

Electro-Magnetic Field Equation and Lorentz gauge, Wave Function, Wave Equation in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime. We treat Lorentz gauge transformation in Rindler spacetime. We find the electro-magnetic wave equation and the electro-magnetic wave function in Rindler space-time. Specially, this article say the uniqueness of the accelerated frame because the accelerated frame can treat electro-magnetic field equation.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic field equation in Rindler space-time.

The Rindler coordinate is

$$\begin{aligned}
 ct &= \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
 x &= \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, \quad y = \xi^2, \quad z = \xi^3
 \end{aligned} \tag{1}$$

In this time, the tetrad $e^a{}_\mu$ is

$$\begin{aligned}
 d\tau^2 &= dt^2 - \frac{1}{c^2} [dx^2 + dy^2 + dz^2] \\
 &= -\frac{1}{c^2} \eta_{ab} \frac{\partial x^a}{\partial \xi^\mu} \frac{\partial x^b}{\partial \xi^\nu} d\xi^\mu d\xi^\nu \\
 &= -\frac{1}{c^2} \eta_{ab} e^a{}_\mu e^b{}_\nu d\xi^\mu d\xi^\nu = -\frac{1}{c^2} g_{\mu\nu} d\xi^\mu d\xi^\nu, \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu}
 \end{aligned} \tag{2}$$

$$e^{\alpha}{}_0(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^0} = \left(\left(1 + \frac{a_0}{c^2} \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right), \left(1 + \frac{a_0}{c^2} \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right), 0, 0 \right) \tag{3}$$

About y -axis's and z -axis's orientation

$$e^{\alpha}{}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0, 0, 1, 0), \quad e^{\alpha}{}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0, 0, 0, 1) \tag{4}$$

The other unit vector $e^{\alpha}{}_1(\xi^0)$ is

$$e^{\alpha}{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = \left(\sinh\left(\frac{a_0 \xi^0}{c}\right), \cosh\left(\frac{a_0 \xi^0}{c}\right), 0, 0 \right) \tag{5}$$

Therefore,

$$\begin{aligned}
 cdt &= c \cosh\left(\frac{a_0 \xi^0}{c}\right) d\xi^0 \left(1 + \frac{a_0}{c^2} \xi^1\right) + \sinh\left(\frac{a_0 \xi^0}{c}\right) d\xi^1 \\
 &= c \cosh\left(\frac{a_0 \xi^0}{c}\right) d\hat{\xi}^0 + \sinh\left(\frac{a_0 \xi^0}{c}\right) d\hat{\xi}^1 \\
 dx &= c \sinh\left(\frac{a_0 \xi^0}{c}\right) d\xi^0 \left(1 + \frac{a_0}{c^2} \xi^1\right) + \cosh\left(\frac{a_0 \xi^0}{c}\right) d\xi^1
 \end{aligned}$$

$$= c \sinh\left(\frac{a_0 \xi^0}{c}\right) d\hat{\xi}^0 + \cosh\left(\frac{a_0 \xi^0}{c}\right) d\hat{\xi}^1, dy = d\xi^2 = d\hat{\xi}^2, dz = d\xi^3 = d\hat{\xi}^3 \quad (6)$$

The vector transformation is

$$V'^{\mu} = \frac{\partial X'^{\mu}}{\partial X^{\alpha}} V^{\alpha}, \quad U'_{\mu} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} U_{\alpha} \quad (7)$$

Therefore, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A}) = A^{\alpha}$ is

$$A^{\alpha} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} A'^{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} A_{\xi}^{\mu} = e^{\alpha}_{\mu} A_{\xi}^{\mu}, \quad e^{\alpha}_{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}}$$

$$dX^{\alpha} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} dX'^{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu} = e^{\alpha}_{\mu} d\xi^{\mu}, \quad e^{\alpha}_{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} \quad (8)$$

Hence, the transformation of the electro-magnetic 4-vector potential (ϕ, \vec{A}) in inertial frame and the

electro-magnetic 4-vector potential $(\phi_{\xi}, \vec{A}_{\xi})$ in uniformly accelerated frame is

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \phi = 4\pi\rho$$

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \vec{A} = \frac{4\pi}{c} \vec{j}$$

$$\text{4-vector } (c\rho, \vec{j}) = \rho_0 \frac{dX^{\alpha}}{d\tau} \quad (9)$$

Lorentz gauge transformation is in Rindler spacetime,

$$A^{\mu} \rightarrow A^{\mu} + \partial^{\mu} \Lambda = A^{\mu} + g^{\mu\nu} \partial_{\nu} \Lambda, \quad \Lambda \text{ is a scalar function.}$$

$$g^{00} = -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2}, \quad g^{11} = g^{22} = g^{33} = 1$$

Hence,

$$\phi_{\xi} \rightarrow \phi_{\xi} - \frac{1}{c} \frac{\partial \Lambda}{\partial \xi^0} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2}, \quad \vec{A}_{\xi} \rightarrow \vec{A}_{\xi} + \vec{\nabla}_{\xi} \Lambda, \quad \Lambda \text{ is a scalar function.} \quad (10)$$

Lorentz gauge fixing condition is in Rindler spacetime,

$$A^\mu{}_{;\mu} = \frac{\partial A^\mu}{\partial \xi^\mu} + \Gamma^\mu{}_{\mu\rho} A^\rho,$$

$$\Gamma^\mu{}_{\mu\rho} = \Gamma^0{}_{01} = \frac{1}{2} g^{00} \left(\frac{\partial g_{00}}{\partial \xi^1} \right) = \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)}$$

$$g^{00} = -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2}, g^{11} = g^{22} = g^{33} = 1 \quad (11)$$

$$0 = \frac{\partial \phi}{\partial t} + \vec{\nabla} \cdot \vec{A} = \frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1} a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)}$$

$$A^\mu \rightarrow A^\mu + \partial^\mu \Lambda = A^\mu + g^{\mu\nu} \partial_\nu \Lambda$$

$$\begin{aligned} A^\mu{}_{;\mu} &= \frac{\partial A^\mu}{\partial \xi^\mu} + \Gamma^\mu{}_{\mu\rho} A^\rho \rightarrow \partial_\mu (A^\mu + g^{\mu\nu} \partial_\nu \Lambda) + \Gamma^0{}_{01} (A^1 + \frac{\partial \Lambda}{\partial \xi^1}) \\ &= \partial_\mu A^\mu + (g^{\mu\nu} \partial_\mu \partial_\nu) \Lambda + \Gamma^0{}_{01} (A^1 + \frac{\partial \Lambda}{\partial \xi^1}) \end{aligned}$$

$$0 = \frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1} a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)}$$

$$\rightarrow \frac{1}{c} \frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi - \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_\xi^2 \right] \Lambda$$

$$+ \frac{A_{\xi^1} a_0}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} + \frac{\partial \Lambda}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_\xi^2 \right] \Lambda - \frac{\partial \Lambda}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0$$

