Quantum Gravitational Relativity
Part III

The theory presented in this paper is the third part of the quantum gravitational formulation of Einstein's special theory of relativity. Based on new and simple 'quantum length to classical length transformations', I shall derive two formulas: (a) the quantum gravitational length contraction formula, introduced without proof in Part I; and (b) the quantum gravitational time dilation formula. In Part I, I showed that the Fitzgerald-Lorentz length contraction formulation violates the space quantization postulate and therefore a new quantum gravitational equation was required. If the second postulate introduced in Part I is correct, then the new length contraction formula should be preferred over the Fitzgerald-Lorentz length contraction counterpart. In contrast I showed that the Einstein's time dilation formula does not violate the time quantization postulate. This is a striking difference between space and time. However, if we apply the 'quantum time to classical time transformations', we obtain a new formula for time dilation. But then the question arises of which of the two time dilation formulas is the correct one? I found that we do not have any arguments to deciding which of the two time dilation formulas should be the preferred one. It seems that only the experiment can answer this question.

by Rodolfo A. Frino

Electronics Engineer
Degree from the National University of Mar del Plata - Argentina
rodolfo_frino@yahoo.com.ar

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1. Introduction

A transformation is a relation between numbers that apply to the same event but that are measured from different coordinate systems. The *Galilean transformations* (see **Table 1**) are common sense relations between the coordinates of two inertial frames of reference that are moving with respect to each other at a constant speed, *v*. These transformations assume that both space and time are absolute. The notions of absolute space and time were introduced by Newton in his *Principia* where he quoted:

"absolute, true and mathematical time, of itself, and from its own nature, flows equably without relation to anything external and is also known as duration".

"absolute, true and mathematical space, of itself, and from its own nature, without relation to

anything external remains constant and motionless".

Newton's laws of mechanics are invariant under a Galilean transformation (For example, acceleration is invariant under a Galilean transformation). The Galilean transformations provide a very good description of our everyday experience, however these transformations are not correct! The inaccuracy of the Galilean transformations becomes significant when the speed of the body is in the order of the speed of light (when v is not *much smaller* than c)

Galilean transformations **Direct Transformations**		**Galilean transformations** **Reverse Transformations**	
$x' = x - v\,t$	(1.1)	$x = x' + v\,t'$	(1.5)
$y' = y$	(1.2)	$y = y'$	(1.6)
$z' = z$	(1.3)	$z = z'$	(1.7)
$t' = t$	(1.4)	$t = t'$	(1.8)

Table 1: The Galilean transformations of classical physics.

There is another set of equations that fix this problem, they are known as: *The Lorentz transformations.* Despite the fact the Lorentz transformations are generally attributed to H. A. Lorentz, they were discovered by W. Voigt in 1887 and published in 1904. This means that the transformations were discovered before the Special Theory of Relativity [1]. These transformations reduce to the Galilean equations when the velocity of the body, v, is small compared to the speed of light, c. A law of physics could be invariant under a given transformation but it could turn out to be not invariant under a different set of equations. Maxwell's equations of electromagnetism, for example, are invariant under a Lorentz transformation, but they are not invariant under a Galilean transformation. Acceleration, for example, is invariant under a Galilean transformation but is not invariant under a Lorentz transformation. The Lorentz transformations are given in **Table 2**:

Lorentz transformations **Direct Transformations**		**Lorentz transformations** **Reverse Transformations**	
$x' = \dfrac{x - v\,t}{\sqrt{1-\beta^2}}$	(1.09)	$x = \dfrac{x' + v\,t'}{\sqrt{1-\beta^2}}$	(1.13)
$y' = y$	(1.10)	$y = y'$	(1.14)
$z' = z$	(1.11)	$z = z'$	(1.15)
$t' = \dfrac{t - \dfrac{v\,x}{c^2}}{\sqrt{1-\beta^2}}$	(1.12)	$t = \dfrac{t' + \dfrac{v\,x'}{c^2}}{\sqrt{1-\beta^2}}$	(1.16)

Table 2: The Lorentz transformations. In order to simplify the equations I shall use the parameter β for the remainder of the article. This variable is defined as: $(\beta \equiv v/c)$.

