A Universe from Itself: The Geometric Mean Expansion of Space and the Creation of Quanta

James A. Tassano

jimtassano@goldrush.com

ABSTRACT

We explore indications that the universe is undergoing a geometric mean expansion. Developing this concept requires the creation of two quantum levels, one being the quantum of our quantum mechanics, and another that is much smaller. The generation of quanta is what allows space to expand. We find that quanta are not static entities, but change with time; for example, the energy of quanta decreases, and the number of quanta increases, with time. The observable universe grows while the quantum levels shrink, giving a simple mechanism to explain the expansion of the universe. The universe does not come from nothing; it comes from itself.

Subject headings: cosmology; creation of quanta; geometric mean expansion of space; initial state of the universe; significance of Planck values; universe from itself

1. Introduction

The core idea of this paper is that the universe is undergoing a geometric mean expansion, an expansion made possible by the creation of quanta.

Following the work reported in [1] and [2], in this paper we take the delta E value calculated in [1], insert it into the time-energy uncertainty relation, and find that delta time for the observable universe is the Planck time squared. This is a remarkable result. Multiplying the largest and smallest values to get a constant is suggestive of a geometric mean relationship. After work on additional calculations that support the idea, we make the assumption that the universe is undergoing a geometric mean based expansion, and find this to be an amazingly productive model.

Many contributions emerge from the geometric mean expansion model.

- We show why, and how, the large and the small of the universe are connected.
- We will derive the initial conditions, such as the age, radius, mass, and energy of the universe when expansion started.
- We calculate the large and small values of the observable universe.
- We show that uncertainty equations are geometric mean relationships.
- The Planck relation is a geometric mean relation.
- The model reveals the true significance of the Planck values.
- We will show that expansion creates two different scales, or levels, of quanta.
- We repeatedly generate the large number reported by Scott Funkhouser, about 10^{122}, and find this number to be a crucial aspect of the geometric mean model of expansion.
- The model produces a possible size for the whole universe, the universe that lies beyond the observable. We only take a cursory look at this.
- We calculate how many times the universe has doubled, and show its relationship to Funkhouser's large number.
- We again see that energy is being continuously created by expansion.
- The creation of quanta explains the addition of space and energy to the universe, giving us a simple way to explain the expansion of space.
- The universe creates itself, from itself, by way of a geometric mean expansion. It is not 'a universe from nothing', but 'a universe from itself'.

1.1. The Large and the Small of the Universe are Related

There are evidences that the large and the small of the universe are deeply related. Joel C. Carvalho [3], using only atomic scale dimensions, calculated a mass, expressed in hyperverse terms, of $\frac{R_H c^2}{2G}$ for the observable universe, off by a factor of two from the mass of the universe equation we developed in [1], shown here in equation (1). Carvalho's work suggests there is a deep connection between the small and the large of the universe. Dimitar Valev [4], using dimensional analysis, discovered a geometric mean relationship, centered on the Planck mass, between the mass of the universe and what he identified as possibly the smallest mass in the universe, a value similar to that reported by Paul S. Wesson [5], who also used dimensional analysis. Hasmukh K. Tank [6] gives a number of potential geometric mean relationships between the small and the large of the universe.

The relation between the small and large appears to have been first noticed by Hermann Weyl, around 1917, and developed further by Dirac [7]. The geometric mean expansion model presented in this paper gives a physical basis for the connection between the large and the small, and for the existence of the large numbers discussed by Weyl, Eddington, Dirac, and others, but without any need for the time variation in the gravitational constant, G, as proposed by Dirac [8].

1.2. The Geometric Mean

To calculate an arithmetic mean of two numbers, the numbers are added and then divided by two. One calculates a geometric mean of two values by multiplying the two numbers and taking the square root of the product.

Figures 1A and 1B display two ways of visually presenting the geometric mean, the most common being the triangle in Figure 9A, where the length of side A, times the length of side B, equals the square of the height: $A \times B = h^2$. As side A increases, side B must decrease in order that their product remains equal to the height squared. A geometric mean preserves the square of the initial height.

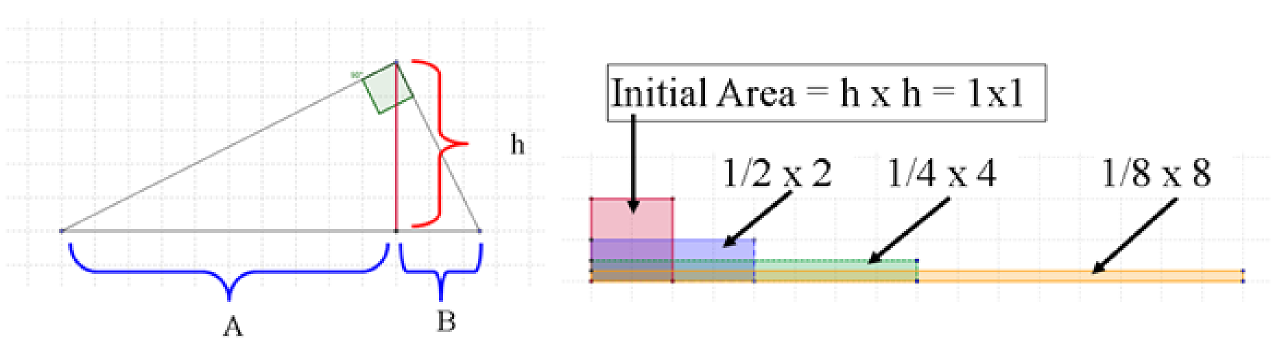

Figure 1A (left) and 1B (right). Two ways of looking at the geometric mean concept.

With the geometric mean, for every large number, there is a corresponding small number.

Given height 'h', if side A in the triangle of Figure 1A were one million units, side B would be one divided by one million. Expand A to one trillion, and B would be one over one trillion, and so on; small, but never zero. Just as there is no limit to how large a number can be, there is no limit to the how small a number can be; the small is as infinite as the large.

The series of rectangles in Figure 1B highlights the preservation of the initial area. As the height of the rectangle shortens, the length increases, their product conserving the initial area. A geometric mean expansion of space would therefore involve the conservation of the initial quantity.

2. The Geometric Mean Expansion of Space

2.1. The Revelation of Delta E

In [1], we gave the mass of the observable universe, M_o, as:

$$M_o = \frac{R_H c^2}{4G} \tag{1}$$

The energy of the observable universe, E_o, is its mass times the speed of light squared, c^2, or:

$$E_o = \frac{R_H c^4}{4G} \tag{2}$$

If we take the rate of change of the energy of the observable universe, ΔE_o, with respect to the radius we get:

$$\frac{\Delta E_o}{\Delta R_H} = \frac{dE_o}{dR_H} = \frac{d\frac{R_H c^4}{4G}}{dR_H} = \frac{c^4}{4G} \tag{3}$$

Rearranging:

$$\Delta E_o = \frac{c^4}{4G} \Delta R_H \tag{4}$$

Since $\Delta R_H = 2c$, [2], we see that ΔE_o is one-half the Planck power:

$$\Delta E_o = \frac{c^5}{2G} = 1.814\,592\,362\,824\,05 \times 10^{52}\,\frac{\mathrm{J}}{\mathrm{s}} \tag{5}$$

As a check, we can multiply ΔE_o by the age of the universe, and we get the energy of the observable universe:

$$\frac{c^5}{2G} \times \frac{R_H}{2c} = \frac{R_H c^4}{4G} \tag{6}$$

The time-energy uncertainty principle is:

$$\Delta E \times \Delta t \geq \frac{\hbar}{2} \tag{7}$$

With our value of ΔE_o, we can calculate Δt_o, the rate of change of time for the observable universe. We get a significant result, the Planck time squared:

$$\Delta t_o = \frac{\hbar}{2\Delta E_o} = \frac{G\hbar}{c^5} \tag{8}$$

We have taken the largest value of delta E, and found what appears to be the smallest delta time value, their product equaling a constant; this is suggestive of a geometric mean relationship.