(12)

$$\phi = \cosh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \sinh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1}$$

$$= \cosh\left(\frac{a_0 \xi^0}{c}\right) \hat{\phi}_\xi + \sinh\left(\frac{a_0 \xi^0}{c}\right) \hat{A}_{\xi^1}$$

$$\begin{aligned}
A_x &= \sinh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_{\xi^1} + \cosh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1} \\
&= \sinh\left(\frac{a_0 \xi^0}{c}\right) \hat{\phi}_{\xi^1} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \hat{A}_{\xi^1} \\
A_y &= A_{\xi^2} = \hat{A}_{\xi^2}, A_z = A_{\xi^3} = \hat{A}_{\xi^3}
\end{aligned} \tag{13}$$

$$g = \begin{pmatrix} -\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e^a{}_{\mu} e_b{}^{\mu} = \delta^a{}_b, \quad e^a{}_{\mu} e_a{}^{\nu} = \delta_{\mu}{}^{\nu}$$

$$e^a{}_{\mu} e_b{}^{\nu} \eta_{ab} = g_{\mu\nu} \rightarrow A^T \eta A = g$$

$$e_a{}^{\mu} e_b{}^{\nu} g_{\mu\nu} = \eta_{ab} \rightarrow (A^T)^{-1} g A^{-1} = (A^T)^{-1} A^T \eta A A^{-1} = \eta$$

$$e^a{}_{\mu} = \eta^{ab} g_{\mu\nu} e_b{}^{\nu} \rightarrow \eta^{-1} (A^T)^{-1} A^T \eta A = A = \eta^{-1} (A^T)^{-1} g \tag{14}$$

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0 \xi^1}{c^2}\right) & \sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ \sinh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0 \xi^1}{c^2}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= A \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) & \sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ \sinh\left(\frac{a_0 \xi^0}{c}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\hat{\xi}^0 \\ d\hat{\xi}^1 \\ d\hat{\xi}^2 \\ d\hat{\xi}^3 \end{pmatrix}$$

(15)

$$e_{\mu}^{\alpha} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} = A^{-1} = \begin{pmatrix} \frac{\partial \xi^0}{\partial t} & \frac{\partial \xi^0}{\partial x} & \frac{\partial \xi^0}{\partial y} & \frac{\partial \xi^0}{\partial z} \\ \frac{\partial \xi^1}{\partial t} & \frac{\partial \xi^1}{\partial x} & \frac{\partial \xi^1}{\partial y} & \frac{\partial \xi^1}{\partial z} \\ \frac{\partial \xi^2}{\partial t} & \frac{\partial \xi^2}{\partial x} & \frac{\partial \xi^2}{\partial y} & \frac{\partial \xi^2}{\partial z} \\ \frac{\partial \xi^3}{\partial t} & \frac{\partial \xi^3}{\partial x} & \frac{\partial \xi^3}{\partial y} & \frac{\partial \xi^3}{\partial z} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} & -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} & 0 & 0 \\ -\frac{\sinh(\frac{a_0 \xi^0}{c})}{c} & \frac{\cosh(\frac{a_0 \xi^0}{c})}{c} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (16)$$

$$\begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^T \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial \xi^1}{\partial} \\ \frac{\partial \xi^2}{\partial} \\ \frac{\partial \xi^3}{\partial} \end{pmatrix} = (A^T)^{-1} \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial \xi^1}{\partial} \\ \frac{\partial \xi^2}{\partial} \\ \frac{\partial \xi^3}{\partial} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} & -\frac{\sinh(\frac{a_0 \xi^0}{c})}{c} & 0 & 0 \\ -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} & \frac{\cosh(\frac{a_0 \xi^0}{c})}{c} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial \xi^1}{\partial} \\ \frac{\partial \xi^2}{\partial} \\ \frac{\partial \xi^3}{\partial} \end{pmatrix}$$

$$= \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) & -\sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ -\sinh\left(\frac{a_0 \xi^0}{c}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \hat{\xi}^0} \\ \frac{\partial}{\partial \hat{\xi}^1} \\ \frac{\partial}{\partial \hat{\xi}^2} \\ \frac{\partial}{\partial \hat{\xi}^3} \end{pmatrix} \quad (17)$$

$$\frac{1}{c} \frac{\partial}{\partial t} = \frac{\partial \xi^0}{\partial t} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial t} \frac{\partial}{\partial \xi^1}$$

$$= \frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1}$$

$$= \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \hat{\xi}^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \hat{\xi}^1}$$

$$\frac{\partial}{\partial x} = \frac{\partial \xi^0}{\partial x} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial x} \frac{\partial}{\partial \xi^1}$$

$$= -\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1}$$

$$= -\sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \hat{\xi}^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \hat{\xi}^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi^2} = \frac{\partial}{\partial \hat{\xi}^2}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi^3} = \frac{\partial}{\partial \hat{\xi}^3}$$

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2 \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 = \frac{1}{c^2} \left(\frac{\partial}{\partial \hat{\xi}^0}\right)^2 - \nabla_{\hat{\xi}}^2$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right), \quad \vec{\nabla}_{\xi} = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}\right), \quad \vec{\nabla}_{\hat{\xi}} = \left(\frac{\partial}{\partial \hat{\xi}^1}, \frac{\partial}{\partial \hat{\xi}^2}, \frac{\partial}{\partial \hat{\xi}^3}\right)$$

(18)

2. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field (\vec{E}, \vec{B}) is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}, \vec{B} = \vec{\nabla} \times \vec{A} \quad (19)$$

$$E_x = -\frac{\partial \phi}{\partial x} - \frac{\partial A_x}{\partial t}$$

$$= -\left[-\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot \left[\cosh(\frac{a_0 \xi^0}{c}) \left(1 + \frac{a_0 \xi^1}{c^2}\right) \phi_\xi + \sinh(\frac{a_0 \xi^0}{c}) A_{\xi^1} \right]$$

$$- \left[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot \left[\sinh(\frac{a_0 \xi^0}{c}) \left(1 + \frac{a_0 \xi^1}{c^2}\right) \phi_\xi + \cosh(\frac{a_0 \xi^0}{c}) A_{\xi^1} \right]$$

$$= -\frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial A_{\xi^1}}{\partial \xi^0} - \left(1 + \frac{a_0 \xi^1}{c^2}\right) \frac{\partial \phi_\xi}{\partial \xi^1} - 2\phi_\xi \frac{a_0}{c^2}$$

$$= -\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^1} \left[\left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \phi_\xi \right] - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial A_{\xi^1}}{\partial \xi^0}$$

$$= -\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \hat{\xi}^1} \left[\left(1 + \frac{a_0}{c^2} \xi^1\right) \hat{\phi}_\xi \right] - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^0} \quad (20)$$

$$E_y = -\frac{\partial \phi}{\partial y} - \frac{\partial A_y}{\partial t} = -\frac{\partial}{\partial \xi^2} \left[\cosh(\frac{a_0 \xi^0}{c}) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \sinh(\frac{a_0 \xi^0}{c}) A_{\xi^1} \right]$$

$$- \left[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] A_{\xi^2}$$

$$= -\left(1 + \frac{a_0 \xi^1}{c^2}\right) \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial \phi_\xi}{\partial \xi^2} - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial A_{\xi^2}}{\partial \xi^0}$$

$$+ \sinh(\frac{a_0}{c} \xi^0) \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right]$$

$$\begin{aligned}
&= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \left[\phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^2}}{\partial \xi^0} \right] \\
&+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right] \\
&= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \hat{\xi}^2} \left[\hat{\phi}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right] - \frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^0} \right] \\
&+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^1} - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^2} \right] \tag{21}
\end{aligned}$$

$$\begin{aligned}
E_z &= -\frac{\partial \phi}{\partial z} - \frac{\partial A_z}{\partial t} = -\frac{\partial}{\partial \xi^3} \left[\cosh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \sinh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1} \right] \\
&\quad - \left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \\
&= -\left(1 + \frac{a_0 \xi^1}{c^2}\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial \phi_\xi}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial A_{\xi^3}}{\partial \xi^0} \\
&\quad + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^3}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^3} \right] \\
&= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \left[\phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^3}}{\partial \xi^0} \right] \\
&+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^3}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^3} \right] \\
&= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \hat{\xi}^3} \left[\hat{\phi}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right] - \frac{\partial \hat{A}_{\xi^3}}{\partial \hat{\xi}^0} \right]
\end{aligned}$$

$$+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial \hat{A}_{\xi^3}}{\partial \hat{\xi}^1} - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^3} \right] \quad (22)$$

