Explorations in Physics

Anamitra Palit Physicist[Freelancer] P154 Motijheel Avenue, Flat C4 Kolkata:700074,India Ph:91-33-25514464 palit.anamitra@gmail.com

Abstract

The article seeks to investigate several issues concerning the topics: Relativity, Quantum Mechanics and classical physics. The following issues have been investigated:1)Relativity suggesting virtual particles 2) Lorentz Transformations Suggestive of a Bypass route 3)Space getting Curved 4) On Higg's Mechanism 5)On Tensor Equations of a Composite Nature 6) General Covariance and Tensor Equations 7) On rocket motion 8) Newton's Law of Gravitation in the Light of GR

1. Introduction

The article invites your attention into the areas stated in the abstract. Certain novel aspects have been highlighted in the sections that follow

2. Relativity and Virtual Particles

Two particles A and B collide in a scattering experiment producing the particles C and D. The relativistic Energy momentum formula^[1] $E^2 = p^2 + m_0^2$ has been used. $E^2 = p^2 + m_0^2$ Before Collision.:

Particle A:

Particle B:

After collision: Particle C:

Particle D:

$$E_D^2 = p_{Dx}^2 + p_{Dy}^2 + p_{Dz}^2 + m_D^2 \quad \dots \dots \quad (4)$$

We assume the total energy is conserved:

$$E_A + E_B = E_C + E_D$$

We also assume that momentum in each direction is conserved and total rest mass is conserved..

[All this is in tune with the calculations used in QED scattering phenomena. The 4Dimensional delta function of the type $\delta(E_f - E_i)\delta(p_{xf} - p_{xi})\delta(p_{yf} - p_{yi})\delta(p_{zf} - p_{zi})$ pops up frequently. It produces a *non zero probability* of transition only if

 $E_i = E_f$, $p_{xi} = p_{xf}$, $p_{yi} = p_{yf}$ and $p_{zi} = p_{zf}$, considering the initial and the final states. The same quantities are off the mass shell in the intermediate stages]

In the scattering process A loses momentum: $\vec{p} \equiv (p_x, p_y, p_z)$ and energy E to particle B that is,

$$E_{C} = E_{A} - E$$
$$E_{D} = E_{B} + E$$
$$p_{cx} = p_{Ax} - p_{x}$$
$$p_{cy} = p_{Ay} - p_{y}$$
$$p_{cz} = p_{Az} - p_{z}$$

$$m_A - m = m_c$$

$$m_A + m = m_D$$

Using the above relations in (3) and (4) we have,

$$(E_A - E)^2 = (p_{Ax} - p_x)^2 + (p_{Ay} - p_y)^2 + (p_{Az} - p_z)^2 + (m_A - m)^2 \dots (5)$$

And

$$(E_B + E)^2 = (p_{Bx} + p_x)^2 + (p_{By} + p_y)^2 + (p_{Bz} + p_z)^2 + (m_B + m)^2 \dots (6)$$

Subtracting (5) from (6) and using(1) and (2) in the result to work out cancellations we have,

$$E(E_B + E_A) = \vec{p}.(\vec{p}_A + \vec{p}_B) + m(m_A + m_B) - (7)$$

The transferred amount of energy generally speaking is not on the mass shell since $E^2 = p^2 + m^2$

may not be valid in the general case for the transferred packet. For the transferred packet it is more likely that we have

 $E^2 \neq p^2 + m^2$

This is indicative of the involvement of virtual particles in the process of energymomentum transfer.

From (8) we have,

$$\|\vec{p}\|\|\vec{p}_{total}\|Cos\theta = EE_{total} - mm_{total}$$

Or,

$$Cos\theta = \frac{EE_{total} - mm_{total}}{|\vec{p}||\vec{p}_{total}|} \quad ----- (9)$$

If the rest mass of each particle does not change we have[for example the scattering of an electron in the field of another electron or a posiron]

$$Cos\theta = \frac{EE_{total}}{|\vec{p}||\vec{p}_{total}|} \quad ----- (10)$$

For a multiple scattering event we may break up the energy and the momentum transferred in the following manner:

$$E = E_1 + E_2 + \dots + E_n$$

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_n$$

For each transfer we have a relation of the type indicated by (9 or (10)

All events in scattering phenomena are consistent with the relativistic energy-momentum relation. Therefore all possible scattering interaction s[known and unknown to us] should be covered by this formula in its broadest capability. The result of the scattering event should depend on the type of interaction concerned, on the involvement of various other veto powered laws in operation, the initial conditions etc.

