Electro-Magnetic Field Equation and Lorentz gauge, Wave Function, Wave Equation in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime. We treat Lorentz gauge transformation in Rindler spacetime. We find the electro-magnetic wave equation and the electro-magnetic wave function in Rindler space-time. Specially, this article say the uniqueness of the accelerated frame because the accelerated frame can treat electro-magnetic field equation.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic field equation in Rindler space-time.

The Rindler coordinate is

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \qquad (1)$$

In this time, the tetrad $e^a{}_\mu$ is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^\mu}\frac{\partial x^b}{\partial \xi^\upsilon}d\xi^\mu d\xi^\upsilon$$

$$= -\frac{1}{c^2}\eta_{ab}e^a{}_\mu e^b{}_\upsilon d\xi^\mu d\xi^\upsilon = -\frac{1}{c^2}g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad , \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \qquad (2)$$

$$e^\alpha{}_0(\xi^0) = \frac{\partial x^\alpha}{c\partial \xi^0} = ((1 + \frac{a_0}{c^2}\xi^1)\cosh(\frac{a_0\xi^0}{c}), (1 + \frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c}), 0, 0) \qquad (3)$$

About y -axis's and z -axis's orientation

$$e^\alpha{}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0,0,1,0) \quad , \quad e^\alpha{}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0,0,0,1) \qquad (4)$$

The other unit vector $e^\alpha{}_1(\xi^0)$ is

$$e^\alpha{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = (\sinh(\frac{a_0\xi^0}{c}), \cosh(\frac{a_0\xi^0}{c}), 0, 0) \qquad (5)$$

Therefore,

$$cdt = c\cosh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \sinh(\frac{a_0\xi^0}{c})d\xi^1$$

$$= c\cosh(\frac{a_0\xi^0}{c})d\hat{\xi}^0 + \sinh(\frac{a_0\xi^0}{c})d\hat{\xi}^1$$

$$dx = c\sinh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \cosh(\frac{a_0\xi^0}{c})d\xi^1$$

$$= c\sinh(\frac{a_0\xi^0}{c})d\hat{\xi}^0 + \cosh(\frac{a_0\xi^0}{c})d\hat{\xi}^1, dy = d\xi^2 = d\hat{\xi}^2,\ dz = d\xi^3 = d\hat{\xi}^3 \quad (6)$$

The vector transformation is

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}, \qquad U'_{\mu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} U_{\alpha} \quad (7)$$

Therefore, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A}) = A^{\alpha}$ is

$$A^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} A'^{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} A_{\xi}^{\ \mu} = e^{\alpha}{}_{\mu} A_{\xi}^{\ \mu} \ , \ e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}}$$

$$dx^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} dx'^{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu} = e^{\alpha}{}_{\mu} d\xi^{\mu} \ , e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} \quad (8)$$

Hence, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A})$ in inertial frame and the eletro-magnetic 4-vector potential $(\phi_{\xi}, \vec{A}_{\xi})$ in uniformly accelerated frame is

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\phi = 4\pi\rho$$

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\vec{A} = \frac{4\pi}{c}\vec{j}$$

4-vector $(c\rho, \vec{j}) = \rho_0 \frac{dx^{\alpha}}{d\tau}$

$$\phi = \cosh(\frac{a_0\xi^0}{c})(1 + \frac{a_0}{c^2}\xi^1)\phi_{\xi} + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$= \cosh(\frac{a_0\xi^0}{c})\hat{\phi}_{\xi} + \sinh(\frac{a_0\xi^0}{c})\hat{A}_{\xi^1}$$

$$A_x = \sinh(\frac{a_0\xi^0}{c})(1 + \frac{a_0}{c^2}\xi^1)\phi_{\xi} + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$= \sinh(\frac{a_0\xi^0}{c})\hat{\phi}_{\xi} + \cosh(\frac{a_0\xi^0}{c})\hat{A}_{\xi^1}$$

$$A_y = A_{\xi^2} = \hat{A}_{\xi^2}, A_z = A_{\xi^3} = \hat{A}_{\xi^3} \quad (9)$$

$$g = \begin{pmatrix} -(1+\frac{a_0\xi^1}{c^2})^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e^a{}_\mu e_b{}^\mu = \delta^a{}_b \ \ , \ e^a{}_\mu e_a{}^\upsilon = \delta_\mu{}^\upsilon$$

$$e^a{}_\mu e^b{}_\upsilon \eta_{ab} = g_{\mu\upsilon} \ \rightarrow \ \ A^T \eta A = g$$

$$e_a{}^\mu e_b{}^\upsilon g_{\mu\upsilon} = \eta_{ab} \rightarrow \ \ (A^T)^{-1} g A^{-1} = (A^T)^{-1} A^T \eta A A^{-1} = \eta$$

$$e^a{}_\mu = \eta^{ab} g_{\mu\upsilon} e_b{}^\upsilon \rightarrow \eta^{-1}(A^T)^{-1} A^T \eta A = A = \eta^{-1}(A^T)^{-1} g \tag{10}$$

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= A \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\hat{\xi}^0 \\ d\hat{\xi}^1 \\ d\hat{\xi}^2 \\ d\hat{\xi}^3 \end{pmatrix}$$

