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How Gravitational Power P_{GW}, and Graviton count from EW Era gives h^{T}_{ij}, and $m_{graviton}$

A. Beckwith[1]

[1]Chongqing University
PRC, 400014

Abstract:

Taking $P_{GW} \sim E^2/\tau^2$, and $E_{GW} \sim P_{GW}\tau \sim E^2/\tau$, where E is an explosion of energy, and $N_{Graviton} \sim E^2/\hbar$, we solve for the evolution of tensor (GW) perturbations from EW era, with a small constant h^{T}_{ij} from the start of inflation. This leads to $10^{-34}\, eV/c^2 < m_g < 10^{-29}\, eV/c^2$ for $|h^T| \approx 10^{-25}$

I. Introduction:

The approximations from $P_{GW} \sim E^2/t^2$, $E_{GW} \sim E^2/\tau$, and $N_{Gravitons} \sim E^2/\hbar$ from dimensional considerations allow us to give dimensional input into ~~[illegible]~~ the evolution equation for tensor (GW) perturbations of the metric given by [1]

$$\ddot{h}^T_{ij} + 2\frac{\dot{a}}{a}\dot{h}^T_{ij} + k^2 h^T_{ij} = 8\pi G a^2 p \Pi_{ij} \quad (1)$$

p = pressure, and Π_{ij} = anisotropic stresses, whereas we have [2]

$$\Pi_B^2 = \Pi_{ij}^2 = f^2(k) \cdot a^{-4} \quad (2)$$

where [3]

$$f(k) \simeq \frac{(2\pi)^{11}}{4} \frac{B_0^4}{k_c^3} \cdot \frac{(3+n)^2}{(3+2n)} \quad \text{when } -\tfrac{3}{2} < n \quad (3)$$

Here k_c is a critical parameter and $B_0^2 \sim 2 T^{00} \propto 2 \cdot$ Energy density

We will commense solving for Eq (1), provided $\Pi_{ij} \sim$ constant in early universe conditions, and

$$H_E = \text{hubble parameter} \sim \frac{\dot{a}}{a} \quad (4)$$

so, then Eq (1) reads

$$\ddot{h}^T_{ij} + 2H_e \dot{h}^T_{ij} + k^2 h^T_{ij} = 8\pi G a^2 p \Pi \quad (5)$$

solving Eq (5) will be the remainder of the manuscript

II $\boxed{\text{Solving Eq (5), with Early universe hypothesis conditions for k space}}$

First, we approximate [4]

$$\ddot{h}^{T}_{ij} + 2 H_c \dot{h}^{T}_{ij} + k^2 h^{T}_{ij} = 0 \qquad (6)$$

by

$$h^{T}_{ij}\Big|_{\text{general}} \propto e^{-H_c\tau} \cdot \left[A e^{i\sqrt{k^2 - H_c^2}\,\tau} + A^* e^{-i\sqrt{k^2 - H_c^2}\,\tau}\right] \qquad (7)$$

as a general ~~___~~ solution,

The particular solution to above is

$$k^2 h^{T}_{ij} = 8\pi\, G a^2 \cdot p_\circ \left[\frac{f(k)}{a^2}\right] \qquad (8)$$

which is

$$h^{T}_{ij}\Big|_{\substack{\text{particular}\\ k\ \text{space}}} \propto \left[\frac{8\pi\, G_\circ p_\circ (2\pi)^{11}}{k^2 \cdot 4}\right] \cdot \frac{B_\circ^4}{k_c^3} \qquad (9)$$

$\therefore$ In Fourier space, we have ~~___~~

$$h^{T}_{ij} \propto e^{-H_c\tau} \cdot \left[A e^{i\sqrt{k^2 - H_c^2}\,\tau} + A^* e^{-i\sqrt{k^2 - H_c^2}\,\tau}\right] \qquad (10)$$
$$+ (2\pi)^{12} \frac{G_\circ p_\circ}{k^2} \cdot \frac{B_\circ^4}{k_c^3}$$