2.1 The Effect of Higher Order Infinitesimals:

Let's again consider the relation:

$$E^2 = p^2 + m_0^2$$
(11)

Differentiating the above relation we have,

$$EdE = pdp$$

Now we choose a frame where the particle was initially at rest. Its momentum was zero and therefore for any dp the RHS of the relation above is also zero. Therefore dE remains zero no matter how much momentum the particle receives. This arises from the fact that we are ignoring he higher order infinitesimals.

Now, in a separate treatment we write the final energy and final momentum of the body as E' and p'

Now,

$$E'^{2} = p'^{2} + m_{0}^{2}$$

[We have assumed here that the rest mass is not changing]

If p happens to be zero we have,

We don't have an anomalous result now.

3. The Lorentz Transformations Suggestive of a "Bypass Route"

This is in relation to the symmetries relating to Special Relativity^[2]. In the derivation of the Lorentz transformation we often consider a set of clocks arranged in a circle round the origin in the transformed y'-z' plane if motion is along the x-x' direction. We assume that they to notify the same time due to the isotropy of space^[3].[Reference:Robert Resnick: An introduction to Special Relativity]

Now suppose there are some marbles and small pieces of stone strewn over the place disturbing the symmetry picture in relation to isotropy and homogeneity of space. Do we expect the Lorentz Transformations^[4] as we know them from the logical point of view?

You could think of coming down from the manifold to the local inertial frame. We work out a transformation from a small region of the manifold to Euclidean space[flat space-time]. We may have this transformation locally only, surrounded by all the anisotropies of curved space. Anisotropy could result from gravity itself or from factors other than gravity. Physically this would mean considering a freely falling frame like a freely falling lift in the context of the earth.

In our LIF we are not considering all the asymmetries round the falling frame/lift . These anisotropies may come into a logical conflict with our familiar Lorentz Transformations.

What transformations should we apply in such a case? Would it be possible to devise a bypass route without hurting/violating Special Relativity?

The Lorentz transformations are of course correct within their own limitations for example in consideration of certain symmetries the homogeneity, isotropy of space etc. What happens if these symmetries fail to hold?

3.1 Maxwell's Equations in a Medium

Two inertial frames K and K' are considered. They are in relative uniform motion along the x-x' direction with relative speed = v. In the frame K' we have a semi infinite dielectric slab [at rest with respect to K'] with a flat face perpendicular to the x-x' direction, that is, this particular face is parallel to the y-z direction. The dielectric is homogeneous and isotropic within itself. We now consider Maxwell's equations [in a medium] with respect to the dielectric in the rest frame of the dielectric, that is ,K' If these equations are transformed, they should retain their form in K[according to the first postulate of SR]. But the individual values of the variables may change

With this information we proceed into the paradox.

Speed of light in the dielectric as observed from K': nc [n is a positive fraction less than 1 that is, Refractive index = 1/n]

Relative speed between the frames, v = cn' [n'] is also a positive fraction less than 1]

For normal incidence: Speed of light in the dielectric as observed from K[Applying the Velocity-Addition Rule of SR]:

$$v = \frac{nc + n'c}{1 + nn'}$$
 ----- (13)

For oblique ray inside the medium at θ degrees with respect to the x' axis in the K' frame:

$$v_x' = ncCos(\theta)$$

 $v_v' = ncSin(\theta)$

$$v_{z}' = 0$$

 $[v'_{z} v'_{z}]$ has been taken to be zero for the convenience of calculations]

Observations from K^[5]:

$$v_x = \frac{ncCos(\theta) + n'c}{1 + nn'Cos(\theta)}$$
$$v_y = \frac{ncSin(\theta)}{1 + nn'Cos(\theta)}\sqrt{1 - n^2}$$
$$v_z = 0$$

$$v = \sqrt{\left(\frac{ncCos(\theta) + n'c}{1 + nn'Cos(\theta)}\right)^2} + \left(\frac{ncSin(\theta)}{1 + nn'Cos(\theta)}\sqrt{1 - n^2}\right)^2 \quad \dots \tag{14}$$

The results from (13) and (14) are not identical, though from the invariant Maxwell's equations[in a medium] we understand that the speed of light should be the same in all directions inside the dielectric as observed from K. What would be the answer to this paradox.

