Fermat's Last Theorem Proved on a Single Page

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

Honorable Pierre de Fermat was truthful. He could have squeezed the proof of his last theorem into a page margin. Fermat's last theorem has been proved on a single page. Five similar versions of the proof are presented, using a single page for each version. The proof is based on the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$, or $\sin^2 x + \cos^2 x = 1$ Using a common sense approach, one will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$ (n an integer), $c^n = a^n + b^n$ does not hold. For the first three versions, one applies a polar coordinate system as follows. Let a, b, and c be three relatively prime positive integers which are the lengths of the sides of a right triangle, where c is the length of the hypotenuse, and a and b are the lengths of the other two sides. Also, let the acute angle between the hypotenuse and the horizontal be denoted by θ. For the fourth and fifth versions of the proof, ratio terms were used to begin the construction of the proof. The fourth and fifth versions confirmed the proofs in the first three versions. It is also exemplified that if some of the lengths are not positive integers but positive radicals, the derived necessary condition for $c^n = a^n + b^n$ to hold is applicable. Each proof version is very simple, and even high school students can learn it. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper. With respect to prizes, if the prize for a 115-page proof is $715,000, then the prize for a single page proof (considering the advantages) using inverse proportion, is $82,225,000.

Proof: Version 1

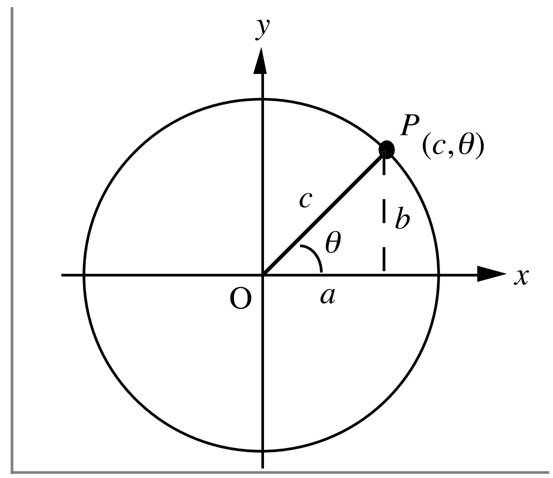
Pythagorean Identity Postulate: There exists only a single fundamental trigonometric identity such that $\cos^n\theta + \sin^n\theta = 1$ (n a positive integer).

Given: $c^n = a^n + b^n$ (n an integer; $a, b,$and c are relatively prime positive integers)

Required: To prove that $c^n = a^n + b^n$ does not hold if $n > 2$.

Plan: Using a common sense approach, one will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$ (n an integer), $c^n = a^n + b^n$ does not hold.

Proof: Let $a, b,$and c be three relatively prime positive integers which are the lengths of the sides of the right triangle in the figure below, where c is the length of the hypotenuse, and a and b are the lengths of the other two sides. Also, let θ denote the acute angle between the hypotenuse and the horizontal.



Then $a = c\cos\theta$ (1)

$b = c\sin\theta$ (2)

$c^n = a^n + b^n$ (3)

$c^n = (c\cos\theta)^n + (c\sin\theta)^n$

$c^n = c^n\cos^n\theta + c^n\sin^n\theta$ (4)

$c^n = c^n(\cos^n\theta + \sin^n\theta)$ (5).

Left-hand side (LHS) of equation (5) equals right-hand side (RHS) of (5) only if

$\cos^n\theta + \sin^n\theta = 1$

That is, a necessary condition for (5) to be true is that

$\boxed{\cos^n\theta + \sin^n\theta = 1}$

If $n = 2$, $c^2 = c^2(\cos^2\theta + \sin^2\theta)$, is true since $\cos^2\theta + \sin^2\theta = 1$ and therefore, equations (5) and (3) hold.

Since there exists only a single Pythagorean identity (a postulate) such that $\cos^n\theta + \sin^n\theta = 1$, and $\cos^2\theta + \sin^2\theta = 1$, with $n = 2$, there are no other positive integers, n, such that $\cos^n\theta + \sin^n\theta = 1$.

Therefore, equations (5) and (3) will be true only if $n = 2$, and there are no other positive integers, $n > 2$ which will make equations (5) and (3) true.