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = \frac{\partial A_{\xi^3}}{\partial \xi^2} - \frac{\partial A_{\xi^2}}{\partial \xi^3} = \frac{\partial \hat{A}_{\xi^3}}{\partial \hat{\xi}^2} - \frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^3} \quad (23)$$

$$\begin{aligned} B_y &= \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = \frac{\partial A_x}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial x} \\ &= \frac{\partial}{\partial \xi^3} \left[\sinh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \cosh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1} \right] \\ &\quad - \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \\ &= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^1}}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial \xi^1} \right] \\ &\quad - \sinh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \left[\phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^3}}{\partial \xi^0} \right] \\ &= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^3} - \frac{\partial \hat{A}_{\xi^3}}{\partial \hat{\xi}^1} \right] \\ &\quad - \sinh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \hat{\xi}^3} \left[\hat{\phi}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right] - \frac{\partial \hat{A}_{\xi^3}}{\partial \hat{\xi}^0} \right] \quad (24) \end{aligned}$$

$$\begin{aligned} B_z &= \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \frac{\partial A_{\xi^2}}{\partial x} - \frac{\partial A_x}{\partial \xi^2} \\ &= \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \end{aligned}$$

$$\begin{aligned}
& -\frac{\partial}{\partial \xi^2} \left[\sinh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \cosh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1} \right] \\
& = \cosh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right] \\
& + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \left[\phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^2}}{\partial \xi^0} \right] \\
& = \cosh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^1} - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^2} \right] \\
& + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \hat{\xi}^2} \left[\hat{\phi}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right] - \frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^0} \right] \quad (25)
\end{aligned}$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler spacetime.

$$\begin{aligned}
\vec{E}_\xi & = -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_\xi \left\{ \phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right\} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial \vec{A}_\xi}{\partial \xi^0} \\
& = -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_\xi \left\{ \hat{\phi}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right\} - \frac{\partial \vec{\hat{A}}_\xi}{\partial \hat{\xi}^0} \\
\vec{B}_\xi & = \vec{\nabla}_\xi \times \vec{A}_\xi = \vec{\nabla}_\xi \times \vec{\hat{A}}_\xi
\end{aligned}$$

In this time, $\vec{\nabla}_\xi = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right)$, $\vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$

$$\vec{\nabla}_\xi = \left(\frac{\partial}{\partial \hat{\xi}^1}, \frac{\partial}{\partial \hat{\xi}^2}, \frac{\partial}{\partial \hat{\xi}^3} \right), \vec{\hat{A}}_\xi = (\hat{A}_{\xi^1}, \hat{A}_{\xi^2}, \hat{A}_{\xi^3}) \quad (26)$$

Lorentz gauge transformation is in Rindler spacetime,

$$\phi_\xi \rightarrow \phi_\xi - \frac{1}{c} \frac{\partial \Lambda}{\partial \xi^0} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2}, \quad \vec{A}_\xi \rightarrow \vec{A}_\xi + \vec{\nabla}_\xi \Lambda, \quad \Lambda \text{ is a scalar function.} \quad (27)$$

$$\begin{aligned}
\vec{E}_\xi &= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})} \vec{\nabla}_\xi \left\{ \phi_\xi \left(1+\frac{a_0\xi^1}{c^2}\right)^2 \right\} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \vec{\nabla}_\xi \frac{\partial\Lambda}{c\partial\xi^0} \\
&\quad - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial\vec{A}_\xi}{c\partial\xi^0} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial}{c\partial\xi^0} \vec{\nabla}_\xi \Lambda \\
&= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})} \vec{\nabla}_\xi \left\{ \phi_\xi \left(1+\frac{a_0\xi^1}{c^2}\right)^2 \right\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial\vec{A}_\xi}{c\partial\xi^0} \\
\vec{B}_\xi &= \vec{\nabla}_\xi \times \vec{A}_\xi + \vec{\nabla}_\xi \times \vec{\nabla}_\xi \Lambda = \vec{\nabla}_\xi \times \vec{A}_\xi
\end{aligned} \tag{28}$$

Lorentz gauge fixing condition is in Rindler spacetime,

$$\begin{aligned}
0 &= \frac{\partial\phi_\xi}{c\partial\xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1}a_0}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \\
&\rightarrow \frac{1}{c} \frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi - \left[\frac{1}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \left(\frac{\partial}{\partial\xi^0}\right)^2 - \nabla_\xi^2 \right] \Lambda \\
&\quad + \frac{A_{\xi^1}a_0}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})} + \frac{\partial\Lambda}{\partial\xi^1} \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})} = 0 \\
&\left[\frac{1}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \left(\frac{\partial}{\partial\xi^0}\right)^2 - \nabla_\xi^2 \right] \Lambda - \frac{\partial\Lambda}{\partial\xi^1} \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0\xi^1}{c^2})} = 0 \\
\phi - \frac{1}{c} \frac{\partial\Lambda}{\partial t} &= \cosh\left(\frac{a_0\xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \left\{ \phi_\xi - \frac{1}{c} \frac{\partial\Lambda}{\partial\xi^0} \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \right\} \\
&\quad + \sinh\left(\frac{a_0\xi^0}{c}\right) \left(A_{\xi^1} + \frac{\partial\Lambda}{\partial\xi^1} \right) \\
A_x + \frac{\partial\Lambda}{\partial x} &= \sinh\left(\frac{a_0\xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \left\{ \phi_\xi - \frac{1}{c} \frac{\partial\Lambda}{\partial\xi^0} \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2} \right\}
\end{aligned}$$

$$+ \cosh\left(\frac{a_0 \xi^0}{c}\right) \left(A_{\xi^1} + \frac{\partial \Lambda}{\partial \xi^1}\right)$$

$$A_y + \frac{\partial \Lambda}{\partial y} = A_{\xi^2} + \frac{\partial \Lambda}{\partial \xi^2}, A_z + \frac{\partial \Lambda}{\partial z} = A_{\xi^3} + \frac{\partial \Lambda}{\partial \xi^3} \quad (29)$$

We obtain the transformation of the electro-magnetic field.

$$\begin{aligned} E_x &= -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^1} \left\{ \phi_{\xi} \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right\} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^1}}{\partial \xi^0} \\ &= -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^1} \left\{ \hat{\phi}_{\xi} \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right\} - \frac{\partial \hat{A}_{\xi^1}}{\partial \xi^0} = E_{\xi^1}, \end{aligned}$$

$$E_y = E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right),$$

$$E_z = E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$B_x = B_{\xi^1},$$

$$B_y = B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$B_z = B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \quad (30)$$

Hence,

$$E_x = E_{\xi^1}, B_x = B_{\xi^1},$$

$$\begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix} = H \begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix}$$

$$H = \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & \sinh\left(\frac{a_0 \xi^0}{c}\right) \\ 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) & -\sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ 0 & -\sinh\left(\frac{a_0 \xi^0}{c}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ \sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) \end{pmatrix} \quad (31)$$