2.2. Delta Time Divided by the Total Time Gives 'Small Time

If we divide delta time for the observable universe, by the age of the universe, we get what we will call 'small time', t_s:

$$\frac{\Delta t_o}{T_o} = \frac{\frac{G\hbar}{c^5}}{\frac{R_H}{2c}} = 2\frac{G\hbar}{R_H c^4} = t_s = 6.639\,859\,798\,408\,22 \times 10^{-105} \text{ seconds} \tag{9}$$

Alternatively, we can express small time as a function of the age of the universe, in a form we will use later:

$$t_s = \frac{T_o}{\left(\frac{R_H}{2l_p}\right)^2} \tag{10}$$

where T_o is the age of the observable universe, $\hbar$ is h-bar (Planck's constant divided by 2 pi), l_p is the Planck length, R_H is the radius of the hyperverse.

Small time, t_s, is viewed here as the smallest unit of time in the observable universe, much less than the Planck time, which is 5.39056×10^{-44} seconds. The ratio of the Planck time to small time is:

$$\frac{5.39056 \times 10^{-44}}{6.639\,859\,798\,408\,22 \times 10^{-105}} = 8.118\,484\,672\,360\,53 \times 10^{60} \tag{11}$$

This value is equal to the ratio of the hyperverse radius to two times the Planck length:

$$\frac{R_H}{2l_p} = \frac{2.62397216 \times 10^{26}\,\mathrm{m}}{3.232\,1 \times 10^{-35}\,\mathrm{m}} = 8.118\,474\,552\,148\,757\,773\,6 \times 10^{60} \tag{12}$$

2.3. Large Energy times Small Time Also Equals h-Bar over Two:

Small time can also be rearranged and expressed as:

$$t_s = \frac{\hbar}{2} \times \frac{1}{E_o} \tag{13}$$

Equation (13) looks like the time-energy uncertainty principle when restated as:

$$E_o \times t_s = \frac{\hbar}{2} \tag{14}$$

In this case we have the largest unit of energy, the energy of the universe, multiplied by what we are defining as the smallest unit of time, 'small time', and the result is h-bar over two, $\frac{\hbar}{2}$. We see that $\frac{\hbar}{2}$ is not just the product of the delta values, but also of the values of time and energy as well, further suggesting a geometric mean relationship exists between energy and time.

$$\Delta E_o \times \Delta t_o = \frac{\hbar}{2} \qquad \rightleftarrows \qquad E_o \times t_s = \frac{\hbar}{2} \tag{15}$$

2.4. The Large and the Small Form Geometric Mean Relationships

We have seen that delta time, divided by the age of the universe, gives small time. Rearranging (9) gives:

$$T_o \times t_s = t_{Planck}^2 \tag{16}$$

Equation (16) is a geometric mean equation as well, as we have the largest and the smallest time values equaling the square of the Planck time, a constant. This implies that as the age of the universe (which is large time) increases, the small unit of time decreases, and the pivot point is a constant, the Planck time squared. Equation (15) speaks similarly, the energy of the universe increases as the unit of small time is decreases.

At this point, let us assume the hyperverse is undergoing a geometric mean expansion, and examine the consequences.

2.5. The Initial State: When the Age of the Universe Equaled Small Time

We will now explore what conditions existed when the large time and small time were identical, by setting the age of the universe equal to small time, and solving first, for the radius:

$$\text{Let the age of the universe equal small time: } \frac{R_H}{2c} = 2\frac{G}{c^4}\frac{\hbar}{R_H} \tag{17}$$

The hyperverse radius at this initial moment was two times the Planck length:

$$\text{initial hyperverse radius} = R_i = 2l_p \tag{18}$$

When the age of the universe was equal to small time, the radius of the hyperverse was two Planck lengths. Let us now substitute $2l_p$ for R_H in other equations.

Substituting the initial length of $2l_p$ for the hyperverse radius in $\frac{R_H}{2c}$, our equation of the age of the universe, gives us the Planck time, t_p, as the initial time:

$$\frac{R_H}{2c} \Rightarrow \frac{2l_p}{2c} = \sqrt{\frac{G}{c^5}\hbar} = t_p = \text{ initial time} \tag{19}$$

Substituting $2l_p$ for R_H in the equations of the energy and mass of the universe, gives the initial energy of the universe as one-half the Planck energy:

$$\frac{R_H c^4}{4G} = \frac{(2l_p)\, c^4}{4G} = \frac{\sqrt{\frac{c^5\hbar}{G}}}{2} = \frac{E_p}{2} = \text{initial energy} \tag{20}$$

The initial mass of the universe was one-half the Planck mass:

$$\frac{R_H c^2}{4G} \Rightarrow \frac{(2l_p)\, c^2}{4G} = \frac{\sqrt{\frac{c\hbar}{G}}}{2} = \frac{M_p}{2} = \text{initial mass} \tag{21}$$

The initial angular momentum of the universe, $L = m v_T r$, was the square root of two times h-bar:

$$\left(\frac{(2l_p)\, c^2}{4G}\right)\left(\sqrt{2}c\right)(2l_p) = \sqrt{2}\hbar = \text{ initial angular momentum} \tag{22}$$

Note that the tangential velocity is taken as a constant, $\sqrt{2}c$, [2].

The rotational kinetic energy, $\frac{1}{2}I\omega^2$, is the same as the initial energy:

$$\frac{1}{2}\left(mr^2\right)\left(\frac{v_T}{r}\right)^2 = \frac{1}{2}mv_T^2 = \frac{1}{2}\left(\frac{(2l_p)\, c^2}{4G}\right)\left(2c^2\right) = \sqrt{\frac{1}{4G}c^5\hbar} = \frac{E_p}{2} \tag{23}$$

The initial circumference was $2\pi 2l_p$

The initial period of rotation of the universe, (with $v_L = 0$), was $2\sqrt{2}\pi t_p$:

$$Period = \frac{\text{Circumference}}{v_T} = \frac{2\pi 2l_p}{\sqrt{2}c} = 2\sqrt{2}\pi\sqrt{\frac{G}{c^5}\hbar} = 2\sqrt{2}\pi t_p \tag{24}$$

The initial frequency (also with $v_L = 0$), is the inverse of the period: $\frac{1}{2\sqrt{2}\pi t_p}$

The initial Schwarzschild radius (using the initial mass) was the Planck length. Even at the initial state, the Schwarzschild radius was one-half of the hyperverse radius.

$$R_{schwarzschild} = \frac{2GM}{c^2} = \frac{2G\sqrt{\frac{c\hbar}{4G}}}{c^2} = l_p \tag{25}$$

Table 1 gives a summary of the initial values. The subscript 'i' represents the initial value, when the age of the universe equaled small time.

$R_i = 2l_p$	$l_i = l_p$
$E_i = \frac{E_p}{2}$	$M_i = \frac{M_p}{2}$
$t_i = t_p$	$L_i = \sqrt{2}\hbar$
Initial $Frequency = \frac{1}{2\sqrt{2}\pi t_p}$	Initial period of rotation $= 2\sqrt{2}\pi t_p$
Initial $KE_{rotational} = M_i c^2 = E_i$	Initial $Circumference = 4\pi l_p$

Table 1

2.6. The Large and Small Values of the Observable Universe

Using the concept of the geometric mean expansion and the initial values, let us determine the large and small values of the universe.

We can calculate the geometric mean counterpart to the radius of the hyperverse, what we will call the small radius, R_s. The hyperverse radius, R_H, multiplied by this small radius, R_s, is equal to the initial radius squared: $R_H \times R_s = (2l_p)^2$. The small radius is:

$$R_s = \frac{(2l_p)^2}{R_H} = \frac{4l_p^2}{R_H} = 3.981\,166\,633\,261\,84 \times 10^{-96}\,\mathrm{m} \tag{26}$$

The small radius can also be expressed as:

$$R_s = \frac{R_H}{\left(\frac{R_H}{2l_p}\right)^2} \tag{27}$$

The energy of the observable universe, times the small energy, should equal the initial energy squared: $E_o \times E_{SEQ} = E_i^2$. Rearranging and solving for the small energy gives us $\frac{c\hbar}{R_H}$. We will refer to this quantum of energy, associated with the small energy, as the 'small energy quantum', E_{SEQ}:

$$E_{SEQ} = \frac{E_i^2}{E_o} = \frac{\left(\frac{\sqrt{\frac{c^5\hbar}{G}}}{2}\right)^2}{\frac{R_H c^4}{4G}} = \frac{c\hbar}{R_H} = 1.204\,863\,888\,041\,40 \times 10^{-52}\,\frac{\mathrm{m}^2}{\mathrm{s}^2}\,\mathrm{kg} \tag{28}$$

The structure of the equation for E_{SEQ} is similar to the Planck relation. Notice that E_{SEQ} decreases with time (because R_H increases with time).

