The Electro-Magnetic Field Equation and Electro-Magnetic Field Transformation in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime.

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1. Introduction

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime.

The Rindler coordinate is

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \tag{1}$$

In this time, the tetrad $e^a{}_\mu$ is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^\mu}\frac{\partial x^b}{\partial \xi^\upsilon}d\xi^\mu d\xi^\upsilon$$

$$= -\frac{1}{c^2}\eta_{ab}e^a{}_\mu e^b{}_\upsilon d\xi^\mu d\xi^\upsilon = -\frac{1}{c^2}g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad , \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \tag{2}$$

$$e^\alpha{}_0(\xi^0) = \frac{\partial x^\alpha}{c\partial \xi^0} = (\cosh(\frac{a_0\xi^0}{c}), \sinh(\frac{a_0\xi^0}{c}), 0, 0) \tag{3}$$

About y -axis's and z -axis's orientation

$$e^\alpha{}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0,0,1,0) \quad , \quad e^\alpha{}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0,0,0,1) \tag{4}$$

The other unit vector $e^\alpha{}_1(\xi^0)$ is

$$e^\alpha{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = (\sinh(\frac{a_0\xi^0}{c}), \cosh(\frac{a_0\xi^0}{c}), 0, 0) \tag{5}$$

Therefore,

$$cdt = c\cosh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \sinh(\frac{a_0\xi^0}{c})d\xi^1$$

$$dx = c\sinh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \cosh(\frac{a_0\xi^0}{c})d\xi^1, dy = d\xi^2, \; dz = d\xi^3 \tag{6}$$

Hence, the transformation of the electro-magnetic potential $(\phi, \vec{A})$ in inertial frame and the eletromagnetic potential $(\phi_\xi, \vec{A}_\xi)$ is

$$\phi = \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$A_x = \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$A_y = A_{\xi^2}, A_z = A_{\xi^3} \tag{7}$$

$$e^a{}_\mu e^b{}_\upsilon \eta_{ab} = g_{\mu\upsilon} \rightarrow \quad A^T\eta A = g$$

$$e_a{}^\mu e_b{}^\upsilon g_{\mu\upsilon} = \eta_{ab} \rightarrow \quad (A^T)^{-1} g A^{-1} = (A^T)^{-1} A^T \eta A A^{-1} = \eta$$

$$e^a{}_\mu = \eta^{ab} g_{\mu\upsilon} e_b{}^\upsilon \rightarrow \eta^{-1}(A^T)^{-1} A^T \eta A = A = \eta^{-1}(A^T)^{-1} g$$

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix}$$

$$= A \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ dz^3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^T \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^T)^{-1} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$
= \begin{pmatrix} \dfrac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ -\dfrac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial\xi^0} \\ \frac{\partial}{\partial\xi^1} \\ \frac{\partial}{\partial\xi^2} \\ \frac{\partial}{\partial\xi^3} \end{pmatrix}
$$

$$
\frac{1}{c}\frac{\partial}{\partial t} = \frac{c\partial\xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{c\partial t}\frac{\partial}{\partial\xi^1}
$$

$$
= \frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}
$$

$$
\frac{\partial}{\partial x} = \frac{c\partial\xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{\partial x}\frac{\partial}{\partial\xi^1}
$$

$$
= -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}
$$

$$
\frac{\partial}{\partial y} = \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial\xi^3} \quad (8)
$$

2. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field $(\vec{E},\vec{B})$ is in the inertial frame,

$$
\vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{c\partial t}, \vec{B} = \vec{\nabla}\times\vec{A} \quad (9)
$$

We define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler spacetime.

$$
\vec{E}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0}
$$

$$
\vec{B}_\xi = \vec{\nabla}_\xi\times\vec{A}_\xi
$$

In this time, $\vec{\nabla}_{\xi} = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3}), \vec{A}_{\xi} = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$ (10)

Hence, we obtain the transformation of the electro-magnetic field.

$$E_x = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}\{\phi_{\xi}(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0} = E_{\xi^1} ,$$

$$E_y = E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$B_x = B_{\xi^1},$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}) \quad (11)$$

3. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \quad \text{(12-i)}$$

$$\vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j} \quad \text{(12-ii)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{(12-iii)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{c\partial t} \quad \text{(12-iv)}$$

Therefore, we obtain the electro-magnetic field equation by Eq (12-i), Eq(12-ii), Eq(12-iii), Eq(12-iv) in Rindler spacetime .

$$\vec{\nabla}_{\xi}\cdot\vec{E}_{\xi} = 4\pi\rho_{\xi} \quad \text{(13-i)}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\xi}\times\{\vec{B}_{\xi}(1+\frac{a_0\xi^1}{c^2})\} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_{\xi}}{c\partial\xi^0} + \frac{4\pi}{c}\vec{j}_{\xi} \quad \text{(13-ii)}$$

$$\vec{\nabla}_{\xi}\cdot\vec{B}_{\xi} = 0 \quad \text{(13-iii)}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \times \{\vec{E}_\xi(1+\frac{a_0\xi^1}{c^2})\} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial \vec{B}_\xi}{c\partial \xi^0} \quad \text{(13-iv)}$$

$$\vec{E}_\xi = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_\xi = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}), \vec{\nabla}_\xi = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3})$$

Hence,

$$\rho = \rho_\xi \cosh(\frac{a_0\xi^0}{c}) + \frac{j_{\xi^1}}{c}\sinh(\frac{a_0\xi^0}{c})$$

$$j_x = j_{\xi^1}\cosh(\frac{a_0\xi^0}{c}) + c\rho_\xi \sinh(\frac{a_0\xi^0}{c}), \quad j_y = j_{\xi^2}, j_z = j_{\xi^3} \quad (14)$$

In this time, the Lorentz gauge is

$$\frac{\partial \phi}{c\partial t} + \vec{\nabla}\cdot\vec{A} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial \xi^1}\{A_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} + \frac{\partial \phi_\xi}{c\partial \xi^0} + \frac{\partial A_{\xi^2}}{\partial \xi^2} + \frac{\partial A_{\xi^3}}{\partial \xi^3}$$

$$= \frac{\partial \phi_\xi}{c\partial \xi^0} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi \cdot \{\vec{A}_\xi(1+\frac{a_0\xi^1}{c^2})\} = 0 \quad (15)$$

4. Conclusion

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime.

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