The four-dimensional spacetime with the mass density

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Abstract: *Until the early twentieth century, the three-dimensional space and one-dimensional time were considered separate beings. In 1909, German mathematician H. Minkowski connected together space and time into single idea, creating a new the four-dimensional spacetime. We proposed the extension of this idea by the connection together the Minkowski four-dimensional spacetime and the mass density into* ***the single idea****, creating* ***a new entity****: the four-dimensional spacetime with the mass density. In this article we discussed the physical consequences of this new idea in terms of the gravitational phenomena.*

1. Introduction

Until the early twentieth century, the three-dimensional space and one-dimensional time were considered separate beings. In 1909, German mathematician H. Minkowski connected together space and time into single idea, creating the four-dimensional spacetime [1]. The idea of the spacetime enjoyed success in the *Special Relativity* (SR) and the *General Relativity* (GR), correctly describing a range of physical phenomena.

In this paper we propose extension of this idea by the connection together the Minkowski four-dimensional spacetime and *the mass density* into **the single idea**, creating a new mathematical structure: *the four-dimensional spacetime with the mass density* or *the spacetime continuum with the mass density*.

The concept of the spacetime continuum and mater was first considered by A. Einstein in 1950. He couldn't imagine an empty spacetime to describe physical phenomena [2].

We expect that idea of the spacetime continuum with the mass density, presented in this paper, will allow us to solve the problem of the sources of inertia.

For our consideration we assume that under influence outer gravitational field *the bare mass density* ρ^{bare} becomes *the effective mass density*. The spacetime continuum with the effective mass density let us call *the effective spacetime*, while the spacetime continuum with the bare mass density we call *the bare spacetime*. The idea of the effective or the bare spacetime continuum we call *the massification of spacetime*.

2. The effective spacetime

The metric of the effective spacetime we mathematically defined

$$ds^2\left(\rho_{\mu\nu}(x)\right) \overset{def}{=} \frac{\rho_{\mu\nu}(x)}{\rho^{bare}} \cdot dx^{\mu} dx^{\nu} \tag{1}$$

where: $\rho_{\mu\nu}(x)$ is the symmetric and position dependent the effective mass density tensor.

This tensor describes the physical and geometrical properties of the effective spacetime and also the mathematical relationship between the effective medium and the bare medium under the influence of the all gravitational and inertial fields.

Note that ρ^{bare} never reaches zero, although it may be very close (see to chapter 7). In the contrast to the vacuum, the effective or the bare spacetime **is a never empty**.

The concept of the effective mass tensor of the body plays important role in the contemporary physics, e.g. is well-known in the solid-state physics. This concept is a very attractive because the equations of the motion includes full information about all existing fields (for example gravitational, electro-magnetic, etc.) surrounding the body, without their exact analysis. The effective mass can be isotropic or anisotropic, positive or negative. The concept of the effective mass tensor to describe gravitational phenomena, instead of usual metric tensor, for the first time was discussed in [3].

Note that, in a some sense, the effective mass density tensor $\rho_{\mu\nu}(x)$ and the metric (1) **are very similar** to the metric tensor $g_{\mu\nu}(x)$ and the metric

$$ds^2(g_{\mu\nu}(x)) = g_{\mu\nu}(x) \cdot dx^{\mu} dx^{\nu} \tag{2}$$

known from GR.

3. The bare spacetime

In absence of the outer gravitational field $\rho_{\mu\nu}(x) \to \rho_{\mu\nu}^{bare}(x)$, where: $\rho_{\mu\nu}^{bare}(x)$ *is the bare mass density tensor* and the metric (1) takes the form

$$ds^2\left(\rho_{\mu\nu}^{bare}\right) \overset{def}{=} \frac{\rho_{\mu\nu}^{bare}(x)}{\rho^{bare}} \cdot dx^{\mu} dx^{\nu} . \tag{3}$$

So determined the bare spacetime is equivalent to *the field of inertia*, which is a special case of the gravitational field. This field is responsible for the inertia of the body and is described by the tensor $\rho_{\mu\nu}^{bare}(x)$.