The small energy quantum can be expressed as:

$$E_{SEQ} = \frac{E_o}{\left(\frac{R_H}{2l_p}\right)^2} \tag{29}$$

Similarly, we can calculate small mass, M_s, using the geometric mean relationship, $M_o \times M_s = M_i^2$.

$$M_{SEQ} = \frac{M_i^2}{M_o} = \frac{\left(\frac{\sqrt{\frac{c\hbar}{G}}}{2}\right)^2}{\frac{R_H c^2}{4G}} = \frac{\hbar}{cR_H} = 1.340\,591\,872\,566\,24 \times 10^{-69}\,\mathrm{kg} \tag{30}$$

$$M_{SEQ} = \frac{M_o}{\left(\frac{R_H}{2l_p}\right)^2} \tag{31}$$

The small volume is:

$$V_s = \frac{(\text{initial volume})^2}{\text{surface volume of hyperverse}} = \frac{\left(2\pi^2\,(2l_p)^3\right)^2}{2\pi^2 R_H^3} = \frac{2\pi^2 R_H^3}{\left(\frac{R_H}{2l_p}\right)^6} \tag{32}$$

The radius for this small volume is the small radius, R_s:

$$\text{Radius of small volume} = \left(R_H^3\left(\frac{2l_p}{R_H}\right)^6\right)^{\frac{1}{3}} = \frac{4l_p^2}{R_H} = R_s \tag{33}$$

To calculate the period of rotation, we calculate the period related to the observable hyperverse: the circumference, C_o, divided by the tangential velocity, v_T. The tangential velocity is a constant, $\sqrt{2}c$. T_o is the age of the universe, $\frac{R_H}{2c}$.

$$\text{Period of rotation of observable universe} = \frac{C_o}{v_T} = \frac{2\pi R_H}{\sqrt{2}c} = \sqrt{2}\pi\frac{R_H}{c} = 2\sqrt{2}\pi\,(T_o) \tag{34}$$

Using the geometric mean relationship and our value for the small radius, we get the small period, which is a factor of the small time:

$$\text{Period of rotation of the small} = \frac{C_s}{v_T} = \frac{2\pi R_s}{\sqrt{2}c} = 2\sqrt{2}\pi\,(t_s) \tag{35}$$

Both periods, as does the initial period, have the same structure, $2\sqrt{2}\pi \times time$.

Frequency is the reciprocal of the period:

$$\text{frequency of rotation of the small} = \frac{1}{2\sqrt{2}\pi\,(t_s)} \tag{36}$$

The angular momentum of the observable hyperverse, L_o. The velocity is the tangential velocity, $\sqrt{2}c$.

$$L_o = mrv = \left(\frac{R_H c^2}{4G}\right)(R_H)\left(\sqrt{2}c\right) = \sqrt{2}\hbar\left(\frac{R_H}{2l_p}\right)^2 = 9.829\,702\,483\,454\,73 \times 10^{87}\,\frac{\mathrm{m}^2}{\mathrm{s}}\,\mathrm{kg} \tag{37}$$

From this we can calculate the geometric mean partner of the large angular momentum:

$$L_s = \frac{L_i^2}{L_o} = \frac{\left(\sqrt{2}\hbar\right)^2}{\sqrt{2}\hbar\left(\frac{R_H}{2l_p}\right)^2} = \sqrt{2}\hbar\left(\frac{2l_p}{R_H}\right)^2 = 2.262\,781\,599\,120\,41 \times 10^{-156}\,\frac{\mathrm{m}^2}{\mathrm{s}}\,\mathrm{kg} \tag{38}$$

Notice that L_s matches the angular momentum derived from using the two quantum quantities, E_s and R_s:

$$L_s = mrv = \left(\frac{\hbar}{cR_H}\right)\left(\frac{4l_p^2}{R_H}\right)\left(\sqrt{2}c\right) = \sqrt{2}\hbar\left(\frac{2l_p}{R_H}\right)^2 \tag{39}$$

2.7. Summary of Values

Table 2 shows the initial, large, and small values for the observable hyperverse. To clarify, 'small' refers to the value generated from the geometric mean expansion. Some quantities are attributable to the small energy quantum and others to the small radius quantum, both of which we will explore next.

	Initial	Large	Small
Radius	$2l_p$	R_H	$R_s = \frac{4l_p^2}{R_H}$
Time	t_p	$T_o = \frac{R_H}{2c}$	$t_s = 2\frac{G\hbar}{c^4 R_H}$
Energy	$\frac{E_p}{2} = \frac{\sqrt{\frac{c^5\hbar}{G}}}{2}$	$E_o = \frac{R_H c^4}{4G}$	$E_s = \frac{c\hbar}{R_H} = \frac{R_s c^4}{4G} = E_{SEQ}$
Mass	$\frac{M_p}{2} = \frac{\sqrt{\frac{c\hbar}{G}}}{2}$	$M_o = \frac{R_H c^2}{4G}$	$\frac{\hbar}{cR_H}$
Angular Momentum	$\sqrt{2}\hbar$	$\sqrt{2}\hbar\left(\frac{R_H}{2l_p}\right)^2$	discussed later
Period	$2\sqrt{2}\pi\,(t_p)$	$2\sqrt{2}\pi\,(T_o)$	$2\sqrt{2}\pi\,(t_s)$
Frequency	$\frac{1}{2\sqrt{2}\pi(t_p)}$	$\frac{1}{2\sqrt{2}\pi(T_o)}$	$\frac{1}{2\sqrt{2}\pi(t_s)}$

Table 2. A summary of some dimensions of a universe undergoing a geometric mean expansion.

2.8. Products of the Large and Small Equal Constants

To further build the case that the universe undergoing a geometric mean expansion, let us start by looking at the product of some of the large and small aspects of the universe, from Table 2. We find they equal constants, as summarized in Table 3.

small \ large	Radius R_H	Time $\frac{R_H}{2c}$	Energy $\frac{R_H c^4}{4G}$	Mass $\frac{R_H c^2}{4G}$
Radius $\frac{4l_p^2}{R_H}$		$2\frac{G}{c^4}\hbar$	$c\hbar$	$\frac{\hbar}{c}$
Time $2\frac{G}{c^4}\frac{\hbar}{R_H}$	$2\frac{G\hbar}{c^4}$		$\frac{\hbar}{2}$	$\frac{\hbar}{2c^2}$
Energy $\frac{c\hbar}{R_H}$	$c\hbar$	$\frac{\hbar}{2}$		$\frac{c^3\hbar}{4G}$
Mass $\frac{\hbar}{cR_H}$	$\frac{\hbar}{c}$	$\frac{\hbar}{2c^2}$	$\frac{c^3\hbar}{4G}$	

Table 3. The product of large and small values equal constants. The left column represents the small values from Table 2, and the top row gives the large values. The product of the same dimensions, such as large and small radius, equal the initial values, squared, and are left out to highlight the other products.

2.9. Large and Small Delta Values

Let us now determine the delta values for the small and large aspects of the geometric mean expansion. We can calculate the rate of change of the small counterpart of ΔR_H , by dividing the square of the initial radius, $(2l_p)^2$, by ΔR_H. We previously [1] calculated ΔR_H to be $2c$. We get:

$$\Delta R_{small} = \frac{4l_p^2}{\Delta R_H} = \frac{4\frac{G\hbar}{c^3}}{2c} = \frac{2G\hbar}{c^4} = 1.742\,280\,725\,732\,64 \times 10^{-78}\,\mathrm{m\,s} \tag{40}$$

Since delta E for the observable hyperverse is $\Delta E_o = \frac{c^5}{2G}$, small delta E comes out as h-bar divided by two, $\frac{\hbar}{2}$:

$$\Delta E_{small} = \frac{(\text{initial energy})^2}{\Delta E_o} = \frac{\left(\frac{\sqrt{\frac{c^5\hbar}{G}}}{2}\right)^2}{\frac{c^5}{2G}} = \frac{\hbar}{2} = 5.272\,863\,3 \times 10^{-35}\,\frac{\mathrm{m}^2}{\mathrm{s}}\,\mathrm{kg} \tag{41}$$