Therefore, $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$. The proof is complete.

Conclusion

Fermat's last theorem has been proved in this paper. Note above that the main criterion is in equation (5) above, which requires that $\cos^n\theta + \sin^n\theta = 1$, if $c^n = c^n(\cos^n\theta + \sin^n\theta)$ and $c^n = a^n + b^n$ are to hold. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper.

Adonten

$a = c\cos\theta$
$b = c\sin\theta$
$c^n = a^n + b^n$
$c^n = (c\cos\theta)^n + (c\sin\theta)^n$
$c^n = c^n\cos^n\theta + c^n\sin^n\theta$
$c^n = .c^n(\cos^n\theta + \sin^n\theta)$. Equation (5) is true only if $\cos^n\theta + \sin^n\theta = 1$
For (5) to be true $\cos^n\theta + \sin^n\theta = 1$. If $n = 2$, $c^2 = c^2(\cos^2\theta + \sin^2\theta)$ is true since $\cos^2\theta + \sin^2\theta = 1$ and therefore, equations (5) and (3) hold. There exists a single identity such that $\cos^n\theta + \sin^n\theta = 1$, and $\cos^2\theta + \sin^2\theta = 1$ with $n = 2$, there are no other positive integers such that $\cos^n\theta + \sin^n\theta = 1$
Therefore, equations (5) and (3) will be true only if $n = 2$, and there are no other integers, $n > 2$ making eqns (5) and (3) true. $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$. QED

Proof: Version 1 in the Margin

Fermat was truthful. He could have squeezed the proof into the page margin.

If Fermat were reincarnated, he would be pleased.

Adonten

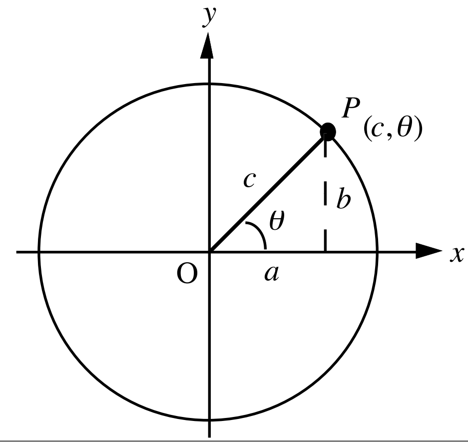
Proof: Version 2

Given: $c^n = a^n + b^n$ (n an integer; $a, b,$and c are relatively prime positive integers)

Required: To prove that $c^n = a^n + b^n$ does not hold if $n > 2$.

Plan: Using a common sense approach, one will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$ (n an integer) **,** $c^n = a^n + b^n$ does not hold.

Proof: Let $a, b,$and c be three relatively prime positive integers which are the lengths of the sides of the right triangle in the figure below, where c is the length of the hypotenuse, and a and b are the lengths of the other two sides. Also, let θ denote the acute angle between the hypotenuse and the horizontal.



Then $a = c\cos\theta$ (1)

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$c^n = a^n + b^n$ (3)

$c^n = (c\cos\theta)^n + (c\sin\theta)^n$

$c^n = c^n\cos^n\theta + c^n\sin^n\theta$ (4)

$c^n = c^n(\cos^n\theta + \sin^n\theta)$ (5).

Left-hand side (LHS) of equation (5) equals right-hand side (RHS) of (5) only if

$\cos^n\theta + \sin^n\theta = 1$

That is, a necessary condition for (5) to be true is that

$\boxed{\cos^n\theta + \sin^n\theta = 1}$

If $n = 2$, $c^2 = c^2(\cos^2\theta + \sin^2\theta)$, is true since $\cos^2\theta + \sin^2\theta = 1$ and therefore, equations (5) and (3) hold. If $n = 3$, $c^3 \neq c^3(\cos^3\theta + \sin^3\theta)$ since $\cos^3\theta + \sin^3\theta \neq 1$and equations (5) and (3) do not hold. ($\cos^3\theta + \sin^3\theta = (\cos\theta + \sin\theta)(\cos^2 - \cos\theta\sin\theta + \sin^2\theta \neq 1)$)

Therefore, if $n = 3$, equations (5) and (3) do not hold..

