# Fermat's Last Theorem Proved on a Single Page

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

## Abstract

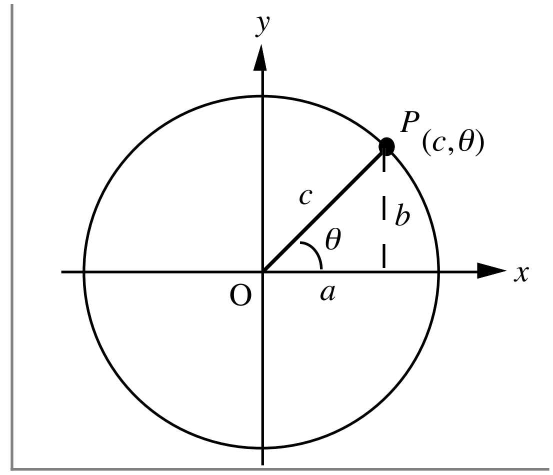
Honorable Pierre de Fermat was truthful. He could have squeezed the proof of his last theorem into a page margin. Fermat's last theorem has been proved on a single page. The proof is based on the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$. One will first show that if $n = 2$. $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$, $c^n = a^n + b^n$ does not hold. Applying a polar coordinate system, let $a, b,$ and $c$ be three relatively prime positive integers which are the lengths of the sides of a right triangle, where $c$ is the length of the hypotenuse, and $a$ and $b$ are the lengths of the other two sides. Also, let the acute angle between the hypotenuse and the horizontal be denoted by $\theta$. Three similar versions of the proof are presented. The proof is very simple, and even high school students can learn it. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper.

# Proof: Version 1

**Pythagorean Identity Postulate:** There exist only a single fundamental trigonometric identity such that $\cos^n\theta + \sin^n\theta \ = 1$.

**Plan**: One will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$, $c^n = a^n + b^n$ does not hold.

Let $a$, $b$,and $c$ be three relatively prime positive integers which are the lengths of the sides of the right triangle in the figure below, where $c$ is the length of the hypotenuse, and $a$ and $b$ are the lengths of the other two sides. Also, let $\theta$ denote the acute angle between the hypotenuse and the horizontal.




Then $a = c\cos\theta$ (1)

$b = c\sin\theta$ (2)

$c^n = a^n + b^n$ (3)

$c^n = (c\cos\theta)^n + (c\sin\theta)^n$

$c^n = c^n\cos^n\theta + c^n\sin^n\theta$ (4)

$c^n = c^n(\cos^n\theta + \sin^n\theta)$ (5).

Left-hand side (LHS) of equation ( 5 ) equals right-hand side (RHS) of (5) only if

$$\cos^n\theta + \sin^n\theta \ = 1$$

That is, a necessary condition for (5) to be true is that

$$\boxed{\cos^n\theta + \sin^n\theta \ = 1}$$

If $n = 2$, $c^2 = c^2(\cos^2\theta + \sin^2\theta)$, is true since $\cos^2\theta + \sin^2\theta = 1$ and therefore, equations (5) and (3) hold.

Since there exists only a single Pythagorean identity such that $\cos^n\theta + \sin^n\theta \ = 1$, and $\cos^2\theta + \sin^2\theta = 1$, with $n = 2$,, there are no other positive integers such that $\cos^n\theta + \sin^n\theta \ = 1$. Therefore, equations (5) and (3) will be true only if $n = 2$, and there are no other integers, $n > 2$ which will make equations (5) and (3) true.

Therefore, $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$. The proof is complete.

## Conclusion

Fermat's last theorem has been proved in this paper. Note above that the main criterion is in equation (5) above, which requires that $\cos^n\theta + \sin^n\theta \ = 1$, if $c^n = c^n(\cos^n\theta + \sin^n\theta)$ and $c^n = a^n + b^n$ are to hold. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper.

