series of various experiments with respect to the motions of the matter and/or fields, we can summarize and deduce many physical laws and equations. For instance, in the approximate inertial frame of reference, Earth, if we do one series of various electromagnetic experiments, just like Coulomb, Faraday and Oersted etc. we can summarize and obtain the following equations:

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho , \tag{1}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} , \tag{2}$$

$$\nabla \cdot \boldsymbol{B} = 0 , \tag{3}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} . \tag{4}$$

Here, $\boldsymbol{E}$ is the electrical field strength; $\boldsymbol{B}$ is the magnetic field strength; ρ is the density of charge; $\boldsymbol{J}$ is the density of current; $c = \sqrt{\varepsilon_0 \mu_0}$ is a constant, ε_0 and μ_0 is the permittivity and permeability of vacuum respectively. These are just Maxwell electromagnetic field equations that we known well. We know that these classic electromagnetic field equations can describe the electromagnetic phenomena that the charges, the electric field and the magnetic field *rest* (static) or *move* (change) relative to an inertial frame of reference. We consider this group of equations to be Special Complete. The “Special” means that we just consider the physical phenomena in the inertial frames of reference. The “Complete” means the physical equations or laws contain some equations which not only can describe the situation that the matter and/or fields rest in an inertial frame of reference, but also can describe the situation that the matter and/or fields are moving (uniform rectilinear motion or accelerating motion) relative to the inertial frame of reference. In an inertial frame of reference, when the matter and/or fields *rest* or *move*, the characters of its behavior are different. The physical equations used to describe the dynamics of matter and/or fields should include the descriptions that not only the matter and/or fields are at rest relative to an inertial frame of reference, but also they move relative to this frame. We call this requirement as the Requirement of Special Completeness (RSC). Physical equations satisfied this requirement have the Special Completeness.

As long as the equations have included the descriptions that matter and/or fields *rest* and *move* in an inertial frame of reference, it can be used to describe the dynamics of the same matter and/or fields in different inertial frames of reference. For example, the charge Q is in space, the observer A who is at rest relative to the charge Q can only use one of Maxwell equations, Gauss law of Electrostatics, to completely describe the electromagnetic field around the charge Q, but the observer B who is moving relative to the charge Q cannot only use Gauss law of Electrostatics to completely describe, he/she has to also use other Maxwell equations. That means Gauss law of Electrostatics alone cannot provide complete descriptions in different inertial frames of reference to the same one series of motion of electromagnetic field, while the group of Maxwell equations can. So Gauss law of Electrostatics alone is not Special Complete, while the group of Maxwell equations is Special Complete. In order to completely describing the *same one* series of the motions of the matter and/or fields in different inertial frames of reference, the physical equations or laws used must include the descriptions that the matter and/or fields *rest and move* relative to an inertial frame of reference. If the physical equations only contain the description of the rest situation of the matter or field M relative to an inertial frame of reference, the observer who is moving relative to M cannot only use the physical equations to completely describe the matter or field M, because now he does not have the physical equation to describe the moving matter or field M. The "Special Completeness" is a requirement for a group of physical equations that makes them be able to describe the *same one* series of the motions of the matter and/or fields in different inertial frame of reference.

If we want to obtain the Special Complete physical equations to describe same sort of physical phenomena, in principle we can obtain them by doing the various experiments with respect to the matter and/or fields. If Faraday had our experience in his time, he would not have spent ten years trying to find the theory of magnetic field producing electric field. He should have thought of letting the magnetic field change (or quickly move) and then observing what occurred. It would not have been by chance to find that changing magnetic field produces a current. The Poisson equation in Newton dynamics is not the Special Complete equation. It can only describe a static gravitational field. We will know this is the reason why it does not satisfy SR.

IV. The Relationship of the Special Complete Equations in Different Inertial Frames of Reference

a. The Universal Applicability of the Special Complete Equations

What is called the universal applicability of the Special Complete equations is that these physical equations describing the motions of matter and/or fields are Special Complete and applicable to all inertial frames of reference.