Small delta mass, ΔM_{small}, is $\frac{\hbar}{2c^2}$:

$$\Delta M_{small} = \frac{(\text{initial mass})^2}{\Delta M_o} = \frac{\left(\frac{\sqrt{\frac{c\hbar}{G}}}{2}\right)^2}{\frac{c^3}{2G}} = \frac{\hbar}{2c^2} = 5.866\,851\,646\,308\,07 \times 10^{-52}\,\mathrm{s\,kg} \tag{42}$$

Delta time is the Planck time squared. The initial time is the Planck time. To get the geometric mean counterpart, we divide the square of the initial time by our value of Δt. Since they are the same, we get one, just the number one, a quantity without any dimensions such as length, energy, mass or time:

$$\Delta t_{small} = \frac{\left(\sqrt{\frac{\hbar G}{c^5}}\right)^2}{\frac{\hbar G}{c^5}} = 1 \tag{43}$$

Table 4 summarizes the large and small delta values, and shows their geometric means, matching those of the large and small from Table 2.

	big Δ	small Δ
$\Delta Radius$	$2c$	$\frac{2G\hbar}{c^4}$
$\Delta Time$	1	$\frac{\hbar G}{c^5}$
$\Delta Energy$	$\frac{c^5}{2G}$	$\frac{\hbar}{2}$
$\Delta Mass$	$\frac{c^3}{2G}$	$\frac{\hbar}{2c^2}$

Table 4. A listing of the large and small delta values of the universe.

2.10. The Products of the Large and Small Delta Values

Here we take the large and small delta values from Table 4, and cross multiply them. Table 5 gives the summary

small \ large	ΔRadius $2c$	ΔTime 1	ΔEnergy $\frac{c^5}{2G}$	ΔMass $\frac{c^3}{2G}$
ΔRadius $\frac{2G\hbar}{c^4}$		$2\frac{G}{c^4}\hbar$	$c\hbar$	$\frac{\hbar}{c}$
ΔTime $\frac{\hbar G}{c^5}$	$2\frac{G}{c^4}\hbar$		$\frac{\hbar}{2}$	$\frac{\hbar}{2c^2}$
ΔEnergy $\frac{\hbar}{2}$	$c\hbar$	$\frac{\hbar}{2}$		$\frac{c^3\hbar}{4G}$
ΔMass $\frac{\hbar}{2c^2}$	$\frac{\hbar}{c}$	$\frac{\hbar}{2c^2}$	$\frac{c^3\hbar}{4G}$	

Table 5. The products of the large and small delta values

2.11. Uncertainty Principles are Geometric Mean Equations

We find that Tables 3 and 5 give identical products. The product of the extreme large and small is the same as the delta values. Besides both products being identical, and equaling constants, as we would expect for a geometric mean relationship, the products give us the uncertainty relationships. We have seen some of this already, in particular, the time-energy uncertainty relation, as in equation (8).

We are using delta radius, not delta length, in these equations. For example, looking at the relation between energy and radius, if we take the radius and divide it by 2, we get a 'length'. A 'length' is one-half a radius. Recall that in the initial state, the radius was two Planck lengths. The Schwarzschild radius was one Planck length. We will define one-half delta radius as delta length and show it as Δx. This is defined here as 'position'.

$$\frac{\Delta R}{2} = \Delta length = \Delta x = position \tag{44}$$

From this, we can derive the position-energy uncertainty equation:

$$\Delta x \times \Delta E = \frac{c\hbar}{2} \text{ the position-energy uncertainty relation} \tag{45}$$

And we can derive the position-mass uncertainty as such:

$$\Delta x \times \Delta M = \frac{\hbar}{2c} \text{ the position-mass uncertainty relation} \tag{46}$$

Line 6 gives the relation between time and radius. Again, using the concept of position as $\frac{\Delta R}{2}$, we find:

$$\Delta x \times \Delta T = \frac{G\hbar}{c^4} \text{ the position-time uncertainty relation} \tag{47}$$

Numerically, the position-time uncertainty is:

$$\frac{G}{c^4}\hbar = 8.711\,403\,628\,663\,192\,761\,2 \times 10^{-79}\,\mathrm{m\,s} \tag{48}$$

The message intended here is that uncertainty relations are geometric mean equations.

2.12. The Planck Relation is a Geometric Mean Equation

We have established the following relations regarding the product of extreme mass and radius:

1. $\Delta R \times \Delta M = \frac{\hbar}{c}$
2. $R_H \times M_{SEQ} = \frac{\hbar}{c}$
3. $R_{SRQ} \times M_o = \frac{4l_p^2}{R_H} \times \frac{R_H c^2}{4G} = \frac{\hbar}{c}$

We can rearrange these equations as such:

1. $\Delta R = \frac{\hbar}{c\Delta M}$
2. $R_H = \frac{\hbar}{cM_{SEQ}}$
3. $R_{SRQ} = \frac{\hbar}{cM_o}$

Notice that these are all Planck relations.

We can do the same thing with radius and energy. Rearranging gives us the Planck relation:

1. $\Delta R \times \Delta E = c\hbar \Rightarrow \Delta E = \frac{c\hbar}{\Delta R}$
2. $R_H \times E_{SEQ} = c\hbar \Rightarrow E_{SEQ} = \frac{c\hbar}{R_H}$

3. $R_s \times E_o = c\hbar \Rightarrow E_o = \frac{c\hbar}{R_s}$

Another way to look at how the Planck relation can be considered a geometric mean relation is considering the Compton wavelength. The Compton radius for a mass the size of the observable universe is given in the Planck relation:

$$R_{Compton} = \frac{\hbar}{M_o c}$$

Rearrange and insert our mass value:

$$\frac{R_H c^2}{4G} \times R_{Compton} = \frac{\hbar}{c}$$

The Compton radius of the observable universe is the small radius, R_{SRQ}.

$$R_{Compton} = \frac{\frac{\hbar}{c}}{\frac{R_H c^2}{4G}} = \frac{4l_p^2}{R_H} = R_{SRQ} = \text{the small radius}$$

The largest mass, times the smallest radius, is a constant, $\frac{\hbar}{c}$. We get the same result if we multiply the largest radius times the smallest mass. And we get the same result multiplying the large and small delta values. This most fundamental quantum equation, the Planck relation, is a geometric mean relationship, supporting the claim that the universe is undergoing a geometric mean expansion.

2.13. The Planck Values Represent the State of the Universe When Expansion Started

The geometric mean expansion model produces Planck values for the initial condition, as seen in Table 1. We defined the initial time as the Planck time. Our initial time is not 'time equals zero', but 'time equals 5.39056×10^{-44} seconds'. This is the starting age of the universe for this model. The initial energy, mass, radius, and angular momentum, etc., are the values when the universe was as old as the initial time, which was the Planck time. These calculations do not tell us about the nature of the universe prior to the Planck time, but they do tell us what the Planck values are, and what the universe was like at that moment.

Although the Planck units are profoundly important in physics, they have no clear, accepted, physical significance. The Planck scale is often said to be "the limit below which the very notions of space and length cease to exist" [9], which is inconsistent, given that the Planck energy and mass are gigantic values. The Planck mass is 10^{19} times greater than the mass of a proton.

An extremely large Planck value is the Planck power, P_p, given as the Planck energy divided by the Planck time:

$$P_p = \frac{E_p}{t_p} = \frac{\sqrt{\frac{c^5\hbar}{G}}}{\sqrt{\frac{\hbar G}{c^5}}} = \frac{c^5}{G} = \frac{(2.99792458 \times 10^8\,\mathrm{m\,s^{-1}})^5}{6.67259 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}} = 3.629\,184\,725\,648\,092\,649\,8 \times 10^{52}\,\frac{\mathrm{J}}{\mathrm{s}} \tag{49}$$

Far from being a lower limit, the Planck power has been said to be the absolute upper limit for power of anything in the universe [10]. Notice that our value of delta energy of the observable universe, ΔE_o, is one-half the Planck power (the initial energy divided by the initial time), and we derived it from a geometric mean equation.

The Planck values represent the initial values of the universe, the geometric means of aspects of the universe, as shown in Table 1. They tell us about the state of the universe at the time expansion started. This explains, for example, why the Planck values have no identifiable physical significance; they are historical values, preserved as geometric means, accounting for their odd range of sizes, and their presence in cosmological calculations.

