# On the Anomalous Oscillation of Newton's Gravitational Constant 

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#### Abstract

Periodic oscillations are observed in Newton's gravitational constant G that are contemporaneous with length of day data obtained from the International Earth Rotation and Reference System. Preliminary research has determined that the oscillatory period of $G$ is $\approx 5.9$ years ( $5.899 \pm 0.062$ years). In this paper, the oscillations are shown to be concomitant with the Earth's distance from the Sun and the angular frequency of its orbit. Implications for space exploration and dark matter are also discussed.


## INTRODUCTION

Ground based measurements of Newton's gravitational constant $\mathbf{G}$ oscillate between $6.672 \times 10^{-11}$ and $6.675 \times 10^{-11} \mathrm{~N} \cdot(\mathrm{~m} / \mathrm{kg})^{2}$ (a difference of $10^{-4} \%$ ) with a periodicity of $\approx 5.9$ years ${ }^{[11]}{ }^{[2]}$. The variations in $\mathbf{G}$ can be predicted from length of day (LOD) data obtained from the International Earth Rotation and Reference System ${ }^{[3]}$ :


Fig. 1: G/LOD synchronicity: The solid curve is a CODATA set of G measurements and the oscillations in LOD measurements are represented by the dashed curve.

The mean motion $\boldsymbol{n}$ of a secondary's orbit is
(l) $n=\omega=\frac{2 \pi}{\mathrm{P}}=\sqrt{\frac{\mathrm{G}(\mathrm{M}+\mathrm{m})}{a^{3}}}$,
where $\boldsymbol{\omega}$ is the angular frequency of the orbit, $\mathbf{P}$ is the sidereal period, $\mathbf{M}$ is the mass of the primary, $\mathbf{m}$ is the mass of the secondary, and $\boldsymbol{a}$ is the secondary's semi-major axis. The mean motion $\boldsymbol{n}$ assumes a circular orbit where the secondary's distance from the origin remains constant and equivalent to $\boldsymbol{a}$. For elliptical orbits, however, the secondary's velocity $\boldsymbol{v}$ and its distance $\boldsymbol{r}$ from the primary varies according to Kepler's $2^{\text {nd }}$ law,
(2) $\frac{d \overrightarrow{\mathrm{~A}}}{d t}=\lim _{\Delta t \rightarrow 0} \frac{\vec{r}(t) \times \vec{r}(t+\Delta t)}{2 \Delta t}=\frac{\vec{r}(t) \times \vec{v}(t)}{2}$,
where $\mathbf{A}$ is the area swept by the secondary during its orbit.
From Kepler/Newton's laws we know
(3) $\mathbf{G}(\mathbf{M}+\mathrm{m})=r v^{2}$.

Combining Kepler's $2^{\text {nd }}$ law with Eq. (3) yields
(4) $\frac{\mathrm{G}(\mathrm{M}+\mathrm{m})}{\vec{v}(t)}=2 \frac{d \overrightarrow{\mathrm{~A}}}{d t}=\vec{h}$,
where the constant $\boldsymbol{h}$ is the secondary's specific relative angular momentum. A definition for $\mathbf{G}$ can then be deduced from Eqs. (1), (2) \& (4) as
(5) $\mathbf{G}=\frac{\omega^{2} r^{3}}{(\mathrm{M}+\mathrm{m})}=2 \frac{d \overrightarrow{\mathrm{~A}} \vec{v}(t)}{d t(\mathrm{M}+\mathrm{m})}=\frac{\vec{h} \vec{v}(t)}{(\mathrm{M}+\mathrm{m})}$.

From the laws of conservation we know the secondary's total angular momentum $\mathbf{L}_{T}$ is
(6) $L_{T}=L_{S}+L_{o}$,
where $\mathbf{L}_{S}$ and $\mathbf{L}_{o}$ are the secondary's spin and orbital angular momentum respectively. Due to spin-orbit coupling, an increase in the Earth's length of day (a decrease in the angular frequency of its spin) must result in an increase in $\boldsymbol{\omega}$ and $\boldsymbol{v}$. Since mass is a conserved quantity, the increase of $\boldsymbol{\omega}$ and $\boldsymbol{v}$ in Eq. (5) results in an increase in G, confirming the G/LOD synchronicity in Fig. $1^{[1],[2],[3]}$.

An alternative method to test if the oscillation of $\mathbf{G}$ is concomitant with the angular frequency of the Earth's orbit is to measure the annual variations in $\mathbf{G}$ relative to an equation of time graph:


Fig. 2: An equation of time graph.
Since it is assumed that the angular frequency of the Earth's orbit varies due to spin-orbit coupling, the annual changes in $\mathbf{G}$ should oscillate proportionately with the red curve graphed in Fig. 2 (assuming the measurements are taken near the Earth's equator).

