

Electro-Magnetic Field Equation and Transformation in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime. We find the electro-magnetic wave equation and the electro-magnetic wave function in Rindler space-time. Specially, this article say the uniqueness of the accelerated frame because the accelerated frame can treat electro-magnetic field equation.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic field equation in Rindler space-time.

The Rindler coordinate is

$$ct = \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$x = \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \quad (1)$$

In this time, the tetrad $e^a{}_\mu$ is

$$d\tau^2 = dt^2 - \frac{1}{c^2} [dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2} \eta_{ab} \frac{\partial x^a}{\partial \xi^\mu} \frac{\partial x^b}{\partial \xi^\nu} d\xi^\mu d\xi^\nu$$

$$= -\frac{1}{c^2} \eta_{ab} e^a{}_\mu e^b{}_\nu d\xi^\mu d\xi^\nu = -\frac{1}{c^2} g_{\mu\nu} d\xi^\mu d\xi^\nu, e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \quad (2)$$

$$e^{\alpha}{}_0(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^0} = \left(1 + \frac{a_0}{c^2} \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right), \left(1 + \frac{a_0}{c^2} \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right), 0, 0 \quad (3)$$

About y -axis's and z -axis's orientation

$$e^{\alpha}{}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0, 0, 1, 0), \quad e^{\alpha}{}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0, 0, 0, 1) \quad (4)$$

The other unit vector $e^{\alpha}{}_1(\xi^0)$ is

$$e^{\alpha}{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = \left(\sinh\left(\frac{a_0 \xi^0}{c}\right), \cosh\left(\frac{a_0 \xi^0}{c}\right), 0, 0\right) \quad (5)$$

Therefore,

$$cdt = c \cosh\left(\frac{a_0 \xi^0}{c}\right) d\xi^0 \left(1 + \frac{a_0}{c^2} \xi^1\right) + \sinh\left(\frac{a_0 \xi^0}{c}\right) d\xi^1$$

$$dx = c \sinh\left(\frac{a_0 \xi^0}{c}\right) d\xi^0 \left(1 + \frac{a_0}{c^2} \xi^1\right) + \cosh\left(\frac{a_0 \xi^0}{c}\right) d\xi^1, dy = d\xi^2, dz = d\xi^3 \quad (6)$$

The vector transformation is

$$V'^{\mu} = \frac{\partial X'^{\mu}}{\partial X^{\alpha}} V^{\alpha}, \quad U'_{\mu} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} U_{\alpha} \quad (7)$$

Therefore, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A}) = A^{\alpha}$ is

$$A^{\alpha} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} A'^{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} A_{\xi}^{\mu} = e^{\alpha}_{\mu} A_{\xi}^{\mu}, \quad e^{\alpha}_{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}}$$

$$dX^{\alpha} = \frac{\partial X^{\alpha}}{\partial X'^{\mu}} dX'^{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu} = e^{\alpha}_{\mu} d\xi^{\mu}, \quad e^{\alpha}_{\mu} = \frac{\partial X^{\alpha}}{\partial \xi^{\mu}} \quad (8)$$

Hence, the transformation of the electro-magnetic 4-vector potential (ϕ, \vec{A}) in inertial frame and the

electro-magnetic 4-vector potential $(\phi_{\xi}, \vec{A}_{\xi})$ in uniformly accelerated frame is

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi = 4\pi\rho$$

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right)\vec{A} = \frac{4\pi}{c} \vec{j}$$

4-vector $(c\rho, \vec{j}) = \rho_0 \frac{dX^{\alpha}}{d\tau}$

$$\phi = \cosh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_{\xi} + \sinh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1}$$

$$A_x = \sinh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_{\xi} + \cosh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1}$$

$$A_y = A_{\xi^2}, A_z = A_{\xi^3} \quad (9)$$

$$g = \begin{pmatrix} -\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e^a_{\mu} e_b^{\mu} = \delta^a_b, \quad e^a_{\mu} e_a^{\nu} = \delta_{\mu}^{\nu}$$

$$e^a_{\mu} e_b^{\nu} \eta_{ab} = g_{\mu\nu} \rightarrow A^T \eta A = g$$

$$e_a^\mu e_b^\nu g_{\mu\nu} = \eta_{ab} \rightarrow (A^T)^{-1} g A^{-1} = (A^T)^{-1} A^T \eta A A^{-1} = \eta$$

$$e^a_\mu = \eta^{ab} g_{\mu\nu} e_b^\nu \rightarrow \eta^{-1} (A^T)^{-1} A^T \eta A = A = \eta^{-1} (A^T)^{-1} g \quad (10)$$