3. Expansion Creates Two Quantum Levels

3.1. The Small Energy Quantum

The energy of the small energy quantum, E_{SEQ}, is the geometric mean counterpart of the energy of the observable universe. If we divide the energy of the hyperverse by the energy of the small energy quantum, we see there are $\left(\frac{R_H}{2l_p}\right)^2$ units of E_{SEQ} within the observable universe:

$$\text{number of small energy quanta} = \frac{\frac{R_H c^4}{4G}}{\frac{c\hbar}{R_H}} = \left(\frac{R_H}{2l_p}\right)^2 \tag{50}$$

The volume and radius of the small energy quantum can be calculated by dividing the volume of the observable universe, V_o, by the number of small energy quanta, $\left(\frac{R_H}{2l_p}\right)^2$, and calculating the radius:

$$\text{volume of a small energy quanta} = \frac{V_o}{\text{number of small energy quanta}} = \frac{2\pi^2 R_H^3}{\left(\frac{R_H}{2l_p}\right)^2} = 2\pi^2 \left(R_H 4l_p^2\right) \tag{51}$$

The radius of the small energy quantum is the cube root of the term $\left(R_H 4l_p^2\right)$:

$$R_{SEQ} = \text{radius of small energy quanta} = \sqrt[3]{\left(R_H 4l_p^2\right)} = 6.495\,953\,894\,227\,41 \times 10^{-15}\,\text{m} \tag{52}$$

This radius can also be expressed as:

$$R_{SEQ} = R_H \left(\frac{2l_p}{R_H}\right)^{\frac{2}{3}} \tag{53}$$

3.2. The Small Radius Quantum

We can calculate the number of small radius volumes within the observable universe by dividing the volume of the observable universe by the volume of a 4D-sphere with radius R_s:

$$\text{number of small volumes} = \frac{2\pi^2 R_H^3}{2\pi^2 \left(\frac{4l_p^2}{R_H}\right)^3} = \left(\frac{R_H}{2l_p}\right)^6 \tag{54}$$

Dividing the energy of the observable universe by the number of small volumes gives the energy per small radius volume:

$$\text{SRQ energy} = \frac{\frac{R_H c^4}{4G}}{\left(\frac{R_H}{2l_p}\right)^6} = \frac{R_H c^4}{4G} \times \left(\frac{2l_p}{R_H}\right)^6 = 2.773\,581\,940\,492\,9 \times 10^{-296}\,\frac{\text{m}^2}{\text{s}^2}\,\text{kg} \tag{55}$$

We will refer to the quantum defined by the small radius as the small radius quantum, or SRQ.

3.3. The Quanta are Distinct

It appears that the geometric mean expansion of space creates these two quantum levels, the small energy quantum, or SEQ, from the energy of the observable universe, and the small radius quantum, or SRQ, from its radius. In addition to both quantum levels being generated by expansion, they are closely related as such:

$$\frac{E_{SEQ}}{R_{SRQ}} = \frac{c^4}{4G} \tag{56}$$

This ratio matches the energy to radius ratio of the observable hyperverse:

$$\frac{E_o}{R_H} = \frac{\frac{R_H c^4}{4G}}{R_H} = \frac{c^4}{4G} \tag{57}$$

Despite the deep relationship between the quantum levels, both the SEQ and the SRQ have distinct identities.

The SRQ is smaller than an SEQ; the small energy quanta consists of $\left(\frac{R_H}{2l_p}\right)^4$ units of small radius quanta:

$$\frac{\text{number of small volume quanta}}{\text{number of SEQ}} = \frac{\left(\frac{R_H}{2l_p}\right)^6}{\left(\frac{R_H}{2l_p}\right)^2} = \left(\frac{R_H}{2l_p}\right)^4 = 4.344\,079\,202\,020\,98 \times 10^{243} \tag{58}$$

A side-by-side comparison of the two quantum levels is shown in Table 6:

	SEQ (small energy quantum)	SRQ (small radius quantum)
Radius	$\left(R_H 4l_p^2\right)^{\frac{1}{3}} \approx 6.495\,95 \times 10^{-15}\,\mathrm{m}$	$\frac{4l_p^2}{R_H} \approx 3.981\,17 \times 10^{-96}\,\mathrm{m}$
Energy	$\frac{c\hbar}{R_H} \approx 1.204\,86 \times 10^{-52}\,\frac{\mathrm{m}^2}{\mathrm{s}^2}\,\mathrm{kg}$	$\frac{c\hbar}{R_H}\left(\frac{2l_p}{R_H}\right)^4 \approx 2.773\,58 \times 10^{-296}\,\frac{\mathrm{m}^2}{\mathrm{s}^2}\,\mathrm{kg}$

Table 6. Comparison of the energy and radii of the two quantum levels

4. The Large Number of the Universe

4.1. The Ratio of the Large to the Small

The term $\left(\frac{R_H}{2l_p}\right)^2$, the ratio of the hyperverse radius to the small radius, reoccurs in the hyperverse model, and will be referred to as the 'large number':

$$\text{The large number} = \frac{\text{large radius}}{\text{small radius}} = \frac{R_H}{\frac{4l_p^2}{R_H}} = \left(\frac{R_H}{2l_p}\right)^2 = 6.590\,962\,905\,388\,70 \times 10^{121} \quad (59)$$

Its reciprocal is:

$$\text{The reciprocal of the large number} = \left(\frac{2l_p}{R_H}\right)^2 = 1.517\,228\,991\,203\,11 \times 10^{-122} \quad (60)$$

The large number is the common ratio of the large and the small in the observable universe.

$$\frac{\text{large time}}{\text{small time}} = \frac{\frac{R_H}{2c}}{2\frac{G}{c^4}\frac{\hbar}{R_H}} = \left(\frac{R_H}{2l_p}\right)^2 \quad (61)$$

$$\frac{\text{large energy}}{\text{SEQ energy}} = \frac{\frac{R_H c^4}{4G}}{\frac{c\hbar}{R_H}} = \left(\frac{R_H}{2l_p}\right)^2 \quad (62)$$

Volume is the cube of the radius and this ratio of large to small is the cube of the large number:

$$\frac{\text{large energy}}{\text{small volume energy}} = \frac{\frac{R_H c^4}{4G}}{2\pi^2 R_H^3 \left(\frac{2l_p}{R_H}\right)^6} = \left(\left(\frac{R_H}{2l_p}\right)^2\right)^3 = \left(\frac{R_H}{2l_p}\right)^6 \quad (63)$$

The small values of the universe can be expressed in terms of the large number as shown in some examples in Table 7:

	Small
Radius	$R_s = R_H \left(\frac{2l_p}{R_H}\right)^2$
Time	$T_s = T_o \left(\frac{2l_p}{R_H}\right)^2$
Energy	$E_{SEQ} = E_o \left(\frac{2l_p}{R_H}\right)^2$
Mass	$M_s = M_o \left(\frac{2l_p}{R_H}\right)^2$

Table 7. Expressing features of the universe in terms of the large number.

4.2. The Large Number

In 2008, Scott Funkhouser [11] reported six cosmological numbers equaling approximately 10^{122}, very close to the 6.59×10^{121} figure that we are seeing repeatedly in our calculations.

The first example Funkhouser gave as producing this large number is the ratio of the initial mass density to the vacuum mass density. Calculating this ratio of mass densities gives us the large number:

$$\frac{\text{initial mass density}}{\text{mass density of universe}} = \frac{\frac{\text{initial mass}}{\text{initial volume}}}{\frac{\text{mass of the observable universe}}{\text{surface volume of hyperverse}}} = \frac{\frac{\frac{\sqrt{\frac{c\hbar}{G}}}{2}}{2\pi^2\left(2\sqrt{\frac{G\hbar}{c^3}}\right)^3}}{\frac{\frac{R_H c^2}{4G}}{2\pi^2 R_H^3}} = \left(\frac{R_H}{2l_p}\right)^2 \tag{64}$$

Funkhouser's second example, deals with the ratio of the mass of the universe to the smallest mass. We will take his particle horizon as the radius of the observable universe. The Wesson mass, which he describes as the smallest physically significant quantum of mass in the universe, is given as about 1.5×10^{-68} kg, a value he derived by dimensional analysis [5]. The Wesson mass is remarkably close to our SEQ value of about 1.341×10^{-69} kg, derived from the geometric mean expansion model. Using our values for the masses, we get the large number:

$$\frac{\frac{R_H c^2}{4G}}{\frac{\hbar}{cR_H}} = \left(\frac{R_H}{2l_p}\right)^2 \tag{65}$$