(11)

$$e_{\mu}{}^{\alpha} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} = A^{-1} = \begin{pmatrix} \frac{c\partial \xi^0}{c\partial t} & \frac{c\partial \xi^0}{\partial x} & \frac{c\partial \xi^0}{\partial y} & \frac{c\partial \xi^0}{\partial z} \\ \frac{\partial \xi^1}{c\partial t} & \frac{\partial \xi^1}{\partial x} & \frac{\partial \xi^1}{\partial y} & \frac{\partial \xi^1}{\partial z} \\ \frac{\partial \xi^2}{c\partial t} & \frac{\partial \xi^2}{\partial x} & \frac{\partial \xi^2}{\partial y} & \frac{\partial \xi^2}{\partial z} \\ \frac{\partial \xi^3}{c\partial t} & \frac{\partial \xi^3}{\partial x} & \frac{\partial \xi^3}{\partial y} & \frac{\partial \xi^3}{\partial z} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & 0 & 0 \\ -\sinh(\frac{a_0 \xi^0}{c}) & \cosh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^{T} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^{T})^{-1} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & -\sinh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & \cosh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \hat{\xi}^0} \\ \frac{\partial}{\partial \hat{\xi}^1} \\ \frac{\partial}{\partial \hat{\xi}^2} \\ \frac{\partial}{\partial \hat{\xi}^3} \end{pmatrix} \tag{13}$$

$$\frac{1}{c}\frac{\partial}{\partial t} = \frac{c\partial\xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{c\partial t}\frac{\partial}{\partial\xi^1}$$

$$= \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$= \cosh(\frac{a_0}{c}\xi^0)\frac{\partial}{c\partial\hat{\xi}^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\hat{\xi}^1}$$

$$\frac{\partial}{\partial x} = \frac{c\partial\xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{\partial x}\frac{\partial}{\partial\xi^1}$$

$$= -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$= -\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{c\partial\hat{\xi}^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\hat{\xi}^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial\xi^2} = \frac{\partial}{\partial\hat{\xi}^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial\xi^3} = \frac{\partial}{\partial\hat{\xi}^3}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_{\xi}{}^2 = \frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2 - \nabla_{\hat{\xi}}{}^2$$

$$\vec{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}), \quad \vec{\nabla}_{\xi} = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3}), \quad \vec{\nabla}_{\hat{\xi}} = (\frac{\partial}{\partial\hat{\xi}^1}, \frac{\partial}{\partial\hat{\xi}^2}, \frac{\partial}{\partial\hat{\xi}^3}) \tag{14}$$

2. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field $(\vec{E}, \vec{B})$ is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{c\partial t}, \vec{B} = \vec{\nabla} \times \vec{A} \tag{15}$$

$$E_x = -\frac{\partial \phi}{\partial x} - \frac{\partial A_x}{c\partial t}$$

$$= -[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]\cdot[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]\cdot[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$= -\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial A_{\xi^1}}{c\partial \xi^0} - (1+\frac{a_0\xi^1}{c^2})\frac{\partial \phi_\xi}{\partial \xi^1} - 2\phi_\xi\frac{a_0}{c^2}$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial \xi^1}[(1+\frac{a_0}{c^2}\xi^1)^2\phi_\xi] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial \xi^0}$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial \hat{\xi}^1}[(1+\frac{a_0}{c^2}\xi^1)\hat{\phi}_\xi] - \frac{\partial \hat{A}_{\xi^1}}{c\partial \hat{\xi}^0} \tag{16}$$

$$E_y = -\frac{\partial \phi}{\partial y} - \frac{\partial A_y}{c\partial t} = -\frac{\partial}{\partial \xi^2}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]A_{\xi^2}$$

$$= -(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial \phi_\xi}{\partial \xi^2} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^2}}{c\partial \xi^0}$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial\xi^0}]$$

$$+ \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^2}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^2}[\hat{\phi}_\xi(1+\frac{a_0\xi^1}{c^2})] - \frac{\partial\hat{A}_{\xi^2}}{c\partial\hat{\xi}^0}]$$

$$+ \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial\hat{A}_{\xi^2}}{\partial\hat{\xi}^1} - \frac{\partial\hat{A}_{\xi^1}}{\partial\hat{\xi}^2}] \tag{17}$$

$$E_z = -\frac{\partial\phi}{\partial z} - \frac{\partial A_z}{c\partial t} = -\frac{\partial}{\partial\xi^3}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$= -(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial\phi_\xi}{\partial\xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^3}}{c\partial\xi^0}$$

$$+ \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^3}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}]$$

$$+ \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^3}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^3}[\hat{\phi}_\xi(1+\frac{a_0\xi^1}{c^2})] - \frac{\partial\hat{A}_{\xi^3}}{c\partial\hat{\xi}^0}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial\hat{A}_{\xi^3}}{\partial\hat{\xi}^1}-\frac{\partial\hat{A}_{\xi^1}}{\partial\hat{\xi}^3}] \qquad (18)$$

$$B_x=\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}=\frac{\partial A_{\xi^3}}{\partial\xi^2}-\frac{\partial A_{\xi^2}}{\partial\xi^3}=\frac{\partial\hat{A}_{\xi^3}}{\partial\hat{\xi}^2}-\frac{\partial\hat{A}_{\xi^2}}{\partial\hat{\xi}^3} \qquad (19)$$

