

# GGU-model Ultra-logic-systems Applied to Developmental Paradigms.

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*Abstract:* The General Grand Unification Model (GGU-model) solves a problem personally presented to this author, in 1979, by John A. Wheeler. It answers the question “Is physics legislated by a cosmogony?” It is a cosmogony that yields “A Theory of Everything” for any descriptive cosmology. Further, it satisfies certain basic requirements. It shows that “the basic structure is something deeper than geometry, something that underlies both geometry and particles (‘pregeometry’).” It should answer “the greatest questions on the books of physics: How did our universe come into being? And of what is it made?” That is, “what is the substance out of which the universe is made?” “But is it really imaginable that the deeper structure of physics should govern how the universe is made? Is it not more reasonable to believe the converse, that the requirement that the universe should come into being governs the structure of physics?” Thus, how the universe comes into being should be distinct from the physical laws that the process yields. It should also satisfy the “observer-participator” principle. That is, we not only observe but change the behavior of our universe through our physical activities. The GGU-model solution to the General Grand Unification Problem has two rather obvious philosophic interpretations. It is due to this fact that this solution is not common knowledge. Although these interpretations need not be applied, many individuals have worked to suppress this solution so as to prevent these interpretations from being made. In this article, a new and more refined approach is presented. It is shown how a predicted ultra-logic-system is directly related to the generation of a developmental and hyper-developmental paradigm. This developmental paradigm portion of the GGU-model corresponds directly to the instruction information form found in Herrmann (2013).

## 1. Introduction.

The requirements for satisfactory cosmogony are found in Patten and Wheeler (1975). For this approach, as a universe develops over observer-time, there is a one-

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to-one correspondence between members of a standard developmental paradigm for a descriptive cosmology and real physical events. A *descriptive cosmology* is any cosmology that can be represented via a comprehensible language, diagrams, images, or any digitized virtual reality sensory impressions. Physical science is presented in a standard language. Specific rational arguments are applied to this language and, where used, corresponding mathematical statements. As physically presented, the patterns displayed by such arguments should follow classical logic. The described entities and behavior can be observed or theoretically assumed. It is from these descriptions that predictions are made and verified. The descriptions and the logical patterns they present are the actual fundamental requirements of physical science. It is a description that corresponds to an observable or assumed physical event. Hence, descriptions form a model, a representation, for real or assumed physical behavior.

**The rationality displayed by developing physical events is mimicked by the rational patterns linguistically displayed by a developmental paradigm as it depicts the conjoining of entities and physical-systems.** Constructing developmental paradigms for a cosmogony, where each is independent from perceived physical laws, and investigating their rational behavior is consistent with the philosophy of science since developmental paradigms correspond to various physical realities.

Based upon observed behavior, GGU-model processes are mathematically predicted. These processes yield a depicted developmental paradigm representation. These predicted processes are fully presented and analyzed in Herrmann (2013). Depending upon GGU-model applications, there are 5, 3 or 2 such processes. Adjoined to these is an additional process that satisfies the Pattern and Wheeler (1975) participator requirements.

Physically each member of a developmental paradigm  $d_q$ , as here defined, yields a “universe-wide frozen-frame.” It is a philosophic stance whether aspects of a descriptive cosmology yield real physically observable behavior or these aspects are but models for behavior that we do not otherwise comprehend. As is customary, the nonstandard model used in all of the articles on the GGU-model is a polysaturated polyenlargements (Lobe and Wolff, 2000; Stroyan and Bayod, 1986). In this paper,  $q = 1, 2, 3, 4$ . These numbers denote the four primitive-time intervals (Herrmann 2006) employed for the GGU-model. The ultimate ultraword approach to generate a universe is replaced with an ultra-logic-system. This is a hyperfinite logic-system where, after application of the extended logic-system algorithm, generates each member of the hyperfinite development paradigm  $d_x^q$  in the proper  $\leq_{d_x^q}$  order such that the embedded developmental paradigm  $\mathbf{d}_q \subset d_x^q \subset {}^* \mathbf{d}_q$ , where  $x = \lambda, \nu\lambda, \mu\lambda, \nu\gamma\lambda$ , respectively.

[Note: By slightly altering the notation presented in this article and changing terms associated with such an alteration, a highly refined developmental paradigm is obtained from the material in Herrmann (2013). This refined developmental paradigm yields not only each universe-wide frozen-frame but yields the development and conjoining of each entity and physical-system contained in each universe-wide frozen-frame.]

Relative to the GGU-model generation of a universe and “instructions” (Herrmann (2013)), an hyperfinite collection of \*instruction-sets  $*\mathbf{I}^q(i, j)$  yields a universe-wide frozen-frame. Each instruction  $x \in *\mathbf{I}^q(i, j)$ , yields a physical or physical-like system. These physical-systems are disjoint.

## 2. Logic-System Generation for the Type-1 Interval.

[For all GGU-model applications as originally presented in Herrmann (1979 - 1994), the developmental paradigm determining functions  $f$  and  $t$ , as discussed below, are defined on  $Z \times \mathbb{N}$  and then the  $q$  notion, where  $q = 1, 2, 3$  indicates a restriction of these functions to  $Z_q \times \mathbb{N}$ . For  $q = 4$ , the indicated functions are the original unrestricted ones. For the  $t$  function, the image is  $R \subset \mathbf{Q}$ . Then  $t^q$  is the appropriate restriction. Hence, the  $F$  domain is  $R$  and maps  $R$  into the informally denoted language  $L$ .]