If $n = 4$,, $c^4 \neq c^4(\cos^4\theta + \sin^4\theta)$ since $\cos^4\theta + \sin^4\theta \neq 1$. $(\cos^4\theta + \sin^4\theta \neq \cos^2\theta + \sin^2\theta)$

$\{\cos^4\theta + \sin^4\theta = (\cos^2\theta + \sqrt{2}\cos\theta\sin\theta + \sin^2\theta)(\cos^2\theta - \sqrt{2}\cos\theta\sin\theta + \sin^2\theta \neq 1\}$

Therefore, if $n = 4$, equations (5) and (3). do not hold.

Replacing n by $k+1$ or by $k+3$ in $\cos^n\theta + \sin^n\theta$, one obtains respectively

A If $n = k+1$ or	**B** If $n = k+3$, $\cos^n\theta + \sin^n\theta = \cos^{k+3}\theta + \sin^{k+3}\theta$
$\cos^n\theta + \sin^n\theta = \cos^{k+1}\theta + \sin^{k+1}\theta$ $\cos^{k+1}\theta + \sin^{k+1}\theta = \cos^2\theta + \sin^2\theta = 1$ only if $k = 1$, and then $n = 2$.	If $\cos^{k+3}\theta + \sin^{k+3}\theta = \cos^2\theta + \sin^2\theta$, then $k+3 = 2$ and $k = -1$ and $n = -1+3 = 2$ Here also, $n = 2$ **. Note**: Necessary condition is in (5) above...

Observe that if $n = k+1$ or $n = k+3$, and one wants to increase the exponent while at the same time maintaining that $\cos^n\theta + \sin^n\theta = 1$ (a necessary condition for (5) and (3) to be true), the results show that one cannot do that simultaneously and that one should keep the exponent at $n = 2$ .Therefore, equations (5) and (3) will be true only if $n = 2$,

Therefore, $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$. The proof is complete.

Conclusion

Fermat's last theorem has been proved in this paper. Note above that the main criterion is in equation (5) above, which requires that $\cos^n\theta + \sin^n\theta = 1$, if $c^n = c^n(\cos^n\theta + \sin^n\theta)$ and $c^n = a^n + b^n$ are to hold. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper. **Adonten**

Proof: Version 3

Given: $c^n = a^n + b^n$ (n an integer; $a, b,$and c are relatively prime positive integers)

Required: To prove that $c^n = a^n + b^n$ does not hold if $n > 2$

Plan:Using a common sense approach, one will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$ (n an integer) **,** $c^n = a^n + b^n$ does not hold.

Proof: Let $a, b,$and c be three relatively prime positive integers which are the lengths of the sides of the right triangle in the figure below, where c is the length of the hypotenuse, and a and b are the lengths of the other two sides. Also, let θ denote the acute angle between the hypotenuse and the horizontal.

Then $a = c\cos\theta$ (1)

$b = c\sin\theta$ (2)

$c^n = a^n + b^n$ (3)

$c^n = (c\cos\theta)^n + (c\sin\theta)^n$

$c^n = c^n\cos^n\theta + c^n\sin^n\theta$ (4)

$c^n = c^n(\cos^n\theta + \sin^n\theta)$ (5).

Left-hand side (LHS) of equation (5) equals right-hand side (RHS) of (5) only if

$\cos^n\theta + \sin^n\theta = 1$

That is, a necessary condition for (5) to be true is that

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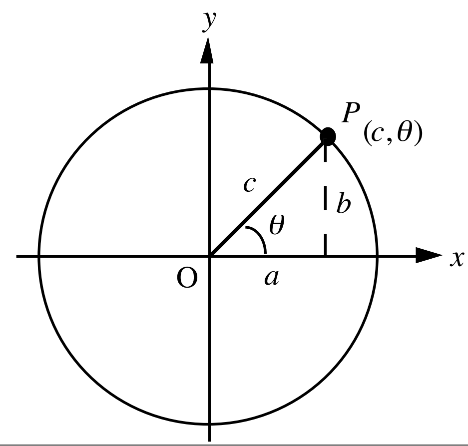

If $n = 2$, $c^2 = c^2(\cos^2\theta + \sin^2\theta)$, is true since $\cos^2\theta + \sin^2\theta = 1$ and therefore, equations (5) and (3) hold.