$$B_y=\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}=\frac{\partial A_x}{\partial\xi^3}-\frac{\partial A_{\xi^3}}{\partial x}$$

$$=\frac{\partial}{\partial\xi^3}[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^1}}{\partial\xi^3}-\frac{\partial A_{\xi^3}}{\partial\xi^1}]$$

$$-\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial\hat{A}_{\xi^1}}{\partial\hat{\xi}^3}-\frac{\partial\hat{A}_{\xi^3}}{\partial\hat{\xi}^1}]$$

$$-\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^3}[\hat{\phi}_\xi(1+\frac{a_0\xi^1}{c^2})]-\frac{\partial\hat{A}_{\xi^3}}{c\partial\hat{\xi}^0}] \qquad (20)$$

$$B_z=\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}=\frac{\partial A_{\xi^2}}{\partial x}-\frac{\partial A_x}{\partial\xi^2}$$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$-\frac{\partial}{\partial \xi^2}[\sinh(\frac{a_0 \xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0 \xi^0}{c})A_{\xi^1}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \xi^2}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial \xi^0}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^1} - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^2}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \hat{\xi}^2}[\hat{\phi}_\xi(1+\frac{a_0\xi^1}{c^2})] - \frac{\partial \hat{A}_{\xi^2}}{c\partial \hat{\xi}^0}] \qquad (21)$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler spacetime.

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial \vec{A}_\xi}{c\partial \xi^0}$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\hat{\xi}}\{\hat{\phi}_\xi(1+\frac{a_0\xi^1}{c^2})\} - \frac{\partial \vec{\hat{A}}_\xi}{c\partial \hat{\xi}^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi = \vec{\nabla}_{\hat{\xi}} \times \vec{\hat{A}}_\xi$$

In this time, $\vec{\nabla}_\xi = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}), \vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$

$$\vec{\nabla}_{\hat{\xi}} = (\frac{\partial}{\partial \hat{\xi}^1}, \frac{\partial}{\partial \hat{\xi}^2}, \frac{\partial}{\partial \hat{\xi}^3}), \vec{\hat{A}}_\xi = (\hat{A}_{\xi^1}, \hat{A}_{\xi^2}, \hat{A}_{\xi^3}) \qquad (22)$$

Lorentz gauge transformation is in Rindler spacetime,

$$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial \Lambda}{\partial \xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, \quad \vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi \Lambda, \quad \Lambda \text{ is a scalar function.} \qquad (23)$$

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\frac{\partial\Lambda}{c\partial\xi^0}$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\Lambda$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi\times\vec{A}_\xi + \vec{\nabla}_\xi\times\vec{\nabla}_\xi\Lambda = \vec{\nabla}_\xi\times\vec{A}_\xi \qquad (24)$$

Lorentz gauge fix condition is in Rindler spacetime,

$$0 = \frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi \rightarrow \frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi - [\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]\Lambda = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]\Lambda = 0 \qquad (25)$$

We obtain the transformation of the electro-magnetic field.

$$E_x = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0}$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\hat{\xi}^1}\{\hat{\phi}_\xi(1+\frac{a_0\xi^1}{c^2})\} - \frac{\partial\hat{A}_{\xi^1}}{c\partial\hat{\xi}^0} = E_{\xi^1} \quad ,$$

$$E_y = E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$B_x = B_{\xi^1},$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}) \qquad (26)$$

Hence,

$$E_x = E_{\xi^1}, B_x = B_{\xi^1},$$

$$\begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix} = H \begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix}$$

$$H = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & \sinh(\frac{a_0\xi^0}{c}) \\ 0 & \cosh(\frac{a_0\xi^0}{c}) & -\sinh(\frac{a_0\xi^0}{c}) & 0 \\ 0 & -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 \\ \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c}) \end{pmatrix} \qquad (27)$$

The inverse-transformation of the electro-magnetic field is

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$\begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix} = H^{-1} \begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & -\sinh(\frac{a_0\xi^0}{c}) \\ 0 & \cosh(\frac{a_0\xi^0}{c}) & \sinh(\frac{a_0\xi^0}{c}) & 0 \\ 0 & \sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c}) \end{pmatrix} \qquad (28)$$

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$E_{\xi^2} = E_y \cosh(\frac{a_0\xi^0}{c}) - B_z \sinh(\frac{a_0\xi^0}{c}),$$

$$B_{\xi^2} = B_y \cosh(\frac{a_0\xi^0}{c}) + E_z \sinh(\frac{a_0\xi^0}{c})$$

$$E_{\xi^3} = E_z \cosh(\frac{a_0\xi^0}{c}) + B_y \sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^3} = B_z \cosh(\frac{a_0\xi^0}{c}) - E_y \sinh(\frac{a_0\xi^0}{c}) \qquad (29)$$

3. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \qquad (30\text{-i})$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j} \qquad (30\text{-ii})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad (30\text{-iii})$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{c\partial t} \qquad (30\text{-iv})$$