The notation in all that follows is from Herrmann (2013). In this article, the set  $W'$  is the informal “words” (the language) and  $\Delta'$  represents the corresponding formal words. One developmental paradigm corresponds to one universe and there can be a vast collection of developmental paradigms. Notice that there are two different  $t$  sequence notations. One  $t$  is in the informal world, while another  $t$  is in the formal standard superstructure. These two sequences are, of course, considered as equivalent since the set of objects that informally yield the informal  $t$  are also formally present within the standard superstructure. The informal composition  $f^q = F^1 \circ t^q$  when embedded relative to the set of equivalence classes  $\mathcal{W}'$  is denoted by  $\mathbf{f}^q = \mathbf{F}^q \circ t^q$  since the  $t^q$  is not embedded relative to  $\mathcal{W}'$  and it merely generates a rational number sequence for the embedded informal paradigm. These different notations are eliminated and only the math-italics font is employed. This is the customary practice throughout Herrmann (1979 - 1993). Notation for informal natural, rational and real numbers, if applicable, is usually the same for the formal superstructure objects. Each  $t^q(i, j)$  is a rational number. (Note: Appropriate results relative to  $\mathcal{W}'$  can be restated in terms of the original equivalence class representation  $\mathcal{E}$ .)

Each member of informal developmental paradigm  $d_q$  is now considered as determined by a function defined on a set  $R_q$  of rational numbers,  $\mathbf{Q}$ . The members of  $R_q$  carry the restricted rational number simple order and the order  $\leq_{d^q}$  for the members of

$d_q$  (the lexicographic order) is order isomorphic to  $R_q$  in the obvious way. Each interval partition is of the form  $[c_i, c_{i+1})$  (with a closed interval in two cases), where  $i \in \mathbf{Z}$  and  $\mathbf{Z}$  is the set of integers, and  $t^q(i, 0) = i$ ,  $t^q(i + 1, 0) = i + 1$ . Then each member of  $(c_i, c_{i+1})$  is a defined rational number  $t^q(i, j)$ , where  $i < j < i + 1$ . For example, consider  $[c_2, c_3)$ . Then  $t^q(2, 1) = 3 - 1/2$ ,  $t^q(2, 2) = 3 - 1/4$ ,  $t^q(2, 3) = 3 - 1/8$ , then, in general,  $t^q(2, j) = 3 - 1/2^j$ . Hence,  $f^q(2, 0) <_{d^q} f^q(2, 1) <_{d^q} f^q(2, 2) <_{d^q} \cdots <_{d^q} f(3, 0)$ .

Let  $d_1$  be an informal developmental paradigm. The first illustrated case is for a developing universe starting with a frozen segment (frame) represented by  $f^1(0, 0)$ . For the other three GGU-model cases, this sequence is appropriately modified. In all cases, the  $(f^q(i, j), f^q(p, k))$  is equivalent to “If  $f^q(i, j)$ , then  $f^q(p, k)$ ”. This notation will be simplified later.

For the type-1 case  $[0, b]$ ,  $b > 0$ , as indicated above, a denumerable developmental paradigm displays a refined form. For  $1 < m \in \mathbb{N}$ ,  $d_1 = \{f^1(i, j) \mid (0 \leq i \leq m) \wedge (i \in \mathbf{Z}) \wedge (j \in \mathbb{N})\}$ .

Due to the simplicity and special nature of the logic-systems used, a simplified algorithm is employed. The basic logic-system algorithm is re-defined for sets of two distinct objects  $\{A, B\}$ . If a deduction yields  $C$  and  $C$  is a member of  $\{A, B\}$ , then the “other” member is a deduction. Hence, if  $A$  is deduced, then from  $\{A, B\}$ ,  $B$  is deduced. This can be written as  $\{A, B\} - \{A\}$  is deduced. In general, this approach is only valid for these special collections of two element sets. This process mimics the proposition-logic modus ponens rule of inference  $\{(X \rightarrow Y, X, Y) \mid X, Y \text{ are propositions}\}$ . However, for both logic-systems only one member of any two element set is deducible. (Mathematical logic can be made very formal in appearance but, to retain intuition, this is rarely done. The symbolic form  $X \rightarrow Y$  represents a binary operation  $\tau$ . This can be written as  $\tau(X, Y)$ , where the domain is composed of ordered pairs. Then this rule of inference is representable as  $\{(\tau(X, Y), X, Y) \mid X, Y \text{ are propositions}\}$ . In this form, the rule of deduction states that if given  $\tau(X, Y)$  and  $X$ , then  $Y$  is a deductive conclusion.)

Using  $d_1$ , consider the following logic-system.

**Definition 2.1** Let  $i \in \mathbf{Z}$ . For each  $n \in \mathbb{N}$ , let  $k_i^1(n) = \{\{f^1(i, j), f^1(i, j + 1)\} \mid (0 \leq j \leq n - 1) \wedge (j \in \mathbb{N})\}$ ,  $K^1(n) = \bigcup\{k_i^1(n) \mid (0 \leq i < m) \wedge (i \in \mathbf{Z})\}$ . Finally, let finite  $\Lambda^1(n) = \{f^1(0, 0)\} \cup K^1(n) \cup \{\{f^1(p - 1, n), f^1(p, 0)\} \mid (0 < p \leq m) \wedge (p \in \mathbf{Z})\}$  and  $\mathcal{L}^1 = \{\Lambda^1(x) \mid x \in \mathbb{N}\}$ . The set  $\{\{f^1(p - 1, n), f^1(p, 0)\} \mid (0 < p \leq m) \wedge (p \in \mathbf{Z})\}$  is called the “jump elements.” Also, each  $\Lambda^1(n)$  is a finite set. (Although  $\mathcal{L}^1$  is not considered as a subset of the language  $W'$ .)

In general, members in  $\mathcal{L}^q$  can be characterized by a first-order sentence. When

the deduction algorithm is applied to  $\Lambda^1(n)$  the result is an ordered set of words from  $W'$  - the ordered developmental paradigm. In accordance with the juxtaposition or join function (a binary operation) that yields words in  $W'$ , this ordered developmental paradigm corresponds to a word in  $W'$ . It can be obtained using the spacing symbol where each member of this paradigm is considered a sentence. For a multi-universe cosmology, each such universe is but a portion of each universe-wide frozen-frame. (As shown in Herrmann 2013, in this model, the informal join process is mimicked by a binary operation  $\circ$  such that  $(\mathcal{W}' \cup \{\emptyset\}, \circ)$  is a monoid.