We have pointed out that Maxwell equations satisfy the RSC. But, are Maxwell equations applicable to all inertial frames of reference? Maybe they can only be applied to a special inertial frame of reference such as Earth. This happened to be the questions which had been ardently discussed in the past and also was the motivation which led Einstein to propose the special relativity. At the end of the nineteen century, the answer to this question was not obvious. But according to the PEIFR and the RSC, the answer is simple and definite. Because according to the PEIFR, in every inertial frame of reference, we can obtain every equation of Maxwell equations, while the group of equation composed of these four equations satisfies the RSC. Therefore, Maxwell equations that are applicable in an inertial frame of reference are also applicable in all other inertial frames of reference. Maxwell equations have universal applicability. They are not only able to be used to describe the electromagnetic phenomena that the charges, electric field and magnetic field rest and move in an inertial frame of reference, but also able to be used in all inertial frame of reference. But single Gauss equation does not have the universal applicability, although, according to the PEIFR, we have the same Gauss equation in all inertial frames of reference. When observers try to describe the charge Q in space, only the observer who is at rest relative to the charge Q can complete describe the physical phenomenon by using Gauss equation, all others who are moving relative to the charge Q cannot. This point of view is not clear in Einstein's special relativity. People do not know why Maxwell equations naturally have universal applicability. We notice that, the RSC makes the physical equations be able to describe the physical phenomena that the matter and/or fields rest and move in an inertial frame of reference, while the PEIFR allows the Special Complete equations in an inertial frame of reference to be applied to all inertial frames of reference.

b. The Constancy of the Speed of Light is only a Deduction

From the universal applicability of Maxwell equations, we can reason that the propagation speed of electromagnetic wave in vacuum is constant in all inertial frames of reference, namely the speed of light is constant. This is an important character of electromagnetic motion. Because there is identical Maxwell equations in all inertial frames of reference, like Maxwell, one can solve this group of equations and obtains the propagation speed of electromagnetic waves $c = 1/\sqrt{\varepsilon_0 \mu_0}$. ε_0 and μ_0 is the permittivity and permeability of vacuum respectively. It is constant and is not related to the motion of the observer and the source of light. For a lightening, all observers who are in the inertial frames of reference which are relatively moving each other can use the same Maxwell equations to describe and obtain the same conclusion that the propagation speed of light is a constant *c*. Regarding the precision of the speed of light *c* is related with the precision of the permittivity and permeability of vacuum. The constancy of the speed of light is a deduction which is obtained from the universal applicability of Maxwell equations, not a premise! It is not necessary to put the constancy of the speed of light in a central position of the theory. Here, we even can conclude: if the propagation speed of a kind of field is always constant in an inertial frame of reference, it must also be constant relative to all inertial frame of reference, because the same propagating motion of the field can be described by the same physical equations in different inertial frame of reference. According to the propagation property of light, light can be used to measure length and synchronize clocks, no worry of the logical circularity!

c. The Essential Significance of Transformations of Space-time

Suppose two inertial frames of reference K and K' which uniformly and rectilinearly move each other, there is a charge rested relative to K'. Two observers in the K and K' observe and describe this phenomenon. The observer in frame K' can correctly describe this phenomenon just by using one of Maxwell equations, Gauss equation (1), while the observer in the frame K cannot, because he observe that the charge is uniformly and rectilinearly moving and there are not only the component of electric field but also the component of magnetic field. To this situation, one has to use other Maxwell equations to describe as well. Maxwell equations are enough and complete. Although the two observers in the frame K and K' can describe multiform electromagnetic motions which take place in space by using Maxwell equations, but the amount and the types of physical quantities observed and measured severally are different. Namely, to the observers, every physical quantity of Maxwell equations in frame K' is not one by one equal to the physical quantity of Maxwell equations in frame K, although both groups of physical quantities satisfy the same Maxwell equations. Certainly, between the frame K and K', there are the transformations relationship between the both groups of physical quantities. Time and length are base quantities. All physical quantities except other base quantities are derived from base quantities and most of them are composed of time and/or length. The differences or transformations of the physical quantities observed in two inertial frames of reference finally come down to or present as the differences or transformations of the space-time between two inertial frames of reference. Here we can also conclude that all physical quantities which are space-time independent are invariant quantities, such as mass, temperature, etc.