The inverse-transformation of the electro-magnetic field is

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$\begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix} = H^{-1} \begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & -\sinh\left(\frac{a_0 \xi^0}{c}\right) \\ 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) & \sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ 0 & \sinh\left(\frac{a_0 \xi^0}{c}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ -\sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) \end{pmatrix} \quad (32)$$

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$E_{\xi^2} = E_y \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_z \sinh\left(\frac{a_0 \xi^0}{c}\right),$$

$$B_{\xi^2} = B_y \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_z \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$E_{\xi^3} = E_z \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_y \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$B_{\xi^3} = B_z \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \quad (33)$$

3. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (34-i)$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j} \quad (34-ii)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (34-iii)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (34-iv)$$

$$1. \vec{\nabla} \cdot \vec{E} = 4\pi\rho$$

$$E_x = E_{\xi^1},$$

$$E_y = E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right),$$

$$E_z = E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$4\pi\rho = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] E_{\xi^1}$$

$$+ \frac{\partial}{\partial \xi^2} \left[E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right]$$

$$+ \frac{\partial}{\partial \xi^3} \left[E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right]$$

$$= \cosh\left(\frac{a_0}{c} \xi^0\right) (\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi}) + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial E_{\xi^1}}{\partial \xi^0} \right]$$

$$= \cosh\left(\frac{a_0}{c} \xi^0\right) (\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi}) + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial B_{\xi^3}}{\partial \hat{\xi}^2} - \frac{\partial B_{\xi^2}}{\partial \hat{\xi}^3} - \frac{\partial E_{\xi^1}}{\partial \hat{\xi}^0} \right] \quad (35)$$

$$2. \vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}$$

$$B_x = B_{\xi^1}$$

$$B_y = B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$B_z = B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$\text{X-component) } \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$$

$$= \frac{\partial}{\partial \xi^2} [B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)]$$

$$- \frac{\partial}{\partial \xi^3} [B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)]$$

$$= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} \right] + \sinh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial E_{\xi^2}}{\partial \xi^2} + \frac{\partial E_{\xi^3}}{\partial \xi^3} \right]$$

$$= \frac{\partial E_x}{\partial t} + \frac{4\pi}{c} j_x$$

$$= \left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] E_{\xi^1} + \frac{4\pi}{c} j_x$$

Hence,

$$\frac{4\pi}{c} j_x$$

$$= \sinh\left(\frac{a_0 \xi^0}{c}\right) (\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi}) + \cosh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial E_{\xi^1}}{\partial \xi^0} \right]$$

$$= \sinh\left(\frac{a_0 \xi^0}{c}\right) (\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi}) + \cosh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial B_{\xi^3}}{\partial \hat{\xi}^2} - \frac{\partial B_{\xi^2}}{\partial \hat{\xi}^3} - \frac{\partial E_{\xi^1}}{\partial \hat{\xi}^0} \right] \quad (36)$$

$$\text{Y-component) } \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}$$

$$\begin{aligned}
&= \frac{\partial B_{\xi^1}}{\partial \xi^3} \\
&- \left[-\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot [B_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) + E_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})] \\
&= \frac{\partial E_y}{\partial t} + \frac{4\pi}{c} j_y \\
&= \left[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot [E_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})] \\
&\quad + \frac{4\pi}{c} j_y \\
\frac{4\pi}{c} j_y &= \frac{\partial B_{\xi^1}}{\partial \xi^3} - \frac{\partial B_{\xi^3}}{\partial \xi^1} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{a_0}{c^2} B_{\xi^3} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial E_{\xi^2}}{\partial \xi^0} \\
&= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^3} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^1} \{B_{\xi^3} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial E_{\xi^2}}{\partial \xi^0} \\
&= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^3} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^1} \{B_{\xi^3} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{\partial E_{\xi^2}}{\partial \xi^0}
\end{aligned}$$

(37)

$$\begin{aligned}
&\text{Z-component) } \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \\
&= \left[-\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot [B_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) - E_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})] \\
&\quad - \frac{\partial B_{\xi^1}}{\partial \xi^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\partial E_z}{\partial t} + \frac{4\pi}{c} j_z \\
&= \left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot [E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)] \\
&\quad + \frac{4\pi}{c} j_z \\
\frac{4\pi}{c} j_z &= \frac{\partial B_{\xi^2}}{\partial \xi^1} - \frac{\partial B_{\xi^1}}{\partial \xi^2} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{a_0}{c^2} B_{\xi^2} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial E_{\xi^3}}{\partial \xi^0} \\
&= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \{B_{\xi^2} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial E_{\xi^3}}{\partial \xi^0} \\
&= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \{B_{\xi^2} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{\partial E_{\xi^3}}{\partial \xi^0}
\end{aligned}$$

(38)

$$3. \vec{\nabla} \cdot \vec{B} = 0$$

$$\begin{aligned}
\vec{\nabla} \cdot \vec{B} &= \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \\
&= \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] B_{\xi^1} \\
&\quad + \frac{\partial}{\partial \xi^2} [B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)] \\
&\quad + \frac{\partial}{\partial \xi^3} [B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)] \\
&= \cosh\left(\frac{a_0 \xi^0}{c}\right) (\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi}) + \sinh\left(\frac{a_0 \xi^0}{c}\right) \left[-\left(-\frac{\partial E_{\xi^2}}{\partial \xi^3} + \frac{\partial E_{\xi^3}}{\partial \xi^2} \right) - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial B_{\xi^1}}{\partial \xi^0} \right] = 0
\end{aligned}$$

$$= \cosh\left(\frac{a_0 \xi^0}{c}\right) (\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi}) + \sinh\left(\frac{a_0 \xi^0}{c}\right) \left[-\left(\frac{\partial E_{\xi^2}}{\partial \xi^3} + \frac{\partial E_{\xi^3}}{\partial \xi^2} \right) - \frac{\partial B_{\xi^1}}{\partial \xi^0} \right] = 0$$

(39)

$$4. \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$E_x = E_{\xi^1},$$

$$E_y = E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right),$$

$$E_z = E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$\text{X-component) } \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$$

$$= \frac{\partial}{\partial \xi^2} [E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)]$$

$$- \frac{\partial}{\partial \xi^3} [E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)]$$

$$= \cosh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3} \right] - \sinh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial B_{\xi^2}}{\partial \xi^2} + \frac{\partial B_{\xi^3}}{\partial \xi^3} \right]$$

$$= -\frac{\partial B_x}{\partial t}$$

$$= -\left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] B_{\xi^1}$$

$$\text{Hence, } -\sinh\left(\frac{a_0 \xi^0}{c}\right) (\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi}) + \cosh\left(\frac{a_0 \xi^0}{c}\right) \left[\left(\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3} \right) + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial B_{\xi^1}}{\partial \xi^0} \right]$$

$$= -\sinh\left(\frac{a_0 \xi^0}{c}\right) (\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi}) + \cosh\left(\frac{a_0 \xi^0}{c}\right) \left[\left(\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3} \right) + \frac{\partial B_{\xi^1}}{\partial \xi^0} \right] = 0 \quad (40)$$

$$\begin{aligned}
& \text{Y-component) } \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\
&= \frac{\partial E_{\xi^1}}{\partial \xi^3} \\
&= - \left[- \frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot [E_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})] \\
&= - \frac{\partial B_y}{\partial t} \\
&= - \left[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot [B_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) - E_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})] \\
&= \frac{\partial E_{\xi^1}}{\partial \xi^3} - \frac{\partial E_{\xi^3}}{\partial \xi^1} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{a_0}{c^2} E_{\xi^3} + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial B_{\xi^2}}{\partial \xi^0} \\
&= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^3} \{E_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^1} \{E_{\xi^3} (1 + \frac{a_0 \xi^1}{c^2})\} + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial B_{\xi^2}}{\partial \xi^0} \\
&= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^3} \{E_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^1} \{E_{\xi^3} (1 + \frac{a_0 \xi^1}{c^2})\} + \frac{\partial B_{\xi^2}}{\partial \xi^0} = 0
\end{aligned}$$