In order to make the notation as simple as possible for the next construction, notice that  $\mathcal{L}^1$  is denumerable. Let  $\mathbb{N} - \{0\} = \mathbb{N}'$ . Thus, there is a bijection  $D^1: \mathbb{N}' \rightarrow \mathcal{L}^1$ . We use the subscript notation for this bijection. Thus, consider  $\mathcal{L}^1 = \{D_i^1 \mid i \in \mathbb{N}'\}$ . For each  $n \in \mathbb{N}'$ , define  $M_n^1 = \{\{D_1^1, \dots, D_n^1\}\}$ . Let  $\mathcal{M}^1 = \{M_n^1 \mid n \in \mathbb{N}'\}$ . The set  $M_n^1 = \{\{D_1^1, \dots, D_n^1\}\}$ , as before, can be considered as a single word from  $L$  formed by replacing comma with an “and.”

(There are a few typographic errors in Herrmann (2006), which is previous version of this paper. For example, in Theorem 4.1,  $m > 0$  should read  $m > 1$ , and  $^*\mathbf{D}$ , should read  $^*\mathbf{D}_1$ .)

A finite consequence operator  $S$  is defined in Herrmann (1979 - 1993, p. 65). (These operators (functions) are also termed as operations. However, the phrase “finite consequence” carries the additional unary operation concept.) In what follows, a new simplified logic-system  $\mathcal{S}^q$ ,  $q = 1, 2, 3, 4$  is employed. When a logic-system is applied, it generates a specific finite consequence operator. It is the logic-system algorithm that does this. In this article, this algorithm is explicitly noted since only logic-systems are used. In general, logic-systems are stated in terms of metamathematics n-tuples. If a set  $\{A, B, C, \dots, D\}$  is used as an hypothesis, then it is word-like since the objects the logical deduction models via the algorithm yields words or word-like objects.

Define  $\mathcal{M}^q$ ,  $q = 2, 3, 4$ , in the same manner as  $\mathcal{M}^1$ , from members of  $\mathcal{L}^q$ . For each  $G^q \in \mathcal{M}^q$ , there exists a unique  $n \in \mathbb{N}'$  such that  $G^q \in M_n^q$ . This  $G^q = \{D_1^q, \dots, D_n^q\}$ ,  $D_i^q \in \mathcal{L}^q$ ,  $1 \leq i \leq n$ .

Define the logic-system  $\mathcal{S}^q = \{\{x, y\} \mid (\exists n(n \in \mathbb{N}')) \wedge (x \in M_n^q) \wedge (y \in \mathcal{L}^q) \wedge (y \in x)\}$ . (This definition can be further described in order to characterize the doubleton set notion and can include all necessary bounds for the quantifiers.) Each member of  $M^q$  is directly related to a corresponding  $\mathcal{S}^q$ . Further, under the simplification used here, each member of  $\mathcal{S}^q$  is a propositional tautology. Notice that  $M^q$  is a function with values a singleton set containing an n-set (i.e. a set of “n” members).

Usually, such a logic-system would use ordered pairs to model the rules of inference. Within these rules, finite conjunctions are displayed as first coordinates via n-sets. Again the simplified doubleton-set approach is used here, where one of these sets is  $\{\{D_1\}, D_1\}$ .

Hypotheses can be considered as members of a set, when part of a logic-system. They are, usually, considered as a list of the members of this set. In general, a logic-system determines a consequence operator that is defined on subsets of the language employed.

From the definitions employed for the logic-systems used here, the properties of the logic-system algorithm  $\mathcal{A}$  can be explicitly described in set-theoretic notation. For these applications,  $\mathcal{A}$  is a function defined on various defined logic-systems and a set of hypotheses. For example, the entire set of deductions or the order in which the deductions are made, among a few other characteristics. In our application to a logic-system, the notation used signifies all of the “deduced” results the algorithm produces when the logic-system is applied to a set of hypotheses. This yields the same results as the corresponding finite consequence operator. What the notation indicates is that the finite consequence operator is being displayed in a more refined and explicit manner. Hence, the algorithm and its relation to the logic-system can be embedded into the formal structure via formalizable characteristics.

When the application characteristics are \*-transferred, then the notation  $\ast\mathcal{A}$  is employed. The process of applying the algorithm to the logic-system  $\mathcal{S}^q$ , that is applied it to a set of hypotheses  $Y$ , is denoted by  $\mathcal{A}((\mathcal{S}^q, Y))$ . Hence,  $\mathcal{A}$  is defined upon a set of ordered pairs. The result of  $\mathcal{A}((\mathcal{S}^q, Y))$  is a set. An additional step can be included for this specific algorithm, where  $Y$  is removed. When this is done the algorithm is denoted by  $\mathcal{A}'$ . The necessary informally and, hence, formally described properties are specifically displayed. In general, the  $q$  notion is not included as part of the  $\mathcal{A}$  or  $\mathcal{A}'$  notation unless confusion would result.

For the denumerable set  $\mathcal{L}^1$ , notice that for any  $\Lambda^1(k)$ ,  $k \in \mathbb{N}$  there exists an  $k' \in \mathbb{N}$  and  $X_{k'}^1 \in M_{k'}^1$ , such that  $\Lambda^1(k) \in \mathcal{A}'((\mathcal{S}^1, \{X_{k'}^1\}))$  and, in this case, finite choice yields the  $\Lambda^1(k)$  logic-system. Then the logic-system algorithm  $\mathcal{A}$  is applied to  $(\Lambda^1(k), \{f^1(0, 0)\})$ , where  $f^1(0, 0)$  is the only hypothesis contained in the logic-system. This yields  $f^1(i, j) \in d_1$  as a deduction from  $f^1(0, 0)$ . Conversely, if  $f^1(i, j) \in d_1$ , then there is an  $X_{k'}^1 \in M_{k'}^1$  and a logic-system  $\Lambda(k) \in \mathcal{A}'((\mathcal{S}^1, \{X_{k'}^1\}))$  such that application of the logic-system algorithm  $\mathcal{A}$  to  $(\Lambda^1(k), \{f^1(0, 0)\})$  yields  $f^1(i, j)$  as a deduction from  $f^1(0, 0)$ .