This paradoxical situation arises from the fact that we have applied SR in an incorrect context. It has been applied in an anisotropic and inhomogeneous configuration. But the overall space being considered is neither homogeneous nor isotropic

In view of anisotropies do we need to upgrade the Lorentz Transformations for a particular problem where the situation is on homogeneous or isotropic? That will change the Lorentz Transformations as well as Maxwell's equations for the problem concerned. Do we have our familiar wave equation for the new equations [Mawxell's equations] now? Does that account for dark energy/dark matter at least in a partial manner?

4. Space Getting Curved!

In general relativity coordinate distances and physical distances are identical only for flat spacetime. But in curved spacetime the coordinate separations and the physical separations are different. They may become radically different if the curvature is strong enough. Lets consider the physical curving of 3D space in view of the above fact. We consider two flat surfaces parallel to the x-y plane at two levels, z=a and z=b in the flat spacetime context. Several points are considered on the two mentioned planes. A gravitational change is now considered. The metric coefficients change and the physical distances of the points lying on each plane change . The points may be considered pair wise on each plane and also pairwise on the two separate planes. Their mutual distances change with changes in the gravitational field and change may occur differently for the different pairs. The planes become undulating surfaces—space gets curved!

Let's consider a spherical planet like the earth. A dense mass approaches it in our thought experiment. The value of the metric coefficients change at each point in the concerned field changes. Due to gravitational effects, even in our classical interpretation, the shape of the earth's surface might change due to an interaction between the changes in the spacetime curvature and non-gravitational factors like the resistance of the earth's crust etc In our "experiment" in the first paragraph we may consider the coordinates as labels---stickers of different colors at different points on the two planes. Initially they were on a flat surface. After the gravitational change they lie on a pair of undulating surfaces. A straight line on some plane becomes a curved line – the path of a light ray bends and the straight line path of a test particle the Minkowski space picks up the curved path of a planet!

5.On Higg's Mechanism

At the LHC the high energy collisions occur in the absence of strong gravity[strong spacetime curvature] that characterized the distant past. The concept of unification has suffered a denial right at the outset. Any discovery whatsoever should stand remote to the strong gravity considerations of the past.

Unification aims at relating the different fields to the effects of spacetime curvature which we call "Gravity". Different types of fields do use the vehicle of spacetime ,"riding" on such curvature in the process of their manifestation. The validity of unification should justify the effects of spacetime curvature on Higg's mechanism.

Referring to the equations on the following link on Higg's Mechanism^[6]: <u>http://en.wikipedia.org/wiki/Higgs_mechanism</u>

You may easily identify the presence of the metric coefficients in the equations provided on the link. Even a scalar function of the type phi(x,y,z,t) is affected by changes in the values of the metric coefficients, $g_{\mu\nu}$. Quantities like Δx , Δy , Δz and Δt are physically meaningful only in the Euclidean context. In curved space time the corresponding separations, from the physical point of view, become $\sqrt{g_{xx}} dx$, $\sqrt{g_{yy}} dy$, $\sqrt{g_{zz}} dz$ and

 $\sqrt{g_{tt}} dt$. Distance between a pair of points a and b lying on the x-axis should be $\int_{x=a}^{x=b} \sqrt{g_{xx}} dx$ in curved space instead of $\int_{a}^{b} dx$ which is valid in the flat spacetime context. The corresponding separation on a line parallel to the x-axis and between the

same coordinate points a and b should be different sin ce the metric tensor $g_{\mu\nu}$ would have different values on a parallel coordinate line. The formul; a of course should remain the same. Differences in the values of $g_{\mu\nu}$ should affect Higg's mechanism since the differential operators themselves are being affected!