In 1905 Einstein published his Special Theory of Relativity using Lorentz transformations. As a consequence of Einstein's theory, Newton's concepts of absolute space and time were abandoned. Now space and time were relative to the observer and they were linked through the equations of **Table 2**. The problem with these equations is that they are classical equations and therefore they are inadequate to describe the quantum world (the real world!). In the next sections I shall introduce four new transformations to overcome this limitation:

(1) **the *quantum length to classical length transformations*:**
(1a) *First length transformation*
(1b) *Second length transformation*
and
(2) **the *quantum time interval to classical time interval transformations*:**
(2a) *First length transformation*
(2b) *Second length transformation*

2. Derivation of the Quantum Gravitational Length Contraction Formula

In order to keep as much of the original Lorentz transformations as possible and as necessary, we have to deal with classical physics and with quantum mechanics at the same time. This means that, in order to derive the quantum gravitational length contraction formula from the quantum gravitational Lorentz transformations, we have to deal with the mathematical descriptions of two different worlds. On the one hand, we have the classical world where bodies (particles and space elements) can have zero length, and, on the other hand, we have the quantum world in which bodies (particles and space elements) are constrained to a minimum length equal to the Planck length, L_P . Thus if x'_2 and x'_1 represent the Cartesian coordinates of the endpoints of a body in certain reference system S' (see Part II, Fig. 1), a classical body of zero length, $x'_2 - x'_1 = 0$, in the macroscopic or classical world, will represent the minimum quantum length, L_P , in the quantum world. In other words we need a formula to transform classical lengths (ideal lengths) to quantum lengths (real lengths). Thus the transformation we need is:

First length transformation: **quantum length to classical length transformation**	
$x'_2 - x'_1 = l_0 - L_P$	(2.1)

This transformation tells us that when the length of the body, l_0 (proper length), is equal to the Planck length, L_P , the length of the body, from the classical physics' point of view, will be exactly zero. Mathematically we express this fact as follows:

$$x'_2 - x'_1 = L_P - L_P = 0 \tag{2.2}$$

Equation (2.1) is a fundamental formula that relates the quantum mechanical length (real length) of a particle (or any other physical entity) with its corresponding classical length (approximate length):

$$\underbrace{x'_2 - x'_1}_{\text{Classical length (approximation)}} = \underbrace{l_0}_{\text{Discrete length (real length)}} - L_P$$

The two equations I shall use are derived from equation (1.09):

$$x'_1 = \frac{x_1 - v\,t_1}{\sqrt{1-\beta^2}} \quad \text{(one \textbf{classical} endpoint of the body)} \tag{2.3 a}$$

$$x'_2 = \frac{x_2 - v\,t_2}{\sqrt{1-\beta^2}} \quad \text{(the other \textbf{classical} endpoint of the body)} \tag{2.3 b}$$

Now I shall calculate the difference $x'_2 - x'_1$ (classical length) between the endpoints of the body:

$$x'_2 - x'_1 = \frac{x_2 - v\,t_2 - \left(x_1 - v\,t_1\right)}{\sqrt{1-\beta^2}} \tag{2.4}$$

This equation can be rewritten as

$$x'_2 - x'_1 = \frac{x_2 - x_1 + v\left(t_1 - t_2\right)}{\sqrt{1-\beta^2}} \tag{2.5}$$

Because both measurements are simultaneous, the time difference between these measurements is zero:

$$t_2 - t_1 = 0 \tag{2.6}$$

It is worthwhile to remark that, from the quantum mechanical point of view, the minimum duration of a given event is equal to the Planck time, T_P . However, equation (2.6) does not represent the duration of an event but the time difference between two different events (two different measurements). Taking this into account, equation (2.5) reduces to:

$$x'_2 - x'_1 = \frac{x_2 - x_1}{\sqrt{1-\beta^2}} \tag{2.7}$$

But according to the first quantum length to classical length transformation, we have:

$$x'_2 - x'_1 = l_0 - L_P \tag{2.8}$$

Thus, from equations (2.7) and (2.8) we can write:

$$l_0 - L_P = \frac{x_2 - x_1}{\sqrt{1-\beta^2}} \tag{2.9}$$

Taking into account that the quantum length of the body, l, for an observer of a reference system S (see Part II, Fig. 1 [3]) is related to its classical length, $x_2 - x_1$, through the second length transformation:

Second length transformation:
quantum length to classical length transformation

$$x_2 - x_1 = l - L_P \tag{2.10}$$

we can rewrite equation (2.9) as follows

$$l_0 - L_P = \frac{l - L_P}{\sqrt{1-\beta^2}} \tag{2.11}$$

And operating algebraically we get the following equations

$$l - L_P = \left(l_0 - L_P\right)\sqrt{1-\beta^2} \tag{2.12}$$

$$l = l_0\sqrt{1-\beta^2} - L_P\sqrt{1-\beta^2} + L_P \tag{2.13}$$

Finally we obtain the formula we were seeking:

Quantum gravitational length contraction formula

$$l = l_0\sqrt{1-\beta^2} + L_P\left(1 - \sqrt{1-\beta^2}\right) \tag{2.14}$$

This is the quantum gravitational length contraction formula we introduced in the first part of this article (See Part I, eq. (4.1) [2]).

3. Derivation of the Quantum Gravitational Time Dilation Formula

For this derivation I shall use the reverse quantum gravitational Lorentz transformation (2.8) of **Table 3**. Let us assume that observers *O* and *O′* of two inertial frames *S* and *S′* measure a given time interval. We assume that reference system *S′* moves at a speed *v* with respect to system *S* in the positive *x* direction as shown in Figure 1, Part II [3]. Thus the two values for the time coordinates of the event are:

$$t_1 = \frac{t'_1 + x'_1 \frac{v}{c^2}}{\sqrt{1-\beta^2}} \quad \text{(beginning of the \textbf{classical} time interval)} \tag{3.1}$$

$$t_2 = \frac{t'_2 + x'_2 \frac{v}{c^2}}{\sqrt{1-\beta^2}} \quad \text{(end of the \textbf{classical} time interval)} \tag{3.2}$$

Now we calculate the time interval $t_2 - t_1$ measured by an observer of system *S*:

$$t_2 - t_1 = \left[t'_2 + \frac{x'_2 v}{c^2} - \left(t'_1 + \frac{x'_1 v}{c^2} \right) \right] \frac{1}{\sqrt{1-\beta^2}} \tag{3.3}$$

Because the time measurements are made at the same location, the difference of the space coordinates must be zero

$$x'_2 - x'_1 = 0 \tag{3.4}$$

Then equation (3.3) reduces to

$$t_2 - t_1 = \frac{t'_2 - t'_1}{\sqrt{1-\beta^2}} \tag{3.5}$$

If t'_1 and t'_2 represent the coordinates of the time interval $t_2' - t_1'$ measured by an observer of system *S′*, then a classical zero time interval, $t'_2 - t'_1 = 0$, in the macroscopic world of system *S′*, will represent the minimum quantum time interval, T_P , in the quantum world. In other words the relation:

First time interval transformation: quantum time interval to classical time interval transformation
$$t'_2 - t'_1 = t_0 - T_P \tag{3.6}$$

tells us that when the duration of an event, t_0 , in the quantum world, is equal to the Planck time, T_P ; the duration of the event, in the macroscopic world (ideal world), will be exactly zero. Mathematically we express this fact as follows:

$$t'_2 - t'_1 = T_P - T_P = 0 \qquad (3.7)$$

Similarly we can formulate the second time interval transformation:

Second time interval transformation: quantum time interval to classical time interval transformation
$t_2 - t_1 = t - T_P \qquad (3.8)$

In virtue of equations (3.6) and (3.8) we can apply substitution to equation (3.5) to get the following relation

$$t - T_P = \frac{t_0 - T_P}{\sqrt{1-\beta^2}} \qquad (3.9)$$

or

$$t = \frac{t_0 - T_P}{\sqrt{1-\beta^2}} + T_P \qquad (3.10\ a)$$

Finally

$$t = \frac{t_0}{\sqrt{1-\beta^2}} + \left(1 - \frac{1}{\sqrt{1-\beta^2}}\right) T_P \qquad (3.10\ b)$$