The informal algorithm  $\mathcal{A}$  is defined on any logic-system that contains an hypothe-

sis and, in this paper, such a logic-system is  $\Lambda^q(x)$  and application is on  $(\Lambda^q(x), Y)$  where  $Y$  is an hypothesis contained in the logic-system and containing but one member. Due to the construction of the  $\Lambda^q(x)$ , this yields a partial sequence of members of  $d_q$ . This sequence is denoted by  $\mathcal{A}[(\Lambda^q, Y)]$ . This sequence represents the steps in the deduction and satisfies the  $\leq_{d_x^q}$  order. Also, for this case,  $\mathcal{A}((\Lambda^q(x), Y)) = d_x^q \subset d_q$ . Significantly, for  $n, k \in \mathbb{N}$ ,  $n \leq k$ ,  $\mathcal{A}((\Lambda^1(n), Y)) \subset \mathcal{A}((\Lambda^1(k), Y))$  and  $\mathcal{A}[(\Lambda^1(k), Y)][[1, n] = \mathcal{A}[(\Lambda^1(n), Y)]$ .

In the usual way, all of the above informally defined objects are embedded relative to  $\mathcal{W}'$ . When the informal set-theoretic expresses are considered as embedded into the standard superstructure, all of the bold font conventions defined in Herrmann (1979-1993) are observed. All other embedded symbols retain their math-italics form. Where script notation is used, an underline is used in place of the bold face font. All the following results are relative to our nonstandard model  ${}^*\mathcal{M}_1$  (Herrmann, (1979 - 1993)).

**Theorem 2.1** *Consider primitive time interval  $1 = [0, b], b > 0$ . It can always be assumed that interval 1 is partitioned into two or more intervals  $[c_0, c_1), \dots, [c_{m-1}, c_m]$ ,  $c_m = b$ ,  $m > 1$ ,  $m \in \mathbf{Z}$ . Let  $\mathbf{d}_1$  be a developmental paradigm order isomorphic to the rational numbers  $R_1 \subset [0, b]$ . For any  $\lambda \in \mathbb{N}_\infty$ , there exists a unique hyperfinite  ${}^*\mathbf{\Lambda}^1(\lambda) \in {}^*\underline{\mathcal{L}}^1$  and a  $\lambda' \in {}^*\mathbb{N}$  such that the ultra-word-like  $X_{\lambda'}^1 \in {}^*\mathbf{M}_{\lambda'}^1$ , and ultra-logic-system  ${}^*\mathbf{\Lambda}^1(\lambda) \in {}^*\underline{\mathcal{A}}'(({}^*\underline{\mathcal{S}}^1, \{X_{\lambda'}^1\}))$  and  $\mathbf{d}_1 \subset {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0, 0)\})) = d_\lambda^1 \subset {}^*\mathbf{d}_1$ . Also the  ${}^*\underline{\mathcal{A}}[({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0, 0)\})]$   ${}^*$ steps satisfy the  $\leq_{d_\lambda^1}$  order and  $({}^*\mathbf{d}_1 - \mathbf{d}_1) \cap {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0, 0)\})) = \text{an infinite set}$ .*

Proof. This follows in the same manner as Theorem 4.1 in Herrmann (2006) by  ${}^*$ -transfer of the appropriate first-order statements that precede this theorem statement. Also note that since for every  $n \in \mathbb{N}'$ , the  $\Lambda(n)$  is finite, then, via the identification process,  ${}^\sigma\mathbf{\Lambda}(n) = \mathbf{\Lambda}(n)$ . It also follows that  ${}^*\mathbf{\Lambda}(n) = \mathbf{\Lambda}(n)$ . Since for any  $n, k \in \mathbb{N}'$ ,  $n \leq k$ ,  $\mathcal{A}((\mathbf{\Lambda}(n), \{\mathbf{f}^1(0, 0)\})) \subset \mathcal{A}((\mathbf{\Lambda}(k), \{\mathbf{f}^1(0, 0)\}))$ , from the above and, via  ${}^*$ -transfer, it follows that  $\mathbf{d}_1 \subset {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0, 0)\})) = d_\lambda^1 \subset {}^*\mathbf{d}_1$ . From the definition of  $\Lambda^1(n)$ , these steps numbers are order isomorphic the set of rational numbers  $R_1$ . Hence,  ${}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0, 0)\}))$  is  ${}^*$ order isomorphic to a hyperfinite subset of  ${}^*\mathbf{Q}$ . Since there are infinitely many  $i < \lambda$  and  $i \in \mathbb{N}_\infty$ , there are infinitely many  ${}^*\mathbf{f}(i, j) \in {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(\lambda), \{{}^*\mathbf{f}^1(0, 0)\})) \subset {}^*\mathbf{d}_1$ , where  ${}^*\mathbf{f}(i, j) \in {}^*\mathbf{d}_1 - \mathbf{d}_1$ . These are interpreted as ultranatural events but in some cases may differ from physical events only in their primitive time identifications. This completes the proof. ■

By considering the definition of  $\mathcal{L}^1$ , it follows that the given  $1 < m \in \mathbb{N}$ ,  ${}^*\mathbf{\Lambda}^1(\lambda)$

is precisely  $\{*\mathbf{f}^1(0,0)\} \cup \{\cup\{*\mathbf{k}_i^1(\lambda) \mid 0 \leq i < m\}\} \cup \{\{*\mathbf{f}^1(p-1,\lambda), *\mathbf{f}^1(p,0)\} \mid (0 < p \leq m) \wedge (p \in *\mathbf{Z})\}$ . Of significance is the fact that the steps in the  $*$ -deduction  $*\underline{\mathcal{A}}(*\Lambda^1(\lambda)(\{*\mathbf{f}^1(0,0)\}))$  preserve the order  $\leq *\mathbf{d}_1$ . Notice that  $*\Lambda^1(\lambda)$  is obtained by hyperfinite choice. Further, any  $*\mathbf{f}^1(i,j) \in \{*\mathbf{f}^1(x,y) \mid (0 \leq x < m) \wedge (0 \leq y \leq \lambda) \wedge (x \in *\mathbf{Z}) \wedge (y \in *\mathbb{N})\} \cup \{*\mathbf{f}^1(m,0)\}$  is a hyperfinite  $*$ -deduction from  $\mathbf{f}^1(0,0) = *\mathbf{f}^1(0,0)$ . And, it also follows that the set of all such  $*$ -deductions yields a hyperfinite set  $d_\lambda^1$  such that  $\mathbf{d}_1 \subset d_\lambda^1 \subset *\mathbf{d}_1$ .