**Adonten**

$a = c\cos\theta$
$b = c\sin\theta$
$c^n = a^n + b^n$
$c^n = (c\cos\theta)^n + (c\sin\theta)^n$
$c^n = c^n\cos^n\theta + c^n\sin^n\theta$
$c^n = .c^n(\cos^n\theta + \sin^n\theta)$. Equation ( 5 ) is true only if $\cos^n\theta + \sin^n\theta = 1$

For (5) to be true $\cos^n\theta + \sin^n\theta = 1$. If $n = 2$, $c^2 = c^2(\cos^2\theta + \sin^2\theta)$ is true since $\cos^2\theta + \sin^2\theta = 1$ and therefore, equations (5) and (3) hold. There exists a single identity such that $\cos^n\theta + \sin^n\theta = 1$, and $\cos^2\theta + \sin^2\theta = 1$ with $n = 2$,, there are no other positive integers such that $\cos^n\theta + . \sin^n\theta = 1$

Therefore, equations (5) and (3) will be true only if $n = 2$, and there are no other integers, $n > 2$ making eqns (5) and (3) true. $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$. QED

**Proof: Version 1**
**in the Margin**

Fermat was truthful. He could have squeezed the proof into the page margin.

If Fermat were reincarnated, he would be pleased.

# Proof: Version 2

**Plan**: One will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$, $c^n = a^n + b^n$ does not hold.

Let $a, b,$and $c$ be three relatively prime positive integers which are the lengths of the sides of the right triangle in the figure below, where $c$ is the length of the hypotenuse, and $a$ and $b$ are the lengths of the other two sides. Also, let $\theta$ denote the acute angle between the hypotenuse and the horizontal.

Then $a = c\cos\theta$ (1)

$b = c\sin\theta$ (2)

$c^n = a^n + b^n$ (3)

$c^n = (c\cos\theta)^n + (c\sin\theta)^n$

$c^n = c^n\cos^n\theta + c^n\sin^n\theta$ (4)

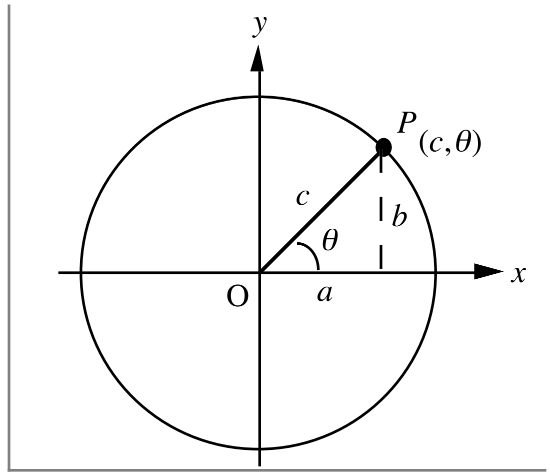
$c^n = c^n(\cos^n\theta + \sin^n\theta)$ (5).

Left-hand side (LHS) of equation ( 5 ) equals right-hand side (RHS) of (5) only if

$\cos^n\theta + \sin^n\theta = 1$

That is, a necessary condition for (5) to be true is that

$\boxed{\cos^n\theta + \sin^n\theta = 1}$




If $n = 2$, $c^2 = c^2(\cos^2\theta + \sin^2\theta)$, is true since $\cos^2\theta + \sin^2\theta = 1$ and therefore, equations (5) and (3) hold.

If $n = 3$, $c^3 \neq c^3(\cos^3\theta + \sin^3\theta)$ since $\cos^3\theta + \sin^3\theta \neq 1$and equations (5) and (3) do not hold.

$(\cos^3\theta + \sin^3\theta = (\cos\theta + \sin\theta)(\cos^2 - \cos\theta\sin\theta + \sin^2\theta \neq 1)$

Therefore, if $n = 3$, equations (5) and (3) do not hold..

If $n = 4$,, $c^4 \neq c^4(\cos^4\theta + \sin^4\theta)$ since $\cos^4\theta + \sin^4\theta \neq 1$. $(\cos^4\theta + \sin^4\theta \neq \cos^2\theta + \sin^2\theta)$

$\{\cos^4\theta + \sin^4\theta = (\cos^2\theta + \sqrt{2}\cos\theta\sin\theta + \sin^2\theta)(\cos^2\theta - \sqrt{2}\cos\theta\sin\theta + \sin^2\theta \neq 1\}$

Therefore, if $n = 4$, equations (5) and (3). do not hold.