1. $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$

$E_x = E_{\xi^1}$,

$$E_y = E_{\xi^2} \cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3} \cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0\xi^0}{c})$$

$$4\pi\rho = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]E_{\xi^1}$$

$$+ \frac{\partial}{\partial\xi^2}[E_{\xi^2} \cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0\xi^0}{c})]$$

$$+ \frac{\partial}{\partial\xi^3}[E_{\xi^3} \cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)(\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial \xi^0}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)(\vec{\nabla}_{\hat{\xi}} \cdot \vec{E}_\xi) + \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial \hat{\xi}^2} - \frac{\partial B_{\xi^2}}{\partial \hat{\xi}^3} - \frac{\partial E_{\xi^1}}{c\partial \hat{\xi}^0}] \qquad (31)$$

2. $\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j}$

$$B_x = B_{\xi^1}$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

X-component) $\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$

$$= \frac{\partial}{\partial \xi^2}[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$- \frac{\partial}{\partial \xi^3}[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3}] + \sinh(\frac{a_0\xi^0}{c})[\frac{\partial E_{\xi^2}}{\partial \xi^2} + \frac{\partial E_{\xi^3}}{\partial \xi^3}]$$

$$= \frac{\partial E_x}{c\partial t} + \frac{4\pi}{c}j_x$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]E_{\xi^1} + \frac{4\pi}{c}j_x$$

Hence,

$$\frac{4\pi}{c}j_x$$

$$= \sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \cosh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial \xi^0}]$$

$$= \sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_{\hat{\xi}} \cdot \vec{E}_\xi) + \cosh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^3}}{\partial \hat{\xi}^2} - \frac{\partial B_{\xi^2}}{\partial \hat{\xi}^3} - \frac{\partial E_{\xi^1}}{c\partial \hat{\xi}^0}] \quad (32)$$

Y-component) $\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}$

$$= \frac{\partial B_{\xi^1}}{\partial \xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}] \cdot [B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \frac{\partial E_y}{c\partial t} + \frac{4\pi}{c}j_y$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}] \cdot [E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+ \frac{4\pi}{c}j_y$$

$$\frac{4\pi}{c}j_y = \frac{\partial B_{\xi^1}}{\partial \xi^3} - \frac{\partial B_{\xi^3}}{\partial \xi^1} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}B_{\xi^3} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^2}}{c\partial \xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \xi^3}\{B_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \xi^1}\{B_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^2}}{c\partial \xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \hat{\xi}^3}\{B_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \hat{\xi}^1}\{B_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} - \frac{\partial E_{\xi^2}}{c\partial \hat{\xi}^0}$$

(33)

Z-component) $\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial B_{\xi^1}}{\partial\xi^2}$$

$$=\frac{\partial E_z}{c\partial t}+\frac{4\pi}{c}j_z$$

$$=[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{4\pi}{c}j_z$$

$$\frac{4\pi}{c}j_z=\frac{\partial B_{\xi^2}}{\partial\xi^1}-\frac{\partial B_{\xi^1}}{\partial\xi^2}+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}B_{\xi^2}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^3}}{c\partial\xi^0}$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{B_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\{B_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^3}}{c\partial\xi^0}$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^1}\{B_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^2}\{B_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\}-\frac{\partial E_{\xi^3}}{c\partial\hat{\xi}^0}$$

(34)

3. $\vec{\nabla}\cdot\vec{B}=0$

$$\vec{\nabla}\cdot\vec{B}=\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}$$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]B_{\xi^1}$$

$$+\frac{\partial}{\partial\xi^2}[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{\partial}{\partial\xi^3}[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$=\cosh(\frac{a_0\xi^0}{c})(\vec{\nabla}_{\xi}\cdot\vec{B}_{\xi})+\sinh(\frac{a_0\xi^0}{c})[-(-\frac{\partial E_{\xi^2}}{\partial\xi^3}+\frac{\partial E_{\xi^3}}{\partial\xi^2})-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^1}}{c\partial\xi^0}]=0$$

$$=\cosh(\frac{a_0\xi^0}{c})(\vec{\nabla}_{\hat{\xi}}\cdot\vec{B}_{\xi})+\sinh(\frac{a_0\xi^0}{c})[-(-\frac{\partial E_{\xi^2}}{\partial\hat{\xi}^3}+\frac{\partial E_{\xi^3}}{\partial\hat{\xi}^2})-\frac{\partial B_{\xi^1}}{c\partial\hat{\xi}^0}]=0$$

(35)

4. $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{c\partial t}$

$E_x=E_{\xi^1}$,

$E_y=E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$,

$E_z=E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$

X-component) $\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}$

$$=\frac{\partial}{\partial\xi^2}[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial}{\partial\xi^3}[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial E_{\xi^3}}{\partial\xi^2}-\frac{\partial E_{\xi^2}}{\partial\xi^3}]-\sinh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^2}}{\partial\xi^2}+\frac{\partial B_{\xi^3}}{\partial\xi^3}]$$

$$=-\frac{\partial B_x}{c\partial t}$$

$$=-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]B_{\xi^1}$$

Hence, $-\sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi \cdot \vec{B}_\xi) + \cosh(\frac{a_0\xi^0}{c})[(\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3}) + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial B_{\xi^1}}{c\partial \xi^0}]$

$$= -\sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_{\hat{\xi}} \cdot \vec{B}_\xi) + \cosh(\frac{a_0\xi^0}{c})[(\frac{\partial E_{\xi^3}}{\partial \hat{\xi}^2} - \frac{\partial E_{\xi^2}}{\partial \hat{\xi}^3}) + \frac{\partial B_{\xi^1}}{c\partial \hat{\xi}^0}] = 0 \quad (36)$$