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0 \xi^1}{c^2}\right) & \sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ \sinh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0 \xi^1}{c^2}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix}$$

$$= A \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix} \quad (11)$$

$$e_\mu^\alpha = \frac{\partial \xi^\alpha}{\partial x^\mu} = A^{-1} = \begin{pmatrix} \frac{\partial \xi^0}{\partial ct} & \frac{\partial \xi^0}{\partial dx} & \frac{\partial \xi^0}{\partial dy} & \frac{\partial \xi^0}{\partial dz} \\ \frac{\partial \xi^1}{\partial ct} & \frac{\partial \xi^1}{\partial dx} & \frac{\partial \xi^1}{\partial dy} & \frac{\partial \xi^1}{\partial dz} \\ \frac{\partial \xi^2}{\partial ct} & \frac{\partial \xi^2}{\partial dx} & \frac{\partial \xi^2}{\partial dy} & \frac{\partial \xi^2}{\partial dz} \\ \frac{\partial \xi^3}{\partial ct} & \frac{\partial \xi^3}{\partial dx} & \frac{\partial \xi^3}{\partial dy} & \frac{\partial \xi^3}{\partial dz} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} & -\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} & 0 & 0 \\ -\sinh\left(\frac{a_0 \xi^0}{c}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (12)$$

$$\begin{aligned}
\begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} &= (A^{-1})^T \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^T)^{-1} \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} \\
&= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} & -\sinh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ \frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} & \cosh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c} \frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} \tag{13}
\end{aligned}$$

$$\begin{aligned}
\frac{1}{c} \frac{\partial}{\partial t} &= \frac{\partial \xi^0}{\partial t} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial t} \frac{\partial}{\partial \xi^1} \\
&= \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial x} &= \frac{\partial \xi^0}{\partial x} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial x} \frac{\partial}{\partial \xi^1} \\
&= -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1}
\end{aligned}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi^2}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi^3}$$

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2 (1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_{\xi}^2$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right), \quad \vec{\nabla}_\xi = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right) \quad (14)$$

2. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field (\vec{E}, \vec{B}) is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A} \quad (15)$$

$$\begin{aligned} E_x &= -\frac{\partial \phi}{\partial x} - \frac{\partial A_x}{\partial t} \\ &= -\left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot \left[\cosh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0 \xi^1}{c^2}\right) \phi_\xi + \sinh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1} \right] \end{aligned}$$

$$-\left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot \left[\sinh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0 \xi^1}{c^2}\right) \phi_\xi + \cosh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1} \right]$$

$$= -\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial A_{\xi^1}}{\partial \xi^0} - \left(1 + \frac{a_0 \xi^1}{c^2}\right) \frac{\partial \phi_\xi}{\partial \xi^1} - 2\phi_\xi \frac{a_0}{c^2}$$

$$= -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^1} \left[\left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \phi_\xi \right] - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^1}}{\partial \xi^0} \quad (16)$$

$$E_y = -\frac{\partial \phi}{\partial y} - \frac{\partial A_y}{\partial t} = -\frac{\partial}{\partial \xi^2} \left[\cosh\left(\frac{a_0 \xi^0}{c}\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \sinh\left(\frac{a_0 \xi^0}{c}\right) A_{\xi^1} \right]$$

$$-\left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] A_{\xi^2}$$

$$= -\left(1 + \frac{a_0 \xi^1}{c^2}\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial \phi_\xi}{\partial \xi^2} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial A_{\xi^2}}{\partial \xi^0}$$

$$+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right]$$

$$\begin{aligned}
&= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \left[\phi_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial A_{\xi^2}}{\partial \xi^0} \right] \\
&+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right] \tag{17}
\end{aligned}$$

$$\begin{aligned}
E_z &= -\frac{\partial \phi}{\partial z} - \frac{\partial A_z}{\partial t} = -\frac{\partial}{\partial \xi^3} \left[\cosh\left(\frac{a_0}{c} \xi^0\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \sinh\left(\frac{a_0}{c} \xi^0\right) A_{\xi^1} \right] \\
&\quad - \left[\frac{\cosh\left(\frac{a_0}{c} \xi^0\right)}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0}{c} \xi^0\right) \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \\
&= -\left(1 + \frac{a_0}{c^2} \xi^1\right) \cosh\left(\frac{a_0}{c} \xi^0\right) \frac{\partial \phi_\xi}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \cosh\left(\frac{a_0}{c} \xi^0\right) \frac{\partial A_{\xi^3}}{\partial \xi^0} \\
&\quad + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^3}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^3} \right] \\
&= \cosh\left(\frac{a_0}{c} \xi^0\right) \left[-\frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \left[\phi_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right)^2 \right] - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial A_{\xi^3}}{\partial \xi^0} \right] \\
&+ \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial A_{\xi^3}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^3} \right] \tag{18}
\end{aligned}$$