It is important to consider the scalar function in the Lagrangian describing any field including the Higg's Field. We may view the scalar function in the light of the section "Space Getting Curved". Straight lines ,comprising coordinate points" become curved lines due to gravitational changes. Flat surfaces become undulating ones due to the same reason. The nature of the scalar function changes physically even if its association with the coordinate points does not change. These facts are of serious consideration for any field mechanism like the Higg's Mechanism, more specially if the mechanism relates to the remote past characterized by immensely curved space

6.General Covariance and Tensor Equations

Quoting Wikipedia^[7]: "In theoretical physics, general covariance (also known as diffeomorphism covariance or general invariance) is the invariance of the form of physical laws under arbitrary differentiable coordinate transformations. The essential idea is that coordinates do not exist a priori in nature, but are only artifices used in describing

nature, and hence should play no role in the formulation of fundamental physical laws." The tensor object itself is defined on and confined to a particular type of manifold. The transformation of a tensor relates to different coordinate systems resting on the same type of manifold. For example the Riemannian curvature tensor has zero value for all components in the flat spacetime context. Any tensor with zero components should transform to a zero valued tensor[components zero individually] in other frames on the same manifold. In the aforesaid example we are always on flat spacetime for all coordinate transformations. We cannot transform the tensor object across different types of manifold.

Tensor equations have the same form[covariant form] in all coordinate systems pertaining to the same manifold. That is mathematically correct.

But what about tensor equations across different/distinct manifolds, for example Maxwell's equations in the covariant form?That Maxwell' s equations[and other fundamental laws] have an unchanging or invariant form[relating to tensor equations] across all types of manifolds can at best an empirical truth , in the present context of physics, having little to do with mathematical rigor.

On the Metric Tensor:

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} ----(15)$$

In a different system

$$ds'^{2} = g'_{pq} dx'^{p} dx'^{q} ---- (16)$$

[prime here does not indicate differentiation]

$$ds^{2} = ds'^{2}$$

Therefore,
$$g'_{pq}dx'^{p}dx'^{q} = g_{\mu\nu}dx^{\mu}dx^{\nu}$$

$$\mathbf{g}_{pq}' \mathbf{d} \mathbf{x}'^{p} \mathbf{d} \mathbf{x}'^{q} = \mathbf{g}_{\mu\nu} \frac{\partial \mathbf{x}^{\mu}}{\partial \mathbf{x}'^{p}} \mathbf{d} \mathbf{x}'^{p} \frac{\partial \mathbf{x}^{\nu}}{\partial \mathbf{x}'^{q}} \mathbf{d} \mathbf{x}'^{q}$$

Or,

$$g'_{pq} dx'^{p} dx'^{q} = g_{\mu\nu} \frac{\partial x^{\mu}}{\partial x'^{p}} \frac{\partial x^{\nu}}{\partial x'^{q}} dx'^{p} dx'^{q}$$
$$g'_{pq} = \frac{\partial x^{\mu}}{\partial x'^{p}} \frac{\partial x^{\nu}}{\partial x'^{q}} g_{\mu\nu} \qquad (17)$$

We conclude from (5) that $g_{\mu\nu}$ is a covariant tensor of rank 2

The idea is based on the invariance of the ds^2 which remains valid on the same type of manifold.

If the invariance of ds^2 remains valid as we pass across different manifolds, it would lead to a violation of Gauss Egregema. We could think of a transformation from one manifold to another preserving the line element.

Transformations across different manifolds that do not preserve the line element are simple enough to devise:

1. You may consider a hemispherical surface standing on a flat surface. Drop perpendiculars from each point of the sphere on to the flat surface. We have a transformation.Draw a curve on the surface of the hemisphere and obtain its projection on the flat surface.The curve lengths are different even for small segments of an arbitrary curve.

2. You may think of a spherical mass providing Schwarzschild's Geometry to a region. There is an annihilation and the energy produced flies off to infinity.Finally we have flat spacetime. The (t, r, θ, φ) coordinate grid is there for us. But the physical separations change since the values of $g_{\mu\nu}$ have changed. [For coordinate time t we have to devise a transformation between the past and future values of t]

 ds^2 is not equal to ds'^2 for the (t, r, θ, φ) and $(t_{transformed}, r, \theta, \varphi)$ points. The manifolds have changed. Incidentally the older and transformed values of r, θ and φ remain the same.

So if you pass across different/distinct manifolds $g_{\mu\nu}$ cannot be regarded as a covariant tensor any more.