## IMPLICATIONS FOR SPACE EXPLORATION

Newton's gravitational force $\mathbf{F}_{\mathrm{g}}$ law is
(7) $\mathrm{F}_{\mathrm{g}}=\mathrm{G} \frac{\mathrm{Mm}}{r^{2}}$.

Combining Newton's force law with the definition of $\mathbf{G}$ in Eq. (5) yields
(8) $\mathrm{F}_{\mathrm{g}}=\frac{\mathrm{Mm} \omega^{2} r^{3}}{(\mathrm{M}+\mathrm{m}) r^{2}}=\mu r \omega^{2}$,
where $\boldsymbol{\mu}$ is the reduced mass of the system $(\boldsymbol{\mu} \approx \mathbf{m}$ when $\mathbf{M} \gg \mathbf{m})$. This version of Newton's force law indicates it may be possible to use spin-orbit coupling to diminish the apparent force of gravity. Increasing a body's spin angular momentum would decrease its orbital angular momentum, decreasing the angular frequency $\boldsymbol{\omega}$ of its orbit. This effect would be greater for contra-rotating systems since their relative angular frequencies are greater. Since the spin of Venus is retrograde from the Sun's spin and the other planets, Eq. (8) also indicates its precession rate should be less than predicted from Eq. (7).

According to General Relativity, a spinning body produces a gravitomagnetic (GM) field ${ }^{[4],[5],[6]}$
(9) $\overrightarrow{\mathrm{B}}_{\mathrm{g}}=\frac{\mathrm{G}}{2 \mathrm{c}^{2}} \frac{\overrightarrow{\mathrm{~L}}_{\mathrm{S}}}{\vec{r}^{3}}=\frac{\vec{h} \vec{v}(t)}{2 \mathrm{mc}^{2}} \frac{\vec{I} \omega_{\mathrm{S}}}{\vec{r}^{3}}$,
where $\mathbf{B}_{\mathrm{g}}$ is the field measured at the body's equator, $\mathbf{c}$ is the velocity of light in a vacuum, $\omega_{\mathrm{s}}$ is the angular frequency of the body's spin, and $\boldsymbol{I}$ is its moment of inertia.

From Special Relativity we know
(10) $\mathrm{mc}^{2}=\mathrm{E} \sqrt{1-(v / \mathrm{c})^{2}}=\mathrm{E} \gamma$,
where $\mathbf{E}$ is total energy and $\gamma$ is the Lorentz factor. A definition for the equatorial GM field can therefore be deduced from Eqs. (5) \& (10) as
(11) $\overrightarrow{\mathrm{B}}_{\mathrm{g}}=\frac{\vec{h} \vec{v}(t)}{2 \mathrm{E} \gamma} \frac{\vec{I} \omega_{\mathrm{S}}}{\vec{r}^{3}}$.

The kinetic temperature of a body is
(12) $\frac{3}{2} k \mathrm{~T}=\mathrm{E}_{\mathrm{K}}$,
where $\boldsymbol{k}$ is the Boltzmann constant, $\mathbf{T}$ is temperature and $\mathbf{E}_{\mathbf{K}}$ is the average kinetic energy. Eqs. (11) \& (12) highlight the possibility that $\mathbf{B}_{g}$ is inversely proportional to a body's temperature. Experiments conducted by Martin Tajmar and Clovis de Matos ${ }^{[7]}$ show that the GM field of rapidly spinning low temperature superconductors are no less than one hundred million trillion times greater than predicted with General Relativity. The fact that superconductors have zero electrical resistance suggests that the GM field is inversely proportional to a body's resistance. Also, since temperature is inversely proportional entropy, $\mathbf{B}_{\mathbf{g}}$ would be directly proportional to a body's entropy, lending credence to Eric Verlinde's entropic theory of gravity ${ }^{[8]}$.

## IMPLICATIONS FOR DARK MATTER

Geological evidence ${ }^{[9]}$ indicates our Sun oscillates vertically about the plane of our galaxy in $31 \pm 1$ Myr cycles during its estimated 225-250 Myr revolution. In effect, the Sun's mean motion is much less than the angular frequency of its orbit. It was shown previously in Eqs. (3) \& (5) that
(13) $\mathbf{G}(\mathbf{M}+\mathbf{m})=\omega^{2} r^{3}=r v^{2}$,
so the relative consistency observed in a stellar orbital speeds can be resolved by
(14) $v=r \omega$.

It is hypothesized that the gravitational lensing effect is caused by the warping of light and gasses near the center of mass points between interstellar $n$-body systems. It may be possible to utilize these "virtual mass" points to increase the efficiency of deep space exploration. This topic will be expanded upon in a subsequent paper.

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