Particles behave in accordance with *the principle of inertia*, i.e. they are at rest or moving in a straight line at constant speed with respect to the bare spacetime (not with respect to the massless spacetime itself).

If we assume that $\rho_{\mu\nu}^{bare}(x) = const$, then our bare spacetime is the homogeneous, isotropic and time independent and the bare mass density tensor we defined

$$\rho_{\mu\nu}^{bare} \overset{def}{=} \rho^{bare} \cdot \eta_{\mu\nu} = \mathrm{diag}(\rho^{bare}, -\rho^{bare}, -\rho^{bare}, -\rho^{bare}) \tag{4}$$

where: $\eta_{\mu\nu}$ is *the Minkowski tensor*, $\mu, \nu = 0, 1, 2, 3$.

According to equation (4) the metric (3) becomes *the Minkowski metric*

$$ds^2(\eta_{\mu\nu}) = \eta_{\mu\nu} \cdot dx^{\mu} dx^{\nu}. \tag{3a}$$

The metric (3a) does not depend *explicitly* on the bare mass tensor and well suited to describe all the physical phenomena occurring in SR. In the homogeneous, isotropic and time independent the bare spacetime the field of inertia is described by the Minkowski tensor $\eta_{\mu\nu}$ [2].

During the motion with respect to the bare spacetime, clocks and roots indicate the different time and length, than at the rest. This difference results is from the change of the bare mass density of these measuring instruments.

Let's analyze the motion of the body in an effective spacetime and we compare this equation with the Newtonian equation.

4. The equation of motion in the effective spacetime

The Lagrangian function for the body in the effective spacetime has form

$$L = \frac{1}{2}\rho_{\mu\nu}(x)\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}$$

The equation of motion

$$\frac{d}{d\tau}\left(\rho_{\mu\gamma}(x) \cdot \frac{dx^{\mu}}{d\tau}\right) - \frac{1}{2}\frac{\partial \rho_{\mu\nu}(x)}{\partial x^{\gamma}}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} = 0 \tag{5}$$

where: $p_{\gamma}(x) = \rho_{\mu\gamma}(x) \cdot \dfrac{dx^{\mu}}{d\tau}$ is *the effective density of the four-momentum*, τ is *the proper time*. This is *the geodesic equation* in the effective spacetime (see also the equation (8)).

The equation of motion (5) *explicitly* refers to the effective spacetime, which is described by the effective mass density tensor $\rho_{\mu\nu}(x)$. So the motion of the body takes place **only** in relation to the effective spacetime, not to the relation of the spacetime itself or all bodies in the Universe (*Mach's Principle* [4], see the Summary). The new quality of the understanding has been reached.

When $\rho_{\gamma\nu}(x)$ does not depends *explicitly* on τ, the equation (5) takes the form

$$\rho_{\mu\gamma}(x)\frac{d^2 x^{\mu}}{d\tau^2} + \Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}(x)) \cdot \frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} = 0 \tag{6}$$

where:

$$\Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}(x)) \overset{def}{=} \frac{1}{2}\left(\frac{\partial \rho_{\gamma\mu}(x)}{\partial x^{\nu}} + \frac{\partial \rho_{\gamma\nu}(x)}{\partial x^{\mu}} - \frac{\partial \rho_{\mu\nu}(x)}{\partial x^{\gamma}}\right) \tag{7}$$

assuming that the condition

$$\frac{\partial\rho_{\mu\gamma}(x)}{\partial x^{\nu}}\cdot\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}=\frac{1}{2}\left(\frac{\partial\rho_{\mu\gamma}(x)}{\partial x^{\nu}}+\frac{\partial\rho_{\nu\gamma}(x)}{\partial x^{\mu}}\right)\cdot\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}$$

is satisfied.

The equation (7) is very similar to *the Christoffel symbols of the first kind*, where instead metric tensor $g_{\mu\nu}(x)$, we apply the effective mass density tensor $\rho_{\mu\nu}(x)$. This is an interesting result because the Christoffel symbols describing *the metric connection*, while the equation

$$\frac{d}{d\tau}\left(g_{\gamma\nu}(x)\frac{dx^{\nu}}{d\tau}\right)-\frac{1}{2}\frac{\partial g_{\mu\nu}(x)}{\partial x^{\gamma}}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}=0 \tag{8}$$

is *the geodesic equation* in GR. Note that equations (5) and (8) are very similar to themselves.