Essentially space and time are not independent of motion of matter or field and are essential characters of motion of matter or field. To talk about space and time without considering the motion of matter or field is meaningless. Any measurement about space and time has to compare the motions of matter or field. All standard ruler and clock are concentrative presence of complex motion of matter. Therefore the transformations of the physical quantities between two inertial frames of reference finally come down to the transformations of the space-time. This is the essential significance of the transformations of space-time. Obviously, the differences of space-time are caused by the differences of measurement of physical quantities in two inertial frames of reference. If the relative speed between two inertial frames of reference equal zero, all differences of space-time also vanish.

d. The Covariance of Special Complete Equations

The Special Complete equations are able to be applied in any inertial frame of reference and can describe the dynamics of the matter and/or fields which rest and move in an inertial frame of reference. To the same one series of motions of the matter and/or fields, although the amount and the type of physical quantities are different in different inertial frames of reference, but the Special Complete equations can be used to describe them and maintain identical form in all inertial frames of reference, namely, the physical quantities measured in different inertial frames of reference obey the same Special Complete equations or laws. The differences or transformations of physical quantities observed in different inertial frames of reference come down to the transformations of space-time between two inertial frames of reference. Thus we can conclude that the Special Complete equations maintain covariant in form under the transformations of space-time between two inertial frames of reference. We call this character as the Covariance of Special Complete Equations.

We see that the Covariance of Special Complete Equations is a deduction deduced from the Principle of the Equality of Inertial Frames of Reference (PEIFR) and the Requirement of Special Completeness (RSC). It is not a supposition! This is easier to let people to understand and accept. Here we must point out that covariant equations maybe not the whole equations that describe the motions of the matter and/or fields. For inertial frame of reference, so long as the equations are the Special Complete equations, they can satisfy the covariance. The RSC is a condition of the covariance of physical equations. The Special Completeness is the physical essence of the covariant equations. If not adopting this RSC, the physical equation cannot be ensured it is covariant, such as Gauss equation. The covariance of physical equations is a more advanced requirement, is not equal to the principle of special relativity (the physical statement), is not suitable to be used as a basic premise and is necessary to explain further! In the past, people thought the both were equal and had same meaning without any thinking.

V. The Coordinate Transformations of Space-time

In Chapter IV, we have concluded: the transformations of the physical quantities between two inertial frames of reference finally come down to the transformations of the space-time, the Special Complete equations are covariant under the transformations of space-time. What are the transformations of space-time between the inertial frames of reference? Whether is it one and only? We will discuss in this chapter.

a. The Lorentz-like Transformations of Space-time Between Inertial Frames of Reference

In arbitrary two inertial frames of reference K' and K, We establish Cartesian three-dimensional coordinate system and use standard clocks to record time by using light to measure distances and synchronize clocks (see Chapter IV, b). Now we try to find the relationship of the coordinate of space-time K(x, y, z, t) and K'(x', y', z', t'). We call a point of space-time (x, y, z, t) as an event. From the definition of inertial frame of reference, we know the space-time is homogenous and space is isotropic, one can directly obtain that this kind of relationship must be linear. Due to this linearity, we can do following standard arrangement to the coordinate systems.

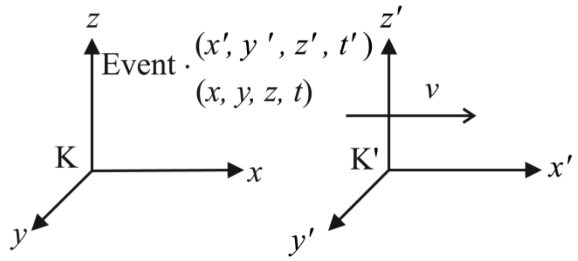

Figure 1. To describe the same event in the inertial frames of reference K and K'

The frame reference K' is moving at uniform speed v along with the x direction. The corresponding coordinate axes of the two coordinate systems always maintain parallel (Fig.1). When $t = t' = 0$, these axes superpose each other. We suppose to use the standard ruler and standard clocks of same structure. It is not difficult to state that the relative speed of the frame of reference K and K' is equal, but the direction is opposite.