Y-component) $\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$

$$= \frac{\partial E_{\xi^1}}{\partial \xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]\cdot[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= -\frac{\partial B_y}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^1}}{\partial \xi^3} - \frac{\partial E_{\xi^3}}{\partial \xi^1} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^3} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial \xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \xi^3}\{E_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \xi^1}\{E_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial \xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \hat{\xi}^3}\{E_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \hat{\xi}^1}\{E_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} + \frac{\partial B_{\xi^2}}{c\partial \hat{\xi}^0} = 0$$

(37)

Z-component) $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial E_{\xi^1}}{\partial\xi^2}$$

$$=-\frac{\partial B_z}{c\partial t}$$

$$=-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^2}}{\partial\xi^1}-\frac{\partial E_{\xi^1}}{\partial\xi^2}+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^2}+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\{E_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\}+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^1}\{E_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\}-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^2}\{E_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\}+\frac{\partial B_{\xi^3}}{c\partial\hat{\xi}^0}=0$$

(38)

Therefore, we obtain the electro-magnetic field equation by Eq (31)-Eq(38) in Rindler spacetime .

$$\vec{\nabla}_{\xi}\cdot\vec{E}_{\xi}=\vec{\nabla}_{\hat{\xi}}\cdot\vec{E}_{\xi}=4\pi\rho_{\xi}(1+\frac{a_0\xi^1}{c^2}) \qquad \text{(39-i)}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\xi}\times\{\vec{B}_{\xi}(1+\frac{a_0\xi^1}{c^2})\}=\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_{\xi}}{c\partial\xi^0}+\frac{4\pi}{c}\vec{j}_{\xi}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\hat{\xi}}\times\{\vec{B}_{\xi}(1+\frac{a_0\xi^1}{c^2})\}=\frac{\partial\vec{E}_{\xi}}{c\partial\hat{\xi}^0}+\frac{4\pi}{c}\vec{j}_{\xi} \qquad \text{(39-ii)}$$

$$\vec{\nabla}_{\xi}\cdot\vec{B}_{\xi}=\vec{\nabla}_{\hat{\xi}}\cdot\vec{B}_{\xi}=0 \qquad \text{(39-iii)}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\xi}\times\{\vec{E}_{\xi}(1+\frac{a_0\xi^1}{c^2})\}=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{B}_{\xi}}{c\partial\xi^0}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\hat{\xi}}\times\{\vec{E}_{\xi}(1+\frac{a_0\xi^1}{c^2})\}=-\frac{\partial\vec{B}_{\xi}}{c\partial\hat{\xi}^0} \qquad (39\text{-iv})$$

$$\vec{E}_{\xi}=(E_{\xi^1},E_{\xi^2},E_{\xi^3}),\vec{B}_{\xi}=(B_{\xi^1},B_{\xi^2},B_{\xi^3}),$$

$$\vec{\nabla}_{\xi}=(\frac{\partial}{\partial\xi^1},\frac{\partial}{\partial\xi^2},\frac{\partial}{\partial\xi^3}),\vec{\nabla}_{\hat{\xi}}=(\frac{\partial}{\partial\hat{\xi}^1},\frac{\partial}{\partial\hat{\xi}^2},\frac{\partial}{\partial\hat{\xi}^3})$$

Hence, the transformation of 4-vector $(c\rho,\vec{j})=\rho_0\frac{dx^{\alpha}}{d\tau}$ is

$$\rho=\rho_{\xi}(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})+\frac{j_{\xi^1}}{c}\sinh(\frac{a_0\xi^0}{c})$$

$$j_x=j_{\xi^1}\cosh(\frac{a_0\xi^0}{c})+c\rho_{\xi}(1+\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c}),\quad j_y=j_{\xi^2},j_z=j_{\xi^3}$$

In this time, 4-vector $(c\rho_{\xi},\vec{j}_{\xi})=\rho_0\frac{d\xi^{\alpha}}{d\tau}$ (40)

We treat Lorentz gauge transformation about the electro-magnetic field equation in Rindler spacetime. Eq(39-i) is

$$\vec{\nabla}_{\xi}\cdot\vec{E}_{\xi}=\vec{\nabla}_{\xi}\cdot\{-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\xi}\{\phi_{\xi}(1+\frac{a_0\xi^1}{c^2})^2\}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_{\xi}}{c\partial\xi^0}\}$$

$$=-\vec{\nabla}_{\xi}\{\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\}\cdot[\vec{\nabla}_{\xi}\{\phi_{\xi}(1+\frac{a_0\xi^1}{c^2})^2\}+\frac{\partial\vec{A}_{\xi}}{c\partial\xi^0}]$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_{\xi}{}^2\{\phi_{\xi}(1+\frac{a_0\xi^1}{c^2})^2\}+\frac{\partial}{c\partial\xi^0}(\vec{\nabla}_{\xi}\cdot\vec{A}_{\xi})]$$