$\cos^2\theta + \sin^2\theta = \frac{b^2}{c^2} + \frac{a^2}{c^2} = \frac{a^2 + b^2}{c^2} = \frac{a^2 + b^2}{a^2 + b^2} = 1$

If $n = 3, c^3 \neq c^3(\cos^3\theta + \sin^3\theta)$ since $\cos^3\theta + \sin^3\theta \neq 1$, and equations (5) and (3) do not hold. $\cos^3\theta + \sin^3\theta =$

$(\cos\theta + \sin\theta)(\cos^2 - \cos\theta\sin\theta + \sin^2\theta \neq 1$

Therefore, if $n = 3$, equations (5) and (3) do not hold..

$\cos\theta = \frac{a}{c}$; $\cos^3\theta = \frac{a^3}{c^3}$ $\sin\theta = \frac{b}{c}$; $\sin^3\theta = \frac{b^3}{c^3}$

$\cos^3\theta + \sin^3\theta = \frac{b^3}{c^3} + \frac{a^3}{c^3} = \frac{a^3 + b^3}{c^3}$

$= \frac{(a + b)(a^2 - ab + b^2)}{c^3} \neq 1$

Replacing n by $k + 1$ or by $k + 3$ in $\cos^n\theta + \sin^n\theta$, one obtains respectively

A If $n = k + 1$ or

$\cos^n\theta + \sin^n\theta = \cos^{k+1}\theta + \sin^{k+1}\theta$

$\cos^{k+1}\theta + \sin^{k+1}\theta = \cos^2\theta + \sin^2\theta = 1$

only if $k = 1$, and then $n = 2$.

B If $n = k + 3$, $\cos^n\theta + \sin^n\theta = \cos^{k+3}\theta + \sin^{k+3}\theta$

If $\cos^{k+3}\theta + \sin^{k+3}\theta = \cos^2\theta + \sin^2\theta$, then $k + 3 = 2$ and $k = -1$ and $n = -1 + 3 = 2$

Here also, $n = 2$ **. Note**: Necessary condition is in (5) above...

Therefore, equations (5) and (3) will be true only if $n = 2$, and there are no other integers, $n > 2$ which will make equations (5) and (3) true.

Therefore, $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$. The proof is complete.

Conclusion

Fermat's last theorem has been proved in this paper. Note above that the main criterion is in equation (5) above, which requires that $\cos^n\theta + \sin^n\theta = 1$, if $c^n = c^n(\cos^n\theta + \sin^n\theta)$ and $c^n = a^n + b^n$ are to hold. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper. **Adonten**

PS Discussion

A If $n = k+1$ $\cos^n\theta + \sin^n\theta = \cos^{k+1}\theta + \sin^{k+1}\theta$ $\cos^{k+1}\theta + \sin^{k+1}\theta = \cos^2\theta + \sin^2\theta = 1$ only if $k = 1$, and then $n = 2$. The k–value implies that n–value , $n = 2$ cannot increase if the necessary condition in (5).is to be maintained.. The necessary condition implies that $n = 2$.	**C** If $n = k+3$, $\cos^n\theta + \sin^n\theta = \cos^{k+3}\theta + \sin^{k+3}\theta$ If $\cos^{k+3}\theta + \sin^{k+3}\theta = \cos^2\theta + \sin^2\theta$, then $k+3 = 2$ and $k = -1$. $n = k-1 = 2$ The negative k–value implies that n–value cannot increase if the necessary condition in (5) is maintained... The necessary condition implies that always, $n = 2$.
B If $n = k+2$ $\cos^n\theta + \sin^n\theta = \cos^{k+2}\theta + \sin^{k+2}\theta$ $\cos^{k+2}\theta + \sin^{k+2}\theta = \cos^2\theta + \sin^2\theta = 1$ $k+2 = 2$ and $k = 0$, and then $n = 2$. k–value implies that n–value cannot increase because of the necessary condition in (5).	**D** If $n = k+4$, $\cos^n\theta + \sin^n\theta = \cos^{k+4}\theta + \sin^{k+4}\theta$ If $\cos^{k+4}\theta + \sin^{k+4}\theta = \cos^2\theta + \sin^2\theta$, then $k+4 = 2$ and $k = -2$. Again $n = -2+4 = 2$ The negative k–value implies that n–value cannot increase because of the necessary condition in (5).