The third example says that the maximum number of degrees of freedom in the universe is one quarter of the surface area of a sphere whose radius is the cosmic event horizon. The event horizon distance, in the hyperverse model, is the radius of the hyperverse, divided by two. The value reported by Funkhouser is approximately 2.5×10^{122}. Our value is:

$$\frac{\pi \left(\frac{R_H}{2}\right)^2}{l_p^2} = \pi \left(\frac{R_H}{2l_p}\right)^2 = 2.070\,612\,064\,365\,197\,093\,6 \times 10^{122} \tag{66}$$

Example 4, Funkhouser equation 2.4, the maximum number of logical operations that could be performed by a mass the size of the observable universe is $\frac{2M_{p0}c^2T_0}{\pi\hbar}$, where M_{p0} is the particle horizon, and T_0 is the age of the universe. The value Scott Funkhouser reports is 2.1×10^{122}. We will define M_{p0} as the mass of the observable universe, M_o, and the age as $\frac{R_H}{2c}$. We get:

$$\frac{2\left(\frac{R_H c^2}{4G}\right) c^2 \frac{R_H}{2c}}{\pi\hbar} = \frac{1}{\pi}\left(\frac{R_H}{2l_p}\right)^2 \approx 2.098 \times 10^{121} \tag{67}$$

The number of nucleon volumes that can be held within the volume of the current event horizon is the subject of example 5, Funkhouser equation 2.5. The answer given is about 1.3×10^{123}. Our value, using $R_H/2$ as the event horizon, is:

$$\frac{2\pi^2 \left(\frac{R_H}{2}\right)^3}{2\pi^2 \left(R_H 4 l_p^2\right)} = \frac{1}{8}\left(\frac{R_H}{2l_p}\right)^2 \tag{68}$$

Had they instead used the observable universe as the volume, we'd find a perfect match to the large number.

Funkhouser's example number six deals with the ratio of the gravitational potential energy of the universe over the gravitational binding energy of a nucleon: $\frac{M_{p0}^2}{R_{p0}} / \frac{m_n^2}{l_n}$ where m_n is the mass of a nucleon and l_n is the length of a nucleon. Funkhouser value is approximately 9.3×10^{119}. Using our values, we get the large number:

$$\frac{\left(\frac{M^2}{R}\right)}{\left(\frac{m_p^2}{r_p}\right)} = \frac{\frac{\left(\frac{R_H c^2}{4G}\right)^2}{R_H}}{\frac{\left(\frac{1}{4G}\frac{\hbar^2}{R_H}\right)^{\frac{2}{3}}}{\left(R_H 4 l_p^2\right)^{\frac{1}{3}}}} = \left(\frac{R_H}{2l_p}\right)^2 \tag{69}$$

It appears our large number, $\left(\frac{R_H}{2l_p}\right)^2$, is the large number reported by Funkhouser.

4.3. The Number of Frame Advances

The hyperverse model incorporates the concept of frame advances [2], the incremental steps of radial expansion, where it was proposed that a frame advance is one radial length. The number of frame advances, since the start of expansion, is the ratio of the hyperverse radius to the small radius, and that works out to be the large number:

$$\text{Number of frame advances} = \frac{\text{radius of hyperverse}}{\text{small radius}} = \frac{R_H}{\frac{4l_p^2}{R_H}} = \left(\frac{R_H}{2l_p}\right)^2 \tag{70}$$

Small time was defined, in equation (10), as $\frac{T_o}{\left(\frac{R_H}{2l_p}\right)^2}$, the age of the universe divided by the number of frame advances. Thus t_s, small time, is the time it takes to make one frame advance. In one frame advance, the universe advances one small radius of distance, in one small unit of time. The velocity is 2c, our rate of radial expansion from [1]:

$$v = \frac{R_s}{t_s} = \frac{\frac{4l_p^2}{R_H}}{2\frac{G}{c^4}\frac{\hbar}{R_H}} = 2c \tag{71}$$

5. Significant Geometric Means for the Derived Quanta

As stated, the SEQ energy and the SRQ radius, are geometric mean counterparts of the energy and radius of the observable universe, respectively. That leaves us to enquire as to what might be the geometric mean counterparts of the SEQ radius and the SRQ energy. The results are profound.

5.1. The Size of the Whole Universe - a First Look

We can take the energy value of the SRQ, and using the geometric mean concept, determine its large partner energy. Dividing the square of the initial energy by the small radius quantum energy gives $E_o\left(\frac{R_H}{2l_p}\right)^4$:

$$E_{\text{large}} = \frac{\left(\frac{\sqrt{\frac{c^5\hbar}{G}}}{2}\right)^2}{\frac{R_H c^4}{4G}\times\left(\frac{2l_p}{R_H}\right)^6} = \left(\frac{R_H c^4}{4G}\right)\left(\frac{R_H}{2l_p}\right)^4 = E_o\left(\frac{R_H}{2l_p}\right)^4 \tag{72}$$

The result hints at the possible energy, and size, of the whole universe, greater by a factor of $\left(\frac{R_H}{2l_p}\right)^4$, or 4.3×10^{243} times the observable. The whole universe is potentially immensely larger than the observable universe. This topic is beyond the scope of this paper, but is too suggestive to not include here.

5.2. The Particle Radius

The small energy quantum radius, R_{SEQ}, is $6.495\,953\,894\,227\,41 \times 10^{-15}$ meters, very close to the reduced Compton wavelength of elementary particles and nucleons. The geometric mean counterpart of the small energy quantum radius, R_{GM_SEQ}, is:

$$R_{GM\_SEQ} = \frac{(2l_p)^2}{\left(R_H 4l_p^2\right)^{\frac{1}{3}}} = \left(\frac{16l_p^4}{R_H}\right)^{\frac{1}{3}} = 1.608\,150\,331\,744\,687\,220\,7 \times 10^{-55}\,\mathrm{m} \tag{73}$$

It will be argued in [12] that the similarity of R_{SEQ} to the Compton radius of an elementary particle is not a coincidence, and that R_{GM_SEQ}, the geometric mean counterpart of R_{SEQ}, is the actual, idealized, particle radius, and leads to a model of both matter and gravity.

6. Creating the Observable Universe from a Planck Scale Vortex

6.1. The Number of Doublings is the Square Root of the Large Number

To calculate the number of times the hyperverse has doubled, we can ask what value of 'n' solves this equation:

$$2^n \times 2l_p = R_H \quad \text{or} \quad 2^n = \frac{R_H}{2l_p} \tag{74}$$

There have been about 202.3 doublings of the hyperverse since expansion started:

$$\log_2\left(\frac{R_H}{2l_p}\right) = \log_2\left(\frac{2.62397216 \times 10^{26}\,\mathrm{m}}{2\,(1.61605 \times 10^{-35}\,\mathrm{m})}\right) = 202.\,336\,894\,366\,120\,619\,5 \tag{75}$$

The value of $2^{202.\,336\,894\,366\,120\,619\,5}$ is the square root of the large number:

$$2^{202.\,336\,894\,366\,120\,619\,5} = 8.\,118\,474\,552\,148\,757\,782\,8 \times 10^{60} \tag{76}$$

We can use the equation $2^n \times 2l_p = R_H$ to check that it produces the hyperverse radius, which it does:

$$\left(8.118\,474\,552\,148\,757\,782\,8 \times 10^{60}\right) \times 2\left(1.61605 \times 10^{-35}\,\mathrm{m}\right) = 2.623\,972\,160\,000\,000\,003\,0 \times 10^{26}\,\mathrm{m} \tag{77}$$

We can apply the doubling concept to energy: $2^{202.336\,894\,366\,120\,619\,50} \times$ initial energy:

$$2^{202.336\,894\,366\,120\,619\,50} \times \frac{1.956\,333\,046\,737\,717\,225\,4 \times 10^{9}}{2}\,\mathrm{J} = 7.941\,226\,863\,350\,439\,148 \times 10^{69}\,\mathrm{J} \tag{78}$$

which is our value for the energy of the universe as derived from $\frac{R_H c^4}{4G}$.