$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}[\frac{\partial}{\partial\xi^1}\{\phi_{\xi}(1+\frac{a_0\xi^1}{c^2})^2\}+\frac{\partial A_{\xi^1}}{c\partial\xi^0}]$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}[\frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0}+\vec{\nabla}_\xi\cdot\vec{A}_\xi],\frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0}+\vec{\nabla}_\xi\cdot\vec{A}_\xi=0$$

$$=-\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}E_{\xi^1}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$=4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \qquad (41)$$

If we apply Lorentz gauge transformation to Eq (41),

$\phi_\xi\rightarrow\phi_\xi-\frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$, $\vec{A}_\xi\rightarrow\vec{A}_\xi+\vec{\nabla}_\xi\Lambda$, Λ is a scalar function.

$$=-\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}E_{\xi^1}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}$$

$$=-\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}E_{\xi^1}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}\{[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\Lambda\} \qquad (42)$$

In this time,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda=0 \qquad (43)$$

Hence, Eq(39-i) is

$$\vec{\nabla}_\xi\cdot\vec{E}_\xi$$

$$= -\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}E_{\xi^1} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2 - \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$= 4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \qquad (44)$$

Eq(39-i) is invariant about Lorentz gauge transformation in Rindler spacetime.

Eq (39-ii) is

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{\nabla}_\xi \times \vec{A}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\{\vec{\nabla}_\xi \times \vec{A}_\xi\} + \vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \vec{A}_\xi$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(1,0,0)\times\vec{B}_\xi + \{-\nabla_\xi{}^2\vec{A}_\xi + \vec{\nabla}_\xi(\vec{\nabla}_\xi \cdot \vec{A}_\xi)\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(0,-B_{\xi^3},B_{\xi^2}) + \{-\nabla_\xi{}^2\vec{A}_\xi + \vec{\nabla}_\xi(\vec{\nabla}_\xi \cdot \vec{A}_\xi)\}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_\xi}{c\partial\xi^0} + \frac{4\pi}{c}\vec{j}_\xi$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial}{c\partial\xi^0}[\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi + \frac{4\pi\vec{j}_\xi}{c}$$

$$= -\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\phi_\xi - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0) - \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi + \frac{4\pi\vec{j}_\xi}{c}$$

(45)

Therefore,

$$\frac{4\pi}{c}\vec{j}_\xi$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(0,-B_{\xi^3},B_{\xi^2}) + \{-\nabla_\xi{}^2\vec{A}_\xi + \vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{A}_\xi)\}$$

$$+\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\phi_\xi + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0) + \frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi$$

$$= \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$+[-\nabla_\xi{}^2 + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi + \vec{\nabla}_\xi[\frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi]$$

$$\frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi = 0 \qquad (46)$$

$$\frac{4\pi}{c}\vec{j}_\xi$$

$$= \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$+[-\nabla_\xi{}^2 + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi \qquad (47)$$

If we apply Lorentz gauge transformation to Eq (47),

$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$, $\vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi\Lambda$, Λ is a scalar function.

$$\frac{4\pi}{c}\vec{j}_\xi$$

$$= \frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{2a_0}{c^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)$$

$$+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{\nabla}_\xi\Lambda$$

(48)

In this time,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda=0$$

$$0=\vec{\nabla}_\xi[\{-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2\}\Lambda]$$

$$=\vec{\nabla}_\xi\{\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\}(\frac{\partial}{\partial\xi^0})^2\Lambda+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{\nabla}_\xi\Lambda$$

(49)

Therefore,

$$\frac{4\pi}{c}\vec{j}_\xi$$

$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{2a_0}{c^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)$$

$$+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi$$

$$+\vec{\nabla}_\xi[\{-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2\}\Lambda]-\vec{\nabla}_\xi\{\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\}(\frac{\partial}{\partial\xi^0})^2\Lambda$$

$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{2a_0}{c^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)$$

$$+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi+\frac{1}{c^2}\frac{2}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{a_0}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)$$

$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi \tag{50}$$

Hence, Eq(39-ii) is invariant about Lorentz gauge transformation in Rindler spacetime.

Eq (39-iii) is

$$\vec{\nabla}_\xi\cdot\vec{B}_\xi=\vec{\nabla}_\xi\cdot(\vec{\nabla}_\xi\times\vec{A}_\xi+\vec{\nabla}_\xi\times\vec{\nabla}_\xi\Lambda)=\vec{\nabla}_\xi\times\vec{\nabla}_\xi\cdot\vec{A}_\xi=0 \tag{51}$$

Eq (39-iv) is

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times[\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}-\vec{\nabla}_\xi(\frac{\partial\Lambda}{c\partial\xi^0})+\frac{\partial\vec{A}_\xi}{c\partial\xi^0}+\frac{\partial}{c\partial\xi^0}(\vec{\nabla}_\xi\Lambda)]$$

$$=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\frac{\partial\vec{A}_\xi}{c\partial\xi^0}=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial(\vec{\nabla}_\xi\times\vec{A}_\xi)}{c\partial\xi^0}=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{B}_\xi}{c\partial\xi^0} \tag{52}$$

Hence, Eq (39-iii), Eq (39-iv) are invariant about Lorentz gauge transformation in Rindler spacetime.

Hence, the electro-magnetic field equations(Maxwell Equations) in Rindler spacetime are invariant about Lorentz gauge transformation.