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = \frac{\partial A_{\xi^3}}{\partial \xi^2} - \frac{\partial A_{\xi^2}}{\partial \xi^3} \tag{19}$$

$$\begin{aligned}
B_y &= \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = \frac{\partial A_x}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial x} \\
&= \frac{\partial}{\partial \xi^3} \left[\sinh\left(\frac{a_0}{c} \xi^0\right) \left(1 + \frac{a_0}{c^2} \xi^1\right) \phi_\xi + \cosh\left(\frac{a_0}{c} \xi^0\right) A_{\xi^1} \right]
\end{aligned}$$

$$\begin{aligned}
& - \left[- \frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \\
& = \cosh(\frac{a_0}{c} \xi^0) \left[\frac{\partial A_{\xi^1}}{\partial \xi^3} - \frac{\partial A_{\xi^3}}{\partial \xi^1} \right] \\
& - \sinh(\frac{a_0}{c} \xi^0) \left[- \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^3} [\phi_\xi (1 + \frac{a_0 \xi^1}{c^2})^2] - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial A_{\xi^3}}{\partial \xi^0} \right] \quad (20)
\end{aligned}$$

$$\begin{aligned}
B_z & = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \frac{\partial A_{\xi^2}}{\partial x} - \frac{\partial A_x}{\partial \xi^2} \\
& = \left[- \frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] A_{\xi^3} \\
& - \frac{\partial}{\partial \xi^2} \left[\sinh(\frac{a_0 \xi^0}{c}) (1 + \frac{a_0}{c^2} \xi^1) \phi_\xi + \cosh(\frac{a_0 \xi^0}{c}) A_{\xi^1} \right] \\
& = \cosh(\frac{a_0}{c} \xi^0) \left[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2} \right] \\
& + \sinh(\frac{a_0}{c} \xi^0) \left[- \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^2} [\phi_\xi (1 + \frac{a_0 \xi^1}{c^2})^2] - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial A_{\xi^2}}{\partial \xi^0} \right] \quad (21)
\end{aligned}$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler spacetime.

$$\vec{E}_\xi = - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \vec{\nabla}_\xi \left\{ \phi_\xi (1 + \frac{a_0 \xi^1}{c^2})^2 \right\} - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial \vec{A}_\xi}{\partial \xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi$$

$$\text{In this time, } \vec{\nabla}_\xi = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right), \vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3}) \quad (22)$$

We obtain the transformation of the electro-magnetic field.

$$\begin{aligned}
E_x &= -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^1} \left\{ \phi_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 \right\} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial A_{\xi^1}}{c \partial \xi^0} = E_{\xi^1}, \\
E_y &= E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right), \\
E_z &= E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_x &= B_{\xi^1}, \\
B_y &= B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_z &= B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)
\end{aligned} \tag{23}$$

Hence,

$$E_x = E_{\xi^1}, B_x = B_{\xi^1},$$

$$\begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix} = H \begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix}$$

$$H = \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & \sinh\left(\frac{a_0 \xi^0}{c}\right) \\ 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) & -\sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ 0 & -\sinh\left(\frac{a_0 \xi^0}{c}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ \sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) \end{pmatrix} \tag{24}$$

The inverse-transformation of the electro-magnetic field is

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$\begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix} = H^{-1} \begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & -\sinh\left(\frac{a_0 \xi^0}{c}\right) \\ 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) & \sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ 0 & \sinh\left(\frac{a_0 \xi^0}{c}\right) & \cosh\left(\frac{a_0 \xi^0}{c}\right) & 0 \\ -\sinh\left(\frac{a_0 \xi^0}{c}\right) & 0 & 0 & \cosh\left(\frac{a_0 \xi^0}{c}\right) \end{pmatrix} \quad (25)$$

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$E_{\xi^2} = E_y \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_z \sinh\left(\frac{a_0 \xi^0}{c}\right),$$

$$B_{\xi^2} = B_y \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_z \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$E_{\xi^3} = E_z \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_y \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$B_{\xi^3} = B_z \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \quad (26)$$

3. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (27-i)$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j} \quad (27-ii)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (27-iii)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (27-iv)$$

$$1. \vec{\nabla} \cdot \vec{E} = 4\pi\rho$$

$$E_x = E_{\xi^1},$$

$$\begin{aligned}
E_y &= E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right), \\
E_z &= E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
4\pi\rho &= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \\
&= \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] E_{\xi^1} \\
&+ \frac{\partial}{\partial \xi^2} \left[E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right] \\
&+ \frac{\partial}{\partial \xi^3} \left[E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right] \\
&= \cosh\left(\frac{a_0}{c} \xi^0\right) (\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \sinh\left(\frac{a_0}{c} \xi^0\right) \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial E_{\xi^1}}{\partial \xi^0} \right] \quad (28)
\end{aligned}$$

$$2. \vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}$$

$$B_x = B_{\xi^1}$$

$$B_y = B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$B_z = B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$\text{X-component) } \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$$

$$= \frac{\partial}{\partial \xi^2} \left[B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right]$$

$$- \frac{\partial}{\partial \xi^3} \left[B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right]$$

$$\begin{aligned}
&= \cosh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} \right] + \sinh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial E_{\xi^2}}{\partial \xi^2} + \frac{\partial E_{\xi^3}}{\partial \xi^3} \right] \\
&= \frac{\partial E_x}{\partial t} + \frac{4\pi}{c} j_x \\
&= \left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] E_{\xi^1} + \frac{4\pi}{c} j_x
\end{aligned}$$

Hence,

$$\begin{aligned}
&\frac{4\pi}{c} j_x \\
&= \sinh\left(\frac{a_0 \xi^0}{c}\right) (\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi}) + \cosh\left(\frac{a_0 \xi^0}{c}\right) \left[\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial E_{\xi^1}}{\partial \xi^0} \right] \quad (29)
\end{aligned}$$

$$\begin{aligned}
\text{Y-component)} \quad &\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \\
&= \frac{\partial B_{\xi^1}}{\partial \xi^3}
\end{aligned}$$

$$\begin{aligned}
&= \left[-\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot \left[B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right]
\end{aligned}$$

$$= \frac{\partial E_y}{\partial t} + \frac{4\pi}{c} j_y$$

$$= \left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot \left[E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right]$$

$$+ \frac{4\pi}{c} j_y$$

$$\frac{4\pi}{c} j_y = \frac{\partial B_{\xi^1}}{\partial \xi^3} - \frac{\partial B_{\xi^3}}{\partial \xi^1} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{a_0}{c^2} B_{\xi^3} - \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial E_{\xi^2}}{\partial \xi^0}$$

$$= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^3} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^1} \{B_{\xi^3} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial E_{\xi^2}}{\partial \xi^0} \quad (30)$$

$$\text{Z-component) } \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}$$

$$= \left[-\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot [B_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) - E_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})]$$

$$- \frac{\partial B_{\xi^1}}{\partial \xi^2}$$

$$= \frac{\partial E_z}{\partial t} + \frac{4\pi}{c} j_z$$

$$= \left[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] \cdot [E_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})]$$

$$+ \frac{4\pi}{c} j_z$$

$$\frac{4\pi}{c} j_z = \frac{\partial B_{\xi^2}}{\partial \xi^1} - \frac{\partial B_{\xi^1}}{\partial \xi^2} + \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{a_0}{c^2} B_{\xi^2} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial E_{\xi^3}}{\partial \xi^0}$$

$$= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^1} \{B_{\xi^2} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial}{\partial \xi^2} \{B_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial E_{\xi^3}}{\partial \xi^0} \quad (31)$$

$$3. \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

$$= \left[-\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1} \right] B_{\xi^1}$$

$$\begin{aligned}
& + \frac{\partial}{\partial \xi^2} [B_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) - E_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})] \\
& + \frac{\partial}{\partial \xi^3} [B_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) + E_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})] \\
& = \cosh(\frac{a_0 \xi^0}{c}) (\vec{\nabla}_\xi \cdot \vec{B}_\xi) + \sinh(\frac{a_0 \xi^0}{c}) [-(-\frac{\partial E_{\xi^2}}{\partial \xi^3} + \frac{\partial E_{\xi^3}}{\partial \xi^2}) - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)} \frac{\partial B_{\xi^1}}{c \partial \xi^0}] = 0
\end{aligned} \tag{32}$$

$$4. \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{c \partial t}$$

$$E_x = E_{\xi^1},$$

$$E_y = E_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0 \xi^0}{c}),$$

$$E_z = E_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})$$

$$\text{X-component) } \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$$

$$= \frac{\partial}{\partial \xi^2} [E_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})]$$

$$- \frac{\partial}{\partial \xi^3} [E_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c} \xi^0) [\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3}] - \sinh(\frac{a_0 \xi^0}{c}) [\frac{\partial B_{\xi^2}}{\partial \xi^2} + \frac{\partial B_{\xi^3}}{\partial \xi^3}]$$

$$= -\frac{\partial B_x}{c \partial t}$$

$$= -[\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{c \partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1}] B_{\xi^1}$$