### 3. Logic-System Generation for the Type-2 Interval

For the type-2 case  $[0, +\infty)$ , a denumerable developmental paradigm displays a refined form. For this case,  $\mathbf{d}_2 = \{\mathbf{f}^2(i,j) \mid (0 \leq i) \wedge (i \in \mathbf{Z}) \wedge (j \in \mathbb{N})\}$ . Using  $\mathbf{d}_2$ , consider the following logic-system.

**Definition 3.1** Let  $0 \leq i \in \mathbf{Z}$ . For each  $n \in \mathbb{N}$ , let  $\mathbf{k}_i^2(n) = \{\{\mathbf{f}^2(i,j), \mathbf{f}^2(i,j+1)\} \mid (0 \leq j \leq n-1) \wedge (j \in \mathbb{N})\}$ . For  $0 < m \in \mathbf{Z}$ , let  $\mathbf{K}^2(m,n) = \cup\{\mathbf{k}_i^2(n) \mid (0 \leq i < m) \wedge (i \in \mathbf{Z})\}$ . Finally, let  $\Lambda^2(m,n) = \{\mathbf{f}^2(0,0)\} \cup \mathbf{K}^2(m,n) \cup \{\{\mathbf{f}^2(p-1,n), \mathbf{f}^2(p,0)\} \mid (0 < p \leq m) \wedge (p \in \mathbf{Z})\} \cup \{\{\mathbf{f}^2(m,j), \mathbf{f}^2(m,j+1)\} \mid (0 \leq j < n) \wedge (j \in \mathbb{N})\}$ , and  $\mathcal{L}^2 = \{\Lambda^2(x,y) \mid (0 \leq x \in \mathbf{Z}) \wedge (y \in \mathbb{N})\}$ . Notice that if  $0 \leq i < k$ ,  $i, k \in \mathbf{Z}$ , then  $\mathcal{A}((\Lambda^2(i,j), \{\mathbf{f}^2(0,0)\})) \subset \mathcal{A}((\Lambda^2(k,n), \{\mathbf{f}^2(0,0)\}))$  for any  $j, n \in \mathbb{N}$ . Also, each  $\Lambda^2(m,n)$  is a finite set. (Notice that members in  $\mathcal{L}^2$  can be characterized by a first-order sentence.)

Consider any  $\Lambda^2(q,k)$ . Then there exists an  $q'k' \in \mathbb{N}'$  ( $q'k'$  is a natural number in  $\mathbb{N}'$ ) and the  $q'k'$ -set  $\mathbf{X}_{q'k'}^2 \in \mathbf{M}_{q'k'}^2$ , such that  $\Lambda^2(q,k) \in \mathcal{S}^2(\{\mathbf{X}_{q'k'}^2\})$  and, in this case, finite choice yields the  $\Lambda^2(q,k)$  logic-system. Then the logic-system algorithm  $\mathcal{A}$  applied to  $(\Lambda^2(q,k), \{\mathbf{f}^2(0,0)\})$  yields  $\mathbf{f}^2(q,k)$  as a deduction from  $\mathbf{f}^2(0,0)$ . Further,  $\mathbf{f}^2(q,k) \in \mathbf{d}_2$ . Conversely, if  $\mathbf{f}^2(q,k) \in \mathbf{d}_2$ , then there exists an  $q'k' \in \mathbb{N}'$  and an  $\mathbf{X}_{q'k'}^2 \in \mathbf{M}_{q'k'}^2$  and a logic-system  $\Lambda(q,k) \in \mathcal{A}'((\mathcal{S}^2, \{\mathbf{X}_{q'k'}^2\}))$  such that application of the logic-system algorithm  $\mathcal{A}$  to  $(\Lambda^2(q,k), \{\mathbf{f}^2(0,0)\})$  yields a deduction of  $\mathbf{f}^2(q,k)$  from  $\mathbf{f}^2(0,0)$ .

**Theorem 3.1** Consider primitive time interval  $2 = [0, +\infty)$ . It can always be assumed that interval 2 is partitioned into intervals  $[c_0, c_1), \dots, [c_{m-1}, c_m)$ ,  $m > 1$ ,  $m \in \mathbf{Z}$ . Let  $\mathbf{d}_2$  be a developmental paradigm order isomorphic to the rational numbers  $R_2 \subset [0, +\infty)$ . For any  $\lambda \in \mathbb{N}_\infty$  and  $\nu \in *\mathbf{Z} - \mathbf{Z}$ ,  $\nu > 0$ , there exists a unique hyperfinite  $*\Lambda^2(\nu, \lambda) \in *\underline{\mathcal{L}}^2$  and  $\nu', \lambda' \in *\mathbb{N}$  such that the ultra-word-like  $\mathbf{X}_{\nu', \lambda'}^2 \in *\mathbf{M}_{\nu', \lambda'}^2$  and ultra-logic-system  $*\Lambda^2(\nu, \lambda) \in *\underline{\mathcal{A}}'(*\underline{\mathcal{S}}^2, \{\mathbf{X}_{\nu', \lambda'}^2\})$  and  $\mathbf{d}_2 \subset *\underline{\mathcal{A}}((*\Lambda^2(\nu, \lambda), \{*\mathbf{f}^2(0,0)\})) = d_{\nu\lambda}^2 \subset *\mathbf{d}_2$ . Also the  $*\underline{\mathcal{A}}[(*\Lambda^2(\nu, \lambda), \{*\mathbf{f}^2(0,0)\})]$   $*$ steps satisfy the  $\leq d_{\nu\lambda}^2$  order and  $(*\mathbf{d}_2 - \mathbf{d}_2) \cap *\underline{\mathcal{A}}((*\Lambda^2(\nu, \lambda), \{*\mathbf{f}^2(0,0)\})) =$  an infinite set.



