YANG-MILLS GAUGE INVARIANT THEORY FOR SPACE CURVED ELECTROMAGNETIC FIELD

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Abstract. It was proposed new gauge invariant Lagrangian, where the gauge field interact with the charged electromagnetic fields. Gauge invariance was archived by replacing of particle mass with new one invariant of the field $F_{\mu\nu}F^{\mu\nu}$ multiplied with calibration constant α_g. It was shown that new proposed Lagrangian generates similar Dirac and electromagnetic field equations. Solution of Dirac equations for a free no massless particle answers to the 'question of the age' why free particle deal in experiments like a de Broil wave. Resulting wave functions of the new proposed Lagrangian will describe quantized list of bespinor particles of different masses. Finally, it was shown that renormalization of the new proposed Lagrangian is similar to QED in case similarity of new proposed Lagrangian to classic QED.

1. Introduction

de Broglie (1924, 1925) formulated the de Broglie hypothesis, claiming that all matter, not just light, has a wave-like nature. Heisenberg (1925) proposes a quantum mechanics, under the form of a mechanics of matrices. Schrödinger (1926) publishes his equation that bases quantum mechanics on the solution of a non-relativistic wave equation. Since relativity theory was already well established when quantum mechanics was formulated, this may surprise. In fact, for accidental reasons, the spectrum of the hydrogen atom is better described by a non-relativistic wave equation than by a relativistic equation without spin, the Klein (1926); Gordon (1926) equation. Dirac (1928) introduces a relativistic wave equation that incorporates the spin 1/2 of the electron, which describes much better the spectrum of the hydrogen atom, and opens the way for the construction of a relativistic quantum theory. In the two following years, Heisenberg and Pauli lay out, in a series of articles, the general principles of quantum field theory. First correct calculation in quantum electrodynamics (Pauli and Weisskopf, 1934) and confirmation of the existence of divergences, called ultraviolet (UV) since they are due, in this calculation, to the short-wavelength photons. Landau (1937a,b) publishes his general theory of phase transitions. At 1947 it was made measurement of the so-called Lamb shift by Lamb and Retherford (1947), which agrees well with the prediction of quantum electrodynamics (QED) after cancellation between infinities. At 1949 it was made construction of an empirical general method to eliminate divergences called renormalization (Dyson, 1952). Yang and Mills (1954) propose a non-Abelian

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generalization of Maxwells equations based on non-Abelian gauge symmetries (associated to non-commutative groups). Maknickas (2013) published its work about curvature of electromagnetic field as ground idea of gravitational mass. So, all this basic ideas listed above are incorporated into new proposed model.

2. Description of Model

Mathematically, Yang-Mills theory is an abelian gauge theory with the symmetry group U(1), where the gauge field interact with the charged spin-1/2 fields. In special case, the Yang-Mills Lagrangian for a spin-1/2 field interacting with the electromagnetic field is given by the real part of

$$\mathcal{L} = \bar{\psi}(i\hbar c\,\gamma^\mu D_\mu - mc^2)\psi - \frac{1}{4\mu_0} F_{\mu\nu}F^{\mu\nu} \tag{2.1}$$

where γ^μ are Dirac matrices; ψ a bispinor field of spin-1/2 particles (e.g. electronpositron field); $\bar{\psi} \equiv \psi^\dagger\gamma^0$, called "psi-bar", is sometimes referred to as Dirac adjoint; $D_\mu \equiv \partial_\mu + ie'\beta_\nu A_\mu = \partial_\mu + ieA_\mu$ is the gauge covariant derivative; e is the coupling constant, equal to the electric charge of the bispinor field; A is the covariant four-potential of the electromagnetic field generated by the electron itself; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor; β_ν is expressed as follow

$$\beta_\nu = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{Vmatrix} \tag{2.2}$$

If we describe m as space time curvature of electromagnetic field, as mentioned in (Maknickas, 2013)

$$R = \frac{24G}{c^2 r^3}\left(M + m\right), \tag{2.3}$$

$$m = \frac{\alpha_{em}V}{2}\left(\frac{B^2}{\mu_0} - \varepsilon_0 E^2\right) = \frac{\alpha_{em}V}{2} F_{\mu\nu}F^{\mu\nu}, \tag{2.4}$$

$$V = \frac{4\pi r^3}{3}, \tag{2.5}$$

where r is average radius of field, we could start substituting the definition of D_μ and m into the Lagrangian

$$\mathcal{L} = i\hbar c\bar{\psi}\gamma^\mu\partial_\mu\psi - e\bar{\psi}\gamma_\mu A^\mu\psi - \frac{\alpha_{em}c^2}{2} F_{\mu\nu}F^{\mu\nu}\bar{\psi}\psi - \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}. \tag{2.6}$$

where volume multiplier V was neglected. So we have invariant Lagrangian to gauge transformations, which form a Lie group. Next, we can substitute this Lagrangian into the Euler-Lagrange equation of motion for a field:

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right) - \frac{\partial\mathcal{L}}{\partial\psi} = 0 \tag{2.7}$$

to find the field equations.