(41)

$$\begin{aligned}
& \text{Z-component) } \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \\
&= \left[- \frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot [E_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})] \\
&= \frac{\partial E_{\xi^1}}{\partial \xi^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\partial B_z}{\partial t} \\
&= -\left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot [B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)] \\
&\quad \frac{\partial E_{\xi^2}}{\partial \xi^1} - \frac{\partial E_{\xi^1}}{\partial \xi^2} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{a_0}{c^2} E_{\xi^2} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial B_{\xi^3}}{\partial \xi^0} \\
&= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \{E_{\xi^2} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \{E_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial B_{\xi^3}}{\partial \xi^0} \\
&= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \{E_{\xi^2} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \{E_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} + \frac{\partial B_{\xi^3}}{\partial \xi^0} = 0
\end{aligned} \tag{42}$$

Therefore, we obtain the electro-magnetic field equation by Eq (35)-Eq(42) in Rindler spacetime .

$$\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi} = \vec{\nabla}_{\xi} \cdot \vec{E}_{\xi} = 4\pi\rho_{\xi} \left(1 + \frac{a_0 \xi^1}{c^2}\right) \tag{43-i}$$

$$\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_{\xi} \times \{\vec{B}_{\xi} (1 + \frac{a_0 \xi^1}{c^2})\} = \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial \vec{E}_{\xi}}{\partial \xi^0} + \frac{4\pi}{c} \vec{j}_{\xi}$$

$$\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_{\xi} \times \{\vec{B}_{\xi} (1 + \frac{a_0 \xi^1}{c^2})\} = \frac{\partial \vec{E}_{\xi}}{\partial \xi^0} + \frac{4\pi}{c} \vec{j}_{\xi} \tag{43-ii}$$

$$\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi} = \vec{\nabla}_{\xi} \cdot \vec{B}_{\xi} = 0 \tag{43-iii}$$

$$\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_{\xi} \times \{\vec{E}_{\xi} (1 + \frac{a_0 \xi^1}{c^2})\} = -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial \vec{B}_{\xi}}{\partial \xi^0}$$

$$\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_{\xi} \times \{\vec{E}_{\xi} (1 + \frac{a_0 \xi^1}{c^2})\} = -\frac{\partial \vec{B}_{\xi}}{\partial \xi^0} \tag{43-iv}$$

$$\vec{E}_\xi = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_\xi = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}),$$

$$\vec{\nabla}_\xi = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right), \vec{\nabla}_{\hat{\xi}} = \left(\frac{\partial}{\partial \hat{\xi}^1}, \frac{\partial}{\partial \hat{\xi}^2}, \frac{\partial}{\partial \hat{\xi}^3} \right)$$

Hence, the transformation of 4-vector $(c\rho, \vec{j}) = \rho_0 \frac{dx^\alpha}{d\tau}$ is

$$\rho = \rho_\xi \left(1 + \frac{a_0 \xi^1}{c^2} \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{j_{\xi^1}}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$j_x = j_{\xi^1} \cosh\left(\frac{a_0 \xi^0}{c}\right) + c\rho_\xi \left(1 + \frac{a_0}{c^2} \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right), \quad j_y = j_{\xi^2}, j_z = j_{\xi^3}$$

$$\text{In this time, 4-vector } (c\rho_\xi, \vec{j}_\xi) = \rho_0 \frac{d\xi^\alpha}{d\tau} \quad (44)$$

Generally, the continuity equation is in Rindler spacetime,

$$0 = j^\mu{}_{;\mu} = \frac{\partial j^\mu}{\partial \xi^\mu} + \Gamma^\mu{}_{\mu\rho} j^\rho,$$

$$\Gamma^\mu{}_{\mu\rho} = \Gamma^0{}_{01} = \frac{1}{2} g^{00} \left(\frac{\partial g_{00}}{\partial \xi^1} \right) = \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1 \right)}$$

$$g^{00} = -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2} \right)^2}, g^{11} = g^{22} = g^{33} = 1$$

$$0 = \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = \frac{\partial \rho_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{j}_\xi + \frac{j_{\xi^1} a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2} \right)} \quad (45)$$

We treat Lorentz gauge transformation about the electro-magnetic field equation in Rindler spacetime.

Eq(43-i) is

$$\begin{aligned} \vec{\nabla}_\xi \cdot \vec{E}_\xi &= \vec{\nabla}_\xi \cdot \left\{ -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2} \right)} \vec{\nabla}_\xi \left\{ \phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2} \right)^2 \right\} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2} \right)} \frac{\partial \vec{A}_\xi}{\partial \xi^0} \right\} \\ &= -\vec{\nabla}_\xi \left\{ \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2} \right)} \right\} \cdot \left[\vec{\nabla}_\xi \left\{ \phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2} \right)^2 \right\} + \frac{\partial \vec{A}_\xi}{\partial \xi^0} \right] \end{aligned}$$

$$\begin{aligned}
& - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} [\nabla_{\xi}^2 \{\phi_{\xi} (1 + \frac{a_0 \xi^1}{c^2})^2\} + \frac{\partial}{c \partial \xi^0} (\vec{\nabla}_{\xi} \cdot \vec{A}_{\xi})] \\
& = \frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} [\frac{\partial}{\partial \xi^1} \{\phi_{\xi} (1 + \frac{a_0 \xi^1}{c^2})^2\} + \frac{\partial A_{\xi^1}}{c \partial \xi^0}] \\
& - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} [\nabla_{\xi}^2 - \frac{1}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} (\frac{\partial}{\partial \xi^0})^2] \{\phi_{\xi} (1 + \frac{a_0 \xi^1}{c^2})^2\} \\
& - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{c \partial \xi^0} [-\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{A_{\xi^1} a_0}{c^2}] \\
& \frac{1}{c} \frac{\partial \phi_{\xi}}{\partial \xi^0} + \vec{\nabla}_{\xi} \cdot \vec{A}_{\xi} = -\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} A_{\xi^1} \\
& = -\frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} E_{\xi^1} - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} [\nabla_{\xi}^2 - \frac{1}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} (\frac{\partial}{\partial \xi^0})^2] \{\phi_{\xi} (1 + \frac{a_0 \xi^1}{c^2})^2\} \\
& + \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} \frac{\partial A_{\xi^1}}{c \partial \xi^0} \frac{a_0}{c^2} \\
& = 4\pi\rho_{\xi} (1 + \frac{a_0 \xi^1}{c^2}) \tag{46}
\end{aligned}$$