Proof. As in Theorem 2.1, the proof follows by \*-transfer of the appropriate formally presented material that appears above in this section 3.

By considering the definition of  $\mathcal{L}^2$ , it follows that the  $*\mathbf{\Lambda}^2(\nu, \lambda)$  is precisely  $\{*\mathbf{f}^2(0, 0)\} \cup \{\bigcup\{*\mathbf{k}_i^2(\lambda) \mid 0 \leq i < \nu\}\} \cup \{(\{*\mathbf{f}^2(p-1, \lambda), *\mathbf{f}^2(p, 0)\} \mid (0 < p \leq \nu) \wedge (p \in *\mathbf{Z})\} \cup \{*\mathbf{f}^2(\nu, j), *\mathbf{f}^2(\nu, j+1)\} \mid (0 \leq j < \lambda) \wedge (j \in *\mathbf{N})\}$ . Of significance is the fact that the steps in the \*-deduction  $*\underline{\mathcal{A}}[( *\mathbf{\Lambda}^2(\nu, \lambda), \{*\mathbf{f}^2(0, 0)\})]$  preserve the order  $\leq_{*\mathbf{d}_2}$ . Notice that  $*\mathbf{\Lambda}^2(\nu, \lambda)$  is obtained by hyperfinite choice. Further, any  $*\mathbf{f}^2(i, j) \in \{*\mathbf{f}^2(x, y) \mid (0 \leq x \leq \nu) \wedge (0 \leq y \leq \lambda) \wedge (x \in *\mathbf{Z}) \wedge (y \in *\mathbf{N})\}$  is a hyperfinite \*-deduction from  $\mathbf{f}^2(0, 0)$ . And, it also follows that the set of all such \*-deductions yield a hyperfinite set  $d_{\nu\lambda}^2$  such that  $\mathbf{d}_2 \subset d_{\nu\lambda}^2 \subset *\mathbf{d}_2$ .

#### 4. Logic-System Generation for the Type-3 Interval

For the type-3 case  $(-\infty, 0]$ , a denumerable developmental paradigm displays a refined form. For this case,  $\mathbf{d}_3 = \{\mathbf{f}^3(i, j) \mid (i \leq 0) \wedge (i \in \mathbf{Z}) \wedge (j \in \mathbf{N})\}$ . Using  $\mathbf{d}_3$ , consider the following logic-system.

**Definition 4.1** Let  $i \in \mathbf{Z}$ ,  $i \leq 0$ . For each  $n \in \mathbf{N}$ , let  $\mathbf{k}_i^3(n) = \{\{\mathbf{f}^2(i, j), \mathbf{f}^1(i, j+1)\} \mid (0 \leq j \leq n-1) \wedge (j \in \mathbf{N})\}$ . For  $m \in \mathbf{Z}$   $m < 0$ , let  $\mathbf{K}^3(m, n) = \bigcup\{\mathbf{k}_i^3(n) \mid (m \leq i < 0) \wedge (i \in \mathbf{Z})\}$ . Finally, let  $\mathbf{\Lambda}^3(m, n) = \{\mathbf{f}^3(m, 0)\} \cup \mathbf{K}^3(m, n) \cup \{\{\mathbf{f}^3(p-1, n), \mathbf{f}^3(p, 0)\} \mid (m < p \leq 0) \wedge (p \in \mathbf{Z})\}$ , and  $\mathcal{L}^3 = \{\mathbf{\Lambda}^2(x, y) \mid (0 \leq x \in \mathbf{Z}) \wedge (y \in \mathbf{N})\}$ . Notice that if  $i < k \leq 0$ ,  $i, k \in \mathbf{Z}$ , then  $\mathcal{A}((\mathbf{\Lambda}^3(i, j), \{\mathbf{f}^3(m, 0)\})) \subset \mathcal{A}((\mathbf{\Lambda}^3(k, n), \{\mathbf{f}^3(m, 0)\}))$  for any  $j, n \in \mathbf{N}$ . Also, each  $\mathbf{\Lambda}^3(m, n)$  is a finite set. (Notice that members in  $\mathcal{L}^3$  can be characterized by a first-order sentence.)

Consider any  $\mathbf{\Lambda}^3(q, k)$ . Then there exists an  $q'k' \in \mathbf{N}$  and  $\mathbf{X}_{q'k'}^3 \in \mathbf{M}_{q'k'}^3$ , such that  $\mathbf{\Lambda}^3(q, k) \in \mathcal{A}'((\mathcal{S}^3, \{\mathbf{X}_{q'k'}^3\}))$  and, in this case, finite choice yields the  $\mathbf{\Lambda}^3(q, k)$  logic-system. Then the logic-system algorithm  $\mathcal{A}$  applied to  $(\mathbf{\Lambda}^3(q, k), \{\mathbf{f}^3(q, 0)\})$  yields  $\mathbf{f}^3(q, k)$  as a deduction from  $\mathbf{f}^3(q, 0)$ . Further,  $\mathbf{f}^3(q, k) \in \mathbf{d}_3$ . Conversely, if  $\mathbf{f}^3(q, k) \in \mathbf{d}_3$ , then there is an  $\mathbf{X}_{q'k'}^3 \in \mathcal{M}_{q'k'}^3$  and a logic-system  $\mathbf{\Lambda}(q, k) \in \mathcal{S}^3(\{\mathbf{X}_{q'k'}^3\})$  such that application of the logic-system algorithm  $\mathcal{A}$  to  $(\mathbf{\Lambda}^3(q, k), \{*\mathbf{f}^3(q, 0)\})$  yields  $\mathbf{f}^3(q, k)$  as a deduction from  $\mathbf{f}^3(q, 0)$ .