The two terms from this Lagrangian are then

$$\begin{aligned}\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right) &= \partial_\mu\left(i\hbar c\bar{\psi}\gamma^\mu\right),\\ \frac{\partial\mathcal{L}}{\partial\psi} &= -e\bar{\psi}\gamma_\mu A^\mu - \frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\bar{\psi}.\end{aligned} \tag{2.8}$$

Substituting these two back into the Euler-Lagrange equation [2.7] results in

$$i\hbar c\partial_\mu\bar{\psi}\gamma^\mu + e\bar{\psi}\gamma_\mu A^\mu + \frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\bar{\psi} = 0 \tag{2.9}$$

with complex conjugate

$$i\hbar c\gamma^\mu\partial_\mu\psi - e\gamma_\mu A^\mu\psi - \frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\psi = 0. \tag{2.10}$$

Bringing the middle term to the right-hand side transforms this second equation into

$$i\hbar c\gamma^\mu\partial_\mu\psi - \frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\psi = e\gamma_\mu A^\mu\psi \tag{2.11}$$

The left-hand side is similar to original Dirac equation and the right-hand side is the interaction with the electromagnetic field.

One further important equation can be found by substituting the Lagrangian into another Euler-Lagrange equation, this time for the field, A_μ:

$$\left(\frac{\partial\mathcal{L}}{\partial(\partial_\nu A_\mu)}\right) - \frac{\partial\mathcal{L}}{\partial A_\mu} = 0 \tag{2.12}$$

The two terms this time are

$$\begin{aligned}\left(\frac{\partial\mathcal{L}}{\partial(\partial_\nu A_\mu)}\right) &= \left(2\alpha_{em}c^2\bar{\psi}\psi + \frac{1}{\mu_0}\right)\partial_\nu\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right),\\ \frac{\partial\mathcal{L}}{\partial A_\mu} &= -e\bar{\psi}\gamma^\mu\psi\end{aligned} \tag{2.13}$$

and these two terms, when substituted back into [2.12] give us

$$\partial_\nu F^{\nu\mu} = \frac{e\bar{\psi}\gamma^\mu\psi}{\left(2\alpha_{em}c^2\bar{\psi}\psi + \frac{1}{\mu_0}\right)} \tag{2.14}$$

Now, if we impose the Lorenz gauge condition, that the divergence of the four potential vanishes

$$\partial_\mu A^\mu = 0 \tag{2.15}$$

then we get

$$\Box A^\mu = \frac{e\bar{\psi}\gamma^\mu\psi}{\left(2\alpha_{em}c^2\bar{\psi}\psi + \frac{1}{\mu_0}\right)} \tag{2.16}$$

which is similar to a wave equation of the four potential of the classical Maxwell equations in the Lorenz gauge.

3. de Broglie waves

According to Banzaitis and Grabauskas (1975) angular momentum of free particle during the time is constant. So, we could choose particle moving direction (for example z) as direction of coordinate system and would get opposite to 3D motion 1D motion. Moreover, motion of free particle do not depend on time. So, we could solve time independent Dirac [2.11]

$$-i\hbar c\gamma^z\partial_z\psi+\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\psi+e\gamma^\nu A^\nu\psi=E\psi \tag{3.1}$$

In general $F_{\mu\nu}F^{\mu\nu}$ depends on space-time coordinates, but in our case let be constant. Now we could start to investigate homogeneous part of equation

$$-i\hbar c\gamma^z\partial_z\psi+\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\psi=E\psi \tag{3.2}$$

in case inhomogeneous equation could be solved using Green function formalism. After including γ^z

$$\gamma^z=\begin{Vmatrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{Vmatrix} \tag{3.3}$$

into [3.1], we get four equations for Dirac wave function

$$-i\hbar c\partial_z\psi_3+\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\psi_1=E\psi_1 \tag{3.4}$$

$$i\hbar c\partial_z\psi_4+\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\psi_2=E\psi_2 \tag{3.5}$$

$$-i\hbar c\partial_z\psi_1-\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\psi_3=E\psi_3 \tag{3.6}$$

$$i\hbar c\partial_z\psi_2-\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\psi_4=E\psi_4 \tag{3.7}$$

So, we get two independent pairs of equations [3.4], [3.6] and [3.5], [3.7] accordantly, where γ_1,γ_3 denotes spin $m_z=\frac{1}{2}$ and γ_2,γ_4 denotes spin $m_z=-\frac{1}{2}$.

