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Increasing of dimention of our space from 4 to 16 through spinors.

Abstract

From the fact, that 2-dimensional basis spinors are complex, there are made two conclusions. The first is that it is possible to change them by 4-dimensional real basis spinors. The second is that it is possible to enter 12 more dimensions in addition to 4 ordinary dimensions of our space. There is found connection of the basis of that 16-dimensional space with the basis of 4-dimensional space of real basis spinors.

1).The inner complex structure of 2-dimensional basis spinors.

Let us consider ordinary 4-dimensional Minkowski space. It's metric tensor is : $n_{\mu\nu} = (\vec{n}_\mu, \vec{n}_\nu)$

	$\vec{n}_1$	$\vec{n}_2$	$\vec{n}_3$	$\vec{n}_4$
$\vec{n}_1$	1	0	0	0
$\vec{n}_2$	0	-1	0	0
$\vec{n}_3$	0	0	-1	0
$\vec{n}_4$	0	0	0	-1

(1.1)

From [1] we'll take the formule (3.1.20) for the connection of basis 4-vectors with the basis complex 2-spinors :

$$\vec{n}_1 = \frac{1}{\sqrt{2}} \cdot (\vec{\xi}_1 \otimes \vec{\xi}_1{}^* + \vec{\xi}_2 \otimes \vec{\xi}_2{}^*)$$

$$\vec{n}_2 = \frac{1}{\sqrt{2}} \cdot (\vec{\xi}_1 \otimes \vec{\xi}_2{}^* + \vec{\xi}_2 \otimes \vec{\xi}_1{}^*) \qquad \vec{n}_\mu = n_\mu{}^{\alpha\beta} \cdot \vec{\xi}_\alpha \otimes \vec{\xi}_\beta{}^* \qquad (1.2)$$

$$\vec{n}_3 = \frac{i}{\sqrt{2}} \cdot (\vec{\xi}_1 \otimes \vec{\xi}_2{}^* - \vec{\xi}_2 \otimes \vec{\xi}_1{}^*)$$

$$\vec{n}_4 = \frac{1}{\sqrt{2}} \cdot (\vec{\xi}_1 \otimes \vec{\xi}_1{}^* - \vec{\xi}_2 \otimes \vec{\xi}_2{}^*)$$

From that formule we get the metric tensors for $\vec{\xi}_\alpha$ and $\vec{\xi}_\alpha{}^*$:

$$(\vec{\xi}_\alpha, \vec{\xi}_\beta) = \varepsilon_{\alpha\beta} \qquad (\vec{\xi}_\alpha{}^*, \vec{\xi}_\beta{}^*) = \varepsilon_{\alpha\beta} \qquad (1.3)$$

	$\vec{\xi}_1$	$\vec{\xi}_2$	$\vec{\xi}_\beta$
$\vec{\xi}_1$	0	1	
$\vec{\xi}_2$	-1	0	
$\vec{\xi}_\alpha$			$\varepsilon_{\alpha\beta}$

It is written in [1] that [(2.5.26)] $\vec{\xi}_\alpha{}^*$ did not decompose along $\vec{\xi}_\beta$. Then we do so. Let us form vectors $\vec{a}_\alpha$ and $\vec{b}_\alpha$ so :

$$\vec{a}_\alpha = \frac{1}{\sqrt{2}} \cdot (\vec{\xi}_\alpha + \vec{\xi}_\alpha{}^*) \qquad \vec{a}_\alpha{}^* = \vec{a}_\alpha \qquad \alpha = 1, 2$$

$$\vec{b}_\alpha = \frac{i}{\sqrt{2}} \cdot (\vec{\xi}_\alpha{}^* - \vec{\xi}_\alpha) \qquad \vec{b}_\alpha{}^* = \vec{b}_\alpha$$

Then

$$\vec{\xi}_\alpha = \frac{1}{\sqrt{2}} \cdot (\vec{a}_\alpha + i \cdot \vec{b}_\alpha)$$

$$\vec{\xi}_\alpha{}^* = \frac{1}{\sqrt{2}} \cdot (\vec{a}_\alpha - i \cdot \vec{b}_\alpha) \qquad (1.5)$$

And the equations (1.3) will take the form :

$$(\vec{a}_\alpha,\vec{a}_\beta)-(\vec{b}_\alpha,\vec{b}_\beta)+i\cdot((\vec{a}_\alpha,\vec{b}_\beta)+(\vec{b}_\alpha,\vec{a}_\beta))=2\cdot\varepsilon_{\alpha\beta}$$
$$(\vec{a}_\alpha,\vec{a}_\beta)-(\vec{b}_\alpha,\vec{b}_\beta)-i\cdot((\vec{a}_\alpha,\vec{b}_\beta)+(\vec{b}_\alpha,\vec{a}_\beta))=2\cdot\varepsilon_{\alpha\beta} \qquad (1.6)$$

From the idea of simplicity we choose the next formule :

$$(\vec{a}_\alpha,\vec{b}_\beta)=(\vec{b}_\alpha,\vec{a}_\beta)=0 \qquad (1.7)$$

From the idea of simplicity we adopt :

$$(\vec{a}_\alpha,\vec{a}_\beta)=\varepsilon_{\alpha\beta} \qquad (\vec{b}_\alpha,\vec{b}_\beta)=-\varepsilon_{\alpha\beta} \qquad (1.8)$$