Many authors have discussed and found that based only on homogeneity of space and time, isotropy of space and the principle of special relativity (both statements) without the light postulate, one can derive Lorentz-like transformations between two arbitrary inertial frame of reference K and K' with an undetermined constant.[4,7,8,15,18-31] Lorentz-like transformations are

$$x' = \gamma(x - vt)\,, \tag{5}$$

$$y' = y\,, \tag{6}$$

$$z' = z\,, \tag{7}$$

$$t' = \gamma(t - \frac{vx}{V^2})\,. \tag{8}$$

The corresponding converse Transformations is

$$x = \gamma(x' + vt')\,, \tag{9}$$

$$y = y'\,, \tag{10}$$

$$z = z'\,, \tag{11}$$

$$t = \gamma(t' + \frac{vx'}{V^2})\,. \tag{12}$$

Here, $0 < \gamma = \left(1 - v^2/V^2\right)^{-1/2}$, V^2 has no relationship with the relative speed and is an undetermined constant to all inertial frames of reference. The velocity addition law is

$$w = \frac{u + v}{1 + vu/V^2}\,, \tag{13}$$

Here we assume u and v are same direction.

We must notice that above deduction is about any event, not one idiographic motion of matter or field, therefore formulas (5) to (8) and (9) to (12) are suitable for any motion of matter or field. The Transformations of space-time between any two inertial frames of reference can be made certain and is one and only.

b. Lorentz Transformations (LT)

In principle, we can make certain the undetermined constant V^2 according to various physical experiments. For example, we can use the experiment data with regard to the lifetime of meson μ when it is moving or is at rest.[35-38] The most direct and convenient method is to adopt the property of constancy speed of light that we have already deduced in Chapter IV. Use the speed of light *c* to replace *u* and w in the formula(13), we obtain

$$V^2 = c^2. \tag{14}$$

This constant *V* is exactly equal to the speed of light *c* in vacuum. Substituting (14) to the formulas (5) to (8) and (9) to (12), we obtain familiar Lorentz transformations and its converse transformations.

Other complex method that makes certain the constant is to require Maxwell equations to be covariant with respect to the Lorentz-like transformations, thereby one can make certain the undetermined constant $V^2 = 1/\varepsilon_0\mu_0 = c^2$. In classic Electromagnetism, Maxwell equations can be wrote as d'Alembert equations (we have to remind these equations are nothing with SR and Minkowski space-time and they are classic expression) with Lorentz gauge,

$$\nabla^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu_0 \boldsymbol{J}\ , \tag{15}$$

$$\nabla^2 \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}, \tag{16}$$

$$(\nabla \cdot \boldsymbol{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} = 0). \tag{17}$$

Here, ***A*** is electromagnetic vector potential, φ is electromagnetic scalar potential. According to the Lorentz-like transformations, we can establish a 4-dimenssional space-time (***x***, *iVt*) and write the physical equations as 4-diamenssional form. We have

$$J_\mu = \left(\boldsymbol{J}, iV\rho\right), \tag{18}$$

$$A_\mu = \left(\boldsymbol{A},\ \frac{i}{V}\varphi\right), \tag{19}$$

$$\Box = \nabla^2 - \frac{1}{V^2}\frac{\partial^2}{\partial t^2} = \frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\nu}, \tag{20}$$

and

$$\Box \boldsymbol{A} = \nabla^2 \boldsymbol{A} - \frac{1}{V^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2}\ , \tag{21}$$

$$\Box\varphi=\nabla^2\varphi-\frac{1}{V^2}\frac{\partial^2\varphi}{\partial t^2}. \tag{22}$$

Combining the formulas (15) and (21), we have

$$\Box\boldsymbol{A}=\left(\frac{1}{c^2}-\frac{1}{V^2}\right)\frac{\partial^2\boldsymbol{A}}{\partial t^2}-\mu_0\boldsymbol{J}\ . \tag{23}$$

Combining the formulas (16) and (22), we have

$$\Box\varphi=\left(\frac{1}{c^2}-\frac{1}{V^2}\right)\frac{\partial^2\varphi}{\partial t^2}-\frac{\rho}{\varepsilon_0}. \tag{24}$$