The doubling applies to time as well, giving us the correct age of the universe:

$$\left(8.118\,474\,552\,148\,757\,782\,8 \times 10^{60}\right) \times 5.39056 \times 10^{-44}\,\mathrm{s} = 4.376\,312\,418\,183\,1 \times 10^{17}\,\mathrm{s} \tag{79}$$

For an example, we can take the initial time, the Planck time, and multiply it by $\frac{R_H}{2l_p}$ to get the current age:

$$\left(\frac{R_H}{2l_p}\right) \times \sqrt{\frac{\hbar G}{c^5}} = \frac{R_H}{2c} \tag{80}$$

6.2. Doubling: 'Now' versus 'Then' for the Observable Universe

An easy way to see the effects of a doubling of the size of the hyperverse is to compare the hyperverse today, or "now", to when its radius was one-half the current size, which we will refer to as "then". Let us start with the radius:

$$\frac{\text{large radius now}}{\text{large radius then}} = \frac{R_H}{\frac{R_H}{2}} = 2 \tag{81}$$

As we would expect, the large radius has doubled with a doubling of the large radius. Next, we see that the small radius, R_s, shrinks by one-half with a doubling of the hyperverse radius:

$$\frac{\text{SRQ radius now}}{\text{SRQ radius then}} = \frac{\frac{4l_p^2}{R_H}}{\frac{4l_p^2}{\frac{R_H}{2}}} = \frac{1}{2} \tag{82}$$

Let us look at the SEQ radius, R_{SEQ}, which we generated as the radius of the small energy quantum. We find that it actually gets a little larger with expansion, growing by $\sqrt[3]{2}$ as the hyperverse radius doubles:

$$\frac{R_{SEQ}\text{ now}}{R_{SEQ}\text{ then}} = \frac{\left(R_H 4l_p^2\right)^{\frac{1}{3}}}{\left(\frac{R_H}{2}4l_p^2\right)^{\frac{1}{3}}} = \sqrt[3]{2} \ = 1.259\,921\,049\,894\,87 \tag{83}$$

The surface volume of the observable hyperverse, which is the volume of the observable universe, increases eight times:

$$\frac{\text{hyperverse surface volume now}}{\text{hyperverse surface volume then}} = \frac{2\pi^2 R_H^3}{2\pi^2\left(\frac{R_H}{2}\right)^3} = 8 \tag{84}$$

while the surface volume of the small volume (based on R_s) shrinks by one-eighth:

$$\frac{\text{small surface volume now}}{\text{small surface volume then}} = \frac{2\pi^2 R_s^3}{2\pi^2\left(\frac{R_s}{2}\right)^3} = \frac{2\pi^2\left(\frac{4l_p^2}{R_H}\right)^3}{2\pi^2\left(\frac{4l_p^2}{\frac{R_H}{2}}\right)^3} = \frac{1}{8} \tag{85}$$

The total energy of the universe doubles with a doubling of the hyperverse radius:

$$\frac{\text{total hyperverse energy now}}{\text{total hyperverse energy then}} = \frac{\frac{R_H c^4}{4G}}{\frac{\frac{R_H}{2}c^4}{4G}} = 2 \tag{86}$$

The energy of the small energy quantum (SEQ) shrinks to one-half per doubling of the hyperverse radius:

$$\frac{\text{the small energy quantum now}}{\text{the small energy quantum then}} = \frac{\frac{c\hbar}{R_H}}{\frac{c\hbar}{\frac{R_H}{2}}} = \frac{1}{2} \tag{87}$$

The number of SEQ increases by four times with a doubling:

$$\frac{\text{the number of SEQ now}}{\text{the number of SEQ then}} = \frac{\left(\frac{R_H}{2l_p}\right)^2}{\left(\frac{\frac{R_H}{2}}{2l_p}\right)^2} = 4 \tag{88}$$

The surface volume of a SEQ increases by two with a doubling:

$$\frac{\text{volume of one SEQ now}}{\text{volume of one SEQ then}} = \frac{2\pi^2\left(R_H 4l_p^2\right)}{2\pi^2\left(\frac{R_H}{2}4l_p^2\right)} = 2 \tag{89}$$

The energy of the small radius quantum (SRQ) shrinks to 1/32:

$$\frac{\text{the small radius quantum energy now}}{\text{the small radius quantum energy then}} = \frac{\frac{c\hbar}{R_H}\left(\frac{2l_p}{R_H}\right)^4}{\frac{c\hbar}{\frac{R_H}{2}}\left(\frac{2l_p}{\frac{R_H}{2}}\right)^4} = \frac{1}{32} \tag{90}$$

while the number of SRQ volumes increases by 64 times:

$$\frac{\text{number of SRQ now}}{\text{number of SRQ then}} = \frac{\left(\frac{R_H}{2l_p}\right)^6}{\left(\frac{\frac{R_H}{2}}{2l_p}\right)^6} = 64 \tag{91}$$

The energy density of the SEQ drops to one quarter:

$$\frac{\text{SEQ energy density now}}{\text{SEQ energy density then}} = \frac{\frac{\frac{c\hbar}{R_H}}{2\pi^2\left(R_H 4l_p^2\right)}}{\frac{\frac{c\hbar}{\frac{R_H}{2}}}{2\pi^2\left(\frac{R_H}{2}4l_p^2\right)}} = \frac{1}{4} \tag{92}$$

The energy density of the SRQ also drops to one quarter:

$$\frac{\text{SRQ energy density now}}{\text{SRQ energy density then}} = \frac{\frac{\left(\frac{R_H c^4}{4G}\right)\left(\frac{2l_p}{R_H}\right)^6}{2\pi^2 R_H^3\left(\frac{2l_p}{R_H}\right)^6}}{\frac{\left(\frac{\frac{R_H}{2}c^4}{4G}\right)\left(\frac{2l_p}{\frac{R_H}{2}}\right)^6}{2\pi^2\left(\frac{R_H}{2}\right)^3\left(\frac{2l_p}{\frac{R_H}{2}}\right)^6}} = \frac{1}{4} \tag{93}$$

And the energy density of the observable hyperverse also drops to one quarter:

$$\frac{\text{hyperverse energy density now}}{\text{hyperverse energy density then}} = \frac{\frac{\frac{R_H c^4}{4G}}{2\pi^2 R_H^3}}{\frac{\frac{\frac{R_H}{2} c^4}{4G}}{2\pi^2 \left(\frac{R_H}{2}\right)^3}} = \frac{1}{4} \tag{94}$$

Time is directly proportional to the hyperverse radius, wherein a doubling of the hyperverse radius is accompanied by a doubling in the time passed:

$$\frac{\text{time now}}{\text{time then}} = \frac{\frac{R_H}{2c}}{\frac{\frac{R_H}{2}}{2c}} = 2 \tag{95}$$

A summary is given in Table 8. An arrow pointing up indicates the quantity increases with time, while a downward pointing arrow means the quantity decreases with time:

	observable	SRQ	SEQ
Radius	$R_H = 2x \uparrow$	$R_{SRQ} = \frac{1}{2}x \downarrow$	$R_{SEQ} = \sqrt[3]{2}x \uparrow$
Volume per unit	$V_o = 8x \uparrow$	$V_{SRQ} = \frac{1}{8}x \downarrow$	$V_{SEQ} = 2x \uparrow$
Energy per unit	$E_o = 2x \uparrow$	$E_{SRQ} = \frac{1}{32}x \downarrow$	$E_{SEQ} = \frac{1}{2} \downarrow$
Number of units	1	$64x \uparrow$	$4x \uparrow$
Total Energy = Energy x Number	$2x \uparrow$	$2x \uparrow$	$2x \uparrow$
Energy Density	$\frac{1}{4} \downarrow$	$\frac{1}{4} \downarrow$	$\frac{1}{4} \downarrow$

Table 8. Summary of the effects of a doubling the hyperverse radius, on the observable hyperverse and the two quantum levels.