**Theorem 4.1** Consider primitive time interval  $\mathfrak{I} = (-\infty, 0]$ . It can always be assumed that interval  $\mathfrak{I}$  is partitioned into intervals  $\dots, [c_{-2}, c_{-1}), [c_{-1}, c_0]$ . Let  $\mathbf{d}_3$  be a developmental paradigm order isomorphic to the rational numbers  $R_3 \subset (-\infty, 0]$ . For any  $\lambda \in \mathbf{N}_\infty$ ,  $\mu \in *\mathbf{Z} - \mathbf{Z}$ ,  $\mu < 0$ , there exists a unique hyperfinite  $*\mathbf{\Lambda}^3(\mu, \lambda) \in *\underline{\mathcal{L}}^3$  and  $\mu', \lambda' \in *\mathbf{N}$  such that the ultra-word-like  $\mathbf{X}_{\mu'\lambda'}^3 \in *\mathbf{M}_{\mu'\lambda'}^3$  and ultra-logic-system  $*\mathbf{\Lambda}^3(\mu, \lambda) \in *\underline{\mathcal{A}}'(( *\underline{\mathcal{S}}^3, \{\mathbf{X}_{\mu'\lambda'}^3\}))$  and  $\mathbf{d}_3 \subset *\underline{\mathcal{A}}(( *\mathbf{\Lambda}^3(\mu, \lambda), *\mathbf{f}^3(\mu, 0))) = d_{\mu\lambda}^3 \subset *\mathbf{d}_3$ .

Also the  $\underline{*A}[(\mathbf{*}\Lambda^3(\mu, \lambda), \{\mathbf{*f}^3(\mu, 0)\})]$   $\ast$ steps satisfy the  $\leq_{d_{\mu\lambda}^3}$  order and  $(\mathbf{*d}_3 - \mathbf{d}_3) \cap \underline{*A}((\mathbf{*}\Lambda^3(\mu, \lambda), \{\mathbf{*f}^3(\mu, 0)\})) = \text{an infinite set.}$

Proof. As in Theorem 3.1, the proof follows by  $\ast$ -transfer of the appropriate formally presented material that appears above in this section 3.

By considering the definition of  $\mathcal{L}^3$ , it follows that the  $\mathbf{*}\Lambda^3(\mu, \lambda)$  is precisely  $\{\mathbf{*f}^3(\mu, 0)\} \cup \{\bigcup\{\mathbf{*k}_i^3(\lambda) \mid \mu \leq i < 0\}\} \cup \{\{\mathbf{*f}^3(p-1, \lambda), \mathbf{*f}^3(p, 0)\} \mid (\mu < p \leq 0) \wedge (p \in \mathbf{*Z})\}$ . Of significance is the fact that the steps in the  $\ast$ -deduction  $\underline{*A}[(\mathbf{*}\Lambda^3(\mu, \lambda), \{\mathbf{*f}^3(\mu, 0)\})]$  preserve the order  $\leq_{\mathbf{*d}_3}$ . Notice that  $\mathbf{*}\Lambda^3(\mu, \lambda)$  is obtained by hyperfinite choice. Further, any  $\mathbf{*f}^3(i, j) \in \{\mathbf{*f}^3(x, y) \mid (\mu \leq x < 0) \wedge (0 \leq y \leq \lambda)\} \cup \{\mathbf{f}^3(0, 0)\}$  is a hyperfinite  $\ast$ -deduction from  $\mathbf{f}^3(\mu, 0)$ . And, it also follows that the set of all such  $\ast$ deductions is a hyperfinite set  $d_{\nu\lambda}^3$  such that  $\mathbf{d}_3 \subset d_{\nu\lambda}^3 \subset \mathbf{*d}_3$ .

## 5. Logic-System Generation for the Type-4 Interval

**Theorem 5.1** Consider primitive time interval  $4 = (-\infty, +\infty)$ . It can always be assumed that interval 4 is partitioned into intervals  $\dots, [c_{-2}, c_{-1}), [c_{-1}, c_0), \dots$ . Let  $\mathbf{d}_4$  be a developmental paradigm order isomorphic to the rational numbers  $R_4 \subset (-\infty, +\infty)$ . For any  $\lambda \in \mathbb{N}_\infty$ ,  $\nu, \gamma \in \mathbf{*Z} - \mathbf{Z}$ , such that  $\nu \leq 0$ ,  $\gamma \geq 0$ , there exists a unique hyperfinite  $\mathbf{*}\Lambda^4(\nu, \gamma, \lambda) \in \mathbf{*}\underline{\mathcal{L}}^4$  and  $\nu', \gamma', \lambda' \in \mathbf{*}\mathbb{N}$  such that the ultra-word-like  $X_{\nu', \gamma', \lambda'}^4 \in \mathbf{*}\mathbf{M}_{\nu', \gamma', \lambda'}^4$  and ultra-logic-system  $\mathbf{*}\Lambda^4(\nu, \gamma, \lambda) \in \underline{*A}'((\mathbf{*}\underline{\mathcal{S}}^4, \{X_{\nu', \gamma', \lambda'}^4\}))$  and  $\mathbf{d}_4 \subset \underline{*A}((\mathbf{*}\Lambda^4(\nu, \gamma, \lambda), \{\mathbf{*f}^4(\nu, 0)\})) = d_{\nu\gamma\lambda}^4 \subset \mathbf{*d}_4$ . Also the  $\underline{*A}[(\mathbf{*}\Lambda^4(\nu, \gamma, \lambda), \{\mathbf{*f}^4(\nu, 0)\})]$   $\ast$ steps satisfy the  $\leq_{d_{\nu\gamma\lambda}^4}$  order and  $(\mathbf{*d}_4 - \mathbf{d}_4) \cap \underline{*A}((\mathbf{*}\Lambda^4(\nu, \gamma, \lambda), \{\mathbf{*f}^4(\nu, 0)\})) = \text{an infinite set.}$