Considering the formulas (18), (19), (23) and $c^2=1/\varepsilon_0\mu_0$, we have

$$\Box A_4=\Box\left(\frac{i}{V}\varphi\right)=\left(\frac{1}{c^2}-\frac{1}{V^2}\right)\frac{\partial^2 A_4}{\partial t^2}+\frac{c^2}{V^2}\left(-\mu_0 J_4\right). \tag{25}$$

Comparing the formulas (23) and (25), if we want to obtain the covariant formula

$$\Box A_\mu=-\mu_0 J_\mu, \tag{26}$$

we must have $V^2=c^2=1/\varepsilon_0\mu_0$, because the Special Complete Maxwell equations should remain covariant with regard to the Lorentz-like transformations.

We emphasize here, the constancy of the speed of light is not the premise of LT, the propagation property of light does not occupy important central position in our approach, only when we try to make certain the undetermined constant *V***,** we just find it is equal to the speed of light *c*. Actually the meaning of the constant *V* is the critical speed of moving matter or field. The speed of light is just equal to the critical speed. This is why $V^2=c^2$ and the speed of light *c* appears in LT.

VI. Lorentz Covariance and the Tensor Expression in Minkowski Space-time

a. Lorentz Covariance

The Special Complete Equations are covariant and the coordinate transformations of space-time between the inertial frames of reference are LT. Therefore, the Special Complete equations are covariant with respect to LT. We call this property as the Lorentz Covariance of Special Complete Equations.

We can see, here has provided a method to find the Special Complete equations, that is to modify the equations to make them covariant with regard to LT. We have to point out that the Special Complete physical equations might not be the whole equations of the motions of the matter and/or fields. As we know in SR, the group of Maxwell equations is Special Complete and can be formulated into the covariant tensor equations, it can be used in different inertial frames of reference to describe the same one series of motions of electromagnetic field, but it does not include the Lorentz force law. The Lorentz force law can be formulated into a covariant Special Complete equation separately. To the phenomenon that a charge is acted on a force in an electromagnetic field, Lorentz force law can completely describe and can be used in different inertial frames of reference, because Lorentz force law has included the descriptions that the charge is at rest and move. It is worthy of notice that the covariance of physical equations with respect to LT is not equal to the PEIFR. The latter is basic and the former is more advanced.

The requirement, the physical equations should be covariant with respect to LT, is adopted in SR. Actually it is forced to use as the physical statement of the principle of special relativity. It has not satisfyingly been explained to the requirement. And just this right requirement brings on Einstein's SR to obtain lots of correct conclusions and become the corner stone of modern physics.

b. The Tensor Expression in Minkowski Space-time

Minkowski proposed that time and three dimensions of space can be seen as equal and can establish a four-dimensional physical space, namely Minkowski space-time.

Using the tensors in Minkowski space-time to formulate the physical equations has completely expressed the whole meanings of the PEIFR and the RSC. The tensor equations are used to describe the dynamics of matter and/or fields and have manifest covariance. Every physical tensor is applicable to multiform moving situations. This character just satisfies the RSC. This shows that the definition of physical tensor is more general. For example, the four-dimensional electric current density is more general than three-dimensional electric current density or electric charge density. To the observer who rest relative to the charges, the charges are described only by the concept of the electric charge density. To the observer who is moving relative to the charges, the charges are described by either the concept of electric charge density or the concept of electric current density. The concept of the four-dimensional electric current density includes these two concepts and is applicable to two situations of rest and move. The covariance of the physical tensor is just corresponding with the requirement of the PEIFR; the generality of physical tensors is just corresponding with the RSC. The tensor equations are the beautiful mathematical expressions of the PEIFR and the RSC.

VII. Summary

The success of Special Relativity is because it has inherited and extended the Galilean principle of relativity from single Mechanical phenomenon to all physical phenomena and it has also correctly adopted LT. However, the foundation of Special Relativity is not beautiful and solid. First, the propagation property of light is placed in a fundamental centre position of the theory. People easily think that the property of space-time is determined by the propagation property of light.[14,15] This approach has covered up the essence of space-time. Also there is a logical circularity between the measurement of speed of light and the synchronization of clocks.[33] Second, there are problems regarding to the statement of the principle of special relativity or the comprehension of the covariance of physical equations and the equality of inertial frames of reference. The main confusion is that people always believe the principle of special relativity (the physical statement) and the covariance of physical equations are the same one principle and express the same meaning. Just as we have discussed in Chapter II (section a) and in Chapter IV (section d), they are different! We must notice the covariance of physical equations is a deduction of more basic premises, not a hypothesis. These problems have also obstructed Einstein to correctly extend the theory of space-time from inertial frame of reference to non-inertial frame of reference, thereby, has led him established the geometric theory of gravitation with many difficulties.