If we apply Lorentz gauge transformation to Eq (46),

$$\begin{aligned}
\phi_{\xi} & \rightarrow \phi_{\xi} - \frac{1}{c} \frac{\partial \Lambda}{\partial \xi^0} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2}, \quad \vec{A}_{\xi} \rightarrow \vec{A}_{\xi} + \vec{\nabla}_{\xi} \Lambda, \quad \Lambda \text{ is a scalar function.} \\
& = -\frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} E_{\xi^1} - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} [\nabla_{\xi}^2 - \frac{1}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} (\frac{\partial}{\partial \xi^0})^2] \{\phi_{\xi} (1 + \frac{a_0 \xi^1}{c^2})^2\} \\
& + \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} [\nabla_{\xi}^2 - \frac{1}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} (\frac{\partial}{\partial \xi^0})^2] \frac{1}{c} \frac{\partial \Lambda}{\partial \xi^0}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \frac{\partial A_{\xi^1}}{\partial \xi^0} \frac{a_0}{c^2} + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^0} \frac{\partial \Lambda}{\partial \xi^1} \\
& = -\frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} E_{\xi^1} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \left[\nabla_{\xi^2}^2 - \frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \right] \left\{ \phi_{\xi^1} \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right\} \\
& + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} \left\{ \left[\nabla_{\xi^2}^2 - \frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \right] \Lambda + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{a_0}{c^2} \frac{\partial \Lambda}{\partial \xi^1} \right\} \\
& + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \frac{\partial A_{\xi^1}}{\partial \xi^0} \frac{a_0}{c^2} \tag{47}
\end{aligned}$$

In this time,

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] \Lambda - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{a_0}{c^2} \frac{\partial \Lambda}{\partial \xi^1} = 0 \tag{48}$$

Hence, Eq(43-i) is

$$\begin{aligned}
& \vec{\nabla}_{\xi^1} \cdot \vec{E}_{\xi^1} \\
& = -\frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} E_{\xi^1} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \left[\nabla_{\xi^2}^2 - \frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \right] \left\{ \phi_{\xi^1} \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right\} \\
& + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \frac{\partial A_{\xi^1}}{\partial \xi^0} \frac{a_0}{c^2} \\
& = 4\pi\rho_{\xi^1} \left(1 + \frac{a_0 \xi^1}{c^2}\right) \tag{49}
\end{aligned}$$

Eq(43-i) is invariant about Lorentz gauge transformation in Rindler spacetime.

Eq (43-ii) is

$$\begin{aligned}
& \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_{\xi^1} \times \left\{ \vec{B}_{\xi^1} \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right\} \\
& = \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_{\xi^1} \times \left\{ \vec{\nabla}_{\xi^1} \times \vec{A}_{\xi^1} \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right\}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \{\vec{\nabla}_\xi \times \vec{A}_\xi\} + \vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \vec{A}_\xi \\
&= \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{a_0}{c^2} (1,0,0) \times \vec{B}_\xi + \{-\nabla_\xi^2 \vec{A}_\xi + \vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{A}_\xi)\} \\
&= \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{a_0}{c^2} (0, -B_{\xi^3}, B_{\xi^2}) + \{-\nabla_\xi^2 \vec{A}_\xi + \vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{A}_\xi)\} \\
&= \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial \vec{E}_\xi}{\partial \xi^0} + \frac{4\pi}{c} \vec{j}_\xi \\
&= -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \frac{\partial}{\partial \xi^0} [\vec{\nabla}_\xi \{\phi_\xi (1 + \frac{a_0 \xi^1}{c^2})^2\}] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{A}_\xi + \frac{4\pi \vec{j}_\xi}{c} \\
&= -\frac{\partial}{\partial \xi^0} \vec{\nabla}_\xi \phi_\xi - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{2a_0}{c^2} \frac{\partial \phi_\xi}{\partial \xi^0} (1,0,0) - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{A}_\xi + \frac{4\pi \vec{j}_\xi}{c}
\end{aligned} \tag{50}$$

Therefore,

$$\begin{aligned}
&\frac{4\pi}{c} \vec{j}_\xi \\
&= \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{a_0}{c^2} (0, -B_{\xi^3}, B_{\xi^2}) + \{-\nabla_\xi^2 \vec{A}_\xi + \vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{A}_\xi)\} \\
&+ \frac{\partial}{\partial \xi^0} \vec{\nabla}_\xi \phi_\xi + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{2a_0}{c^2} \frac{\partial \phi_\xi}{\partial \xi^0} (1,0,0) + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{A}_\xi \\
&= \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} (0, -B_{\xi^3}, B_{\xi^2}) + \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{2a_0}{c^2} \frac{\partial \phi_\xi}{\partial \xi^0} (1,0,0)
\end{aligned}$$

$$\begin{aligned}
& + [-\nabla_{\xi}^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \bar{A}_{\xi} + \vec{\nabla}_{\xi} [-\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} A_{\xi^1}] \\
& \frac{1}{c} \frac{\partial \phi_{\xi}}{\partial \xi^0} + \vec{\nabla}_{\xi} \cdot \bar{A}_{\xi} = -\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} A_{\xi^1} \tag{51}
\end{aligned}$$

$$\begin{aligned}
& \frac{4\pi}{c} \vec{j}_{\xi} \\
& = \frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} (0, -B_{\xi^3}, B_{\xi^2}) + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{2a_0}{c^2} \frac{\partial \phi_{\xi}}{c \partial \xi^0} (1, 0, 0) \\
& + [-\nabla_{\xi}^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \bar{A}_{\xi} \\
& + \vec{\nabla}_{\xi} [-\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} A_{\xi^1}] \tag{52}
\end{aligned}$$

If we apply Lorentz gauge transformation to Eq (52),

$$\phi_{\xi} \rightarrow \phi_{\xi} - \frac{1}{c} \frac{\partial \Lambda}{\partial \xi^0} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})}, \quad \bar{A}_{\xi} \rightarrow \bar{A}_{\xi} + \vec{\nabla}_{\xi} \Lambda, \quad \Lambda \text{ is a scalar function.}$$

$$\begin{aligned}
& \frac{4\pi}{c} \vec{j}_{\xi} \\
& = \frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} (0, -B_{\xi^3}, B_{\xi^2}) + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{2a_0}{c^2} \frac{\partial \phi_{\xi}}{c \partial \xi^0} (1, 0, 0) \\
& \quad - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^3} \frac{2a_0}{c^2} \frac{1}{c^2} (\frac{\partial}{\partial \xi^0})^2 \Lambda (1, 0, 0) \\
& + [-\nabla_{\xi}^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \bar{A}_{\xi} + [-\nabla_{\xi}^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \vec{\nabla}_{\xi} \Lambda \\
& + \vec{\nabla}_{\xi} [-\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} A_{\xi^1}] + \vec{\nabla}_{\xi} [-\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial \Lambda}{\partial \xi^1}]
\end{aligned}$$

$$\begin{aligned}
&= \frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} (0, -B_{\xi^3}, B_{\xi^2}) + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{2a_0}{c^2} \frac{\partial \phi_\xi}{c \partial \xi^0} (1, 0, 0) \\
&\quad - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^3} \frac{2a_0}{c^2} \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 \Lambda (1, 0, 0) \\
&+ [-\nabla_\xi^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2] \vec{A}_\xi + [-\nabla_\xi^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2] \vec{\nabla}_\xi \Lambda \\
&+ \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} \frac{a_0^2}{c^4} A_{\xi^1} (1, 0, 0) - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \vec{\nabla}_\xi A_{\xi^1} \\
&+ \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} \frac{a_0^2}{c^4} \frac{\partial \Lambda}{\partial \xi^1} (1, 0, 0) - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1} \vec{\nabla}_\xi \Lambda
\end{aligned}$$