By considering the definition of  $\mathcal{L}^4$ , it follows that the  $\mathbf{*}\Lambda^4(\nu, \gamma, \lambda)$  is precisely  $\{\mathbf{*f}^4(\nu, 0)\} \cup \{\bigcup\{\mathbf{*k}_i^4(\lambda) \mid (\nu \leq i < \gamma) \wedge (i \in \mathbf{*Z})\}\} \cup \{\{\mathbf{*f}^4(p-1, \lambda), \mathbf{*f}^4(p, 0)\} \mid (\nu < p \leq \gamma) \wedge (p \in \mathbf{*Z})\} \cup \{\{\mathbf{*f}^4(\gamma, j), \mathbf{*f}^4(\gamma, j+1)\} \mid (0 \leq j < \lambda) \wedge (j \in \mathbf{*}\mathbb{N})\}$ . Of significance is the fact that the steps in the  $\ast$ -deduction  $\underline{*A}[(\mathbf{*}\Lambda^4(\nu, \gamma, \lambda), \{\mathbf{*f}^4(\nu, 0)\})]$  preserve the order  $\leq_{\mathbf{*d}_4}$ . Notice that  $\mathbf{*}\Lambda^4(\nu, \gamma, \lambda)$  is obtained by hyperfinite choice. Further, any  $\mathbf{*f}^4(i, j) \in \{\mathbf{*f}^4(x, y) \mid (\nu \leq x \leq \gamma) \wedge (0 \leq y \leq \lambda)\}$  is a hyperfinite  $\ast$ -deduction from  $\mathbf{f}^4(\nu, 0)$ . And, it also follows that the set of all such  $\ast$ deductions is a hyperfinite set  $d_{\nu\gamma\lambda}^4$  such that  $\mathbf{d}_4 \subset d_{\nu\gamma\lambda}^4 \subset \mathbf{*d}_4$ .

## 6. GGU-model Scheme for Developmental Paradigms

The following schemes are not expressed in complete composition form. In what follows, for  $q = 1, 2, 3, 4$ , the  $a, b, c$  take the appropriate value for a specific  $q$ . The relations (operators) such as  $\underline{*A}$  and the others presented in the following left-to-right

sequential form, represent processes. There is also the process, here denoted by Ch, that, depending upon an interpretation, represents a characterizable choice process.

For the multi-complexity cosmogony with processes Ch,  $*\underline{\mathcal{A}}'$ , and  $*\underline{\mathcal{A}}$ , the scheme is

$$(M) \quad *M_{a'}^q \Rightarrow \text{Ch}(*M_{a'}^q) = (*\underline{\mathcal{S}}^q, \{X_{a'}^q\}) \Rightarrow \text{Ch}(*\underline{\mathcal{A}}'(*\underline{\mathcal{S}}^q, \{X_{a'}^q\})) = \\ (*\underline{\Lambda}^q(a), \{*\mathbf{f}^q(b, c)\}) \Rightarrow *\underline{\mathcal{A}}(*\underline{\Lambda}^q(a), \{*\mathbf{f}^q(b, c)\}) = d_a^q.$$

The operators  $\underline{\mathcal{A}}$  and  $\underline{\mathcal{A}}'$  have characterizing first-order statements. These statements need not capture all of the intuitive statements that describe the algorithms. The results of application of  $\underline{\mathcal{A}}'$  as formalized can show major aspects of the algorithm's selection process. For example, for a multi-complexity cosmogony

$$\forall x \forall y \forall z \forall w ((w \in \underline{\mathcal{M}}^q) \wedge (y \in \mathcal{F}(\underline{\mathcal{L}}^q)) \wedge (y \in w) \wedge (x \in \underline{\mathcal{A}}'((\underline{\mathcal{S}}^q, y))) \rightarrow \\ \exists p ((p \in \underline{\mathcal{S}}^q) \wedge (y \in p) \wedge (x \in p) \wedge (y \neq x)).$$

The complexity is the value of the  $a$  chosen. For the single-complexity universe, a specific  $\lambda = a$  is used in each of the four cases. In these cases, the scheme is considerably simplified and becomes

$$(S) \quad (*\underline{\Lambda}^q(a), \{*\mathbf{f}^q(b, c)\}) \Rightarrow *\underline{\mathcal{A}}(*\underline{\Lambda}^q(a), \{*\mathbf{f}^q(b, c)\}) = d_a^q.$$

The results in this section, as they refer to instruction-information, have been refined to include the rational generation of each universe-wide frozen-frame. By a mere substitution of notion, those portions of section 6 in Herrmann, (2013) that do not refer to the gathering operator also yield the more refined \*developmental paradigm that corresponds to a complete \*development. The results there represent an ordered application of the GGU-model processes that correspond to the developmental paradigm concept. However, only the \*instruction-sets need to be considered, for in general, such a developmental paradigm approach need not be employed. Its use depends upon a specific application of the GGU-model.

## 7. Other Results.

For the GGU-model, one of the most difficult requirements is to include the concept of the “participator” universe. As stated at the May 1974 Oxford Symposium in Quantum Gravity, Patton and Wheeler describe how existence of human beings alter the universe to various degrees. “To that degree the future of the universe is changed. We change it. We have to cross out that old term ‘observed’ and replace it with the

new term ‘participator.’ In some strange sense the quantum principle tells us that we are dealing with a participator universe.” (Patton and Wheeler (1975, p. 562).) This aspect of the GGU-model is only descriptively displayed in section 4.8 in Herrmann (2002). It is now possible to obtain formally the collection of developmental paradigm universes that satisfies this participator requirement.

The complete Participator Universe Model appears in Herrmann (2013) and if developmental paradigms are employed, the information there can be transferred to the corresponding developmental paradigm requirements.

There is also a necessary refinement to the above that appears in Herrmann (2013). This refinement can be restated for the developmental paradigm notation and yields a development paradigm for each  $\mathbf{f}^q(i, j)$ . There are also a few other GGU-model schemes presented in Herrmann (2013).

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