If the surrounding bodies consist only with the bare masses, i.e. $\rho_{\mu\nu}(x)=\rho_{\mu\nu}^{bare}$, $\Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}^{bare})=0$ then the equation of motion (6) takes the form:

$$\rho_{\mu\nu}^{bare}\frac{d^2x^{\nu}}{d\tau^2}=0\,. \tag{9}$$

The body with the bare mass density $\rho_{\mu\nu}^{bare}$ is in the rest or moves in a straight line with the constant velocity in the respect to the bare spacetime. The principle of inertia has gained a new meaning and the equation (9) determines the new inertial reference frame – *the bare medium reference frame*. This reference frame is determined by the bare spacetime property only. The measuring instruments: clocks and roots now have the mass density. Old traditional reference system consisting with the massless rods and clocks loses its physical sense.

During any change in state of motion of the body appears the inertia, which source is the massification of spacetime. The inertia becomes an intrinsic property of the massification of spacetime. The magnitude of the inertia of any body is also determined by the massification of spacetime. This is the opposite of that, than previously thought. Until now it was thought that inertia is determined by the masses of the Universe and by their distribution [5]. In our model an isolated the body in the Universe always has inertia, because the spacetime with the bare mass density ρ^{bare} formed an inseparable whole. The spacetime ceased to be empty. In the particular case the equation (9) becomes

$$\rho^{bare}\frac{d^2x^{i}}{d\tau^2}=0\,. \tag{9a}$$

where: i = 1, 2, 3.

5. The weak gravitational field

In the weak gravitational field we can decompose the effective mass density tensor $\rho_{\mu\nu}(x)$ to the following simple form: $\rho_{\mu\nu}(x)=\rho_{\mu\nu}^{bare}+\rho_{\mu\nu}^{*}(x)$, where: $\rho_{\mu\nu}^{*}(x)<<1$ is a very small perturbation.

In a weak gravitational field the metric (1) takes form:

$$ds^2\left(\rho^*_{\mu\nu}(x)\right)=\left(\eta_{\mu\nu}+\frac{\rho^*_{\mu\nu}(x)}{\rho^{bare}}\right)\cdot dx^\mu dx^\nu \tag{10}$$

5.1. The equation of motion

At slow motion speeds, in a static, a weak and the spherically symmetric field the equation of motion (6) reduces to

$$\rho^{bare}\left(1+\frac{\rho^*_{rr}(r)}{\rho^{bare}}\right)\cdot\frac{d^2r}{dt^2}\cong-\frac{c^2}{2}\cdot\nabla\rho^*_{00}(r) \tag{11}$$

where: c is the speed of light.

The equation (11) is a **different** than well-known Newton's equation of the motion for the gravity. It what currently we consider to be *the inertial mass density*, really is the sum of the bare mass density ρ^{bare} and $\rho^*_{rr}(x)$ - *rr*-component of the very small perturbation in the effective mass density. Note that *the gravitational mass density* does not appear *explicitly* in the equation (10).

Does it mean that during massification of the spacetime *the Equivalence Principle* lost its *raison d'être*?

Dividing both sides of the equation (11) by ρ^{bare} (on the assumption that $\rho^*_{rr}(r)\to 0$), we get

$$\frac{d^2r}{dt^2}\cong-\frac{c^2}{2}\cdot\nabla\left(\frac{\rho^*_{00}(r)}{\rho^{bare}}\right) \tag{11a}$$

This is a new form of the equation of motion for the body in the gravitational field. This equation is very similar to the equation known from GR

$$\frac{d^2r}{dt^2}\cong-\frac{c^2}{2}\cdot\nabla h_{00}(r) \tag{11b}$$

where $h_{00}(r)$ is the small perturbation in the metric tensor $g_{\mu\nu}(x)$.