Let us designate :

$$\vec{c}_1=\vec{a}_1 \qquad \vec{c}_2=\vec{a}_2 \qquad \vec{c}_3=\vec{b}_1 \qquad \vec{c}_4=\vec{b}_2 \qquad (1.9)$$

Then $$(\vec{c}_\mu,\vec{c}_\nu)=c_{\mu\nu} \qquad (1.10)$$

	$\vec{a}_1$	$\vec{a}_2$	$\vec{b}_1$	$\vec{b}_2$	$\vec{c}_\mu$
$\vec{a}_1$	0	1	0	0	$\vec{c}_1$
$\vec{a}_2$	-1	0	0	0	$\vec{c}_2$
$\vec{b}_1$	0	0	0	-1	$\vec{c}_3$
$\vec{b}_2$	0	0	1	0	$\vec{c}_4$
$\vec{c}_\nu$	$\vec{c}_1$	$\vec{c}_2$	$\vec{c}_3$	$\vec{c}_4$	$c_{\mu\nu}$

And we derive from this for the complex 2-dimensional basis spinors :

	$\vec{\xi}_\beta$	$\vec{\xi}_\beta^{\,*}$	$\vec{u}_\beta$
$\vec{\xi}_\alpha$	$\varepsilon_{\alpha\beta}$	0	
$\vec{\xi}_\alpha^{\,*}$	0	$\varepsilon_{\alpha\beta}$	
$\vec{u}_\alpha$			$(\vec{u}_\alpha,\vec{u}_\beta)$

(1.11)

2). The 16-dimensional vector space.

Let us consider the following four 4-dimensional spaces (determine their bases) :

$$\vec{m}_\mu = n_\mu{}^{\alpha\beta} \cdot \vec{\xi}_\alpha \otimes \vec{\xi}_\beta$$
$$\vec{n}_\mu = n_\mu{}^{\alpha\beta} \cdot \vec{\xi}_\alpha \otimes \vec{\xi}_\beta{}^* \qquad \mu = 1, 2, 3, 4 \quad (2.1)$$
$$\vec{p}_\mu = n_\mu{}^{\alpha\beta} \cdot \vec{\xi}_\alpha{}^* \otimes \vec{\xi}_\beta$$
$$\vec{q}_\mu = n_\mu{}^{\alpha\beta} \cdot \vec{\xi}_\alpha{}^* \otimes \vec{\xi}_\beta{}^*$$

From (1.1), (1.2), (1.11) there is following this scalar product of these basis vectors :

	$\vec{m}_\nu$	$\vec{n}_\nu$	$\vec{p}_\nu$	$\vec{q}_\nu$
$\vec{m}_\mu$	$n_{\mu\nu}$	0	0	0
$\vec{n}_\mu$	0	$n_{\mu\nu}$	0	0
$\vec{p}_\mu$	0	0	$n_{\mu\nu}$	0
$\vec{q}_\mu$	0	0	0	$n_{\mu\nu}$

(2.2)

Let us designate

$$\vec{e}_1 = \vec{m}_1 \quad \vec{e}_2 = \vec{m}_2 \quad \vec{e}_3 = \vec{m}_3 \quad \vec{e}_4 = \vec{m}_4$$

$$\vec{e}_5 = \vec{n}_1 \quad \vec{e}_6 = \vec{n}_2 \quad \vec{e}_7 = \vec{n}_3 \quad \vec{e}_8 = \vec{n}_4 \qquad (2.3)$$

$$\vec{e}_9 = \vec{p}_1 \quad \vec{e}_{10} = \vec{p}_2 \quad \vec{e}_{11} = \vec{p}_3 \quad \vec{e}_{12} = \vec{p}_4$$

$$\vec{e}_{13} = \vec{q}_1 \quad \vec{e}_{14} = \vec{q}_2 \quad \vec{e}_{15} = \vec{q}_3 \quad \vec{e}_{16} = \vec{q}_4$$

And, using (1.2), (1.5), (1.9), (2.1), we derive the "c**onnection o**f **bases"** (**cobases**) - $e_\mu{}^{\alpha\beta}$:

$$\vec{e}_\mu = e_\mu{}^{\alpha\beta} \cdot \vec{c}_\alpha \otimes \vec{c}_\beta \qquad \mu = 1, 2, \ldots, 16 \quad \alpha = 1, 2, 3, 4 \quad (2.4)$$

Now we see that the space W with basis $\vec{e}_\mu$ is the squared space V with the basis $\vec{c}_\alpha$:

$$W = V \otimes V \qquad (2.5)$$

What is the metric tensor in W ?

$$g_{\mu\nu} = (\vec{e}_\mu, \vec{e}_\nu)$$

From (1.1), (2.2), (2.3) it follows, that along the main diagonal stand numbers :

1, -1, -1, -1, 1, -1, -1, -1, 1, -1, -1, -1, 1, -1, -1, -1
And the other values of $g_{\mu\nu}$ are zeros. The space W we'll call the basis space.
It is possible to represent the formula (2.5) in other views :

$$W = V^2 \qquad V = W^{\frac{1}{2}} \qquad (2.6), (2.7)$$

That means, that the spinor space V is the power ½ from the basis space W.

Literature:
1). R. Penrose, W. Rindler "Spinors and space-time." Volume 1, 1984

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