Hence,

$$-\sinh\left(\frac{a_0\xi^0}{c}\right)(\vec{\nabla}_\xi \cdot \vec{B}_\xi) + \cosh\left(\frac{a_0\xi^0}{c}\right)\left[\left(\frac{\partial E_{\xi^3}}{\partial \xi^2} - \frac{\partial E_{\xi^2}}{\partial \xi^3}\right) + \frac{1}{\left(1 + \frac{a_0\xi^1}{c^2}\right)} \frac{\partial B_{\xi^1}}{\partial \xi^0}\right] = 0 \quad (33)$$

$$\text{Y-component) } \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$$

$$= \frac{\partial E_{\xi^1}}{\partial \xi^3}$$

$$-\left[-\frac{\sinh\left(\frac{a_0\xi^0}{c}\right)}{\left(1 + \frac{a_0\xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0\xi^0}{c}\right) \frac{\partial}{\partial \xi^1}\right] \cdot [E_{\xi^3} \cosh\left(\frac{a_0\xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0\xi^0}{c}\right)]$$

$$= -\frac{\partial B_y}{\partial t}$$

$$= -\left[\frac{\cosh\left(\frac{a_0\xi^0}{c}\right)}{\left(1 + \frac{a_0\xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0\xi^0}{c}\right) \frac{\partial}{\partial \xi^1}\right] \cdot [B_{\xi^2} \cosh\left(\frac{a_0\xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0\xi^0}{c}\right)]$$

$$\frac{\partial E_{\xi^1}}{\partial \xi^3} - \frac{\partial E_{\xi^3}}{\partial \xi^1} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{a_0}{c^2} E_{\xi^3} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial B_{\xi^2}}{\partial \xi^0}$$

$$= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^3} \left\{E_{\xi^1} \left(1 + \frac{a_0}{c^2} \xi^1\right)\right\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \left\{E_{\xi^3} \left(1 + \frac{a_0}{c^2} \xi^1\right)\right\} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial B_{\xi^2}}{\partial \xi^0}$$

$$= 0$$

(34)

$$\text{Z-component) } \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$$

$$= \left[-\frac{\sinh\left(\frac{a_0\xi^0}{c}\right)}{\left(1 + \frac{a_0\xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0\xi^0}{c}\right) \frac{\partial}{\partial \xi^1}\right] \cdot [E_{\xi^2} \cosh\left(\frac{a_0\xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0\xi^0}{c}\right)]$$

$$- \frac{\partial E_{\xi^1}}{\partial \xi^2}$$

$$\begin{aligned}
&= -\frac{\partial B_z}{\partial t} \\
&= -\left[\frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \right] \cdot [B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)] \\
&\quad \frac{\partial E_{\xi^2}}{\partial \xi^1} - \frac{\partial E_{\xi^1}}{\partial \xi^2} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{a_0}{c^2} E_{\xi^2} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial B_{\xi^3}}{\partial \xi^0} \\
&= \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^1} \{E_{\xi^2} (1 + \frac{a_0}{c^2} \xi^1)\} - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial}{\partial \xi^2} \{E_{\xi^1} (1 + \frac{a_0}{c^2} \xi^1)\} + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)} \frac{\partial B_{\xi^3}}{\partial \xi^0} \\
&= 0 \tag{35}
\end{aligned}$$

Therefore, we obtain the electro-magnetic field equation by Eq (28)-Eq(35) in Rindler spacetime .

$$\vec{\nabla}_\xi \cdot \vec{E}_\xi = 4\pi\rho_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \tag{36-i}$$

$$\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_\xi \times \left\{ \vec{B}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right\} = \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial \vec{E}_\xi}{\partial \xi^0} + \frac{4\pi}{c} \vec{j}_\xi \tag{36-ii}$$

$$\vec{\nabla}_\xi \cdot \vec{B}_\xi = 0 \tag{36-iii}$$

$$\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \vec{\nabla}_\xi \times \left\{ \vec{E}_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \right\} = -\frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial \vec{B}_\xi}{\partial \xi^0} \tag{36-iv}$$

$$\vec{E}_\xi = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_\xi = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}), \vec{\nabla}_\xi = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right)$$