III $\boxed{\text{FT to (Real x space) soln to Eq(10)}}$

We use,

$$\frac{1}{(2\pi)^{3/2}}\int_{-\infty}^{\infty}(2\pi)\cdot k^2 e^{-ik\cdot x}\,dk \quad \text{integration}$$

of Eq (10), to obtain

$$h^{T}_{ij}(x) \propto \frac{e^{-H_0 \tau}}{\sqrt{2\pi}}\int_{-\infty}^{\infty} k^2 e^{-ik\cdot x}\cdot\left[A e^{i\sqrt{k^2-H_0^2}\,\tau} + A^{*} e^{-i\sqrt{k^2-H_0^2}\,\tau}\right]\cdot dk \tag{11}$$

$$+ \frac{(2\pi)^{11}}{(2\pi)^{3/2}} G\cdot\rho\cdot\frac{B_0^4}{k_c^3}\cdot\int_{-\infty}^{\infty} e^{-ik\cdot x}\,dk$$

The 1st term of Eq (11) is comparable to [5]

$$h^{T}_{ij}(x)\Big|_{\text{1st term}} \propto \begin{pmatrix} \begin{pmatrix} h_+ & h_x \\ h_x & -h_+ \end{pmatrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{pmatrix}_{ij} \cos(\omega[t - \tfrac{z}{c}]) \tag{12}$$

As predicted in pre-inflation models, h_+, h_x are $\sim 10^{-36}$ (13)

These are far below the threshold of detector sensitivity due to $e^{-H_0 \tau} \to 0^+ \sim 10^{-36} - 10^{-40}$ for very large τ values

The 2nd term is where we will put our attention to

IV Solving 2nd term of Eq (11) for present day:

$$\left. h^{T}_{ij}(x) \right|_{\text{effective}} \sim \frac{(2\pi)^{11}}{(2\pi)^{3/2}} \cdot \frac{G \cdot \rho \cdot B_0^4}{k_c^3} \cdot \left[\int_{-\infty}^{\infty} e^{-ikx} \, dk \right] \quad (13)$$

Here, $(2\pi) \cdot \int_{-\infty}^{\infty} e^{-ik \cdot x} \, dk \sim \delta(-x) \equiv \delta(x)$

But, we use a 'tempered $\hat{\delta}(x)$ distribution' [5] approximation to the delta fctn, and we also use $a_{EW} \sim 10^{-60}$, or so if EW was $10^{-36} - 10^{-10}$ seconds in duration, ~~leading~~ after BB to get

$$\left. h^{T}_{ij}(x) \right|_{\text{Effective — todays conditions}} \propto \frac{(2\pi)^{10}}{(2\pi)^{3/2}} \cdot \frac{G \cdot \rho \cdot B_0^4}{k_c^3} \cdot \frac{\hat{\delta}(x)}{10^{60}} \quad (14)$$

so, if $\frac{\hat{\delta}(x)}{10^{60}} \sim \Theta(1)$

$$\left. h^{T}_{ij} \right|_{\text{Todays Condt}} \propto \frac{(2\pi)^{10}}{(2\pi)^{3/2}} \cdot G \cdot \rho \cdot \frac{B_0^4}{k_c^3} \quad (15)$$

V Conclusion: Present $|h^T_{ij}|$ magnitude:

Eq (15) can have its B_0 term estimated as follows:

$$\text{If } N_{\substack{EW \\ \text{Gravitons}}} \sim \frac{E^2/\tau}{\hbar/\tau} \sim \frac{P_{GW}^2 \tau}{\hbar/\tau} \qquad (16)$$

$\ni$

$$P_{GW} \sim \frac{\sqrt{\hbar N_{EW}}}{T_{time}} \sim \frac{MV^2}{T_{time}} \qquad (17)$$

$$\text{If } N_{\substack{GW \\ EW}} \sim 10^{50} \Rightarrow P_{GW} \sim 10^{29} \sqrt{\hbar} \quad (T_{time} \sim 10^{-32} \text{sec}) \qquad (18)$$

$$\& \; E^2 \sim P_{GW}^2 \tau^2 \sim (m v^2)^2 \qquad (19)$$
$$\sim \frac{1}{2} (E^2 + B_0^2)^2$$

Get upper bound to B_0^2 this way:

Durrer [1], [3], estimates $B_0 \sim 10^{-11}$ Gauss.

Our effective B_0 is higher

Then, after algebra:

If $N \sim 10^{50}$ after 10^{-20} seconds

(a) $10^{-31}\, eV/c^2 \leq m_{graviton} \leq 10^{-29}\, eV/c^2$

(b) $\omega_c \gtrsim 10^7$ hz

(c) $|h|_{today} \sim 10^{-25} - 10^{-26}$

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