4. Electro-magnetic wave equation in Rindler space-time

The electro-magnetic wave function is

$$E_x=E_{x0}\sin\Phi, E_y=E_{y0}\sin\Phi, E_z=E_{z0}\sin\Phi$$

$$B_x=B_{x0}\sin\Phi, B_y=B_{y0}\sin\Phi, B_z=B_{z0}\sin\Phi$$

$$E_{\xi^1} = E_x , B_{\xi^1} = B_x$$

$$E_{\xi^1} = E_{X0} \sin \Phi' , B_{\xi^1} = B_{X0} \sin \Phi'$$

$$E_{\xi^2} = E_y \cosh(\frac{a_0 \xi^0}{c}) - B_z \sinh(\frac{a_0 \xi^0}{c}),$$

$$= (E_{y0} \sin \Phi') \cosh(\frac{a_0 \xi^0}{c}) - (B_{z0} \sin \Phi') \sinh(\frac{a_0 \xi^0}{c})$$

$$B_{\xi^2} = B_y \cosh(\frac{a_0 \xi^0}{c}) + E_z \sinh(\frac{a_0 \xi^0}{c})$$

$$= (B_{y0} \sin \Phi') \cosh(\frac{a_0 \xi^0}{c}) + (E_{z0} \sin \Phi') \sinh(\frac{a_0 \xi^0}{c})$$

$$E_{\xi^3} = E_z \cosh(\frac{a_0 \xi^0}{c}) + B_y \sinh(\frac{a_0 \xi^0}{c})$$

$$= (E_{z0} \sin \Phi') \cosh(\frac{a_0 \xi^0}{c}) + (B_{y0} \sin \Phi') \sinh(\frac{a_0 \xi^0}{c})$$

$$B_{\xi^3} = B_z \cosh(\frac{a_0 \xi^0}{c}) - E_y \sinh(\frac{a_0 \xi^0}{c})$$

$$= (B_{z0} \sin \Phi') \cosh(\frac{a_0 \xi^0}{c}) - (E_{y0} \sin \Phi') \sinh(\frac{a_0 \xi^0}{c}) \qquad (53)$$

$$\Phi = \omega(t - l \frac{x}{c} - m \frac{y}{c} - n \frac{z}{c}),$$

$$\Phi' = \omega' (\hat{\xi}^0 - l' \frac{\hat{\xi}^1}{c} - m' \frac{\hat{\xi}^2}{c} - n' \frac{\hat{\xi}^3}{c})$$

In this time,

$$ct = (\frac{c^2}{a_0} + \xi^1) \sinh(\frac{a_0 \xi^0}{c}) \quad , \quad x = (\frac{c^2}{a_0} + \xi^1) \cosh(\frac{a_0 \xi^0}{c}) - \frac{c^2}{a_0}$$

$$y = \xi^2 , z = \xi^3$$

$$\xi^0 = \frac{c}{a_0} \tanh^{-1}(\frac{ct}{x + \frac{c^2}{a_0}}), \xi^1 = \sqrt{(x + \frac{c^2}{a_0})^2 - c^2 t^2} - \frac{c^2}{a_0}$$

$$\lim_{a_0\to 0}\xi^0 = \lim_{a_0\to 0} c\tanh^{-1}(\frac{cta_0}{a_0 x + c^2})/a_0 = \lim_{a_0\to 0} c\tanh^{-1}(\frac{cta_0}{c^2})/a_0 = \lim_{a_0\to 0} c\frac{1}{1-(\frac{a_0 t}{c})^2}\frac{t}{c} = t$$

$$\lim_{a_0\to 0}\xi^1 = \lim_{a_0\to 0} c^2(\sqrt{(1+\frac{a_0}{c^2}x)^2 - \frac{{a_0}^2 t^2}{c^2}} - 1)/a_0 = \lim_{a_0\to 0} c^2(\sqrt{(1+\frac{a_0}{c^2}x)^2} - 1)/a_0$$

$$= \lim_{a_0\to 0} c^2(\frac{a_0}{c^2}x)/a_0 = x$$

Hence,

$$\lim_{a_0\to 0}\hat{\xi}^0 = \lim_{a_0\to 0}\int d\hat{\xi}^0 = \lim_{a_0\to 0}\int(1+\frac{a_0}{c^2}\xi^1)d\xi^0 = \lim_{a_0\to 0}\int d\xi^0 = \lim_{a_0\to 0}\xi^0 = t$$

$$\lim_{a_0\to 0}\hat{\xi}^1 = \lim_{a_0\to 0}\xi^1 = x,\quad y = \xi^2 = \hat{\xi}^2, z = \xi^3 = \hat{\xi}^3$$