Hence, the transformation of 4-vector $(c\rho, \vec{j}) = \rho_0 \frac{dx^\alpha}{d\tau}$ is

$$\begin{aligned}
\rho &= \rho_\xi \left(1 + \frac{a_0 \xi^1}{c^2}\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) + \frac{j_{\xi^1}}{c} \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
j_x &= j_{\xi^1} \cosh\left(\frac{a_0 \xi^0}{c}\right) + c\rho_\xi \left(1 + \frac{a_0}{c^2} \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right), \quad j_y = j_{\xi^2}, j_z = j_{\xi^3}
\end{aligned}$$

$$\text{In this time, 4-vector } (c\rho_\xi, \vec{j}_\xi) = \rho_0 \frac{d\xi^\alpha}{d\tau} \tag{37}$$

4. Electro-magnetic wave equation in Rindler space-time

The electro-magnetic wave function is

$$E_x = E_{x0} \sin \Phi, B_x = B_{x0} \sin \Phi,$$

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$E_{\xi^1} = E_x = E_{x0} \sin \Phi = E_{x0} \sin \Phi' - C_{\xi^1}, B_{\xi^1} = B_x = B_{x0} \sin \Phi = B_{x0} \sin \Phi' - C_{\xi^1}$$

$$E_{\xi^2} = E_y \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_z \sinh\left(\frac{a_0 \xi^0}{c}\right),$$

$$= (E_{y0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) - (B_{z0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$= (E_{y0} \sin \Phi' - C_{\xi^2}) \cosh\left(\frac{a_0 \xi^0}{c}\right) - (B_{z0} \sin \Phi' - C_{\xi^3}) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$B_{\xi^2} = B_y \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_z \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$= (B_{y0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) + (E_{z0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$= (B_{y0} \sin \Phi' - C_{\xi^2}) \cosh\left(\frac{a_0 \xi^0}{c}\right) + (E_{z0} \sin \Phi' - C_{\xi^3}) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$E_{\xi^3} = E_z \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_y \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$= (E_{z0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) + (B_{y0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$= (E_{z0} \sin \Phi' - C_{\xi^3}) \cosh\left(\frac{a_0 \xi^0}{c}\right) + (B_{y0} \sin \Phi' - C_{\xi^2}) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$B_{\xi^3} = B_z \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_y \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$= (B_{z0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) - (E_{y0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$= (B_{z0} \sin \Phi' - C_{\xi^3}) \cosh\left(\frac{a_0 \xi^0}{c}\right) - (E_{y0} \sin \Phi' - C_{\xi^2}) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

In this time,

$$\left(\frac{\partial}{\partial \xi^0}\right)^2 \Phi' \cdot \frac{\partial(E_{x0} \sin \Phi')}{\partial \xi^0} / \frac{\partial \Phi'}{\partial \xi^0} = \left(\frac{\partial}{\partial \xi^0}\right)^2 C_{\xi^1},$$

$$\left(\frac{\partial}{\partial \xi^0}\right)^2 \Phi' \cdot \frac{\partial(E_{y0} \sin \Phi')}{\partial \xi^0} / \frac{\partial \Phi'}{\partial \xi^0} = \left(\frac{\partial}{\partial \xi^0}\right)^2 C_{\xi^2},$$