According to *the Correspondence Principle* (*CP*) we expect that there is a relationship between the component $\rho^*_{00}(r)$ and the gravitational potential $V(r)$ in the following form [6]

$$\frac{\rho^*_{00}(r)}{\rho^{bare}}\cong\frac{2V(r)}{c^2} \tag{12}$$

where: $V(r)=\dfrac{GM}{r}$, G is the gravitational constant, M is the mass and r is the distance. After substituting (12) to (11), we obtain

$$\frac{d^2r}{dt^2}=-\frac{\partial V(r)}{\partial r} \tag{13}$$

the well-known Newtonian equation of motion in the gravitational potential $V(r)$.

5.2. The rotating body

Let' s consider the slowly rotating body in a static and weak gravitational field. The equation of motion have the form

$$\rho^{bare} \cdot \frac{d^2 x^i}{dt^2} = -\frac{c^2}{2} \frac{\partial \rho_{00}^*(x)}{\partial x^i} + c \cdot \left(\frac{\partial \rho_{0k}^*(x)}{\partial x^j} - \frac{\partial \rho_{0j}^*(x)}{\partial x^k} \right) \frac{dx^i}{dt} \tag{14}$$

The following mathematical expressions are responsible for **the real** sources of inertia: $\frac{\partial \rho_{00}^*(x)}{\partial x^i}$ and $\frac{\partial \rho_{0k}^*(x)}{\partial x^j} - \frac{\partial \rho_{0j}^*(x)}{\partial x^k}$. The first term is a force, which is appearing due to the existence of a gradient in the small perturbation of the 00-component in the effective mass tensor $\frac{\partial \rho_{00}^*(x)}{\partial x^i}$. The second one is velocity-dependent $\frac{dx^i}{dt}$ and the rotation $\frac{\partial \rho_{0k}^*(x)}{\partial x^j} - \frac{\partial \rho_{0j}^*(x)}{\partial x^k}$.

For the Newtonian approximation the suitable components we can determine from the matrix [7]

$$\rho_{\mu\nu}^*(x) = \rho^{bare} \cdot \begin{pmatrix} -\frac{\omega^2 (x^2 + y^2)}{c^2} & \frac{\omega y}{c} & -\frac{\omega x}{c} & 0 \\ \frac{\omega y}{c} & 0 & 0 & 0 \\ -\frac{\omega x}{c} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

where $r^2 = x^2 + y^2$.

Finally, we get well-known equation for the slowly rotating body in a static and weak gravitational field, which includes *the centrifugal* and *Coriolis acceleration*.

$$\frac{d^2 r}{dt^2} = -\omega^2 r - 2\omega \frac{dr}{dt} \tag{15}$$

In the Newtonian approximation the equations of motion (13) and (15) does not depend, *explicitly*, on the mass density and the centrifugal and Coriolis forces are the fictitious forces.

6. The rotating bucket with water problem

There are two entirely different measurements of the Earth's angular velocity, *astronomical* (from upper culmination to upper culmination of the star) and *dynamic* (by means of Foucault's pendulum experiment), which give *the same results* (in the limit of the experimental errors). In both cases the motion of the body is described with respect to the effective spacetime and the coincidence of these measurements is the result of massification of the spacetime.

In the famous experiment with *the rotating bucket with water* [8, 9] the motion of water takes place also to relative of the effective spacetime, therefore the surface of water takes the shape of the parabolic. So, massification of the spacetime explains both these physical phenomena.

7. What is the bare and effective mass density?

Each theoretical model must correspond with the real of the physical world. We suppose that the bare mass density ρ^{bare} **probably corresponds** with *the critical density* $\rho_c = \frac{3H^2}{8\pi G}$ [7], where: H is the Hubble constant. This term is use in the modern cosmology to determine the spatial geometry of the Universe, where ρ_c is the critical density for which the spatial geometry is flat (or Euclidean).

The flat spatial geometry in SR corresponds with the bare spacetime in our model. The curved spacetime in GR corresponds with the effective spacetime. The Universe around us is almost homogeneous and isotropic (we omitted concentrations forming a hierarchy of stars, galaxies and clusters of galaxies). The Universe around us is flat.