6.3. The Speed of Light Remains Constant

Like the tangential velocity, the speed of light remains constant. The speed of light can be described as the distance light travels in one unit of time. With our geometric mean values, we see that light travels one small length, which is one half the small radius, in one small unit of time:

$$\frac{\text{small radius now}}{\text{small time now}} = \frac{\frac{R_s}{2}}{t_s} = \frac{\frac{\frac{4l_p^2}{R_H}}{2}}{2\frac{G}{c^4}\frac{\hbar}{R_H}} = c \tag{96}$$

Using a 'now and then' approach, even a doubling of the hyperverse radius, which alters both the value of the small length and small time, the speed of light remains constant, so the speed of light is unchanged by expansion:

$$\frac{\text{small radius then}}{\text{small time then}} = \frac{\frac{\left(\frac{4l_p^2}{\frac{R_H}{2}}\right)}{2}}{2\frac{G}{c^4}\frac{\hbar}{\frac{R_H}{2}}} = c \tag{97}$$

6.4. The Continuous Creation of Energy

With a doubling of the hyperverse radius, we find that the radius of the small radius quantum (SRQ) shrinks by one-half, accompanied by a decrease in the energy per small radius quantum, to 1/32 of the starting value, while the number of SRQ increases from one to sixty four, producing a net increase in energy of two times, matching the increase of the universe. At the SEQ level, the total energy doubles as well.

The first doubling was like any other. Energy doubled from one-half the Planck energy to the Planck energy, while the doubling of this initial hyperverse radius produced 64 hypervortices comprising the surface of the now rapidly expanding hyperverse. Approaching this a little differently, since rotational kinetic energy of a single vortex is $\frac{1}{2}I\omega^2$, or$\frac{1}{2}mr^2\left(\frac{\sqrt{2}c}{r}\right)^2$, which reduces to mc^2, multiplying this energy by the number of vortices, 64, we get the doubled energy. The initial energy was one half the Planck energy, so the first doubled energy was the Planck energy:

$$\frac{1}{2}mv_T^2 = \frac{1}{2}\left(\left(\frac{\frac{4l_pc^2}{4G}}{\left(\frac{4l_p}{2l_p}\right)^6}\right)\left(\sqrt{2}c\right)^2\right) \times \left(\frac{4l_p}{2l_p}\right)^6 = \sqrt{\frac{1}{G}c^5\hbar} = E_{Planck} \tag{98}$$

The expansion created energy from its very start. From an initial energy of one half the Planck energy, or about 9.78×10^8 joules, the current energy of the observable universe has increased by 8.1186×10^{60} times, or $\frac{R_H}{2l_p}$, to its current value of about 7.94×10^{69} joules.

As discussed in [12], the ratio of mass or energy to the radius is conserved with expansion, the relationship being quite visible in our equation of the mass of the universe, $E_o = \frac{R_H c^4}{4G}$. The ratio of energy to radius is a constant, $\frac{c^4}{4G}$:

$$E_o = R_H \left(\frac{c^4}{4G} \right) \tag{99}$$

From this equation, we see that a doubling of the radius would produce a doubling of the energy; they are closely bound; increasing the radius increases the energy, supporting the 'now vs. then' calculations. All of this also tells us that the universe is continually creating energy.

In equation 17 of [2], we saw that the Hubble constant could be defined as the ratio of the change in the energy of the observable universe to its total energy,

$$H = \frac{\Delta E_o}{E_o} \tag{100}$$

implying that the Hubble constant is a measurement of the increase in energy in the universe, a continual process.

6.5. The Quanta were United in the Initial State

In the initial state, both the SEQ and SRQ were identical quanta; they were as one with the initial state. As the following calculations show, there was no separation between the quantum levels or between them and the initial state. We can take our initial values for energy and radius, plug in the energy and radius values for both the SEQ and SRQ, and we get the initial values. Both quantum levels were united at the initial state; there were no quanta.

SEQ energy: $\left(\frac{R_H c^4}{4G}\right)\left(\frac{2l_p}{R_H}\right)^2 \Rightarrow \left(\frac{2l_p c^4}{4G}\right)\left(\frac{2l_p}{2l_p}\right)^2 = \frac{\sqrt{\frac{c^5\hbar}{G}}}{2}$

SEQ radius: $\left(R_H 4l_p^2\right)^{\frac{1}{3}} \Rightarrow \left(2l_p 4l_p^2\right)^{\frac{1}{3}} = 2l_p$

SRQ energy: $\frac{c\hbar}{R_H}\left(\frac{2l_p}{R_H}\right)^4 \Rightarrow \frac{c\hbar}{2l_p}\left(\frac{2l_p}{2l_p}\right)^4 = \frac{\sqrt{\frac{c^5\hbar}{G}}}{2}$

SRQ radius: $\frac{4l_p^2}{R_H} \Rightarrow \frac{4l_p^2}{2l_p} = 2l_p$

7. A Universe From Itself: The Geometric Mean Expansion Creates Space and Energy

We will now present insight into how the geometric mean expansion of space creates space and energy, starting from our initial condition.

7.1. Adding Space to the Universe

Let us look at the two quantum levels, starting with the small radius quantum, or SRQ. With a doubling of the hyperverse radius, each SRQ turns into 64 SRQ, each individual quantum having a radius of one-half the starting value. Sixty four SRQ would make a cube $4 \times 4 \times 4$ per side. After a doubling, the length of a side contains four SRQ, each with a new radius of one-half the starting SRQ radius, so the side has grown from one length to two lengths. The distance between two points has doubled, as expected with a doubling of the hyperverse radius.

With a doubling of the hyperverse radius, one SEQ turns into four SEQ, while the SEQ radius has grown by $2^{\frac{1}{3}}$ times. The four SEQ would make a cube $4^{\frac{1}{3}}$ (equivalent to $2^{\frac{2}{3}}$) units per side, with the radius of each quantum being $2^{\frac{1}{3}}$ times larger. The product of the two also gives us the expected increase of two times:

$$2^{\frac{2}{3}} \text{ quanta per side} \times 2^{\frac{1}{3}} \text{ length increase per quanta} = 2 \tag{101}$$

Or we can take the cube root of the product of the final volume, after a doubling, and see that the radius has grown by two times:

$$\sqrt[3]{4 \times \left(2^{\frac{1}{3}}\right)^3} = 2 \tag{102}$$

We can argue that adding volume, or distance, to space is just as much of an issue as adding energy. The geometric mean expansion, acting on the quanta of space, allows the three dimensional volume of space to expand, and the 'price' is shrinkage of the small radius.

7.2. Adding Energy to the Universe

The energy equation for the observable universe is:

$$E_O = \frac{R_H c^4}{4G} \tag{103}$$

The equation is the same for the initial condition, when we use the initial radius, $2l_p$, as the hyperverse radius, as seen in equation (20), above.

Centripetal force, F_C, discussed at length in [12], is

$$F_C = m\alpha = m\frac{v_T^2}{r} = \frac{R_H c^2}{4G}\frac{2c^2}{R_H} = \frac{c^4}{2G} \tag{104}$$

Energy, $\frac{R_H c^4}{4G}$, can be expressed as the product of centripetal force and distance. In our case, we have:

$$E_O = F_C \times distance = \frac{c^4}{2G} \times \frac{R_H}{2} \tag{105}$$

where the distance is one-half the radius. Energy, then, is a function of the radius. With a doubling of the radius, we have a doubling of energy. And as with space itself, the trick is in the shrinkage of the quantum radii.

7.3. A Universe from Itself

The large and the small of the universe are connected to one another because the universe is undergoing a geometric mean expansion. The geometric mean model takes the universe back to the Planck time, when the universe was about 5.39×10^{-44} seconds of age. There were no separate quanta when expansion started.

A geometric mean expansion allows us to explain how to grow the hyperverse, going from an initial vortex, with a radius of two Planck lengths, 1.61605×10^{-35} meters, to the observable universe, at $2.62397216 \times 10^{26}$ meters, to even what appears to be the whole universe, at 4.281×10^{107} meters (unpublished data), without any need to create something from nothing.

It appears that the creation and continued shrinkage, or relative shrinkage, of quantum levels allows the whole to expand. The whole gets larger while the components shrink, the process following a geometric mean relationship. A geometric mean expansion conserves certain values, and in the case of the expanding universe, it is the Planck values that are conserved.

Many cosmologists believe the universe came from nothing; where else could the vast increase in energy come from? Nothingness seems the only source. There is a simple alternative. The geometric mean expansion of space gives us a way to grow a universe without added energy, eliminating the need to figure out how nothing can become something, at least starting from our initial state.

Expansion is self-contained, the whole expanding while the quanta shrink. It is not a universe from nothing; the universe creates itself, out of itself. The universe comes from itself.

In [12] we develop the hyperverse concept further, and based on the creation of quanta, develop a model of matter. We will see that particles of matter, like quanta, are not static entities, and this immediately leads to a model of quantum gravity.

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