Formula (3.10) tells us that when $t_0 = T_P$, the "dilated" time t must be equal to the Planck time, T_P . In contrast, the original Einstein time dilation equation will give us a "dilated" time greater than T_P . Naturally one can ask: which of the two time dilation formulas is the correct one? On the one hand, we know that the Einstein time dilation formula does not violate the *time quantization postulate* introduced in Part 1 [2]. On the other hand, we know that when we derived the quantum gravitational length contraction formula we used the *quantum length to classical length transformation*. Symmetry would indicate that we should treat time the same way we treat space. But this is only a desire that suggests that we should use the *quantum time interval to classical time interval transformations* so that space and time are on equal footing. However our desire for symmetry, in this case, does not prove that equation (3.10) is the correct formula. So we cannot answer this question until we have more solid evidence (see **Table 4**, Summary section).

4. Summary

Table 4 shows both the Lorentz length contraction formula and its quantum gravitational counterpart (the QGLC formula). The time dilation relations from both formulations are identical.

For the quantum gravitational length contraction formula I have assumed that:

$$L_{MIN} = L_P \tag{4.1}$$

For the quantum gravitational time dilation formula I have assumed that:

$$T_{MIN} = T_P \tag{4.2}$$

Name of the Formula	Relativistic Formula (SR)	Quantum Gravitational Formula (QGR)
Length contraction	$l = l_0 \sqrt{1-\beta^2}$	$l = l_0 \sqrt{1-\beta^2} + \left(1-\sqrt{1-\beta^2}\right) L_P$
Time dilation	$t = \frac{t_0}{\sqrt{1-\beta^2}}$	Which is the correct formula: $t = \frac{t_0}{\sqrt{1-\beta^2}}$ or $t = \frac{t_0}{\sqrt{1-\beta^2}} + \left(1-\frac{1}{\sqrt{1-\beta^2}}\right) T_P$?

Table 4: The formulas for the Lorentz length contraction and time dilation. Note that L_{MIN} has been replaced by the Planck length, L_P, and T_{MIN} by the Planck time, T_P.

5. Conclusions

The physical significance of the quantum length to classical length transformations is that these equations explain the phenomena of quantum gravitational length contraction which cannot be explained by Einstein's Special Theory of Relativity. The significance of the quantum time interval to classical time interval transformations is unclear because we were unable to decide which is the correct time dilation equation. A final important point to make is that this formulation incorporates both the Planck's constant, *h*, and Newton's gravitational constant, *G*. Consequently, The new transformation equations might be suitable to provide better descriptions of nature in areas such as particle physics and quantum gravity generally.

Appendix 1
Nomenclature

The following are the symbols, abbreviations and terminology used in this paper

Symbols

$h =$ Planck's constant
$c =$ speed of light in vacuum
$G =$ Newton's gravitational constant
$l_0 =$ proper length
$l =$ "contracted" length
$t_0 =$ proper time
$t =$ "dilated" time
$T_{MIN} =$ Minimum time with physical meaning
$L_{MIN} =$ Minimum length with physical meaning
$T_P =$ Planck time
$L_P =$ Planck length
$v =$ speed of a massive body with respect to certain inertial observer
$\beta =$ ratio between the speed, v, of a massive body and the speed of light, c

Abbreviations

$SR =$ Special relativity
$QGR =$ Quantum gravitational relativity
$QGLC =$ Quantum gravitational length contraction

Terminology

The terms macroscopic world, ideal world and classical world correspond to a classical description of nature in which lengths and time intervals can be zero. On the other hand, the terms microscopic world, real world and quantum world correspond to a quantum mechanical description of nature in which lengths and time intervals are discrete and cannot be zero.

Version Notes

Version 1: published on line on June 12, 2015. Version 1 should be discarded as the quantum gravitational Lorentz transformations published there are incorrect.

REFERENCES

[1] A. Einstein, *On the Electrodynamics of Moving Bodies*, Annalen der Physik, 17:891, (1905)

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[3] R. A. Frino, *Quantum Gravitational Relativity – Part II*, vixra.org: viXra 1505.0004, (2015).