In A, B, C and D, above, the n–value remains constant at $n = 2$, despite attempt s to increase it. because of the necessary condition , $\cos^n\theta + \sin^n\theta = 1$ in (5).

Note: Apart from $\cos^2\theta + \sin^2\theta = 1$, if there were another trigonometric identity such that $\cos^n\theta + \sin^n\theta = 1$, that n–value would be another n–value satisfying $c^n = a^n + b^n$.

Analogy 1:
If ones feet are on the second rung of a ladder, and one wants to move one's feet to a higher rung while at the same time maintaining one's body on the second rung, one will be forced to return to the second rung, implying that one cannot move ones feet to a higher rung while maintaining ones body on the second rung.

Analogy 2:
If one lives on the second floor of a building, and one wants to move to a higher floor, say, the third floor, and the second floor has a necessary health-care facility, which is not available on the higher floors, one would not be able to move and live on the third or a higher floor. Even if one tried hard to live on the third floor, one would be compelled to move back to the second floor because of the necessary health-care facility on the second floor.

Under bad financial conditions, the best financial decision should be based on what is necessary and not on what is wanted.

The next version of the proof, Version 4 of the proof, "miraculously" confirms the proofs in versions 1, 2, and 3; and was motivated by the author's success, using ratio terms, in attacking the Navier-Stokes equations (viXra: 1405.0251 as well as the Riemann hypothesis (viXra: 1411.0075) of the CMI Prize problems,

Miraculous Confirmation of the Proofs in Versions, 1, 2, and 3 of the Proof

Proof: Version 4 (Using ratios)

Plan: Using a common sense approach, one will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$ (n an integer), $c^n = a^n + b^n$ does not hold.

One will begin by applying ratio terms.

$c^n = a^n + b^n$ (1) (Given)

$a^n + b^n = c^n$ (2) (rewriting)

$a^n = rc^n$ (3) (r is a ratio term)

$b^n = sc^n$ (4) (s is a ratio term) ($r + s = 1$)

$rc^n + sc^n = c^n$ (5) (substitute for a^n and b^n from (3) and (4)

$c^n(r + s) = c^n$ (6)

Now, by the substitution axiom, since $r + s = 1$, $r + s$ can be replaced by any quantity = 1. One can therefore replace $r + s$ by $\sin^2 x + \cos^2 x$ since $\sin^2 x + \cos^2 x = 1$. Then equation (6) becomes

$c^n(\sin^2 x + \cos^2 x) = c^n$ (7)

If $n = 2$, (7) becomes $c^2(\sin^2 x + \cos^2 x) = c^2$ (8)

$c^2 = c^2(\sin^2 x + \cos^2 x)$ (8) (rewriting)

Equation (8) is true since $\sin^2 x + \cos^2 x = 1$. Consequently, equations (8) and (1) hold. Therefore, if $n = 2$, $c^n = a^n + b^n$.

Generalizing equation (7), one obtains $c^n(\sin^n x + \cos^n x) = c^n$ (9)

in which the necessary condition for (9) to hold is that $\sin^n x + \cos^n x = 1$.

If $n = 3$, $\sin^3 x + \cos^3 x \neq 1$, and one cannot replace $r + s$ by $\sin^3 x + \cos^3 x$ in (6).

Therefore, if $n = 3$, equations (9) and (1) do not hold.

Similarly, If $n = 4$, $\sin^4 x + \cos^4 x \neq 1$, and one cannot replace $r + s$ by $\sin^4 x + \cos^4 x$ in (6), and equations (9) and (1) do not hold.

Example on ratio terms

If $4 + 8 = 12$, and the ratio terms are $\frac{1}{3}$ and $\frac{2}{3}$, then

$4 = \frac{1}{3} \bullet 12$,

$8 = \frac{2}{3} \bullet 12$; and the sum of the ratio terms is

$\frac{1}{3} + \frac{2}{3} = 1$

Other equivalent identities
Note: "magic "number, 2.