We start with Einstein's field equations^[8] now:

[Lambda is the cosmological constant]

Multiplying both sides of (18) by $A^{\alpha}A^{\beta}$ [A^{α} is the vector EM potential or some other physical quantity] and with subsequent contraction we gave a scalar equation derived from (18). If the form of (18) is preserved across all manifolds, the scalar equation derived will hold over all manifolds. We may think of multiplying equation (18) by the electromagnetic tensor $F^{\alpha\beta}$ but that will produce a trivial identity since the product of a symmetric and an antisymmetric tensor is always zero. equations

You may consider Maxwell's equation's in the covariant form

and multiply both the sides of (19)by $G_{\nu\alpha}$ or by $W_{\nu\alpha}$ to get a tensor equation involving different physical quantities.

The above tensor equation may be multiplied on both sides by $A_{\alpha\beta}$ or $A^{\alpha\beta}$ which

represents some arbitrary physical quantity[but not anti symmetric in nature]. The new equation will preserve its form[tensor-form] in all coordinate transformations worked out on the same manifold.

What about other types of manifolds?

There are two *logical alternatives*::

A) Tensor equations are preserved for all manifolds

B) Tensor equations are preserved for coordinate transformations over a particular type of manifold and not across all distinct manifolds.

Let me consider the second option.

We may write relation (19) as:

The $Extra^{v}$ additional term should become zero in the flat spacetime context. In curved space it may not be zero.

Now we multiply both sides of (20) by $A_{\alpha\beta}$ or by $A^{\alpha\beta}$ representing some physical

quantity to get a tensor equation that should remain true across different/distinct manifolds.

This gives us an indication that quantities like $F^{\mu\nu}$ or $A^{\alpha\beta}$ are richer in physical content than what we are inclined to believe in, such richness becoming observable in the context of curved space or strongly curved space.

Interestingly option (B) includes option (A) as a particular case if the extra additional term is always zero.

In relation to option (B):

Invariance of tensor relations is based on the invariance of the line element: $ds^2 = ds'^2$ To make a proper fit of this concept into the invariance of relation (2) across different types of manifolds we have to assume the extra dimensions that should figure into ds^2 . The addition terms like *Extra*^v may become visible only in the strongly curved multidimensional space but otherwise remain concealed for example in the flat spacetime context.

A Suggested Model:

 $ds^{2} = g_{tt}dt^{2} - g_{xx}dx^{2} - g_{yy}dy^{2} - g_{zz}dz^{2} - \sum_{\mu\nu}g_{\mu\nu}dx^{\mu}dx^{\nu} - \cdots$ (21)

The last term on the RHS of (21) pertains to the extra dimensions. The quantities $g_{\mu\nu}$ in the last summation term may be positive or negative. We could also use imaginary quantities for some dx^{j} , replacing dx^{j} by $i \times dx^{j}$. The "*i*" in the term $i \times dx^{j}$ is the imaginary "*i*"[$i^{2} = -1$] and not an index.

The quantity dx^2 should be an invariant covering all the dimensions to facilitate the transformation of tensors.

On the Fourier domain we have the variables: E, p and other p_j s pertaining to the extra dimensions.

The formula should be

$$E^{2} = p^{2} + C_{i} \sum_{i} p_{i} + m_{o}^{2}$$
 ------ (22)

The suffix "*i*" refers to the extra or the hidden dimensions.

We could think of flows of " p_i " between the visible and the hidden dimensions.

In relation (21) we have a corresponding effect between the metric coefficients covering the visible and the hidden dimensions in the preservation of ds^2 .

We are always on the same manifold, getting a partial view of it in with our known senses or gadgets.

In relation to the equation(20), that follows:

$$\frac{\partial F^{\mu\nu}}{\partial x^{\mu}} = j^{\nu} + Extra^{\nu}$$

It would be convenient to replace it by:

$$\frac{\partial F^{\mu\nu}}{\partial x^{\mu}} = j^{\nu} + A \times Extra^{\nu}$$

Where A is a constant with dimensions. The units/dimensions of A should be adjusted to get new physical quantities for

Extra^{\vee} Extra^{\vee}

We could also think of:

to provide greater scope to the mentioned formulation.