Let start to investigate meaning meaning of indexes $1,3$. Equations [3.4], [3.6] could be rewritten as follow

$$\partial_z\psi_3=\frac{\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}-E}{-i\hbar c}\psi_1 \tag{3.8}$$

$$\psi_3=-\frac{i\hbar c\partial_z}{\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}+E}\psi_1 \tag{3.9}$$

Differentiation of [3.9] and inserting into [3.8] gives second order differential equation for ψ_1

$$\partial_z^2\psi_1+\frac{E^2-\frac{\alpha_{em}^2c^4}{4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2}{\hbar^2c^2}\psi_1=0 \tag{3.10}$$

Solution of [3.10] is

$$\psi_1=N_1\exp\left(ikz\right)+N_2\exp\left(-ikz\right) \tag{3.11}$$

where

$$k = \frac{\sqrt[2]{E^2 - \frac{\alpha_{em}^2 c^4}{4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2}}{\hbar c} \tag{3.12}$$

Now we could express ψ_3 from [3.9] as follow

$$\psi_3 = \sqrt[2]{\frac{E - \frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}}{E + \frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}}}\left(N_1 \exp\left(ikz\right) - N_2 \exp\left(-ikz\right)\right) \tag{3.13}$$

So, we get two independent solutions

$$\psi_{\pm k} = \begin{Vmatrix} 1 \\ 0 \\ \pm\sqrt[2]{\frac{E-\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}}{E+\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}}} \\ 0 \end{Vmatrix} N_{1,2}\exp\left(\pm ikz\right) \tag{3.14}$$

where N_1 and N_2 norm multipliers for ψ_{-k} and ψ_{+k} accordantly.

The same way could be found and wave functions ψ_2, ψ_4. Using this functions second two independent solutions could be found

$$\psi_{\pm k'} = \begin{Vmatrix} 0 \\ 1 \\ 0 \\ \pm\sqrt[2]{\frac{E-\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}}{E+\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}}} \end{Vmatrix} N_{1,2}\exp\left(\pm ik'z\right) \tag{3.15}$$

General solution is superposition of the corresponding wave functions with the same k.

Since z range boundaries are in $(-\infty, +\infty)$, wave functions [3.14] and [3.15] are defined $\forall z$ just for real k. Thus

$$E^2 \geq \frac{\alpha_{em}^2 c^4}{4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 \tag{3.16}$$

This condition will be true if one of equalities will be true

$$E \geq \frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu} \tag{3.17}$$

$$E \leq -\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu} \tag{3.18}$$

for a positive $F_{\mu\nu}F^{\mu\nu} > 0$. So we have a gap $\left(-\frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}, \frac{\alpha_{em}c^2}{2}F_{\mu\nu}F^{\mu\nu}\right)$ in the range $(-\infty < E < \infty)$. For a negative $F_{\mu\nu}F^{\mu\nu} < 0$ we obtain the same but reversed gap, where $F_{\mu\nu}F^{\mu\nu} < 0$ denotes antigravity mass. The negative mass this time could be interpreted not only as a positron or anti-proton but also antigravity positron ant antigravity anti-proton. Moreover, every particle describes four components field and it easy explain why every particle deal as wave in experiments.

4. General solution

Let rewrite wave function equation using notation $\not{a} = \gamma^\mu a_\mu$ in units $c = 1$

$$i\hbar\not{\partial}\psi = \frac{\alpha_{em}}{2}F_{\mu\nu}F^{\mu\nu}\psi + e\not{A}\psi \tag{4.1}$$