$\sec^2 x - \tan^2 x = 1$

$\csc^2 x - \cot^2 x = 1$

$\cos 2x + 2\sin^2 x = 1$

$2\cos^2 x - \cos 2x = 1$

$\sec^2 x - \tan^2 x = 1$

$\csc^2 x - \cot^2 x = 1$

Elimination of the ratio terms r and s

The author was impressed and gratified by the substitution axiom which permitted the introduction of the much needed necessary condition in Versions 1, 2, and 3 of the proof.

Replacing n by $k + 1$ or by $k + 3$ in $\sin^n x + \cos^n x$, one obtains respectively,

A	**B**
If $n = k + 1$ or $\sin^n x + \cos^n x = \sin^{k+1} x + \cos^{k+1} x$ $\sin^{k+1} x + \cos^{k+1} x = \sin^2 x \cos^2 x = 1$ only if $k = 1$, and then $n = 2$.	If $n = k + 3$, $\sin^n x + \cos^n x = \sin^{k+3} x + \cos^{k+3} x$ If $\sin^{k+3} x + \cos^{k+3} x = \sin^2 x + \cos^2 x$, then $k + 3 = 2$ and $k = -1$ and $n = -1 + 3 = 2$ Here also, $n = 2$. **Note**: Necessary condition is in (9) above...

Observe that if $n = k + 1$ or $n = k + 3$, and one wants to increase the exponent while at the same time maintaining that $\cos^n \theta + \sin^n \theta = 1$ (a necessary condition for (9) and (1) to be true), the results show that one cannot do that simultaneously and that one should keep the exponent at $n = 2$. Therefore, equations (9) and (1) will be true only if $n = 2$.

Therefore, $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$. The proof is complete.

Conclusion

Fermat's last theorem has been proved in this paper. Note above that the main criterion is that $\sin^n x + \cos^n x = 1$, if $c^n = c^n(\sin^n x + \cos^n x)$ and $c^n = a^n + b^n$ are to hold. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper.

Proof: Version 5 (Using ratios)

Pythagorean Identity Postulate: There exists only a single fundamental trigonometric identity such that $\sin^n x + \cos^n x = 1$ (n a positive integer).

Given: $c^n = a^n + b^n$ (n an integer; a, b, and c are relatively prime positive integers)

Required: To prove that $c^n = a^n + b^n$ does not hold if $n > 2$

Plan: One will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$ (n an integer), $c^n = a^n + b^n$ does not hold. One will begin by applying ratio terms.

$c^n = a^n + b^n$ (1) (Given)

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If $n = 2$, (7) becomes $c^2(\sin^2 x + \cos^2 x) = c^2$ (8)

$c^2 = c^2(\sin^2 x + \cos^2 x)$ (8) (rewriting)

Since the left-hand side of equation (8) equals right-hand side of (8), equations (8) and (1) hold. Therefore, if $n = 2$, $c^n = a^n + b^n$.

Generalizing equation (7), one obtains $c^n(\sin^n x + \cos^n x) = c^n$ (9)

in which the necessary condition for (9) to hold is $\sin^n x + \cos^n x = 1$.

Since there exists only a single fundamental Pythagorean identity (a postulate) such that $\cos^n x + \sin^n x = 1$, and $\cos^2 x + \sin^2 x = 1$, with $n = 2$, there are no other positive integers, n, such that $\cos^n x + \sin^n x = 1$. Therefore, equations (9) and (1) will be true only if $n = 2$, and there are no other positive integers, $n > 2$ which will make equations (9) and (1) true. Therefore, $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$. The proof is complete.

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Elimination of the ratio terms r and s

The author was impressed and gratified by the substitution axiom which permitted the introduction of the much needed necessary condition $\sin^n x + \cos^n x = 1$ in Versions 1, 2, and 3 of the proof.

Conclusion

Fermat's last theorem has been proved in this paper. Note above that the main criterion is that $\sin^n x + \cos^n x = 1$, if $c^n = c^n(\sin^n x + \cos^n x)$ and $c^n = a^n + b^n$ are to hold. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper.

Question for a mathematics final exam for the 2016 Fall semester.