$$\begin{aligned}
\left(\frac{\partial}{\partial \xi^0}\right)^2 \Phi' \cdot \frac{\partial(E_{z0} \sin \Phi')}{\partial \xi^0} / \frac{\partial \Phi'}{\partial \xi^0} &= \left(\frac{\partial}{\partial \xi^0}\right)^2 C_{\xi^3} \\
\left(\frac{\partial}{\partial \xi^0}\right)^2 \Phi' \cdot \frac{\partial(B_{x0} \sin \Phi')}{\partial \xi^0} / \frac{\partial \Phi'}{\partial \xi^0} &= \left(\frac{\partial}{\partial \xi^0}\right)^2 C_{\xi^1}, \\
\left(\frac{\partial}{\partial \xi^0}\right)^2 \Phi' \cdot \frac{\partial(B_{y0} \sin \Phi')}{\partial \xi^0} / \frac{\partial \Phi'}{\partial \xi^0} &= \left(\frac{\partial}{\partial \xi^0}\right)^2 C_{\xi^2}, \\
\left(\frac{\partial}{\partial \xi^0}\right)^2 \Phi' \cdot \frac{\partial(B_{z0} \sin \Phi')}{\partial \xi^0} / \frac{\partial \Phi'}{\partial \xi^0} &= \left(\frac{\partial}{\partial \xi^0}\right)^2 C_{\xi^3} \\
\left(\frac{\partial}{\partial \xi^1}\right)^2 \bar{C}_\xi &= \left(\frac{\partial}{\partial \xi^2}\right)^2 \bar{C}_\xi = \left(\frac{\partial}{\partial \xi^3}\right)^2 \bar{C}_\xi = \bar{0} \\
, \left(\frac{\partial}{\partial \xi^1}\right)^2 \bar{C}'_\xi &= \left(\frac{\partial}{\partial \xi^2}\right)^2 \bar{C}'_\xi = \left(\frac{\partial}{\partial \xi^3}\right)^2 \bar{C}'_\xi = \bar{0}
\end{aligned} \tag{38}$$

$$\Phi = \omega t - l \frac{X}{c} - m \frac{Y}{c} - n \frac{Z}{c},$$

$$\Phi' = \omega' \left[\frac{c}{a_0} \left(1 + \frac{a_0 \xi^1}{c^2}\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) - l' \frac{c}{a_0} \left(1 + \frac{a_0 \xi^1}{c^2}\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - m' \frac{\xi^2}{c} - n' \frac{\xi^3}{c} + l' \frac{c}{a_0} \right]$$

$$= \omega' \left[\frac{c}{a_0} \left(1 + \frac{a_0 \xi^1}{c^2}\right) \left\{ \sinh\left(\frac{a_0 \xi^0}{c}\right) - l' \cosh\left(\frac{a_0 \xi^0}{c}\right) \right\} - m' \frac{\xi^2}{c} - n' \frac{\xi^3}{c} + l' \frac{c}{a_0} \right]$$

$$ct = \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right), x = \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3$$

$$, l'^2 + m'^2 + n'^2 = 1, l'^2 + m'^2 + n'^2 = 1 \tag{39}$$

Hence,

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] E_{\xi^1}$$

$$= \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] [E_{\xi^1} \sin \Phi' - C_{\xi^1}] = 0$$

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] B_{\xi^1}$$

$$\begin{aligned}
&= \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] [B_{\xi^1 0} \sin \Phi - C_{\xi^1}] = 0 \\
&\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] E_y \\
&= \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] [E_{y0} \sin \Phi - C_{\xi^2}] = 0 \\
&\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] B_y \\
&= \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] [B_{y0} \sin \Phi - C_{\xi^2}] = 0 \\
&\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] E_z \\
&= \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] [E_{z0} \sin \Phi - C_{\xi^3}] = 0 \\
&\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] B_z \\
&= \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi}^2 \right] [B_{z0} \sin \Phi - C_{\xi^3}] = 0 \tag{40}
\end{aligned}$$

The electro-magnetic wave equation is in vacuum

$$\begin{aligned}
&\bar{\nabla}_{\xi} \times \left(1 + \frac{a_0}{c^2} \xi^1\right) \bar{\nabla}_{\xi} \times \left\{ \bar{E}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} \\
&= \bar{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \bar{\nabla}_{\xi} \times \left\{ \bar{E}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} + \left(1 + \frac{a_0}{c^2} \xi^1\right) \bar{\nabla}_{\xi} \times \bar{\nabla}_{\xi} \times \left\{ \bar{E}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \right\} \\
&= \bar{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \bar{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \bar{E}_{\xi} \\
&+ \left(1 + \frac{a_0}{c^2} \xi^1\right) \bar{\nabla}_{\xi} \left(1 + \frac{a_0}{c^2} \xi^1\right) \times \bar{\nabla}_{\xi} \times \bar{E}_{\xi}
\end{aligned}$$