Solution of this equation could be expressed using Green function described as follow

$$i\hbar\partial\!\!\!/ G(x-x') = \delta^4(x-x') \tag{4.2}$$

The explicit form of the Green function can be written as a Fourier transform

$$G(x-x') = \frac{1}{(2\pi)^4}\int d^4p\hat{G}(p)\exp\left(i(x-x')p\right) \tag{4.3}$$

where

$$\hat{G}(p) = \frac{p\!\!\!/}{p^2} \tag{4.4}$$

The general solution of the inhomogeneous equation [4.1] reads

$$\psi(x) = \phi(x) + e\int G(x-x')A\!\!\!/\,(x')\,\psi(x')d^4x' \tag{4.5}$$

$$+\,\alpha_{em}\int G(x-x')F_{\mu\nu}\,(x')\,F^{\mu\nu}\,(x')\,\psi(x')d^4x' \tag{4.6}$$

On the other hand, using the same Green function formalism could be found and solutions of [2.16]

$$A^{\mu}(x) = e\int \frac{K(x-x')\bar{\psi(x')}\gamma^{\mu}\psi(x')}{\left(2\alpha_{em}c^2\bar{\psi}(x')\psi(x')+\frac{1}{\mu_0}\right)}d^4x' \tag{4.7}$$

where $K(x-x')$ is a Green function of wave equation

$$\Box K(x-x') = \delta^4(x-x') \tag{4.8}$$

and is known too

$$K(r-r',t-t') = \frac{1}{8\pi^3(r-r')}\int_{-\infty}^{+\infty} d\omega\; e^{i\omega((r-r')/c-(t-t'))} \tag{4.9}$$

For weak enough coupling constants e and α_{em} one can use the perturbation theory

$$\left\{\begin{array}{l} \psi^{(0)}(x) = \phi(x) \\ \quad A^{(0)\mu} = e\displaystyle\int \frac{K(x-x')\bar{\phi(x')}\gamma^{\mu}\phi(x')}{\left(2\alpha_{em}c^2\bar{\phi}(x')\phi(x')+\frac{1}{\mu_0}\right)}d^4x' \\ \psi^{(1)}(x) = \phi(x) + \displaystyle\int G(x-x')\left(eA\!\!\!/^{(0)}\,(x') + \alpha_{em}F^{(0)}_{\mu\nu}F^{(0)\mu\nu}\right)\psi(x')d^4x' \\ \quad A^{(1)\mu} = e\displaystyle\int \frac{K(x-x')\bar{\psi^{(1)}}(x')\gamma^{\mu}\psi^{(1)}(x')}{\left(2\alpha_{em}c^2\bar{\psi^{(1)}}(x')\psi^{(1)}(x')+\frac{1}{\mu_0}\right)}d^4x' \\ \qquad \dots \end{array}\right. \tag{4.10}$$

So, resulting wave functions will describe quantized list of bespinor particles of different masses. Quantization of masses ensures due to term $F_{\mu\nu}(x)F^{\mu\nu}(x)\psi(x)$ inserted into model Lagrangian.

5. Renormalization

If we choose β_ν as follow

$$\beta_\nu = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix} \tag{5.1}$$

we get 'massless' version of QED with additional term

$$F_{\mu\nu}F^{\mu\nu} \sim \frac{1}{p^2} \tag{5.2}$$

in momentum space after Fourier transformation of equation [2.14]. The meaning 'massless' denotes in this case that term $m\psi = 0$. Since A^μ is the same order after Fourier transformation

$$A^\mu \sim \frac{1}{p^2} \tag{5.3}$$

too, according to equation [2.16] renormalization is applicable and could be done using steps descibed in (Soper, 2001).

6. Conclusions

It was proposed gauge invariant Lagrangian of a fully renormalized quantum version of Yang-Mills theory of $\mathbb{R}^4$ based on Lie group, where the gauge field interact with the charged spin-1/2 electromagnetic fields. Gauge invariance was archived by replacing of particle mass with new one invariant of the field $F_{\mu\nu}F^{\mu\nu}$ multiplied with calibration constant α_{em}. It was shown that new proposed Lagrangian generates similar Dirac and electromagnetic field equations. Solution of Dirac equations for a free no massless particles answers to the 'question of the age' why free particle deal in experiments like a de Broglie waves. The answer is they are space curved field waves. The negative mass in proposed model could be interpreted not only as a positron or anti-proton but also antigravity positron and antigravity anti-proton, where magnetic moment of antigravity positron is less than magnetic moment of electron to meet condition $F_{\mu\nu}F^{\mu\nu} < 0$. Resulting wave functions of the new proposed Lagrangian will describe quantized list of bespinor particles of different masses. Finally, it was shown that renormalization of the new proposed Lagrangian is similar to QED in case similarity of new proposed Lagrangian to classic QED.

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