(53)

In this time,

$$\begin{aligned}
&\left[\frac{1}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_\xi^2 \right] \Lambda - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial \Lambda}{\partial \xi^1} = 0 \\
0 &= \vec{\nabla}_\xi \left[\left\{ -\nabla_\xi^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1} \right\} \Lambda \right] \\
&= \vec{\nabla}_\xi \left\{ \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \right\} \left(\frac{\partial}{\partial \xi^0} \right)^2 \Lambda + [-\nabla_\xi^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2] \vec{\nabla}_\xi \Lambda \\
&\quad - \vec{\nabla}_\xi \left\{ \frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \right\} \frac{\partial \Lambda}{\partial \xi^1} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1} \vec{\nabla}_\xi \Lambda \\
&= -\frac{2}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^3} \frac{a_0}{c^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 \Lambda (1, 0, 0) + [-\nabla_\xi^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2] \vec{\nabla}_\xi \Lambda
\end{aligned}$$

$$+ \frac{a_0^2}{c^4} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \frac{\partial \Lambda}{\partial \xi^1} (1,0,0) - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1} \vec{\nabla}_\xi \Lambda \quad (54)$$

Therefore,

$$\begin{aligned} & \frac{4\pi}{c} \vec{j}_\xi \\ &= \frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} (0, -B_{\xi^3}, B_{\xi^2}) + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{2a_0}{c^2} \frac{\partial \phi_\xi}{\partial \xi^0} (1,0,0) \\ & \quad - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^3} \frac{2a_0}{c^2} \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \Lambda (1,0,0) \\ & + [-\nabla_\xi^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2] \vec{A}_\xi + \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} \frac{a_0^2}{c^4} A_{\xi^1} (1,0,0) \\ & - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \vec{\nabla}_\xi A_{\xi^1} + \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} \frac{a_0^2}{c^4} \frac{\partial \Lambda}{\partial \xi^1} (1,0,0) - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1} \vec{\nabla}_\xi \Lambda \\ & + \vec{\nabla}_\xi \left[\left\{ -\nabla_\xi^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1} \right\} \Lambda \right] \\ & + \frac{2}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^3} \frac{a_0}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \Lambda (1,0,0) - \frac{a_0^2}{c^4} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \frac{\partial \Lambda}{\partial \xi^1} (1,0,0) \\ & + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1} \vec{\nabla}_\xi \Lambda \\ & = \frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} (0, -B_{\xi^3}, B_{\xi^2}) + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{2a_0}{c^2} \frac{\partial \phi_\xi}{\partial \xi^0} (1,0,0) \\ & \quad + [-\nabla_\xi^2 + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2] \vec{A}_\xi \end{aligned}$$

$$+ \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} \frac{a_0^2}{c^4} A_{\xi^1}(1,0,0) - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \vec{\nabla}_{\xi} A_{\xi^1} \quad (55)$$

Hence, Eq(43-ii) is invariant about Lorentz gauge transformation in Rindler spacetime.

Eq (43-iii) is

$$\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi} = \vec{\nabla}_{\xi} \cdot (\vec{\nabla}_{\xi} \times \vec{A}_{\xi} + \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \Lambda) = \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \cdot \vec{A}_{\xi} = 0 \quad (56)$$

Eq (43-iv) is

$$\begin{aligned} & \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \vec{\nabla}_{\xi} \times \{ \vec{E}_{\xi} (1 + \frac{a_0 \xi^1}{c^2}) \} \\ &= - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \vec{\nabla}_{\xi} \times [\vec{\nabla}_{\xi} \{ \phi_{\xi} (1 + \frac{a_0 \xi^1}{c^2})^2 \} - \vec{\nabla}_{\xi} (\frac{\partial \Lambda}{\partial \xi^0}) + \frac{\partial \vec{A}_{\xi}}{\partial \xi^0} + \frac{\partial}{\partial \xi^0} (\vec{\nabla}_{\xi} \Lambda)] \\ &= - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \vec{\nabla}_{\xi} \times \frac{\partial \vec{A}_{\xi}}{\partial \xi^0} = - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial (\vec{\nabla}_{\xi} \times \vec{A}_{\xi})}{\partial \xi^0} = - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial \vec{B}_{\xi}}{\partial \xi^0} \quad (57) \end{aligned}$$

Hence, Eq (43-iii), Eq (43-iv) are invariant about Lorentz gauge transformation in Rindler spacetime.

Hence, the electro-magnetic field equations(Maxwell Equations) in Rindler spacetime are invariant about Lorentz gauge transformation.

4. Electro-magnetic wave equation in Rindler space-time

The electro-magnetic wave function is

$$E_x = E_{x0} \sin \Phi, E_y = E_{y0} \sin \Phi, E_z = E_{z0} \sin \Phi$$

$$B_x = B_{x0} \sin \Phi, B_y = B_{y0} \sin \Phi, B_z = B_{z0} \sin \Phi$$

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$E_{\xi^1} = E_{x0} \sin \Phi', B_{\xi^1} = B_{x0} \sin \Phi'$$

$$E_{\xi^2} = E_y \cosh(\frac{a_0 \xi^0}{c}) - B_z \sinh(\frac{a_0 \xi^0}{c}),$$

$$\begin{aligned}
&= (E_{y_0} \sin \Phi') \cosh\left(\frac{a_0 \xi^0}{c}\right) - (B_{z_0} \sin \Phi') \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_{\xi^2} &= B_y \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_z \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
&= (B_{y_0} \sin \Phi') \cosh\left(\frac{a_0 \xi^0}{c}\right) + (E_{z_0} \sin \Phi') \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
E_{\xi^3} &= E_z \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
&= (E_{z_0} \sin \Phi') \cosh\left(\frac{a_0 \xi^0}{c}\right) + (B_{y_0} \sin \Phi') \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_{\xi^3} &= B_z \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
&= (B_{z_0} \sin \Phi') \cosh\left(\frac{a_0 \xi^0}{c}\right) - (E_{y_0} \sin \Phi') \sinh\left(\frac{a_0 \xi^0}{c}\right) \quad (58)
\end{aligned}$$

$$\begin{aligned}
\Phi &= \omega \left(t - l \frac{x}{c} - m \frac{y}{c} - n \frac{z}{c} \right), \\
\Phi' &= \omega' \left(\hat{\xi}^0 - l' \frac{\hat{\xi}^1}{c} - m' \frac{\hat{\xi}^2}{c} - n' \frac{\hat{\xi}^3}{c} \right)
\end{aligned}$$

In this time,

$$ct = \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right), \quad x = \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}$$

$$y = \xi^2, \quad z = \xi^3$$

$$\xi^0 = \frac{c}{a_0} \tanh^{-1}\left(\frac{ct}{x + \frac{c^2}{a_0}}\right), \quad \xi^1 = \sqrt{\left(x + \frac{c^2}{a_0}\right)^2 - c^2 t^2} - \frac{c^2}{a_0}$$

$$\lim_{a_0 \rightarrow 0} \xi^0 = \lim_{a_0 \rightarrow 0} c \tanh^{-1}\left(\frac{cta_0}{a_0 x + c^2}\right) / a_0 = \lim_{a_0 \rightarrow 0} c \tanh^{-1}\left(\frac{cta_0}{c^2}\right) / a_0 = \lim_{a_0 \rightarrow 0} c \frac{1}{1 - \left(\frac{a_0 t}{c}\right)^2} \frac{t}{c} = t$$

$$\begin{aligned}
\lim_{a_0 \rightarrow 0} \xi^1 &= \lim_{a_0 \rightarrow 0} c^2 \left(\sqrt{\left(1 + \frac{a_0}{c^2} x\right)^2 - \frac{a_0^2 t^2}{c^2}} - 1 \right) / a_0 = \lim_{a_0 \rightarrow 0} c^2 \left(\sqrt{\left(1 + \frac{a_0}{c^2} x\right)^2} - 1 \right) / a_0 \\
&= \lim_{a_0 \rightarrow 0} c^2 \left(\frac{a_0}{c^2} x \right) / a_0 = x
\end{aligned}$$