In present paper, we have proposed a new approach to SR. The main premises of this approach are bellow:

1. The homogeneousness of space-time and the isotropy of space (These are obtained by the definition of inertial frame of reference);
2. The Requirement of the Special Completeness to the physical equations or laws;
3. The equality of inertial frames of reference;
4. The experiments of Physics (for instance, the measurement of meson’s life-time and the experiments of classic Electromagnetism).

For the logical clue of the new approach, we can summary here. The premises 1 and 3 can deduce the Lorentz-like transformation. The premise 2 and 3 can deduce the covariance of physical equations or laws. The premises 2, 3 and 4 (Maxwell equations) can deduce that the propagation speed of light must be constancy. So the premise 1, 2, 3 and 4 (Maxwell equations) can deduce Lorentz Transformations. The premise 1, 3 and 4 also can deduce Lorentz transformations, but, at this time, we are still no reason to ask the physical equations or laws covariant under Lorentz transformations and not clear what is the signification of the transformations of space-time. We must add the RSC. That will be perfect. The new foundation is simple and obvious, so can avoid the confusions and arguments, and let us profoundly understand the significance of space-time.

In Chapter III, we have reiterated the definitions of space, time and inertial frame of reference, renamed the principle of special relativity as the principle of the equality of inertial frames of reference in order to avoid the confusion, and proposed a new Requirement of Special Completeness (RSC) to the physical equations in order to make the physical equations can describe the same one series of motion of matter and/or fields (for instance, an charge or a magnetic field) in different inertial frames of reference. In Chapter IV, according to the equality of inertial frames of reference and the RSC, we have deduced that the Special Complete equations have the universal applicability and the covariance. Further, combining Maxwell equations, we conclude that the speed of light must be constant in all inertial frames of reference. As for what are the transformations of space-time, in Chapter V, we have introduced the Lorentz-like transformations with an undetermined constant that many authors have discussed and obtained only using the homogeneousness, isotropy and the principle of special relativity (both statements) without light. Further, we have proposed several ways to determine the undetermined constant. In Chapter VI, we have elaborated that using the tensors of Minkowski space-time to formulate the physical equations has completely expressed the whole meanings of the PEIFR and the RSC.

The covariant equations are not always the laws of Physics, and the laws of Physics maybe not covariant. The Special Complete equations are the covariant physical equations, but maybe not whole equations of the dynamics of the matter and/or fields. The RSC is the physical condition of the covariance of physical equations. The Special Completeness is the physical essence of the covariant equations. If not adopting this RSC and only using the PEIFR alone, the physical equation, such as Gauss equation, cannot be ensured it can be used to describe the same one series of motion of matter and/or fields (like a charge or a magnetic field) in different inertial frames of reference, and taking about the transformations of the physical quantities and the covariance of equations is out of the question. The differences or transformations of the physical quantities observed in two inertial frames of reference finally come down to or present as the differences or transformations of the space-time. This is the physical significance of Lorentz Transformations. LT is none of business of the constancy of speed of light! The constancy of speed of light is a deduction, not a premise! Maxwell Electromagnetism automatically satisfies LT without any modification, while Newton law of gravity does not, because Newton law of gravity is not the Special Complete and Maxwell's is.

The new approach to SR has paved a road for establishing the theory of space-time in non-inertial frames of reference. It is not difficult to extend the Special Completeness to the General Completeness that has considered the structural characters of the space-time of non-inertial frame of reference. According to this extension and the principle of general equality that has considered the structural characters of the space-time of frames of reference, it is not difficult to establish the theory of space-time of non-inertial frame of reference without considering with gravitation. We will publish this theory after present paper.

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