Therefore, electro-magnetic wave function is

$$\lim_{a_0\to 0}\Phi' = \lim_{a_0\to 0}\omega'(\hat{\xi}^0 - l'\frac{\hat{\xi}^1}{c} - m'\frac{\hat{\xi}^2}{c} - n'\frac{\hat{\xi}^3}{c}) = \omega'(t - l'\frac{x}{c} - m'\frac{y}{c} - n'\frac{z}{c})$$

$$= \omega(t - l\frac{x}{c} - m\frac{y}{c} - n\frac{z}{c}) = \Phi$$

$$\omega = \omega', l' = l, m' = m, n' = n$$

$$l^2 + m^2 + n^2 = 1 \qquad , l'^2 + m'^2 + n'^2 = 1 \tag{54}$$

Hence,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]E_{\xi^1}$$

$$= [\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2 - \nabla_{\hat{\xi}}{}^2]E_{\xi^1} = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]B_{\xi^1}$$

$$= [\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2 - \nabla_{\hat{\xi}}{}^2]B_{\xi^1} = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]E_y$$

$$= [\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2 - \nabla_{\hat{\xi}}{}^2]E_y = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_y$$

$$=[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2-\nabla_{\hat{\xi}}{}^2]B_y=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]E_z$$

$$=[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2-\nabla_{\hat{\xi}}{}^2]E_z=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_z$$

$$=[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2-\nabla_{\hat{\xi}}{}^2]B_z=0 \qquad (55)$$

The electro-magnetic wave equation is in vacuum

$$\vec{\nabla}_\xi\times(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0}{c^2}\xi^1)\}$$

$$=\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0}{c^2}\xi^1)\}+(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi\times\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0}{c^2}\xi^1)\}$$

$$=\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{\nabla}_\xi\times\vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi\times\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2\vec{\nabla}_\xi\times\vec{\nabla}_\xi\times\vec{E}_\xi$$

$$=\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{E}_\xi+(1+\frac{a_0}{c^2}\xi^1)^2\vec{\nabla}_\xi\times\vec{\nabla}_\xi\times\vec{E}_\xi$$

$$=[\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{E}_\xi]\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)-[\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)]\vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2[\vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{E}_\xi)-\nabla_\xi{}^2\vec{E}_\xi]$$

$$=-\frac{1}{c}\frac{\partial}{\partial\xi^0}[\vec{\nabla}_\xi\times\{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\}]=-\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{E}_\xi,$$

In this time, $\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)=(\frac{a_0}{c^2},0,0)$ (56)

Hence,

$$\vec{\nabla}_{\xi}\times(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_{\xi}\times\{\vec{E}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\}+\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{E}_{\xi}$$

$$=[\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{E}_{\xi}]\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)-[\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)]\vec{E}_{\xi}$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2[\vec{\nabla}_{\xi}(\vec{\nabla}_{\xi}\cdot\vec{E}_{\xi})-\nabla_{\xi}{}^2\vec{E}_{\xi}]+\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{E}_{\xi}$$

$$=\frac{a_0{}^2}{c^4}(E_{\xi^1},0,0)-\frac{a_0{}^2}{c^4}(E_{\xi^1},E_{\xi^2},E_{\xi^3})+(1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_{\xi}{}^2]\vec{E}_{\xi}$$

$$=\frac{a_0{}^2}{c^4}(0,-E_{\xi^2},-E_{\xi^3})+(1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_{\xi}{}^2]\vec{E}_{\xi}$$

$$=\frac{a_0{}^2}{c^4}(0,-E_{\xi^2},-E_{\xi^3})+(1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2-\nabla_{\hat{\xi}}{}^2]\vec{E}_{\xi}$$

$$=\vec{0} \tag{57}$$

Hence, the magnetic wave equation is in vacuum

$$\vec{\nabla}_{\xi}\times(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_{\xi}\times\{\vec{B}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\}+\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{B}_{\xi}$$

$$=[\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{B}_{\xi}]\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)-[\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)]\vec{B}_{\xi}$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2[\vec{\nabla}_{\xi}(\vec{\nabla}_{\xi}\cdot\vec{B}_{\xi})-\nabla_{\xi}{}^2\vec{B}_{\xi}]+\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{B}_{\xi}$$

$$=\frac{a_0{}^2}{c^4}(0,-B_{\xi^2},-B_{\xi^3})+(1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_{\xi}{}^2]\vec{B}_{\xi}$$

$$=\frac{a_0{}^2}{c^4}(0,-B_{\xi^2},-B_{\xi^3})+(1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2-\nabla_{\hat{\xi}}{}^2]\vec{B}_{\xi}$$

$$=\vec{0} \tag{58}$$

The electromagnetic wave function, Eq(53),Eq(54) satisfy the electromagnetic wave equation, Eq(57),Eq(58).

5. Conclusion

We find the electro-magnetic field transformation and the electro-magnetic equation in uniformly accelerated frame.

Generally, the coordinate transformation of accelerated frame is

(I) $$ct = \left(\frac{c^2}{a_0} + \xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right)$$

$$x = \left(\frac{c^2}{a_0} + \xi^1\right)\cosh\left(\frac{a_0\xi^0}{c}\right) - \frac{c^2}{a_0} \quad , y = \xi^2 , z = \xi^3 \qquad (59)$$

(II) $$ct = \frac{c^2}{a_0}\exp\left(\frac{a_0}{c^2}\xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right)$$

$$x = \frac{c^2}{a_0}\exp\left(\frac{a_0}{c^2}\xi^1\right)\cosh\left(\frac{a_0\xi^0}{c}\right) - \frac{c^2}{a_0} , y = \xi^2 , z = \xi^3 \qquad (60)$$

Hence, this article say the accelerated frame is Rindler coordinate (I) that can treat electro-magnetic field equation.

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