Bonus Question: Prove Fermat's Last Theorem

Overall Conclusion

Fermat's last theorem has been proved in this paper. In the first three versions of the proof, one began with reference to a polar coordinate system; but in the fourth and fifth versions of the proof, the proof construction began with ratio terms without reference to any polar coordinate system. The ratio terms were later on "miraculously" eliminated from the equations. The necessary condition for the relevant equations involved to be true is that $\sin^n x + \cos^n x = 1$ (or $\cos^n\theta + \sin^n\theta = 1$). Thus, if $c^n = c^n(\sin^n x + \cos^n x)$ and $c^n = a^n + b^n$ are to hold, $\sin^n x + \cos^n x = 1$ or $\cos^n\theta + \sin^n\theta = 1$ must be satisfied. First, the author determined, why the equation, $c^n = a^n + b^n$ is true if $n = 2$. It was determined that the necessary condition is $\sin^n x + \cos^n x = 1$ or $\cos^n\theta + \sin^n\theta = 1$, and this condition is satisfied only if $n = 2$. If $n = 3,4,5,...$, this necessary $\sin^n x + \cos^n x = 1$ or $\cos^n\theta + \sin^n\theta = 1$ is never satisfied. Any attempts by induction to use larger exponents while satisfying the necessary condition involved resulted in being returned to $n = 2$. Moreover. there exists only a single fundamental Pythagorean identity (a postulate) such that $\cos^n\theta + \sin^n\theta = 1$, and $\cos^2\theta + \sin^2\theta = 1$, with $n = 2$, there are no other positive integers, n, such that $\cos^n\theta + \sin^n\theta = 1$. From the proof, the only condition for $c^n = a^n + b^n$ to hold is the necessary condition derived in this paper.

Therefore, $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$.

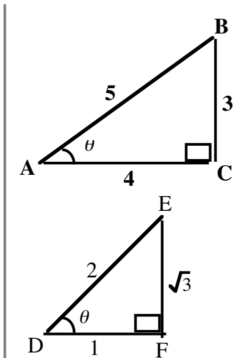
One should note above that versions 4 and 5 confirm the proofs in versions 1, 2 and 3 of the proof. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper.

About the numbers, *a, b,* c, being positive integers

The equation $c^n = a^n + b^n$ will be true if $\cos^n\theta + \sin^n\theta = 1$

If $n = 2$, $\cos^n\theta + \sin^n\theta = 1$ becomes $\cos^2\theta + \sin^2\theta = 1$.

Let one apply the necessary condition to the dimensions of the triangles in the figures to the right.



The lengths of the sides of triangle ABC are $3,\ 4,\ 5$ (pos.integers)

$\cos\theta = \frac{4}{5};\ \cos^2\theta = \frac{16}{25};\ \sin\theta = \frac{3}{5};\ \sin^2\theta = \frac{9}{25}$ $\boxed{\cos^2\theta + \sin^2\theta = \frac{16}{25} + \frac{9}{25} = 1}$

The lengths of the sides of triangle DEF are $1, \sqrt{3},\ 2$ (one radical)

$\cos\theta = \frac{1}{2};\ \cos^2\theta = \frac{1}{4};\ \sin\theta = \frac{\sqrt{3}}{2};\ \sin^2\theta = \frac{3}{4}$ $\boxed{\cos^2\theta + \sin^2\theta = \frac{1}{4} + \frac{3}{4} = 1}$

If $n = 3$, for triangle ABC, $\boxed{\cos^3\theta + \sin^3\theta = \frac{64}{125} + \frac{27}{125} = \frac{91}{125} \neq 1}$

If $n = 3$, for triangle DEF, $\boxed{\cos^3\theta + \sin^3\theta = \frac{1}{8} + \frac{3\sqrt{3}}{8} = \frac{1+3\sqrt{3}}{8} \neq 1}$

If $n = 2$, each of the sets of the dimensions of the two triangles satisfies the necessary condition, $\cos^n\theta + \sin^n\theta = 1$; but if $n = 3$ ($n > 2$), the necessary condition is not satisfied, that is, $\cos^n\theta + \sin^n\theta \neq 1$

The lengths of the sides of triangle ABC are all positive integers, but not all the lengths of triangle DEF are positive integer (one length is a radical).

Therefore, the necessary condition is applicable even if some of the numbers involved are positive radicals.

Adonten