Replacing $n$ by $k+1$ or by $k+3$ in $\cos^n\theta + \sin^n\theta$ , one obtains respectively

| | |
|---|---|
| **A** If $n = k+1$ or | **B** If $n = k+3$, $\cos^n\theta + \sin^n\theta = \cos^{k+3}\theta + \sin^{k+3}\theta$ |
| $\cos^n\theta + \sin^n\theta = \cos^{k+1}\theta + \sin^{k+1}\theta$ | If $\cos^{k+3}\theta + \sin^{k+3}\theta = \cos^2\theta + \sin^2\theta$, then |
| $\cos^{k+1}\theta + \sin^{k+1}\theta = \cos^2\theta + \sin^2\theta = 1$ | $k+3 = 2$ and $k = -1$ and $n = -1+3 = 2$ |
| only if $k = 1$, and then $n = 2$. | Here also, $n = 2$. **Note**: Necessary condition is in (5) above... |

Observe that if $n = k+1$ or $n = k+3$, and one wants to increase the exponent while at the same time maintaining that $\cos^n\theta + \sin^n\theta = 1$ ( a necessary condition for (5) and (3) to be true), the results show that one cannot do that simultaneously and that one should keep the exponent at $n = 2$.

Therefore, equations (5) and (3) will be true only if $n = 2$,

Therefore, $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$.

**Conclusion**

Fermat's last theorem has been proved in this paper. Note above that the main criterion is in equation (5) above, which requires that $\cos^n\theta + \sin^n\theta = 1$, if $c^n = c^n(\cos^n\theta + \sin^n\theta)$ and $c^n = a^n + b^n$ are to hold. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper. **Adonten**

# Proof: Version 3

**Plan**: One will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$, $c^n = a^n + b^n$ does not hold.

Let $a, b,$and $c$ be three relatively prime positive integers which are the lengths of the sides of the right triangle in the figure below, where $c$ is the length of the hypotenuse, and $a$ and $b$ are the lengths of the other two sides. Also, let $\theta$ denote the acute angle between the hypotenuse and the horizontal.

Then $a = c\cos\theta$ (1)

$b = c\sin\theta$ (2)

$c^n = a^n + b^n$ (3)

$c^n = (c\cos\theta)^n + (c\sin\theta)^n$

$c^n = c^n\cos^n\theta + c^n\sin^n\theta$ (4)

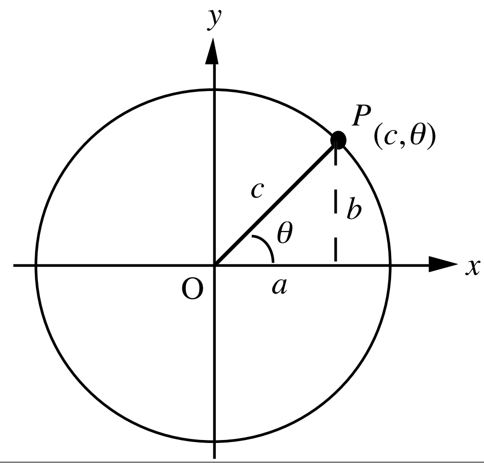
$c^n = c^n(\cos^n\theta + \sin^n\theta)$ (5).

Left-hand side (LHS) of equation ( 5 ) equals right-hand side (RHS) of (5) only if

$\cos^n\theta + \sin^n\theta = 1$

That is, a necessary condition for (5) to be true is that

$\boxed{\cos^n\theta + \sin^n\theta = 1}$




If $n = 2$, $c^2 = c^2(\cos^2\theta + \sin^2\theta)$, is true since $\cos^2\theta + \sin^2\theta = 1$ and therefore, equations (5) and (3) hold.

$\cos^2\theta + \sin^2\theta = \frac{b^2}{c^2} + \frac{a^2}{c^2} = \frac{a^2 + b^2}{c^2} = \frac{a^2 + b^2}{a^2 + b^2} = 1$

If $n = 3, c^3 \neq c^3(\cos^3\theta + \sin^3\theta)$ since $\cos^3\theta + \sin^3\theta \neq 1$ and equations (5) and (3) do not hold.

$\cos^3\theta + \sin^3\theta =$

$(\cos\theta + \sin\theta)(\cos^2 - \cos\theta\sin\theta + \sin^2\theta \neq 1$

Therefore, if $n = 3$, equations (5) and (3) do not hold..

$\cos\theta = \frac{a}{c}$; $\cos^3\theta = \frac{a^3}{c^3}$ $\sin\theta = \frac{b}{c}$; $\sin^3\theta = \frac{b^3}{c^3}$

$\cos^3\theta + \sin^3\theta = \frac{b^3}{c^3} + \frac{a^3}{c^3} = \frac{a^3 + b^3}{c^3}$

$= \frac{(a+b)(a^2 - ab + b^2)}{c^3} \neq 1$

Replacing $n$ by $k+1$ or by $k+3$ in $\cos^n\theta + \sin^n\theta$ , one obtains respectively

**A** If $n = k+1$ or

$\cos^n\theta + \sin^n\theta = \cos^{k+1}\theta + \sin^{k+1}\theta$

$\cos^{k+1}\theta + \sin^{k+1}\theta = \cos^2\theta + \sin^2\theta = 1$

only if $k = 1$, and then $n = 2$.