Hence,

$$\lim_{a_0 \rightarrow 0} \hat{\xi}^0 = \lim_{a_0 \rightarrow 0} \int d\hat{\xi}^0 = \lim_{a_0 \rightarrow 0} \int (1 + \frac{a_0}{c^2} \xi^1) d\xi^0 = \lim_{a_0 \rightarrow 0} \int d\xi^0 = \lim_{a_0 \rightarrow 0} \xi^0 = t$$

$$\lim_{a_0 \rightarrow 0} \hat{\xi}^1 = \lim_{a_0 \rightarrow 0} \xi^1 = x, \quad y = \xi^2 = \hat{\xi}^2, \quad z = \xi^3 = \hat{\xi}^3$$

Therefore, electro-magnetic wave function is

$$\begin{aligned} \lim_{a_0 \rightarrow 0} \Phi^1 &= \lim_{a_0 \rightarrow 0} \omega^1 (\hat{\xi}^0 - l' \frac{\hat{\xi}^1}{c} - m' \frac{\hat{\xi}^2}{c} - n' \frac{\hat{\xi}^3}{c}) = \omega^1 (t - l' \frac{x}{c} - m' \frac{y}{c} - n' \frac{z}{c}) \\ &= \omega (t - l' \frac{x}{c} - m' \frac{y}{c} - n' \frac{z}{c}) = \Phi \\ \omega &= \omega^1, \quad l' = l, \quad m' = m, \quad n' = n \\ l'^2 + m'^2 + n'^2 &= l^2 + m^2 + n^2 = 1 \end{aligned} \tag{59}$$

Hence,

$$\begin{aligned} &[\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_{\xi}^2] E_{\xi^1} \\ &= [\frac{1}{c^2} (\frac{\partial}{\partial \hat{\xi}^0})^2 - \nabla_{\hat{\xi}}^2] E_{\xi^1} = 0 \\ &[\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_{\xi}^2] B_{\xi^1} \\ &= [\frac{1}{c^2} (\frac{\partial}{\partial \hat{\xi}^0})^2 - \nabla_{\hat{\xi}}^2] B_{\xi^1} = 0 \\ &[\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_{\xi}^2] E_y \\ &= [\frac{1}{c^2} (\frac{\partial}{\partial \hat{\xi}^0})^2 - \nabla_{\hat{\xi}}^2] E_y = 0 \\ &[\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_{\xi}^2] B_y \\ &= [\frac{1}{c^2} (\frac{\partial}{\partial \hat{\xi}^0})^2 - \nabla_{\hat{\xi}}^2] B_y = 0 \end{aligned}$$

$$\begin{aligned}
& \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] E_z \\
&= \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] E_z = 0 \\
& \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] B_z \\
&= \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] B_z = 0 \tag{60}
\end{aligned}$$

The electro-magnetic wave equation is in vacuum

$$\begin{aligned}
& \vec{\nabla}_{\xi} \times \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_{\xi} \times \left\{ \vec{E}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} \\
&= \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_{\xi} \times \left\{ \vec{E}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \times \left\{ \vec{E}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} \\
&= \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{E}_{\xi} \\
&+ \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_{\xi} \times \vec{E}_{\xi} \\
&+ \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{E}_{\xi} \\
&+ \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \times \vec{E}_{\xi} \\
&= \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \vec{E}_{\xi} + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \vec{\nabla}_{\xi} \times \vec{\nabla}_{\xi} \times \vec{E}_{\xi} \\
&= \left[\vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{E}_{\xi} \right] \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) - \left[\vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right] \vec{E}_{\xi} \\
&\quad + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\vec{\nabla}_{\xi} (\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi}) - \nabla_{\xi}^2 \vec{E}_{\xi} \right] \\
&= -\frac{1}{c} \frac{\partial}{\partial \xi^0} \left[\vec{\nabla}_{\xi} \times \left\{ \vec{B}_{\xi} \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right\} \right] = -\frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{E}_{\xi},
\end{aligned}$$

$$\text{In this time, } \vec{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) = \left(\frac{a_0}{c^2}, 0, 0\right) \tag{61}$$

Hence,

$$\begin{aligned}
& \vec{\nabla}_\xi \times \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_\xi \times \left\{ \vec{E}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{E}_\xi \\
&= \left[\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{E}_\xi \right] \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) - \left[\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \right] \vec{E}_\xi \\
&\quad + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{E}_\xi) - \nabla_\xi^2 \vec{E}_\xi \right] + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{E}_\xi \\
&= \frac{a_0^2}{c^4} (E_{\xi^1}, 0, 0) - \frac{a_0^2}{c^4} (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}) + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_\xi^2 \right] \vec{E}_\xi \\
&= \frac{a_0^2}{c^4} (0, -E_{\xi^2}, -E_{\xi^3}) + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_\xi^2 \right] \vec{E}_\xi \\
&= \frac{a_0^2}{c^4} (0, -E_{\xi^2}, -E_{\xi^3}) + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_\xi^2 \right] \vec{E}_\xi \\
&= \vec{0}
\end{aligned} \tag{62}$$

Hence, the magnetic wave equation is in vacuum

$$\begin{aligned}
& \vec{\nabla}_\xi \times \left(1 + \frac{a_0}{c^2} \xi^1\right) \vec{\nabla}_\xi \times \left\{ \vec{B}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{B}_\xi \\
&= \left[\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{B}_\xi \right] \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) - \left[\vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \cdot \vec{\nabla}_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \right] \vec{B}_\xi \\
&\quad + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{B}_\xi) - \nabla_\xi^2 \vec{B}_\xi \right] + \frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 \vec{B}_\xi \\
&= \frac{a_0^2}{c^4} (0, -B_{\xi^2}, -B_{\xi^3}) + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_\xi^2 \right] \vec{B}_\xi \\
&= \frac{a_0^2}{c^4} (0, -B_{\xi^2}, -B_{\xi^3}) + \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \left[\frac{1}{c^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_\xi^2 \right] \vec{B}_\xi \\
&= \vec{0}
\end{aligned} \tag{63}$$

The electromagnetic wave function, Eq(58),Eq(59) satisfy the electromagnetic wave equation, Eq(62),Eq(63).

5. Conclusion

We find the electro-magnetic field transformation and the electro-magnetic equation in uniformly accelerated frame.

Generally, the coordinate transformation of accelerated frame is

$$(I) \quad ct = \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$x = \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \quad (64)$$

$$(II) \quad ct = \frac{c^2}{a_0} \exp\left(\frac{a_0}{c^2} \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$x = \frac{c^2}{a_0} \exp\left(\frac{a_0}{c^2} \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \quad (65)$$

Hence, this article say the accelerated frame is Rindler coordinate (I) that can treat electro-magnetic field equation.

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