7. On Rocket Motion

A rocket receives a forward thrust from ejected fuel Equation:

$$\mathbf{F} - mg = m\frac{dv}{dt} + v\frac{dm}{dt}$$
(22)

F is the forward thrust from ejected fuel, m the mass of the rocket. m,g, v etc are variable quantities and dm/dt is negative Multiplying bothy sides of (1) by dr we have,

$$Fdr - mgdr = m\frac{dv}{dt}dr + v\frac{dm}{dt}dr$$
Or,

Fdr
$$-mgdr = m\frac{dv}{dt}\frac{dr}{dt} dt + v\frac{dm}{dt}\frac{dr}{dt} dt$$

$$Fdr - mgdr = mvadt + v^2 \frac{dm}{dt} dt$$

Dividing both sides of the above by dt we have,

$$F\frac{dr}{dt} - mg \ \frac{dr}{dt} = mgva - v^2 K(t) \qquad (23)$$

K(t) is the rate at which fuel is ejected.[dm/dt=-K(t)]

We have,

P(t)-mgv = $mva - v^2 K(t)$ ------ (24) P(t) is the instantaneous power All quantities in the above equation are variables. Now for constant velocity, a=0 Relation (24) reduces to: P(t)-mgv = $-v^2 K(t)$ ------ (25)

For power to be zero,

$$mg = K(t)v \qquad -----(26)$$

$$mg = \left(-\frac{dm}{dt}\right)v$$
$$\frac{dm}{m} = -\frac{g}{v}dt$$

v in the above is constant but g is not constant

g [One should never use conservation of linear momentum in analyzing rocket motion since the force of gravity is quite strong near the earth's surface]

Intuitive Analysis: If some mass is just released from the rocket without thrusting it downwards with a force power is required is zero. Reaction force on the rocket is also zero. The ejected initially has the same speed as the rocket wrt the ground frame. Loss of mass of the rocket allows it to maintain the speed despite gravity. But the mass has to be

released in a certain manner given by the differential equation: $\frac{dm}{m} = -\frac{g}{v}dt$; v is a

constant while g is a variable here. The rocket has to be accelerated to some height to get a suitable value of v. Then the zero powered propelling has to be used.

We may consider a chunk of mass m being ejected from the rocket having mass M. The mass m is ejected with an interaction F, in presence of gravity. Let the initial speed of the ejected mass be v and the speed of the rocket due to ejection be V

If a larger chunk m'>m is released with the same force F, the speed of the ejected mass, v', will be less ie, v'<v since the mass in this case is larger. Acceleration would be less since the mass is larger , force remaining the same.

In each case the left over mass experiences the same forward reaction/interaction. But in the second case there is a larger gain in speed since the left over mass (M-m') is smaller than M-m. The rocket gains greater power . Reason: $F \times$ increase of speed is greater in the second case. Interestingly power used by the rocket is less since $F \times$ speed of ejected mass is less in the second case.

It is important to note that F is same in each situation.

We rewrite equation (22) below:

For a given v if there is sudden reduction of mass there is an increase in the value of K(t)Consequently dv/dt increases in accordance with relation (27). Now F can be controlled independently by controlling the interaction applied on the ejected mass/fuel. Even if F is zero we may have a zero or positive value for dv/dt depending on the value of K(t).

For zero F and constant v, the condition is $K(t) = \frac{mg}{v}$ which we got earlier in relation

(26).

[The amount of mass ejected in time dt and the speed with which it is ejected may be controlled independently. A mass of higher density may be thrown out at a slower rate using the same interaction.]

8. Newton's Law of Gravitation in the light of GR The Geodesic Equation^[9]:

Schwarzschild's metric^[10] for radial motion:

Dividing both sides of (30) by dt^2 we have,

Using (31) in relation (29) we have,

$$\frac{d^2r}{d\tau^2} = -\frac{M}{r^2} \qquad (32)$$

In the above relations

 $\begin{aligned} \tau \to c\tau \\ t \to ct \end{aligned}$

 $M \rightarrow \frac{GM}{c^2}$

Relation (32) gives us the familiar for of Newton's Law of Gravitation as an exact derivation

9. Conclusion

The article, thus has considered certain ideas in physics in a new way to encourage rethinking and research for the future

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