$$\begin{aligned}
& + (1 + \frac{a_0}{c^2} \xi^1) \vec{\nabla}_\xi \times \vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) \times \vec{E}_\xi \\
& + (1 + \frac{a_0}{c^2} \xi^1)^2 \vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \vec{E}_\xi \\
& = \vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) \times \vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) \times \vec{E}_\xi + (1 + \frac{a_0}{c^2} \xi^1)^2 \vec{\nabla}_\xi \times \vec{\nabla}_\xi \times \vec{E}_\xi \\
& = [\vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) \cdot \vec{E}_\xi] \vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) - [\vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) \cdot \vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1)] \vec{E}_\xi \\
& \quad + (1 + \frac{a_0}{c^2} \xi^1)^2 [\vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{E}_\xi) - \nabla_\xi^2 \vec{E}_\xi] \\
& = -\frac{1}{c} \frac{\partial}{\partial \xi^0} [\vec{\nabla}_\xi \times \{\vec{B}_\xi (1 + \frac{a_0 \xi^1}{c^2})\}] = -\frac{1}{c^2} (\frac{\partial}{\partial \xi^0})^2 \vec{E}_\xi,
\end{aligned}$$

$$\text{In this time, } \vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) = (\frac{a_0}{c^2}, 0, 0) \quad (41)$$

Hence,

$$\begin{aligned}
& \vec{\nabla}_\xi \times (1 + \frac{a_0}{c^2} \xi^1) \vec{\nabla}_\xi \times \{\vec{E}_\xi (1 + \frac{a_0}{c^2} \xi^1)\} + \frac{1}{c^2} (\frac{\partial}{\partial \xi^0})^2 \vec{E}_\xi \\
& = [\vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) \cdot \vec{E}_\xi] \vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) - [\vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) \cdot \vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1)] \vec{E}_\xi \\
& \quad + (1 + \frac{a_0}{c^2} \xi^1)^2 [\vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{E}_\xi) - \nabla_\xi^2 \vec{E}_\xi] + \frac{1}{c^2} (\frac{\partial}{\partial \xi^0})^2 \vec{E}_\xi \\
& = \frac{a_0^2}{c^4} (E_{\xi^1}, 0, 0) - \frac{a_0^2}{c^4} (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}) + (1 + \frac{a_0}{c^2} \xi^1)^2 [\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_\xi^2] \vec{E}_\xi \\
& = \frac{a_0^2}{c^4} (0, -E_{\xi^2}, -E_{\xi^3}) + (1 + \frac{a_0}{c^2} \xi^1)^2 [\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_\xi^2] \vec{E}_\xi \\
& = \vec{0} \quad (42)
\end{aligned}$$

Hence, the magnetic wave equation is in vacuum

$$\begin{aligned}
& \vec{\nabla}_\xi \times (1 + \frac{a_0}{c^2} \xi^1) \vec{\nabla}_\xi \times \{\vec{B}_\xi (1 + \frac{a_0}{c^2} \xi^1)\} + \frac{1}{c^2} (\frac{\partial}{\partial \xi^0})^2 \vec{B}_\xi \\
& = [\vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) \cdot \vec{B}_\xi] \vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) - [\vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1) \cdot \vec{\nabla}_\xi (1 + \frac{a_0}{c^2} \xi^1)] \vec{B}_\xi \\
& \quad + (1 + \frac{a_0}{c^2} \xi^1)^2 [\vec{\nabla}_\xi (\vec{\nabla}_\xi \cdot \vec{B}_\xi) - \nabla_\xi^2 \vec{B}_\xi] + \frac{1}{c^2} (\frac{\partial}{\partial \xi^0})^2 \vec{B}_\xi
\end{aligned}$$

$$\begin{aligned}
&= \frac{a_0^2}{c^4} (0, -B_{\xi^2}, -B_{\xi^3}) + (1 + \frac{a_0}{c^2} \xi^1)^2 \left[\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_{\xi^2}^2 \right] \vec{B}_{\xi} \\
&= \vec{0}
\end{aligned} \tag{43}$$

The electromagnetic wave function, Eq(38),Eq(39) satisfy the electromagnetic wave equation, Eq(42),Eq(43).

5. Conclusion

We find the electro-magnetic field transformation and the electro-magnetic equation in uniformly accelerated frame.

Generally, the coordinate transformation of accelerated frame is

$$\begin{aligned}
\text{(I)} \quad ct &= \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c} \right) \\
x &= \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c} \right) - \frac{c^2}{a_0}, \quad y = \xi^2, \quad z = \xi^3
\end{aligned} \tag{44}$$

$$\begin{aligned}
\text{(II)} \quad ct &= \frac{c^2}{a_0} \exp\left(\frac{a_0}{c^2} \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c} \right) \\
x &= \frac{c^2}{a_0} \exp\left(\frac{a_0}{c^2} \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c} \right) - \frac{c^2}{a_0}, \quad y = \xi^2, \quad z = \xi^3
\end{aligned} \tag{45}$$

Hence, this article say the accelerated frame is Rindler coordinate (I) that can treat electro-magnetic field equation.

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