**B** If $n = k+3$, $\cos^n\theta + \sin^n\theta = \cos^{k+3}\theta + \sin^{k+3}\theta$

If $\cos^{k+3}\theta + \sin^{k+3}\theta = \cos^2\theta + \sin^2\theta$, then $k + 3 = 2$ and $k = -1$ and $n = -1 + 3 = 2$

Here also, $n = 2$. **Note**: Necessary condition is in (5) above...

Therefore, equations (5) and (3) will be true only if $n = 2$, and there are no other integers, $n > 2$ which will make equations (5) and (3) true.

Therefore, $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$.

**Conclusion**

Fermat's last theorem has been proved in this paper. Note above that the main criterion is in equation (5) above, which requires that $\cos^n\theta + \sin^n\theta = 1$, if $c^n = c^n(\cos^n\theta + \sin^n\theta)$ and $c^n = a^n + b^n$ are to hold. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin for it in his paper. **Adonten**

## PS Discussion

| | |
|---|---|
| **A** If $n = k+1$<br>$\cos^n\theta + \sin^n\theta = \cos^{k+1}\theta + \sin^{k+1}\theta$<br>$\cos^{k+1}\theta + \sin^{k+1}\theta = \cos^2\theta + \sin^2\theta = 1$<br>only if $k = 1$, and then $n = 2$.<br>The $k$–value implies that $n$–value, $n = 2$ cannot increase if the necessary condition in (5).is to be maintained..<br>The necessary condition implies that $n = 2$. | **C** If $n = k+3$, $\cos^n\theta + \sin^n\theta = \cos^{k+3}\theta + \sin^{k+3}\theta$<br>If $\cos^{k+3}\theta + \sin^{k+3}\theta = \cos^2\theta + \sin^2\theta$, then $k+3 = 2$ and $k = -1$. $n = k-1 = 2$<br>The negative $k$–value implies that $n$–value cannot increase if the necessary condition in (5) is maintained...<br>The necessary condition implies that always, $n = 2$. |
| **B** If $n = k+2$<br>$\cos^n\theta + \sin^n\theta = \cos^{k+2}\theta + \sin^{k+2}\theta$<br>$\cos^{k+2}\theta + \sin^{k+2}\theta = \cos^2\theta + \sin^2\theta = 1$<br>$k+2 = 2$ and $k = 0$, and then $n = 2$.<br>$k$–value implies that $n$–value cannot increase because of the necessary condition in (5). | **D** If $n = k+4$, $\cos^n\theta + \sin^n\theta = \cos^{k+4}\theta + \sin^{k+4}\theta$<br>If $\cos^{k+4}\theta + \sin^{k+4}\theta = \cos^2\theta + \sin^2\theta$, then $k+4 = 2$ and $k = -2$. Again $n = -2+4 = 2$<br>The negative $k$–value implies that $n$–value cannot increase because of the necessary condition in (5). |

In A, B, C and D, above, the $n$–value remains constant at $n = 2$, despite attempt s to increase it. because of the necessary condition , $\cos^n\theta + \sin^n\theta = 1$ in (5).

**Note:** Apart from $\cos^2\theta + \sin^2\theta = 1$, if there were another trigonometric identity such that $\cos^n\theta + \sin^n\theta = 1$, that $n$–value will be another $n$–value satisfying $c^n = a^n + b^n$.

**Analogy 1:**
If ones feet are on the second rung of a ladder, and one wants to move one's feet to a higher rung while at the same time maintaining one's body on the second rung, one will be forced to return to the second rung, implying that one cannot move ones feet to a higher rung while maintaining ones body on the second rung.

**Analogy 2:**
If one lives on the second floor of a building, and one wants to move to a higher floor, say, the third floor, and the second floor has a necessary health-care facility, which is not available on the higher floors, one would not be able to move and live on the third or a higher floor. Even if one tried hard to live on the third floor, one would be compelled to move back to the second floor because of the necessary health-care facility on the second floor.

In a financial decision, base your decision on what is necessary and not what you want.