Fermat's Last Theorem Proved on a Single Page

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

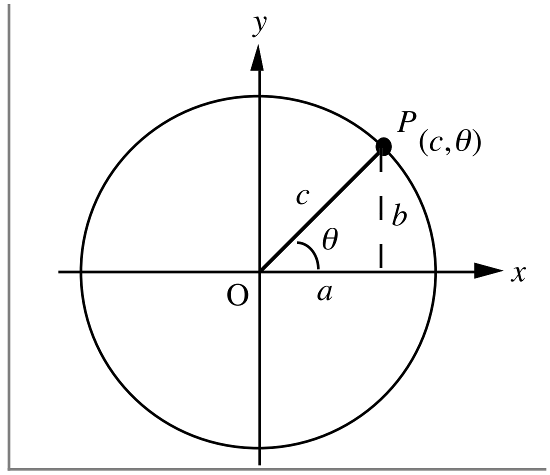
"The simplest solution is usually the best solution"---Albert Einstein

Abstract

Fermat's last theorem has been proved on a single page. The proof is based on the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$. One will first show that if $n = 2$. $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$, $c^n = a^n + b^n$ does not hold. Applying a polar coordinate system, let $a, b,$ and c be three relatively prime positive integers which are the lengths of the sides of a right triangle, where c is the length of the hypotenuse, and a and b are the lengths of the other two sides. Also, let the acute angle between the hypotenuse and the horizontal be denoted by θ. The proof is very simple, and even high school students can learn it. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin space for it in his paper.

Proof

Plan: One will first show that if $n = 2$, $c^n = a^n + b^n$ holds, followed by showing that if $n > 2$, $c^n = a^n + b^n$ does not hold.
Let $a, b,$and c be three relatively prime positive integers which are the lengths of the sides of the right triangle in the figure below, where c is the length of the hypotenuse, and a and b are the lengths of the other two sides. Also, let θ denote the acute angle between the hypotenuse and the horizontal.



Then $a = c\cos\theta$ (1)

$b = c\sin\theta$ (2)

$c^n = a^n + b^n$ (3)

$c^n = (c\cos\theta)^n + (c\sin\theta)^n$

$c^n = c^n\cos^n\theta + c^n\sin^n\theta$ (4)

$c^n = c^n(\cos^n\theta + \sin^n\theta)$ (5).

Left-hand side (LHS) of equation (5) equals right-hand side (RHS) of (5) only if

$$\cos^n\theta + \sin^n\theta = 1$$

That is, a necessary condition for (5) to be true is that

$$\boxed{\cos^n\theta + \sin^n\theta = 1}.$$

If $n = 2$, $c^2 = c^2(\cos^2\theta + \sin^2\theta)$, is true since $\cos^2\theta + \sin^2\theta = 1$ and therefore, equations (5) and (3) hold.

If $n = 3$, $c^3 \neq c^3(\cos^3\theta + \sin^3\theta)$ since $\cos^3\theta + \sin^3\theta \neq 1$and equations (5) and (3) do not hold.
$(\cos^3\theta + \sin^3\theta = (\cos\theta + \sin\theta)(\cos^2 - \cos\theta\sin\theta + \sin^2\theta \neq 1)$
Therefore, if $n = 3$, equations (5) and (3) do not hold..
If $n = 4$,, $c^4 \neq c^4(\cos^4\theta + \sin^4\theta)$ since $\cos^4\theta + \sin^4\theta \neq 1$. $(\cos^4\theta + \sin^4\theta \neq \cos^2\theta + \sin^2\theta)$
$(\cos^2\theta + \sin^2\theta = 1)$
$\{\cos^4\theta + \sin^4\theta = (\cos^2\theta + \sqrt{2}\cos\theta\sin\theta + \sin^2\theta)(\cos^2\theta - \sqrt{2}\cos\theta\sin\theta + \sin^2\theta \neq 1\}$
Therefore, if $n = 4$, equations (5) and (3). do not hold.
Replacing n by $k+1$ in $\cos^n\theta + \sin^n\theta$, one obtains $\cos^{k+1}\theta + \sin^{k+1}\theta$.

$\cos^{k+1}\theta + \sin^{k+1}\theta = \cos^2\theta + \sin^2\theta = 1$, only if $k = 1$, and then $n = 2$.

Therefore, equations (5) and (3) will be true only if $n = 2$, and there are no other integers, $n > 2$ which will make equations (5) and (3) true.
Therefore, $c^n = a^n + b^n$ holds only if $n = 2$, and does not hold if $n > 2$.

Conclusion

Fermat's last theorem has been proved in this paper. Note above that the main criterion is in equation (5) above, which requires that $\cos^n\theta + \sin^n\theta = 1$, if $c^n = c^n(\cos^n\theta + \sin^n\theta)$ and $c^n = a^n + b^n$ are to hold. Perhaps, the proof in this paper is the proof that Fermat wished there were enough margin space for it in his paper.

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