8. In the search of a field equations

In his book [4] A. Einstein wrote: *This is the reason why E. Mach was led to make the attempt to eliminate space as an active cause in the system of mechanics. According to him, a material particle does not move in not accelerated motion relatively to space, but relatively to the centre of all the other masses in the Universe; in this way the series of causes of mechanical phenomena was closed, in contrast to the mechanics of Newton and Galileo. In order to develop this idea within the limits of the modern theory of action through a medium,* ***the properties of the space-time continuum which determine inertia must be regarded as field properties of space****, analogous to the electromagnetic field. The concepts of classical mechanics afford no way of expressing this. For this reason Mach's attempt at a solution failed for the time being.*

In this paper we presented that the inertia is shown as field properties of the effective spacetime. Tensor $\rho_{\mu\nu}(x)$ is also a geometric and the physical object. Geometrically it determines the relations metric (1) of spacetime with the mass density and physically responsible for gravitational field.

Component of $\rho_{00}(x)$, in a weak field, is expressed by the equation

$$\nabla\left(\frac{\rho_{00}(x)}{\rho^{bare}}\right) = \frac{8\pi G}{c^2} \cdot \rho(x) \tag{16}$$

where $\rho(x)$ is the mass density of the body under influence the gravitational field. According to the *CP* (the equation (12)), the equation (16) becomes the Poisson equation for the gravity.

$$\nabla V(x) = 4\pi G \rho(x) \tag{17}$$

With the effective mass tensor $\rho_{\mu\nu}(x)$, we can define *the curvature tensor* $R_{\mu\nu\lambda\eta}(\rho_{\mu\nu}(x))$ of this spacetime with the mass density. The influence of the gravitational field on the spacetime with the bare mass can be very complicated, thus the mathematical structure of $R_{\mu\nu\lambda\eta}(\rho_{\mu\nu}(x))$ can be a very sophisticated. We must therefore find the differential equations that govern of the spacetime with the mass density under the influence of the variable gravitational fields.

Here is the field equation for $\rho^{bare} = 1$, without mathematical proof

$$R_{\mu\nu}(\rho_{\mu\nu}) - \frac{1}{2}\rho_{\mu\nu} \cdot R(\rho_{\mu\nu}) \cong \frac{8\pi G}{c^4} \cdot T_{\mu\nu}(\rho_{\mu\nu}) \tag{18}$$

where: tensor $R_{\mu\nu}(\rho_{\mu\nu})$ and scalar $R(\rho_{\mu\nu})$ are the essentially unique contractions of the curvature tensor, $R_{\mu\nu}(\rho_{\mu\nu}) \equiv \rho^{\lambda\kappa} \cdot R_{\mu\lambda\nu\kappa}(\rho_{\mu\nu})$, $R(\rho_{\mu\nu}) \equiv \rho^{\mu\nu} \cdot R_{\mu\nu}(\rho_{\mu\nu})$, $T_{\mu\nu}(\rho_{\mu\nu})$ *the energy-momentum tensor*, which now depends on the effective mass $\rho_{\mu\nu}$ and never reaches zero.

The left side of the equation (18) describes the geometry of the effective spacetime, and the right side describes the distribution of the effective sources, being under influence of a gravitational fields.

9. Summary

In this paper was applied an alternative attempt to describe gravitational phenomena, using a new idea of the massification of spacetime, which provides the following benefits:

1. During any change in state of motion of the body appears the inertia, which source is the spacetime with the effective mass density.
2. The inertia becomes an intrinsic property of massification of the spacetime.
3. The magnitude of the inertia of any body is determined by massification of the spacetime.
4. Inertial forces, appearing in the non-inertial frames of reference, there are no longer fictitious forces.
5. In the gravitational field clocks and roots indicate the different time and length, than in the absence of the field. This difference results from the change of the bare mass density in a gravitational field [10].

10. Conclusion

The idea of massification of spacetime, although is a very an attractive, requires **the experimental confirmation**. According to our model, each body moving in the gravitational field of a star in an elliptical orbit, should demonstrate a change in the effective mass. Predicted the annual relative change of the fluctuation in the effective mass as resulting from ellipticity of the orbit for the Earth, is equal to 6.6 x 10^{-10} [10].

GR does not predicts a such fluctuations.

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